Neural Network and Applications
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Lecture - 28
Use of Green's Function in Regularization Networks

Hello everybody, now one of the things, which I have to do at the beginning of every lecture, is to write down the title of that lecture. And you remember that, last time we had written the title as solution of Regularization Equation and to talk about Green's Function. Now, unfortunately, we could not complete that whole process, because solution as such is pretty much involved. And that is the reason, why it could not be completed in one lecture and we are spelling over to this one.

So, in this class, actually what we will be doing is the plan that we had for the last lecture, that is to really solve the regularization equation and it be introducing the green's function. In fact, we will be introducing in the green's function from the beginning of this lecture itself. And later on, what I want to do is to talk about the regularization network, which is expressible in terms of the green's function So, that is going to be the title of this lecture, that we want to talk about green's function in regularization network.

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So, let us begin with this topic now, before we do that, just a very quick summary as to, what we have been doing so far in our discussions pertaining to the radial basis function chapter. Now, as I told the beginning of the discussions related to this chapter, that we have been looking at it, purely from the point of view of the interpolation of a function.

So, what we have is that at the input we are having, input in the multidimensional space in m 0 dimensional precisely where, we are having a non-linearly separable problem, which we are classifying ultimately by building up non-linear interpolation function. And in fact in the last class, we had seen that rather than treating it as a non-linear interpolation function entirely, because if it is treated an entire as an interpolation function. That means to say, that the surface the multi dimensional surface, that we are trying to reconstruct.

Has to necessarily pass through the given set of points, given as the training samples, but we said that instead of making it like a rigid requirement, that it has to pass through the given set of points. We will be treating the problem, not exactly like at interpolation, but like regularization. In the sense that, the cost function that we considered, that included two terms.

The first term was the standard error term, which is in the mean square error sense and there was a second term, which minimized the derivatives of the function, which means to say the day we had imputed, a smoothness of the function that we are trying to interpolate. So, these two things, that is to say, the standard error term and the regularization term, related to each other. The relative proportions decided by the factor lambda, which being the regularization parameter.

So, we had these two terms combinedly leading to the error functional and we had said that, this error functional is to be minimized. And the approach that we had taken the, mathematical approach that we had taken was that, we took the differential, that is to say the fret differential, we had considered. And then, the fret differential, we had expressed in the form of the inner products. And by expressing it as inner products, the condition that we had ultimately derived is that, in order that differential works of to be 0.

The essential condition that we have got was in the form of Euler Lagrange equation, which I had presented in the last class. In fact, our last class, if you remember ended there only.

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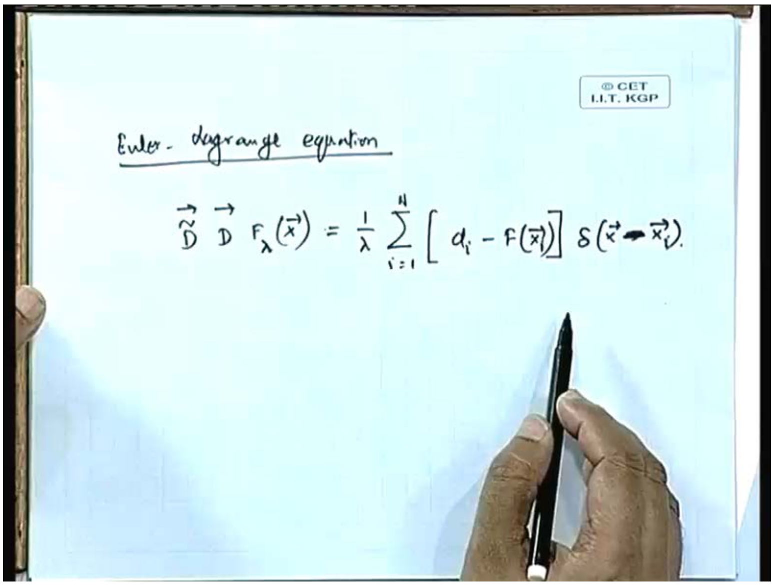

So, we had an equation, which we were calling as the Euler Lagrange equation, which is of this form; that it was D tilde, everything, D is of course in the matrix notation, D is the derivate operator and D tilde is the adjoint derivative operator of that. And then, if lambda is the optimal function that we had found for F, so optimal function is written in form of F lambda of x. So, F lambda x, although we are not writing exact expression for F lambda s, but we are writing the expression in terms of D tilde D, which means to say that it is expressible in the form of the partial differential equations involving F lambda.

And this was equal to 1 upon lambda i is equal to 1 to N, d i minus F x i and this multiplied by the delta function that we have got delta x minus x i vector. So, this is what we had got, this one is not exactly the solution. Because, in order to get the actual solution, what we have to do is to get some equation in the form of F lambda of x, but what we had got is, F lambda operated with this D tilde D operator.

So, it is not exactly, what we are looking for, but definitely in a better form than, what it was initially. So, if out of this equation, it is possible for us to make a solution of F lambda, then we have got what we want to do. So, in order to solve this problem, in order to solve this partial differential equation, this is now very straight forward. What, we have to do is to solve it by making use of a class of functions, which are known as green’s function. And green’s functions are in fact defined, always with respective some differential operator.

