Digital Image Processing.
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Lecture-19.
Interpolation With Examples-II.

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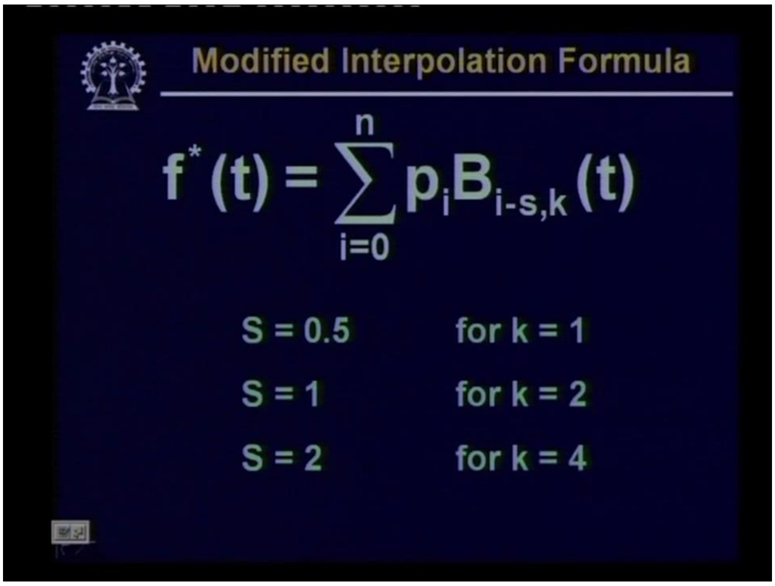

Lecture series on digital image processing, formula we make say f*(t) is equal to piBi -s,k(t), where again i varies from 0 to n. And the value of S we decide that for k equal to 1, that is when you go for constant interpolation, we assume value of S to be 0.5, for k equal to 2 that is for linear interpolation we assume value of S to be 1. And for k equal to 4 that is for cubic interpolation we assume value of S to be 2.

So here again we find that I have not considered k equal to 3, because as we said that k equal to 3 gives you the quadratic interpolation. And in case of quadratic interpolation, the interpolation is not symmetric. So what we effectively do by changing Bi,k(t) to Bi a I -s,k(t) is that the interpolation function or the B-Spline interpolation function. We give this a shift by S in the left direction, while we consider the contribution of the sample pi to the point t the for interpolation purpose.

Now let us see what we get after doing this. So as we said that with k equal to 1, I take the value of S is equal to 0.5, so in the formula that pi Bi,k(t) if I when I consider the contribution of sample p0 in the earlier formulation we had to use the B-Spline function B0,k(t). Now if k equal to 1, this is a constant interpolation so I have to consider B0,1(t). Using this modified formulation when I consider the contribution of point p0, I do not consider the B-Spline function to be B0,1(t), but rather I consider the B-Spline function to be B -0.5,1(t).

Similarly for the linear operation when I take the contribution of point p0 to any arbitrary point along with B p0 I had to consider as per the initial formulation B0,2(t). Now using this modified formulation I will use – B -1,2(t). Similarly for the cubic interpolation again with p0, I will consider the B-Spline function to be B - 4(t) instead of B0,4(t). So here you find that in this particular diagram using this formulation effectively what we are doing is we are shifting the B-Spline functions by the value of S in the leftward direction.

