Adaptive Signal Processing

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Lecture – 23
Lattice Recursions

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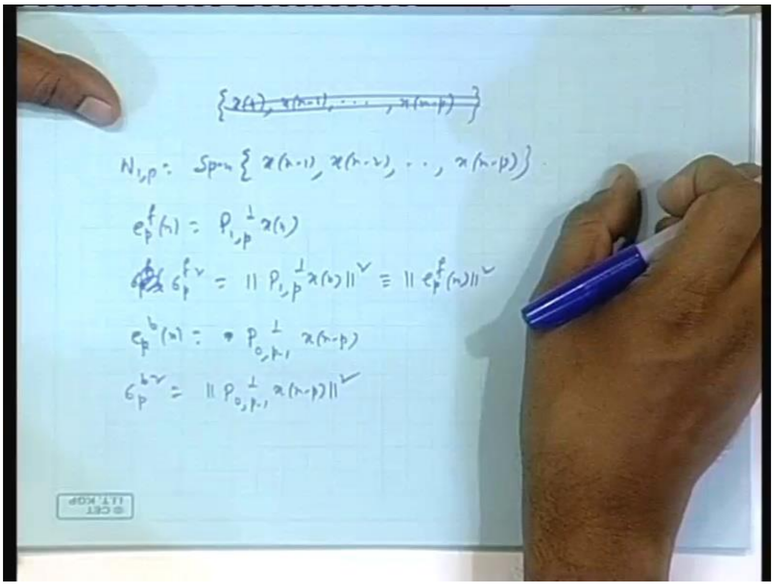

So, this is a quick recap that is what we are doing yesterday. We are considering space span by this people x n, x n minus p. When, I mean, let me make it more general. We had actually, the basic space was this. We can identify this actually. Span of this; in 1 case we had x n in the beginning, in other case we had x n minus p minus 1 towards the right. We call it w 1 p; we call w 1 p this span.

Then, very quickly, we have this definitions; e p f n was nothing but project x n on the space span by this; and take the error P 1 p perpendicular x n. Sigma p f, sorry; sigma p f square, n goes because this is a variants of this; variants of what a after all x n minus linear combination of this. There is a projection error. Those combiner coefficients are independent of n because of seasonality, you know.

It is to the variants of that again because of seasonality, only the correlation values comma, from which n disappears; that is way this variants is independent of n; this is nothing but mod square of this quantity; p this mod square of p f n, e p f n that is equivalent to mod square of e p f n, right? Mod square means variants; expected value of mod square because that is our definition of inner product. Inner product of x and y is, to random variables is x and y value is e of x star, correlation. And therefore, normally means variants p of x into x star.

Similarly, for the backward prediction we had, e p, what? E p b n. That time x n minus p was projected on the space, this was not space that time; this unfortunately only for the forward case; for the backward case, the space starts from what? I mean, this is a span of which elements? X n, x n minus 1, upto x n minus p plus 1. Those are the p future terms if you are standing at n minus p th point. Is it not? N minus p th point, the p future terms.

So, that means what? N minus p, x n minus p needs to be projected; and on what? 0 to p minus1. Are you following this? N minus p, it was standing there; this is backward prediction standing here at p th order. So, p future terms; what are those future terms? X n minus 0, x n minus 1, dot, dot, x n minus p plus 1. It starts at 0 degree, upto p minus 1 degree; this minus space corresponding projection, the corresponding error.

And again, the projection was same thing. Then, I found out an update relation; update recursion actually it is called order of the recursion because update was in terms of p; you get e p f a and e p b n; question is get me e p plus 1 f n and e p plus 1 b n. So, I am updating it in p. P is the order; it is called order updating. Finding out the same thing this for higher order, that means x n must be projected on the space span by, not just by x n minus 1 to x n minus p, one more element, but I did that element is x n minus p minus 1.

So, net space is what? If you take x n minus p minus 1, project it orthogonally on this, take the error. Then, this net subspace will be direct sum of or orthogonal sum of span of this and that error space span by that error term. And then I projected x n on this; projection on this part, projection on other part, then take the error. And, you got an expression for e p plus 1 f, e p plus 1 f n; similarly, we did for e p plus 1 b n. Just for your sake I am writing the recursion and we draw the lattice stage also.

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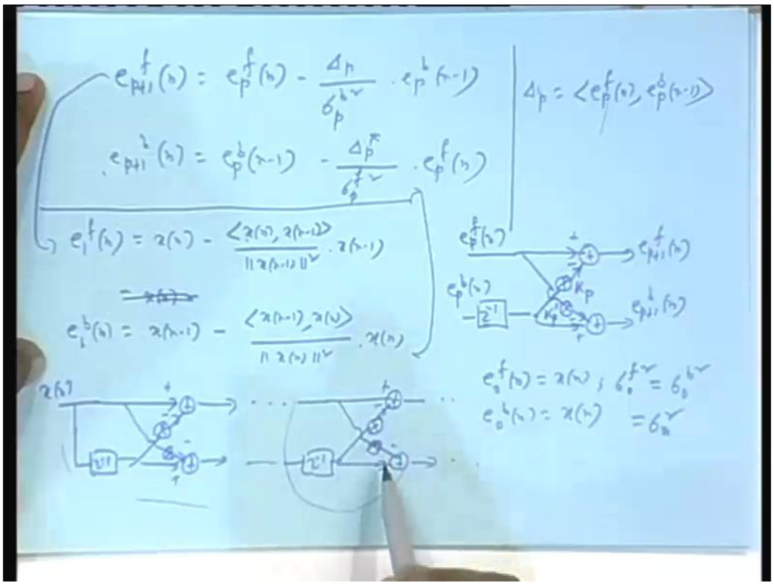