Now, before we actually define what green's function is, look at again this expression, you see that this F lambda is operated upon with D tilde D and both D tilde as well as D happen to be linear operators. So, the combined operator, if we treat D tilde D as combined differential operator, then this combined differential operator is of course say, linear differential operator that we are talking of. So, this is a linear differential equation that we had got.

So, we have to now made a solution of this and to solve this, we introduce a class of function called as the green's function. And let us see that, what green's function definition has to tell for us.

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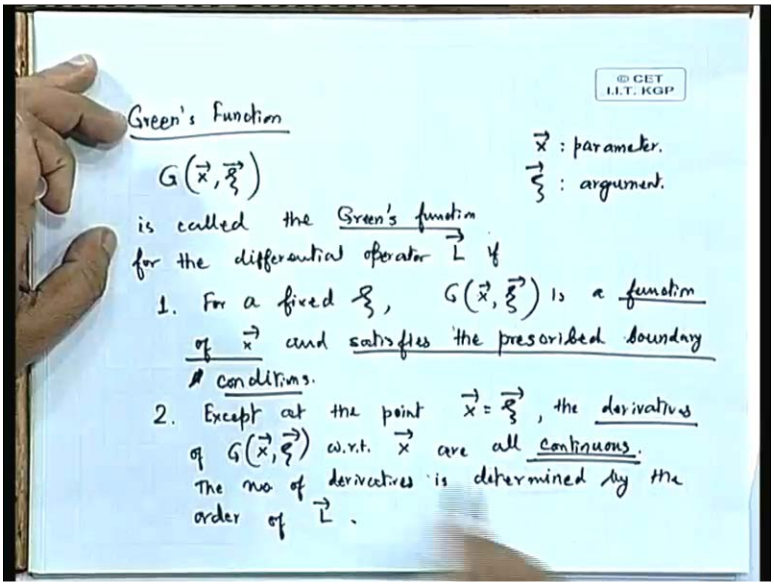

Now, green's function says that, if we take a function G, written as, it is arguments are G x comma zeta, both are vector arguments. In fact, both these arguments are having the same quantity, this x will be treated as a input as you can will understand and zeta is also repeat treated as input. So, actually quantity wise both are inputs, but in functional definition, we will be treating x as a parameter and zeta as an argument. So, will be treated as a parameter and zeta vector will be treated as an argument, but essentially they are the same quantity.

Now, this as G of x zeta, this is called the green's function for the differential operator. Now, green's function, mind you is always is define with respective some differential operator, so we call this to be a green's function, for the differential operator L. So, we consider a differential operator, a linear differential operator and we are designating that linear differential operator by L. So, G is green’s function for differential operator L, if it satisfies three conditions.

So, the three conditions, that we are putting forward is that, number 1; that for a fixed zeta, for a fixed zeta because zeta happens to be the argument. In fact, you will later on find that the green’s function, that we develop. The green’s function will be centered around this zeta, it will be a function of x, but it will be centered around zeta. Something very similar to what, we had already considered for the radial basis functions.

You remember that radial basis function also, what we had considered was that, radial basis function was a function of the x vector, which is the input vector, but radial basis functions also had some definite centers. And we had to consider the centers of the radial basis function to be the individual patterns. So, you remember that we had consider the radial basis functions, center around x 0, center around x 1 like that, center around x N. So, very similarly, we take a function G of x, which is center around zeta.

So, there be first condition is that, for a fixed zeta, G of x comma zeta, that our green’s function, that we are considering is a function of x. Definitely, it has to be a function of x vector and it has to satisfy the prescribed boundary condition of what is specified. So, and it has to satisfy and it satisfies the prescribed boundary conditions. So, this is the number 1; condition for G of x to qualify as a green’s function.

And the point number 2; is about it is continuity, it says that except at the point x equal to zeta, where, x is equal to means that, were variable that you are considering is at the center of the function. That means, to say that at the weather parameter, it was to the argument. So, then, there the derivatives, expect at that point, everywhere else, the derivatives of G of x comma zeta with respective x are all continuous.

The number of derivatives that we have to consider, because we are saying that, the derivatives of G x zeta, so that means to say, how many derivatives, that will be decided by this operator L. Because, I only say that it is a differential operator, I have and said that differential operator of what order. It can be order 2, it can be order 3 and it can be order n, so the number of derivatives, which has to fulfill this condition of continuity.

The number of derivatives is determining by the order of this operator n is determined by the order of L, so this is condition number 2. So, condition number 1, is about it as a function and satisfying the boundary condition. Condition number 2, is the continuity condition that expect of the point, x is equal to zeta, this function G of x zeta has to be continuous.