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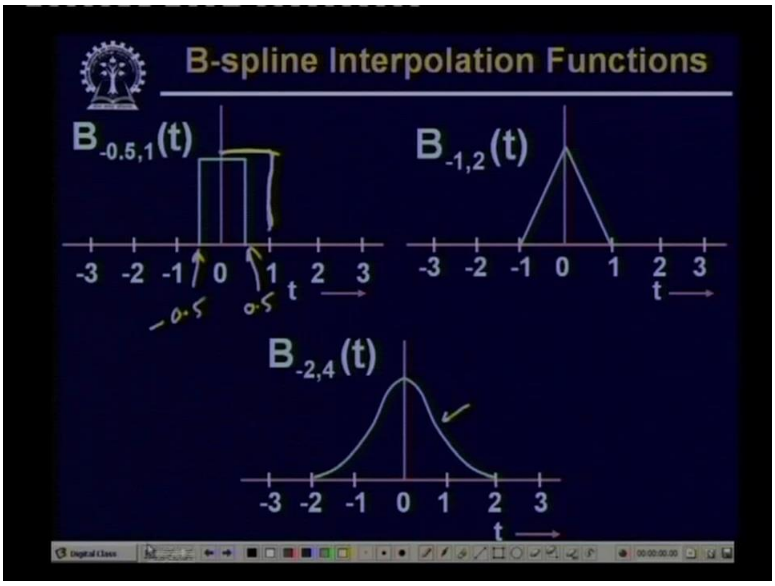

So B0,1(t) in the earlier case we had B0,1(t) to be 1 between 0 and 1. So B0,1(t) was something like this, now along with p0, I do not consider B0,1(t) but I will consider B - 0.5,1(t) and – 0.5,1(t) is equal to 1 for values of t between - 0.5 and +0.5 and value of B - 0.5,1(t) will be equal to 0 beyond this range. Similar is also the case for the linear interpolation the linear B-Spline and it is also similar for the cubic B-Spline that is Bi,4(t) and in this case it will be i - 2,4(t).

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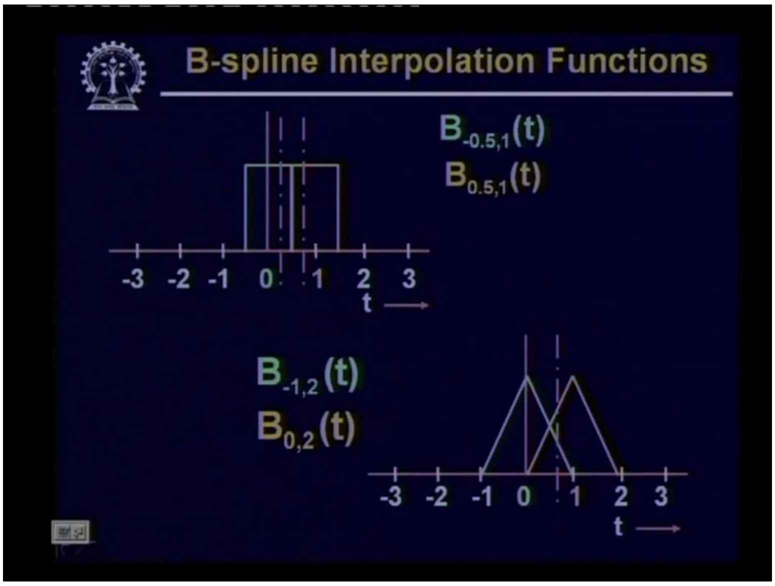

So using this let us see that how it helps us in the interpolation operation. So as I said that for interpolation when I consider the contribution of p0 to any particular point along with p0 I will consider the B-Spline function to be my B -0.5,1(t) for constant interpolation. So it appears like this that for contribution of p0 I consider B - 0.5,1(t) to find out the contribution of p1 I consider the B-Spline function to be B 0.5,1(t).

Similarly in case of linear interpolation to find out the contribution of point p0, I consider the B-Spline function to be B-1,2(t) and to find out the contribution of p1, I consider the B-Spline function to be B0,2(t) and so on.