The recursions were like this. These are all, they are delta p by sigma p b square, I think. I tell you, rewrite what is delta p. So, you all know what it is; e p b n minus 1; and minus delta p star where delta p was in a, I mean, e p f n and e p b n minus 1, inner product between the 2 that is the variants, sorry, the correlation. But the correlation is independent of n again because of the stationarity because this error is nothing but originally x n minus a projection; projection coefficient independent of x n.

This also x n minus p minus 1, minus the projection, and you multiply those 2 expressions; all products of some x n minus k into x n minus n, that kind of thing will come; n will disappear because of stationarity; you get a net term in terms of correlation of the process, but independent of n; we call it delta p; it is called parcor coefficient. In one case you have delta p, another case delta p star.

We denominated these 2 quantities are real and in other case not negative because no, there is no linear relation between x n, I mean all the samples of x n; no x n, no sample. Remains in the space span by the rest which means the corresponding orthogonal projection error cannot have a 0 to a variance, cannot have 0 mod. There is a division by the 0 does not occur, have seen this; division never occurs because no sample of x n, no sample is remaining in the space span by the rest.

That is the projection error has got it non 0; that is the strictly positive non square. That is why this things; it will give rise to division by 0, but they are real, positive and real. So far, so good, I said that time that I will start this business assuming that p is greater than equal to 1. So that, if we knowing the extent case p equal to 1 considered now; p equal to 1; w 1 comma 1 of span of these; x n was projected on this; the error was found; that error we start with; then you want to go for 1 order up. So, introduce x n minus 2, the same process.

But, how about the case, p equal to 0? That is, we are starting with the case where, you are starting, I mean you go for the projection on the space span by these 2, you are starting with what? P 1 p n, but who gives you these? Again, these also has to be obtained recursively. Is it not? E p plus 1, sorry, e p plus 1 f n is obtained recursively from e p f n, and so on and so forth.

You started at p 1 f n, then you go to e 2 f n, so on and so forth, but who will give you e 1 f n? To get e 1 f n, you have started e 0 f n; e 0 f n means what? That is what we do not know; e 0 f n means extent projected on nothing. When p equal to 1 there is something; p cannot be less than 1. So, x n projection nothing than the error, then nothing than error; and this are all very qualitative high terms. Is it not?

So, we have to see what they are? We have to really define separately, what is, I mean, e 0 f n? What is e 0 b n or n minus 1? This has to be defined separately. Using them I can find out e 1 f n, e 1 b n. E 0 b n or e 0 b n minus 1, you define using these 2. I have to find out e 1 f n, e 1 b n, but I have defined them. You understand the problem? Because I started at p equal to 1, and then I can go for p equal to 2, p equal to 3, p equal to 4, like that.

But to get p equal to 1 case, recursive regions have to get that is for the p equal to 0 also. Is it not? But, p equal to 0 case has got actually no mathematical meaning because then there is no element; yes.

Student: voice not clear.

No, no, no, no. This is a set. This one is the space. I am not questioning this space; this space minus this space; apply one more element set again, disperse another space. But from the set you have, suppose eliminating one element to another, you are left with these. And now you are saying no I am not even taking that. You understand? You have to find out the projection error when x n is projected on the space span by this fellow only, that you call e 1 f n.

Now, to get the recursively you have to start at e 0 f n; now e 0 f n means what? 0 th order; there is no term; n p is a set span by space span by n p set, and all that there is no meaning actually in terms of maximum that I gave; is it not? That is why this is to be, this is my, why you are reading books, write some qualitative things and all that, but that is mathematically incorrect, n value. There is something. If you are doing 2 2 mathematics, you have to consider those cases separately; you have to give separate definitions.

Now, this we know. We can also draw the structure here quickly; k p i I gave; I forget notations; and the other one I gave, yeah, in the previous class I called it k p f and k p v, forward prediction, backward prediction; anyway k p, k p s that is fine; another conjugate of the other prediction. This is k p prime, right? And, here you get your e p. You go on cascading it till you have e 1 f n, e 1 b n; before that what happens, that you have my problems, is it not?

So, now instead of doing all this is by vector space method, let me work out directly. After all what is e 1 f n, means able to take x n, project it on the space span by x n minus 1, sorry, x n minus 1 e 1 f n, after all; and take the error that is x n minus; when I use this notation I treat them as vectors. When I bring in e operator and all that, then they are back to their physical forms, that is under variables.

Please understand the, this slide mathematical thing actually, in this what you say in incorrectness in notation because I am using the, repeating the same x n thing in the vector space form also. I should put a bar, underscore, or something to differentiate vectors, but I suppose by now you are predict on conversion. So, when I am doing that, writing this, using this notation, I am basically meaning the inner product of vectors; I am looking at that vector.