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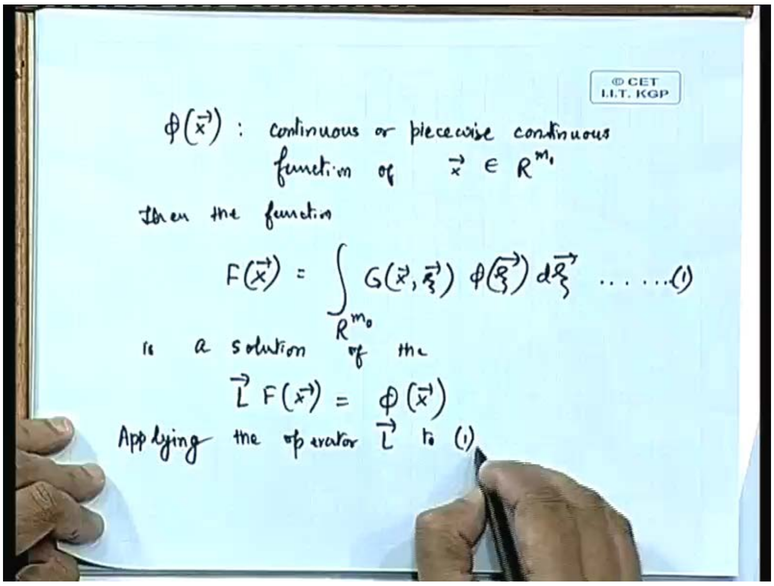

And there is a point number 3: which says that with G of x zeta, consider as a function of x. It has to satisfy, it satisfies the partial differential equations, I am writing that as P D E, so it satisfies the partial differential equation of L G, argument x zeta is equal to 0. So, the differential equation that we are getting, the partial differential equation that we are getting, by applying the operator L on this G function, that is equal to 0. Everywhere, expect that x is equal to zeta, so expect at the center point. The partial differential equation that we get out of this G is equal to 0.

Now; that means, to say that there are two things, involved about the derivative constrains and the partial differential equation. Number 1 is that, it is individual derivatives must be continuous, number 2 that when you are considering all the derivatives together, expressible in the partial differential equation. That partial differential equation involving L, involving G should be equitable to 0. So, if these three conditions are satisfies, that mean to say that G x zeta being a function of x satisfying boundary condition.

Number 2 the continuity of G of x and I am the continuity of it is derivatives and number 3 that it is partial differential equation is equal to is equated to 0. If all these three functions, if all these three conditions as satisfies, then only we can say that G of x zeta is the green's function for the differential operator L. And this is a very important definition that we have to keep in mind. Because, the solution that we will be now attempting for the Euler Lagrange equation will be best upon the green's function.

Now, one think, there is a different way of writing this, because here I wrote the statement, G x zeta operated on this L, is equal to 0, expect at this. Now, in mathematical language, I should be writing it more preciously as L G x zeta to be equal to delta of x minus zeta, this means one on the same thing. Because, when x is equal to zeta, then this delta is giving us value, other than 0 and when x is not equal to zeta, then delta gives us the values of 0.

So, these are the three things and now based on this green's function definition, we attempt the solution of the Euler Lagrange equation. Euler Lagrange equation is as I have already told you, has been derived out of the original Tikhonov functional, which involved both the standard error term and the regularization term. Now, let us consider function, which we call it as phi of x, we consider a function phi of x, which is a continuous or piecewise continuous function of x.

Continuous or piecewise continuous function of x vector and x vector as I have told again and aging will be belonging to the m 0 dimensional real space, so it lies in a R m 0. So, we define a function phi of x, as follows a as define just now, then the function, if we consider a function of this nature, that is to say what, we consider a green's function G of x zeta, which is a green's function with respective, let say an operator L. Mind you green's function, every time we have to define a green's function.

We have to say, it is green's function with respective what operator, so it is a green's function with respective operator L and then, we take an integral of this, that is we multiply this greens function with phi of zeta, D of zeta. And then, we integrate this over the R m 0 space and if we do that, then f x vector, we get a function involving x. Because, here we are integrating it in the zeta space, but ultimately this integral quantity as a whole, becomes function of x, so we are calling that as a function of x.

So, if we write a function of x like this, function of x in the form of this integral, where G is a green's function. In that case, this F x is equal to this, this is a solution of the differential equation, yes

Student: ((Refer Time: 21:15))

L, F x is equal to 5 x, not L G x. In fact, it originates from L G x, you are not 100 percent wrong, because what you can do is that, I can understand that, what thinking he had in mind, that if we now operate both the sides with the operator L. Then, what you are getting is that, on the left hand side, you are getting L F x. And on the right hand side, you are getting, L operated on with this entire integral expression.

Of course, that means to say, that the L operator could be taken inside the integral sign also, so it could be treated as an integral over R m 0 space, L G x zeta and L G x zeta, we very much know, just now we got L x zeta, L x zeta means what, it is delta of x minus zeta. In fact, if it is a delta function multiplied by phi function, then delta has got a value, that expect at the point of x equal to zeta. Everywhere is the delta function is having a value equal to 0, so ultimately we get what?

Student: ((Refer Time: 22:28))

Very good very, ultimately we get phi of x. So, ultimately what you are getting is that, this is a solution of L F x is equal to phi x. So, the very fact the L F x is equal to phi x, has been already worked out by some of the students here. But, just for the benefit of those who could not follow, this done very horridly. Because, we did not write down the every steps, but for the benefit of those who are little slow in understanding this aspect of the equation.