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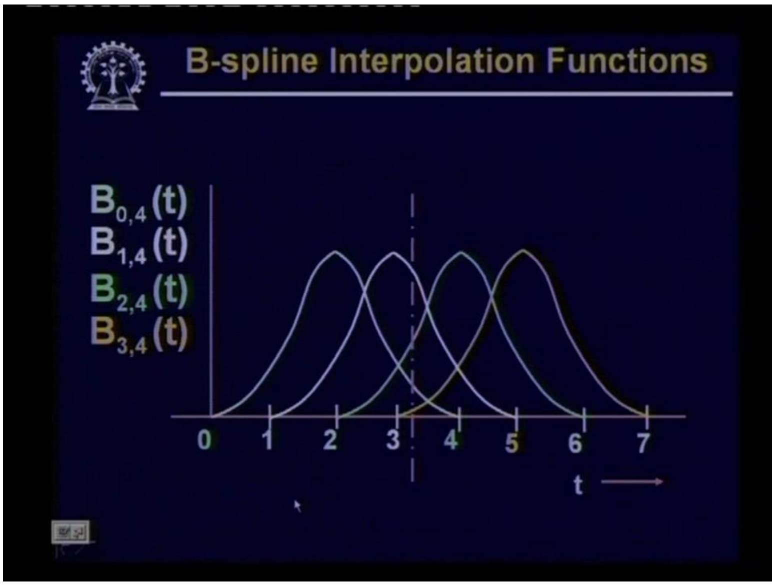

Similar is also case for the cubic interpolation, here again to find out the contribution of say p0 I have to consider the B-Spline function of B - 2,4(t). To find out the contribution of p1, I have to consider the B-Spline function of B - 1,4(t).

Find out the contribution of p2, I have to consider the B-Spline function of B 0,4(t) and so on. Now let us see that using this kind of using this modified formulation whether our interpolation is going to improve or not. So let us take this particular case again we go for constant interpolation.

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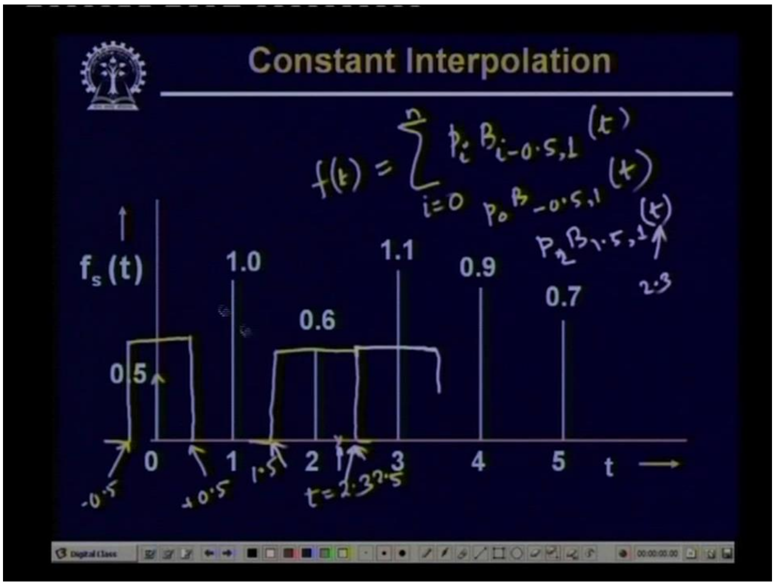

Here again I have shown the same set of samples and now suppose I want to find out what will be the value of the function at say t equal to 2.3 and for considering this I will consider the equation to be pi into Bi - 0.5 for constant interpolation value of k is equal to 1, t and I will take the sum for i equal to 0 to n.

So this will give me the approximate or interpolated value at t. So again you find that coming to this diagram as we have said that when I consider Bi - 0.5(t), ok. In that case B - 0.5,1(t) is equal to 1 between the range -0.5 to +0.5 and beyond this range B -0.5(t) will be equal to 0. So when I compute p0 B -0.5,1(t) for computation of this particular component along with this sample p0 which is equal to 0.5, I have to consider the Bezier function, the Bezier sorry B-Spline interpolation function has this which is equal to 1from -0.5 to +0.5.