Finally, when I replace by e or something, then their actual physical form will be brought in operations. This times, and we now bring in those forms. What is this? I can keep it like this. But physically I know, what it is? It is expected value of x n, x n minus 1 star; that is correlation with lag 1. And, what is this? This is nothing but variants, independent of n, so that is n minus, n minus 1, that is does not matter.

And, what is e 1 b n directly? X n minus 1 is to be projected on x n; this has to be taken, right; x n minus 1 has to be projected on x n; n minus 1 that has to be projected on x n, one term; only one future term; it is standing at n minus 1, x n and the error. Now, you compare the 2; this with this and this with this. You can say, I can define, if p is 0 you put. I define now. I say, I am mathematically correct. I define, e 0 f n, as x as, everything will fall in craze; you will just see. And according to be, this is how it should be thought.

Some books have there where they have taken I am projecting in space span by n p is nothing; if I take the error I get the same thing; this is all nonsense, you know; there is no meaning, is it not? Space span by an n p set, what is the meaning of that, mathematically?

Student: voice not clear.

No. Space span by n p set means what? There is no definition. Did I give you that? Was it part of assumes? It is not. Or, it is a physical way argument, you know, that I spacing on projector nothing, and therefore I get back everything; this is, these are books, right, but this has to be this way. Then, you can just see that I am defining that I am really defining; now looking at this I have to bring them in the same fold of recursion.

What I am doing? This I compute separately. And, preparing the same recursive formula by the same recursive formula where this x n if you put x n equal to f n, and as again x n, instead of b n x n, so that means, this is instead of b n minus 1; I am saying instead of b n also x n; that means, e 0 b n minus 1 coming. Now, delta p, you should look at this, it should be p f n, e b n minus 1; you see, e 0 f n, regular b n minus 1, all this things are coming, right.

Sigma p b square; that means, mod square of what? E 0 b n minus 1 that is x n minus 1 only, is it not? Are you seeing this thing that means, in those recursions I can now start at p equal to 0 and go ahead, and I will give the initial condition like this; and, 2 more conditions, the corresponding variances also. No I have not done anything for variances; that is recursive way of doing things is concerned. I have done only for this sequences, this signals, not for variances.

Variances are independent of n. Nevertheless, the sigma 0 f square is a variance of this guy, and sigma 0 b square is a variance of again this guy, both are same. So, at 0 th stage, both are same. There is a reason why they are same for any order, stationarity plus that, is that was missing that; that at the 0 th stage both are same; this is nothing but sigma x square. So, now I have got this perfect lattice structure.

So, what is this lattice? This kind of stages you see, initially you have got x n, this is one stage. You know the values; I am not writing the values; here you can put the corresponding values with the order, p will be 0 in these case and all that, you know, delta 0 and all those; dot, dot, dot, in general let us take typical lattice structure. So, understand this order recursive structure; it is recursive in order. If you want to carry out, if you want to find out projection for one extra order, you do not have to destroy this previous part, you just to add one more stage.

Each is, each section is called stage, one stage. Structurally from circuit point of view they are identical sections. Values are the multiplies and all different, but structurally they are same. See, just go on casketing more and more structure in this thing, it is very easy for, very nice for well assigned; just you design the mass for this, and go on repeating it. You understand.

This order recursive is very nice for well assigned. It is absolutely fantastic for relation. This one stage is to be repeated. Structurally, same thing you are repeating; multiply values will be different, this values. So, remember, you can draw this structure and all this, provided you know the coefficients; is it not? This is the thing which works in real time. This values e p f n, e p n, b n, and all those are real time; this is the function of n.

This is the filter, lattice filter. It gives you all those things, but provided you know the coefficients. But, coefficients depended on what? Those inner product, there is correlation between the respective projection errors and the corresponding variances. The entire lattices defined by these coefficients; is it not? There inverse is the constant thing. Plus, what determines the lattice? Only the values of the coefficient. And, what is the coefficient? Delta p by and the 2 variances, right.

So, then we should then try to develop computational ways, but what it will be giving? It will be giving only this random process and it is not random process, but its correlation. I will be giving you the auto correlation, just stationary process 0 mean say; and this auto correlation values for sufficient lakh, 0 lakh, 1 lakh, 2 lakhs, sufficient number of lakh will be given to you. You will be asked to construct this filter that is develop this multipliers with proper value.

Once you have that you start filtering x n through that, you will get those projection errors, as theory suggests. But, how will you compute them? Because then that means, you have to compute them. How can you compute? Just smartly, where we are not given this values. You are given only correlation values. Whereas, what you are getting here in this multiplied coefficients are these inner products of errors and the corresponding norm square of the errors.

So, there is a next step; that is a non real time of operation mind you; this only a computational back I mean back to whom off line composition; we mean a set up of correlation value; how to generate the synthesized the lattice filter. There is only that computation. Once you synthesize it, implement the filter is just coefficient, and work for real time, say, procedure the same extent you will get those corresponding projection errors accurately.

Still we do not know why is the utility of all this, all this, will, you will see what all things it can do. You know it is one of the most beautiful things that have come up in electric inner product lattice; anyway. So, let us target these fellows; there is 2 variances because I told you and there also kind of inner I mean a different of opinion with some of you that is to how to prove that these 2 are same. That is for the backward and forward case that 2 norms are same. So, let us tackle that first; then we will come to this delta fellows.