So, for the benefit of them, what I say is that, by applying is call this one as equation number 1, so I can say that, applying the operator L, the differential operator in fact, to 1. When, I say applying the operator L to 1 that means to say, write it will be operated on the sides, left hand side as well as the right hand side. So, by applying that, what we get is that L F x.

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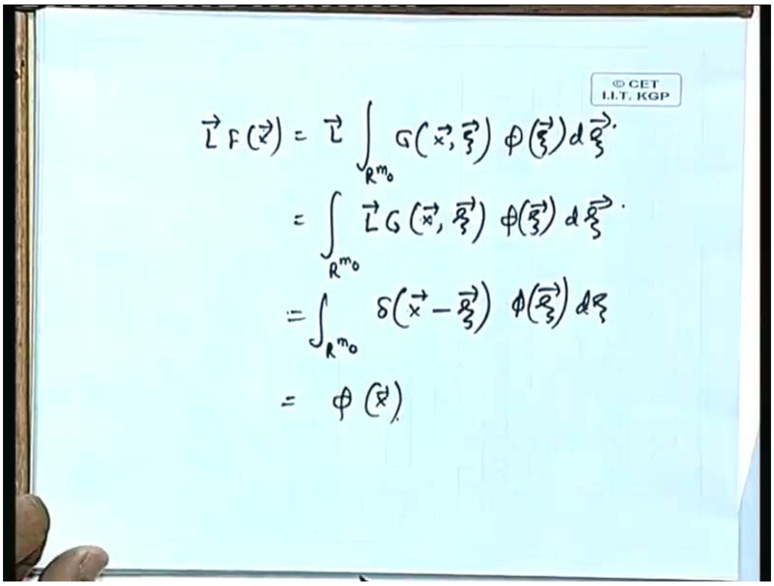

$$\vec{L} F(\vec{x}) = \vec{L} \int_{R^{m_0}} G(\vec{x}, \vec{\xi})\, \phi(\vec{\xi})\, d\vec{\xi}$$
$$= \int_{R^{m_0}} \vec{L} G(\vec{x}, \vec{\xi})\, \phi(\vec{\xi})\, d\vec{\xi}$$
$$= \int_{R^{m_0}} \delta(\vec{x} - \vec{\xi})\, \phi(\vec{\xi})\, d\xi$$
$$= \phi(\vec{x})$$

L F x is equal to L, integral R m 0, G x zeta, phi zeta and d zeta and this, as I told you just now can be written as R m L G, x zeta, x comma zeta phi zeta, D zeta and this being equal to the delta function. It is R integral over R m 0, delta x minus zeta, phi zeta, d zeta. And this is simply equal to phi x, from the where the definition of the delta function, so we have got this, so that means to say 1.

If we have got a green’s function G, then G is actually a green’s function with respect to the derivative and then, if this is expressible as an integral equation like this. Then, effectively the solution of this becomes L F x is equal to phi x. So, this is the very important relation that we will be making use of that. Here, we are getting L F x is equal to phi x.

Now, ultimately what we are trying to do, we trying to solve the Euler Lagrange equation. Now, Euler Lagrange equation says, D tilde D, F lambda is equal to this quantity. Now, D tilde D itself, combinedly is a linear differential operator, so I can call this D tilde D together to be the operator n. So, in that case, it becomes L F lambda is equal to this.

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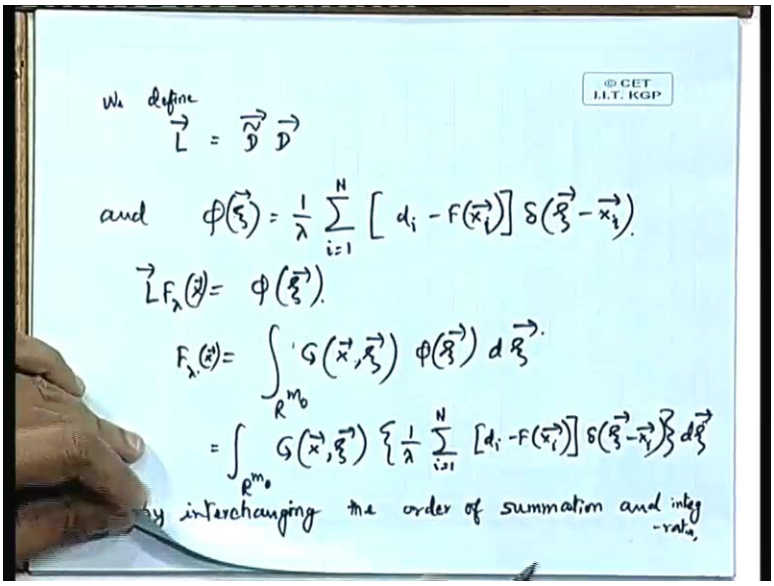

So, in order to shows Euler Lagrange equation, simply what we will be doing is to define my L operator as D tilde D. So, we define the operator L like this and then, we it consider phi of zeta to be the term that we have got in the Euler Lagrange equation. So, here you see that, ultimately I am going to express the Euler Lagrange equation in exactly this form, if I call this to be equation number 2.

Then, I want to represent Euler Lagrange equation in the form given by 2, so 2 means that L operated on, F operated with L and L we are taking to be the D tilde D. So, left hand side is obtained and right hand side this whole thing phi of x, we are treating like this. So, here what we doing is that, we defining phi zeta, then to be equal to 1 by lambda, summation i equal to 1 to N, d i minus F x i delta zeta minus x i, prefer to Euler Lagrange equation once again.

Here, what we have simply done is that instead of x, we had put zeta, does not matter. So, we are calling this whole function to be, here it becomes phi of x. And by the finding it like this, it becomes phi of zeta, so phi of zeta is define in this format. If that is so, that means to say that L F lambda is equal to this. In that case, in what form can expressed F of lambda, because you see L F lambda is equal to phi zeta, answer it is this.

Then, F lambda is expressible as F lambda of x of course, so F lambda of x is going to be the integral equation, integral involving what

Student: ((Refer Time: 28:18))

Green's function and then, phi function, so it will be green's function that is G of x zeta and then, the phi term and the phi is phi zeta. Because, the argument, the variable that we are considering and that the integral sign is the variable is zeta. So, we will be writing this phi zeta, d zeta and this will be integrated in the R m 0 spaces. So, now looks like that our solution is getting closure.

So, the solution of F lambda that is what we have been looking from is equal to this and this can be written simply as integral R m 0. Just I substitute the expression for phi zeta, x zeta. And here, I am going to write 1 by lambda that is whole substitution of this, summation i is equal to 1 to N, d i minus F x i, delta, zeta minus x i and then, d zeta is there.

Now, here actually first comes integral and then comes summation, the order of integration and summation can be interchanged in this case. So, if we interchange the order of summation and integration. So, by inter changing the order of summation and integral and integration, what we get is, I will show in the next page.

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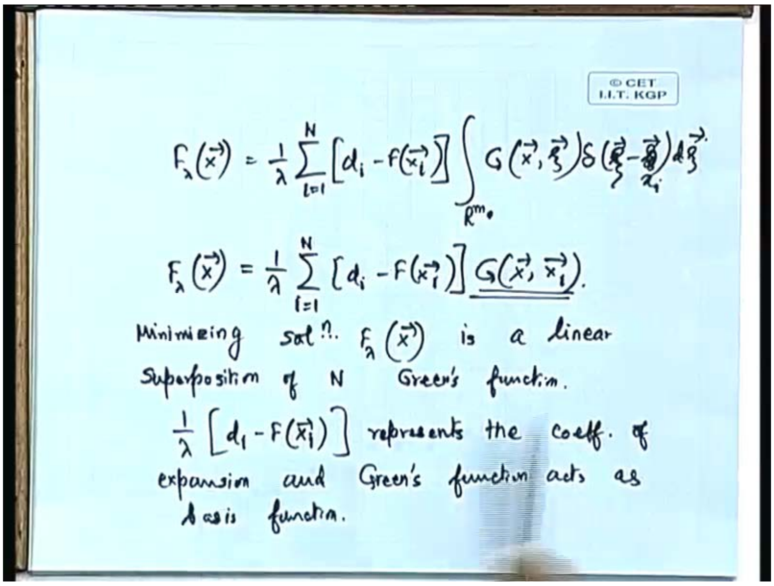

That is to say F lambda of x is equal to 1 by lambda, summation i is equal to 1 to N, d i minus F x i. This comes out, the summation I am taking out and then what remains as the integral is G x zeta, delta zeta minus x i, d zeta. So, then we had the left over as the integral R m 0, G x zeta, delta x minus zeta, d zeta. Because, here this is zeta minus x, zeta minus x I, that is part, it is.

So, now this one is what, this one this term remains as it is, that 1 by lambda, summation i is equal to 1 to N, d i minus F x i and then, what will be this

Student: ((Refer Time: 31:52))

Correct, G of x comma x I, this is the interesting expression, so that means to say that the minimizing solution, what is after all F lambda x, F lambda x is the minimizing solution to this interpolation problem. To this not interpolation, strictly speaking, regularization problem. Now, this is actually obtained as a solution of, this is actually obtained as a super position of N different green's function. And what is N, N is the number of training patterns that we are feeding.

So, corresponding to the number of training patterns N, we have to constitute N number of different green's function. So, it is a linear super position, so the solution is obtained as a linear super position of N number of green's function. So, I can say, that the minimizing solution, which is F lambda x is a linear superposition of N green's function and what are these green's functions.

Let me see, that how are we getting N different green's function, quite simple, because we have got G x comma x i and what is x I, x i is the center of that function and we have got i varying from 1 to N. So, that mean to say, that we have got G x comma x 1, x comma x 2, x comma x 3, like that up to x comma x N and we have got N such different green's function possible, so for every input x.

So, x is our input and we already have got the green's function, which as centered around the N number of training inputs, which we already got. So, x will be mapped into G x, x i and there are N such mappings, which will be possible and all this N green's functionally mapped outputs, have to be linearly combined. So, you see that effectively, what is this G functions acting for, this G function is acting as a basis. And what is this d i minus F x I, this d i minus x i is represented in the form of coefficients of expansion.