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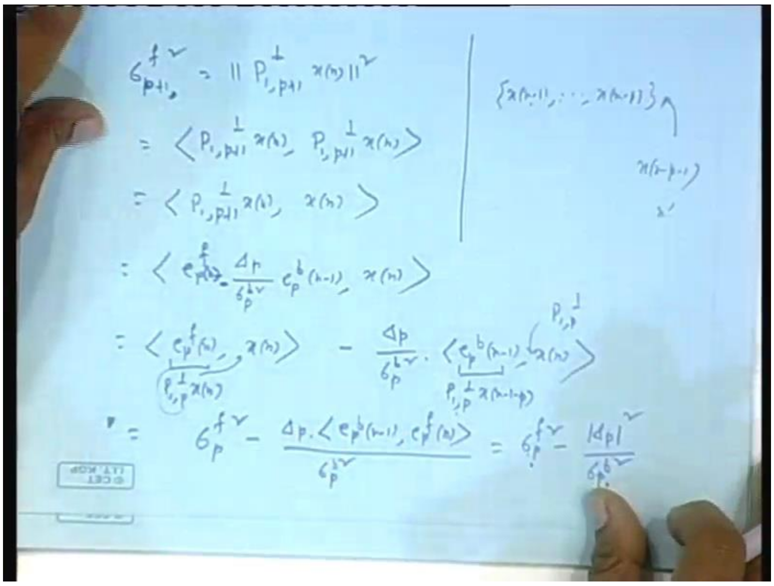

So, sigma, again I will say, I will show that I can have order recursion for those variances also. That is, suppose you give b sigma p f square and sigma p b square for some order p, I can get after, of course delta p, then I can get you same thing for order p plus 1 also. If we will try it and found out, and this may claim and I make I am making a claim and I will show this to others.

So, sigma p plus 1 f square, I want to find out. Now, what is that? That means, p, forward prediction, x n minus 1, upto x n minus p minus 1. They are spreading a space, project on that, take the error project whom project the correct guy take the norm square. And now, I have to bring some beautiful inner variances, the kind of thing that you are already used to. Inner norm square means inner product itself. But, I am sure you all permit me to drop this from 1 of the 2, I drop this here, I keep it x n as it is. I have done it many times. This is what forward prediction error; forward prediction error for p plus 1 th order. For this I already know the recursion, the lattice recursion.

So, I put that here. This is e p f n minus, the delta p thing; delta p you know; I am not defining it again. Remember that this recursion I am brining back here; e p plus 1 is same is equal to this. We know what is delta p and all. So, I am not defining it. This is n comma x n; and now inner product with this, inner product with the rest. First one, again e p f n comma x n, minus something, I am coming to that; minus what? Delta p and delta p by sigma p b square; they can come out.

This part with this now; this inner product coefficients are coming in the first coordinate; they do not get conjugated; they just come out as it is. So, delta p by sigma p b square. Now, what is this quantity? This is p 1 p. And you do not mind if I repeat this here, p 1 p perpendicular on this. I told you I can always bring that back here after all what is the physical, what is happening, just to remind you; x n can be written as what?

As a summation of 2 components - one projection on the space span by x n minus 1 to x n minus p, and the error. That projection part is orthogonal to this fellow. So, that part will be 0. So, only that error part will be there. See that I, that I did many times again for your sake when you do not forget it, I am reminding. What I am saying, just I can take this operator and repeat it here. Physically what does it mean?

X n, I am writing as a summation of 2 component - one the projection, another projection error; projection on space is same span by x n minus 1 to x n minus p. But the projection part and this, this orthogonal, they are cancelling the inner product. So, again that error part will be there; is it not? And, if I bring it here, p 1 p perpendicular x n, this will be this, same as this; this is p 1 p perpendicular x n; this fellow will be brought it here; so the same thing p 1 p perpendicular x n; that means, norm square of this fellow; are you understanding these? I do not know how many? All are understanding or not?

Same thing is repeating here; p 1 p perpendicular x n this fellow; I will take, I apply the operator here. So, 2 persons are same; so the norm square; norm square means, but for order p; so sigma p f square. See, this becoming recursive minus; here again what is this guy? P; Please see, this is little trickier than that. For backward prediction is always e p b n minus 1. Again, in case of confused, what is the, at what index P will be standing, n minus 1 minus p. And, look at p future terms; you call n minus 1 as n prime, like yesterday; call n minus 1 as n prime. So, n minus 1 minus n, n prime minus p is your index, from that you are looking at p future ones.

So, what are the terms? N prime, n prime minus 1, n prime minus 2, dot, dot, dot, n prime minus p plus 1; they are perpendicular space on which your projecting. Now, replace n prime by n minus 1; immediately you get this operator. Are you understanding these? This fellow, what was this fellow actually? Either you remember it always or what are these guy?

This guy is, my claim is if you take this, and you have got an extra component coming here, x n minus p minus 1, you project this on this span, on the span of this and take the error; that fellow is this; is it not? Because, you can always call n minus 1 as n prime. So, n prime minus p; n prime is the connecting it; n prime minus p is standing at looking at p future terms; so n prime minus 1, n prime minus 2, n prime minus p, plus 1;.

Then if you project, by definition that is the backward projection error, p th order for index n prime; that is what it is. But, index n prime is n minus 1. So, this is for x n. I am giving this definition time and again, but in case you get confused I am just repeating, so that, you know, you get pre used to it. And now, this I can repeat on this. I can bring this p 1 p perpendicular here; this operator I can repeat by the same logic. Now, do not ask me to repeat those again, 2 minutes back I, p 1 p perpendicular on this means what? Forward prediction, right.

So, delta p; and this becomes what? Inner product between e p b n minus 1, e p f n which is delta p starts, delta p was inner product between this and this; now it is become reverse. This is nothing but; so you see how nicely recursive it is. If you know these 2 guys and this guy, for that lattice stage, you know this guy. But then I have to find out same for b also. After all you need this fellow also, you need this fellow also, you need this fellow also, then only you can get sigma p plus 1 f square.

But then I must get this fellow also, this fellow also, for p plus 1 th order, is it not? Otherwise I cannot proceed recursively. So, for this also is the same way you can find out. So, quickly I go through that; absolutely similar way. Just the spaces will be slightly different, as you know. So, let us carry that out.

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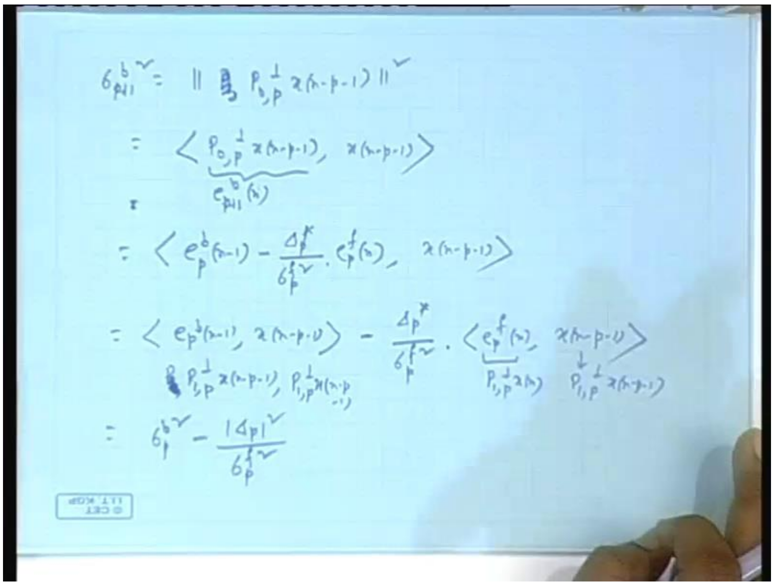

That is, is your norm square of, what? Sigma p plus 1 b square; that means, x n minus p minus 1, that is what your standing, looking at p future, p plus 1 future terms. P plus 1 means starting at current 1 x n, x n minus 1, dot, dot, dot, upto x n minus p, there it becomes plus 1, right. That means, p 0; this means I will not jump over some step; this is a inner product itself; but in from one of them I will remove this operator; from all these I will remove this operator.

So, let it be like this. And this is what? This is backward here, after all backward, this is backward prediction error, x n minus p minus 1, p plus, 1 th order backward prediction error for index n. Index n is the current index. This is p plus 1, that is why this is actually e p plus 1 b n; n minus p minus 1 is the index where were it standing, looking at p, p plus 1 future terms, is it not?

This is this. And I now for its recursive formula; recursive formula I know in a lattice computation, simply replace that; that was e p b n minus 1 minus; minus what? Delta p star know; you have delta p star that time. I just replacing this by the that lattice recursion, lower one. I have delta p star for the end. And then these 2 inner product; same operation. What is this?

Again write, after sometime it will become very mechanically; you know, similar kind of steps I am doing again and again; e p b n minus 1, what was that? P 0 2, p th order, sorry, p 1 2; very good. N minus 1 is the current index, n minus 1 minus p; from there we are looking at p future fellows. This p future fellows are x n minus 1, x n minus 2, upto x n minus p, that projection; and this fellows, and again I will repeat this on this guy and I get back this guy. These 2 are same.

So, this part is simple; whether this is the backward prediction error for this is I mean norm square for this fellow because of stationarity this is simply sigma p b square, norm square of this fellow, p th order. Here again, this is pretty simple; this is p 1 p perpendicular x n. So, this also I will make it p 1 p perpendicular; this I have already seen, what is this, this guy is, this is nothing but e p b n minus 1. So, this inner product becomes delta p, is it not?

This is not delta p. So, this is again mode delta p square. So, this is backward; first term is backward; in the denominator forward. If it is forward, first term is forward, is the denominator backward, that way. Now, I can easily prove that both sigma p f square and sigma b f square are same; very easily proveD by induction. Suppose, upto the order p, it is true; upto the order p it is true; that is sigma p f square is equal sigma b f square.