You see that, if we treat G or in this summation equation, this is the summation equation. In this summation equation, there are N such green's function, so if we treat all these N different green’s functions to be the basis. Then, the coefficients which are associated with this, becomes the coefficients of expansion, means as if to say the treating this as basis functions. We are realizing the ultimate function F lambda as a summation of basic functions.

Just like the way, we express any function as a summation of basic functions, taking the complex exponential to be the basis functions as we do in the case of the Fourier transforms. Taking the cosine to be the basis functions, in the case of discrete or cosine transforms like that. So, this will be form the basis functions, so green’s function acting as the basis.

And this term that is d i minus F x I, nothing exactly d i minus F x i, I should say 1 by lambda d i minus F x i, they represent the coefficients of expansion. And green’s functions acts as basis function. Now, we have to determine all this expansion coefficients, means as if to say that we can also say, that if this whole thing 1 by lambda d i minus F x i, the coefficients of expansion that we have got.

If this whole thing 1 by lambda d i minus F x i, we call as w i, in that case w i is the coefficient of expansion. So, then F lambda x is manageable in a form of summation of w i times G of x, x i, so if we treated that way.

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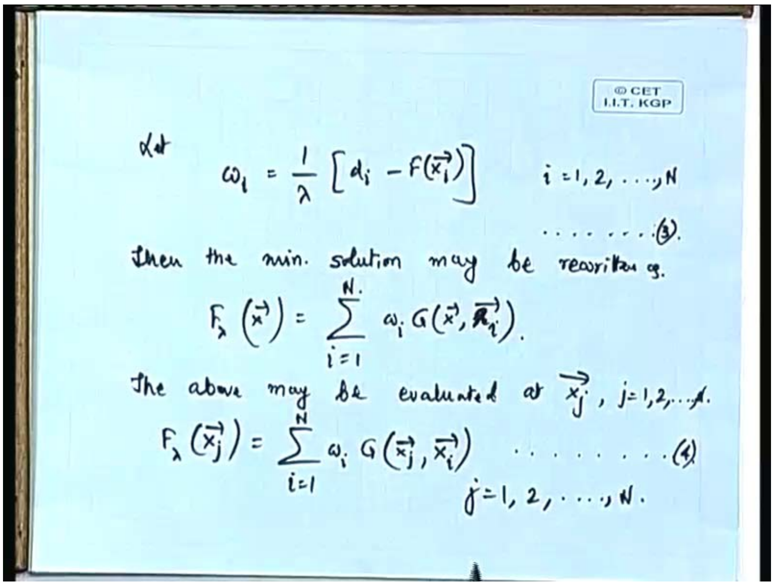

So, if we let, that w i is equal to 1 by lambda d i minus F x i, which we have to define for i is equal to 1, 2 up to N, so this should be the equation number 3 for us. So, here there are N such w's, which one can define. Then, we may recast the minimization solution, then the minimization solution can be re written as what F lambda x equal to summation w i G x comma x comma x i and this has to be summed up for i is equal to 1 to N.

Now, this is a general term x is the input, x is any input. So, you take x as any input and then you have to operate this through this green's connection G x comma xi's, there are N, such xi's, which are available and such centers, which are available. So, as and input to it, as the input to this, we can feed any of this xi's, we can feed x 1 as input, we can feed x as input, we can feed up to x N as input. That means to say, that this function this minimize solution, we can evaluate at all these N different points.

So, the above may be evaluated at x of j, we can say, because we are using this as the index i, we can say, this may be evaluated as at x of j, where j is equal to 1, 2 up to N. So, if we do that, then we can write F lambda of x j vector is equal to summation i is equal to 1 to N, w i G of x j, yes x j comma x i and this we can compute for j is equal to 1, 2 up to N.

So, this and this are basically the same, only thing is that, this is a general solution and these are all these N different, the general solution evaluated at N different points. Call it as equation number 4. So, now what we can do is that, this equation number 3 and the equation number 4, which can actually be computed for N different points. Equation number 3 also can be computed for N different points. Equation number 4 also can be computed for N different points with j are equal to 1 to N.

So, then we can represent the whole equation, the entire minimization equation, that we have got. Instead of writing it in this form, we can express in the matrix form, if we define the matrices like this.

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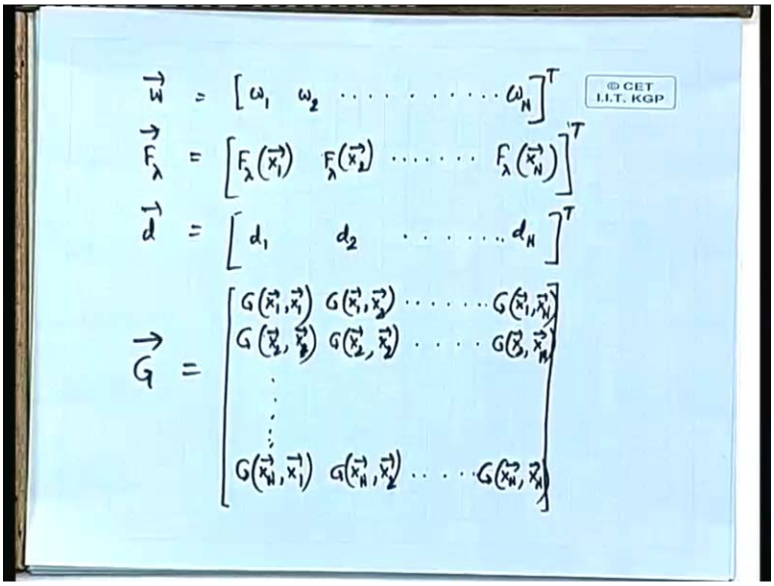