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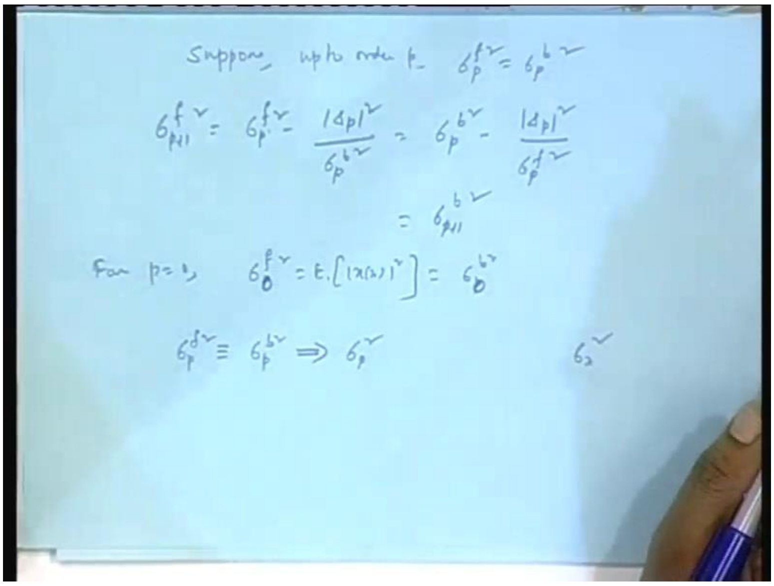

That is, suppose upto order p, sigma p f square is sigma p b square, then we already found out this formula, this we already derived, just now we derived, we have just now derived, and this 2 are same. So, you replace this by this, again this guy by this. So, this is same as what you got. The question is, in induction this is a major stage, but you have to show that it is prove for that is p equal to 0 or somebody.

For p equal to 0 case, both 0 th order forward and backward prediction error are same as x n. So, corresponding norm square was some variants of simply variants of x n. So, it was atleast p equal to 0. For p equal to 0, f square is E which is also same as sigma p b square, sorry, this, because e p e 0 b n was x n again; this is true for p equal to 0; so therefore, this true for all orders.

So, henceforth, I simply write sigma p square is that innovator; neither sigma p f square nor sigma p b square. So, my notation will be sigma p f square and sigma p b square will go to, we have sigma p square. Just like incursive station notation because when I come to x n, is a variants I write as sigma x square, is it not? So, as though it seems the quantities in the same domain, they are not; here p b means in order, here x means signal.

So, will you please permit me to do this? Because, do not, p 2 be a signal and x will be; do not try to see equivalent between the 2. Here it only means, because we will be, will not be mixing up; very rarely will that question come off as to the variants of x n and all that, or sigma p square means this.

Then, comes another beautiful property that is each of the reflection coefficients has magnitudes less than 1. Reflection coefficients are what? Delta p by sigma p square and delta p star by sigma p square; obviously, both are same magnitude; you understand know. The denominator is same; just now proved; numerator is delta p or delta p star. So, magnitude of this reflection coefficient in both the braches in each lattice instant, they are same, is it not? Numerator conjugate coefficient that the denominator same. I show that they are not only same, they are same of course, but they are all having magnitude less than 1.

How many know of you know carchy-selwar inequality? Have you heard of carchy-selwar inequality, how many? And, how is it proved in this book? Just tell. Do not have to process the entire thing. Just tell me how is it proved usually? They do all those things, and square as, and all those things, is it not? But, I will prove in the, you see the proof for today and they never forget it in life, because this is what, what we show in the equivalent is; the proof is in vector space.

I do not know. No, this proof is in books, maths books; linear algebra books, definitely. Yeah, I am, I have not constructed the proof; I would not thing that; no way; but this is the actual history instead of carchy-selwar inequality. Say, suppose x and y are 2 vectors, the real numbers can be see seen it written as real vectors in the vector space of real numbers, is it not, that way. So, this extents to even numbers also. You understand what I said just now.

You mean the real numbers can be this, because 2 real number added, and it is another real number, so fourth standard addition; real number into the scalar times, a real number scalar is a real itself; it is again another real number. So, set of real number is a vector space, so what itself. So, whatever I prove in the context of I am using vectors space of those they are all valid there.

But, in the case of real number they are much more structured than this; they have got in this stand, all those of things, you know; this stands they are many other things. Whether this only obstructs symbol of very few degrees of this is just 2 operations - one obstructs lost and other obstructs skill multiplication, other things. Real numbers you can do just; there anything you consider you can do.

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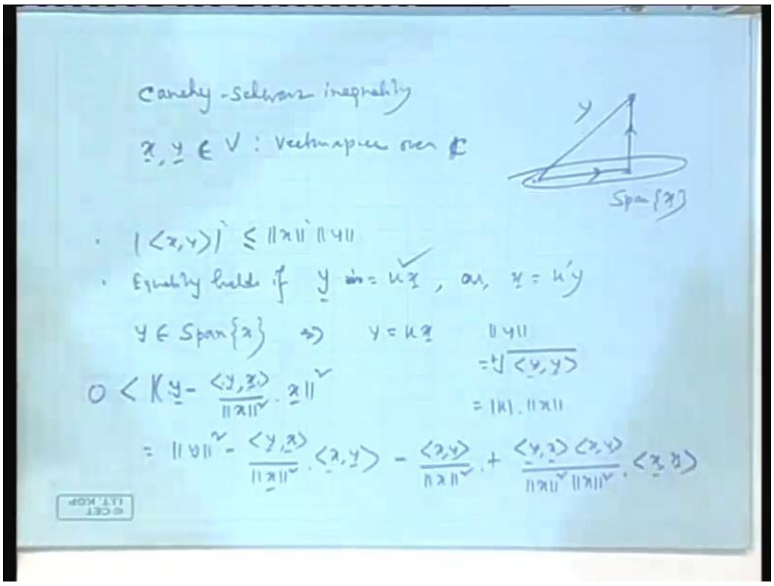

First it says, carchy-selwar inequality, suppose x, y element of V where V is a vector space; I mean it prove for infinite dimension of vectors space also. What I prove is, I am not using the finite dimensional mixture of the vector space V, V over this is c, in general; but of course, this c is required; c over r, r is part of c only. Real field is a part of such c only. So, whatever I do for c that is vary for r.

But, if you change the field into, say, binary field; consider 2 numbers, 1 and 0, that also a field modulative field, then this will not work. That is the problem, you know. I have to define a more, you know, then that is, I mean, high stuff mathematics. Better, I have to define more generalized carchy-selwar inequality for an arbitrary field and all that, that is mathematics. This says that if you take the inner product between them and take the mod, this is always less than equal to norm of this into norm of this.

Some times books put a square on this, and therefore, square and square which I am still in this form, where both are equivalent actually, because you are taking positive, mod means positive number only. So, this is less than equal to this, obviously, square also is less than or equal to, and vice versa. This says this is always less than equal to, then equality holds if y is, if y equal to k x, or x equal to say k y, some k prime y that will be the cost.

Y equal to k x because, know, that says, that means says x is equal to 1 by k y, but if you take k 2 is 0 then I cannot write 1 by 0 times y. So, just to take up that type mathematically, this is a mathematician as 1. I told this is say, if y equal to k x equality holds, but I am adding this extra statement y; mathematically y equal to k x means, simply I can say, suppose I put k equal to 0 this prove; it does not mean x equal to then 1 by 0 y, that is no meaning. How to prove it?

This is true means, or this is true means, y and x they are in the same space; y in the space span by x, or x is in the space span by y. Suppose, y is in the space span by x, suppose y that is this speaks then only; obviously, in this case if you really, that means, y equal to some k x; you put that back here, equal to this, you can easily see. After all, what is norm y? Norm y means square root of, positive square root of inner product itself.

Norm square is inner product itself; norm is positive square root of inner product itself. And, we replace y by k x, y by k x; k, k; both k, k star; not k square; x k with itself; norm square of x; positive square root. So, you get mod k times, is it not? Put that back here; mode k times mode x square. On the other hand, left hand side you put y equal to k x; k star goes out; mod k; and x with itself; mode of that; x with itself, no need to put a mod on that; norm square. So, that is clearly satisfied; understood or not? This is trivial.

Now suppose, it is not. You have some question? Now suppose if it is not; that is y does not belong to, for this case you have to, whole thing we have seen. Now, suppose this is not, then you have to, we have to show that this is strictly less than. So, in this case suppose y does not remaining span of x; that is suppose span of x is this, space span of x is this, y is here; I then project y on this.

When you read carchy-selwar inequality books all this projection are in, nothing is told because people do not, I mean; they do not expect people to know; but now we know all that. Suppose I project orthogonally on this. Since y is outside, this error, projection error vector we have norm greater than 0; norm cannot be less than 0; if norm equal to 0 means, this is 0 means y itself is here.

But there is containing; when I say that y is outside the span of x, logically y is outside the span of x, that means, if you take the orthogonal projection, projection exists; it is a unique; take the error, that error is orthogonal to this, this span. Now my claim is just error cannot be 0 vector because if it is 0 vector then y remains in the span of x only which is quantum mix because I am quite question that is where is outside the span of x.

So, that means, error and if it is not a 0 vector norms has to be greater than 0, right. So, that means, I am doing this side; instead of greater than 0 on the right hand side that the norm square of this vector. What is the projection of y on x? Y inner product it takes given by norm square of x into x; and, projection error is y minus that norm square, sorry, right; this is a projection; this is the original; this is a error; norm square of error simply greater than because y is outside the span of x.

Because this is just a error. This particular projection y we take here divided by norm square f x into x; y minus z this is a error norm square that has to be greater than 0 because y is outside this span. Now, norm square means inner product this itself. See, imagine that I am writing inner product, this component comma, again this n component; I am not writing that line; and then term wise inner product.

So, what do that means? Y to, y with itself one term; then this, again put a comma repeat the same thing; this term will come out x with y; this with itself, so this second term into first term; x with y; then again this with its, right, y with x; and this time this will come out with a conjugate because using second coordinate. I am not showing that extra line, the extra, following?

Y with this; from the first term y, from the second term this; this is the second coordinate; so conjugate will come up. This is the problem. So, y, x conjugate means it will become x, y; denominator real. Of course, I should have said that x and y, now it is a 0 vector because this is trivially obvious for 0 vector, what is, if there is a 0 vector, y is definitely k times 0 vector; I mean, if this 0 vector this trivially object I will take that aside; I will I will take that case later.

Now, there is a 0 vector; otherwise, you could, anybody could found and this, that I am dividing by 0 vector; none of them was 0 vector that I should have mentioned; further 0 vector case it is trivially obvious, you can just directly put and see. In this side vector 0 and norm of the 0, both side norm of the 0, 0 vector is 0, on the left hand side inner product of vector is 0 that also is 0. So, equality holds for 0 vectors.

It becomes (y, x) and then conjugate (x, y); and, however, this is in the second part; y and the second, this second, this term from the second. So, this comes out, that is why I made x, y; and last one is plus, this with itself. So, y, x and x, y; y, x will come up and x, y will come up; y, square again this square, and x with itself; this is simply like that, this part is simply like that.