So, we make some definitions, we introduce some definitions where, all this F lambda x 1, F lambda x 1 means, that it is the F lambda evaluated at the point x 1. Like that, one can evaluate F lambda at the point x 2 and up to F lambda can be evaluated at x N. This, if we treat as a vector, column vector, then we can call this combinedly by F lambda vector. So, the definition of F lambda vector is that, it consists of N different elements, whose elements are the functions evaluated at N such different points.

And like wise, we have got the desired outputs for the pattern x 1, the desire output is d 1, for the pattern x 2, the desire output is d 2, for the pattern x N, the desire output is d N. So, now I can write everything together in the form of a vector and call this as the d vector and now you see that, I have got this G functions. This green’s function and here it becomes G of x j comma x i.

So, which means to say, that I can treat any of this, I can have N different centers to that green point, to this green’s function and this can be evaluated again at N different points. So, for every center, it can evaluated for N different points, every center it can be evaluated for N different points, meaning that, for N centers, it can be evaluated at n by N number of point. So, in effect out of these G x j comma x i, we are going to get a matrix of size N by N.

So, we are going to define that matrix, which translator in row terms, would mean that we got function G of x 1 comma x 1, which means to say, evaluated at x 1, centered around x 1, then we can take G of x 1 comma x 2. The green’s function centered at x 2, evaluated at x 1 and likewise, green’s function having center at x N, but evaluated at x 1. And then, again we can say green’s function, whose center is at x 2, whose center is at x 1, but which is evaluated at x 2.

In the first row, you see that everything is evaluated at the point x 1, in the second row; everything is evaluated at the point x 2. So, evaluated at x 2 with center x 2, here evaluated there x 2 with center as x N and here it is G evaluated at x N with center at x 1, evaluated at x N center as x 2. So, this column effectively corresponds to the center. So, this is the center 1, this is the center 2, last one is center N and the row corresponds to at what point it is evaluated.

So, it is the first point of evaluation, that is x 1, this is second point of evaluation, this one is nth point of evaluations, so this is G x N comma x N. So, in effect we are getting an N by N matrix and this matrix combinedly, we are calling as the G matrix and what else is to be defined, this w i’s, you see, w i also we can define at N different points. So, I can compute w 1, w 2 and w 3 up to w N.

So, combinedly, we should write another vector, which because, when I may not be having place in the bottom, so I am writing on top, so this one is w 1, w 2 up to w N. All these different w’s are possible N different, such w’s are possible and combinedly, we are going to call it as the w vector. So, now I think, this definition is nothing but just putting all these N together in one representation.

Earlier, we had N different representations of this equation, N different instance of this equation, with i is equal to 1 to etcetera up to N. And now, the same equation, that is equation number 3, what we can write down in the combined form.

Student: ((Refer Time: 45:59))

W vector, equation number 3; I am saying equation number 3; becomes w vector is equal to 1 by lambda, d vector minus F lambda vector. So, that is the matrix form of representation of equation number 3. And likewise, the matrix form of equation number 4; will be F lambda vector or F lambda matrix rather will be equal to G w, G matrix multiplied by w vector.

So, the two equations, equations 3 and equations 4 in the vector matrix form, this is representing in the scalar of form, individual scalar form. So, in the vector matrix Notation equation number 3 and 4; may be re written as.

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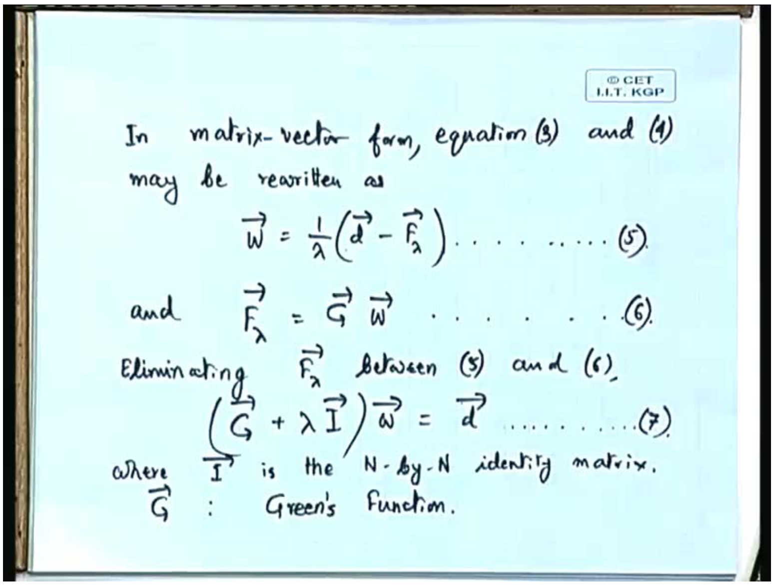