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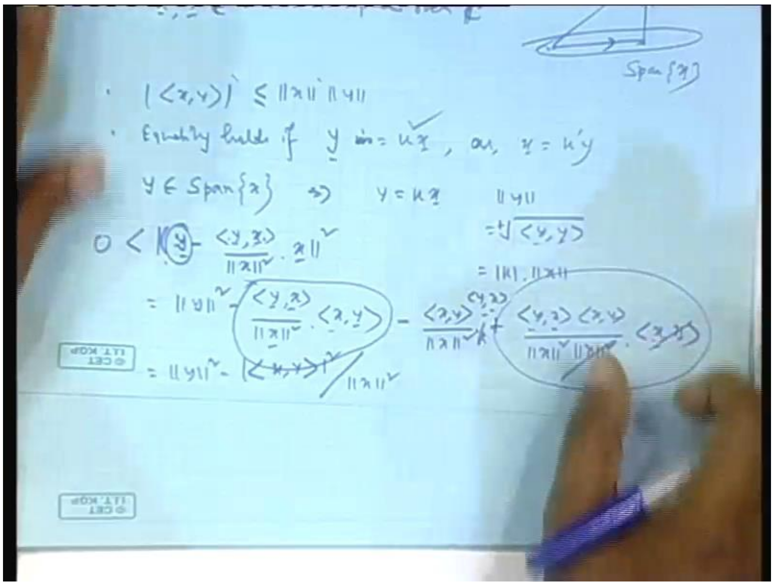

This term with itself; so from the second thing this will get conjugated becomes x, y; denominator does not change because it is a real; if it is a norm x square, repeat twice; and x with itself. Now, you see one thing, consider this term and this term, they are same because inner product with itself is a norm square; this cancels with this; y x, x y by norm x square; y x is norm square; so left with this. And, this is greater than 0; now the proof is obvious.

Third term, I have left out one term here; third term here missed the one more terms, I missed out; how the, how did the third term come? This came out; no; how did it third term, y, x; yes, sorry; I only looked at the coefficient, but the inner product part I did not looked at it. This fellow is the first case, this fellow is the second case. So, this coordinate, this thing came out, this conjugate and it becomes x y, but the inner product part with y and x I left out, sorry.

So, there what, what do get here? Norm of y square divided and mod square divided by norm of x square; this is greater than the 0. So, proof is obvious now. Take this guy on the left hand side; and this is the prove we done.

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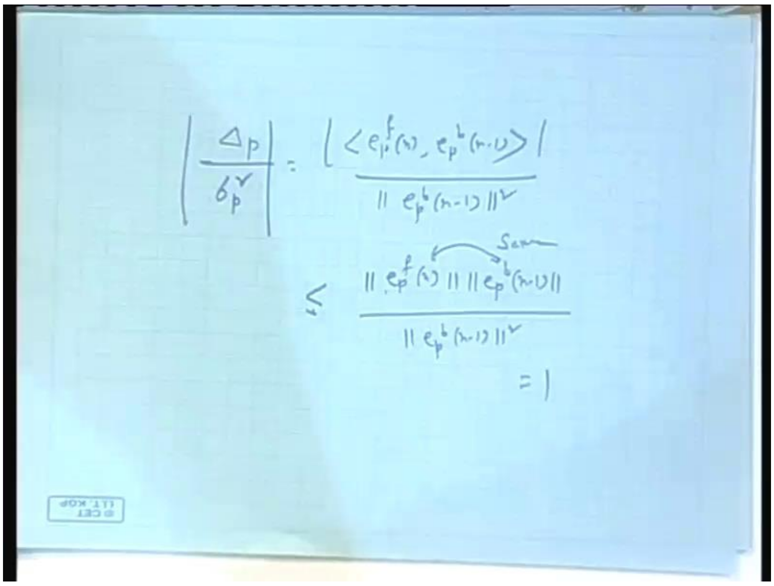

Now, look at quickly. R terms was delta p by sigma p square; delta p is mod I mean interested. What is delta p? Delta p was e p f n, e p b n minus 1. And, what is this? Norm square of e p b, n or n minus 1, does not matter, right. I am interested in finding the mod, mod of this. Mod of this means this only because denominator is already real, no pointing real, and positive no point of taking mod and all that.

Now, I apply carchy-selwar. Since, I am assuming, since there is no linear relation you can see yourself, this is not a multiple of this, they will not become; that is x n projected on this fellow the error, and x n minus p projected on the space error, they will not be linearly, you know, linearly related, in general.

So, in that case, still I am putting less than equal to; actually it is strictly less than because that will never happen, that x n projected on the past and the error, and the x n minus p projected on the future and the error, they become, one become the multiplier of the other; it does not, it is not happening in our case because there is no linear relation actually, leaving the sample or leaving the projection errors and all that.

But, you mean otherwise this is always true less than or equal to. So, this is what this norm is by carchy-selwar; and this 2 norms are same; this 2 norms are same. We have seen this 2 norms are same, is it not? These 2 same we have proved it. So, this numerator and denominator both are same now, is equal to 1.

Remember this; this will be required because I will be using auto replacing modeling there will be all pole filters; the filter is stable because of this property. This poles will lie within in its circle, itself stable recursion because this property of the reflection coefficient; that is all for today.

Thank you.