So, in matrix vector form, equation 3 and equation 4, they are not new, it is just another way of representing in the mat matrix vector form. So, this may be re written as w vector is equal to 1 by lambda, d vector minus F lambda vector

Student: ((Refer Time: 47:33))

F lambda is the minimum 1, but minimum one is the 1, that we are seeking, mind you

Student: ((Refer Time: 47:43))

You are correct, see that means to say that

Student: ((Refer Time: 47:48))

You see that, when we obtain the final solution, the final solution to it when we got, the final w i is that we are going to get is d i minus F lambda of x i

Student: ((Refer Time: 48:08))

F lambda is w times G, yes

Student: ((Refer Time: 48:14))

Let me clear your confusion, you see, what you are trying to tell me

Student: ((Refer Time: 48:28))

See, if what we have got for the F lambda is this, this equation F x i, we got and the minimized solution, we are getting as F lambda x, I accept, so that means to say that given the function F x i, we have got F lambda x. Now what we have been doing is that, if ultimately we get the solution as F lambda, when we got the ultimate solution, minimize solution F lambda.

And at that point, if we want to calculate that what is the coefficient; when we have obtained the solution that means to say that if we are solving for w. If we are obtaining the solution of w, under the minimized functional case, in that case, I should not be taking that as F any mode, I should replace F by F lambda. Because, already we will be getting F lambda to be the solution of that, followed.

So, this becomes the minimized solution case, so what I have to do is that, this one is F lambda, which I can call as the equation number 5 and this F lambda should be equal to G w, which you have set, only thing is that, you had some confusion about putting F lambda over here. Now, F lambda, we can only put, when this is the final solution of w, because ultimately, that is the w; that we are going to solve at the end.

So, now, if from equation number 5 and 6, we eliminate F lambda, so by eliminating F lambda between 5 and 6, what we get here, because it is G w here. We can have this to be lambda times w, lambda times w to be equal to d minus F lambda. But, that make it that, that is one on the same as writing, that I can write it as G plus lambda, if I treat i to be an identify matrix.

So, I can write it as G plus lambda I, w to be equal to d, where I is the N by N identity matrix. So, this is what we get and we call G matrix as, G is refer to as green's matrix, because it is matrix realized out of green's function. So, if we have this, now can you see one thing, this is equation, this equation that we have got and that is equation number 7.

Does not it remind us of the equation, which we had got, why solving the interpolation equation involving radial basis function. What was the interpolation equation, remember, that was phi w, phi matrix w is equal to d. And here, we are getting G plus lambda I times, w equal to d. As if to say, that whatever role, the phi functions, the radial basis functions at plate for interpolation, the similar type of action is being taken by the G function to solve the regularization..

So, what was phi for interpolation becomes G plus lambda I, for the regularization. Now, for regularization, you again see, that it is not G alone. It is G plus a term lambda I, where there is a definite involvement with this term, just why we introduced this I, because we could have satisfied the equation without introducing this I, also. But, just to write in a consistent the form, that if we write these things in the parenthesis form, then there has to be this lambda, lambda is a scalar quantity.

There lambda has to be multiplied by matrix, so that together this G plus lambda I, becomes a matrix, because what we have to do is that, if you now want to solve for this w. What we indeed are doing is to take the inverse of G plus lambda I. So, whatever arguments we had put forward about the existence of inverse functions for the case of phi, will now come into being for the G functions as well.

That means to say, that they are should be only a certain class of functions, for which this G plus lambda I, this matrix will be positive definite. And only, if it is positive definite, then G plus lambda I, inverse should exists and then, we can solve for w. And what are the G functions, that fulfill this criterion, in fact it is called as Mitchell’s criteria. The Mitchell’s criteria’s be saw that, the positive definite criteria is fulfill by certain class of functions for phi, what were they, we took as examples.

The Gaussian functions, the Gaussian functions could do and then, the multi quadrics and the inverse multi quadrics, this three functions could fulfill that condition. Now, here for the case of G plus lambda I, we find that the Gaussian as well as the multi quadrics, which are essentially local functions. They become positive definite and they can be used as green’s function.

And another point, which should be noted for the green’s function is that, that we have define the differential operator, how be defined that differential operator L, in this definition it is L is equal to D tilde D and what is D tilde, D tilde is nothing but the adjoint of this D. So, since it is defined as D tilde D, you will find that the adjoint of L.

Now, if we take the adjoint of L, we will find that they adjoint of L, becomes equal to the operator L itself.

That means to say, that the operator L, it can proved that, since it is define as D tilde D, the adjoint of L, if you take, the adjoint becomes same operator L, which means to say that L is self adjoint. And if L is self adjoint, then G matrix becomes a symmetric matrix; that means to say that G x i, x j whatever we have as G x i, x j becomes equal to G x j, x i.

Now, I could discuss further, in fact the title of the talk, said that, it is using this function we want to realize a regularization network. Now, this is the equation, which will form the basis for explaining the regularization network. Now, that we have got his equation, equation number 4 or better in the matrix form as equation number 6. It is easy for us to concept the regularization network, which I will show, till then bye.

Thank you.