So here again you find that because this B -0.5,1(t) is equal to 0 beyond t equal to 1.5, so this p0 does not have any contribution to point t equal to 2.3, because at this point the contribution of this, or the this particular product p0B -0.5,1(t) will be equal to 0. Similarly, if I consider the effect of point p1 to t equal to 2.3, the effect of point p1 will also be equal to 0. Because the product p1 B -1 p1 B 0.5,1(t) is equal to 0 at t equal to 2.3.

But, if I consider the effect of p2 which is equal to 0.6 here you find that this the B-Spline interpolation function has a range something like this, the region of support. So this is equal to 1 for t equal to 1.5 to t equal to 2.5. And it is equal to 0 beyond this range. To find out the contribution of p3 which is equal to 1.1 here again I you can see that to find out the contribution of this particular point. The corresponding B-Spline function that is B 2.5(t) is equal to 1 in the range 2.5 to 3.5 and it is equal to 0 outside.

So even p3 does not have any contribution to this particular point t equal to 2.3, so at t equal to 2.3 if I expand this I will have a single term which is equal to p1 into B sorry p2 into B 1.5, 1(t), so where t is equal to 2.3 and the same will be applicable for any value of t in the range t equal to 1.5 to t equal to 2.5. So I can say that using this formulation what I am getting is, I am getting the interpolation something like this.

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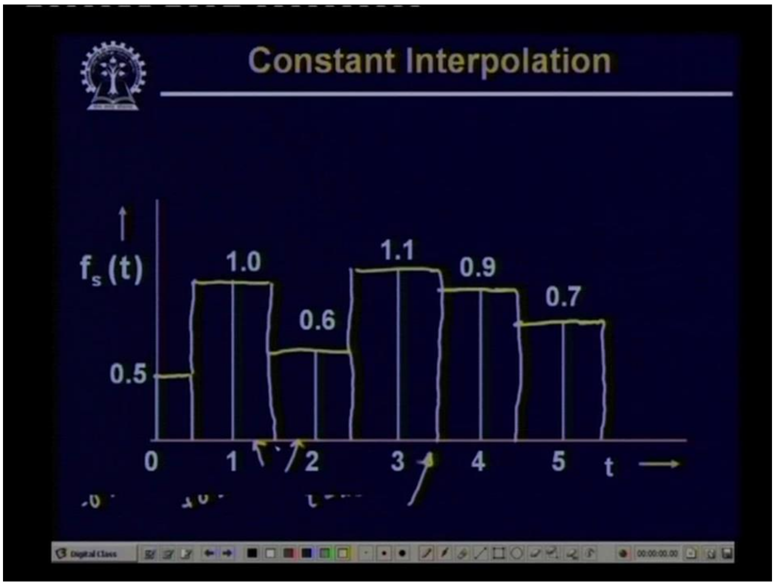

The after interpolation using this constant interpolation function after or modified formulation the value of the interpolated function will be say from t equal to 0 to t equal to 1.5, the f(t) value of f(t) will be equal to f(0). Between 1.5 to between 0.5 to 1.5, value of f(t) will be equal to f(1), from point 1.5 to 2.5, the value of f(t) will be equal to f(2). From 2.5 to 3.5 the value of f(t) will be equal to f(3), from 3.5 to 4.5 the value of f(t) will be equal to f(4). And from 4.5 to 5.5 the value of f(t) will be equal to f(5).

And this appears to be a more reasonable approximation, because what we are doing is whenever we are trying to interpolate at a particular value of t what we are doing is we are trying to find out what is the nearest sample to that particular location of that particular value of t and whatever is the value of the nearest sample we are simply copying that value to this desired value to t.

So here for any point within this range that is for t equal to 1 to t equal to 1.5, the nearest sample is f(1). For any sample from 1.5 to 2, the nearest sample is p2, or f(2) so this f(2) is copied to this particular location t, where t is from 1.5 to 2. So this appears to be a more logical interpolation than the original formulation of interpolation. Similar is also case for linear interpolation.

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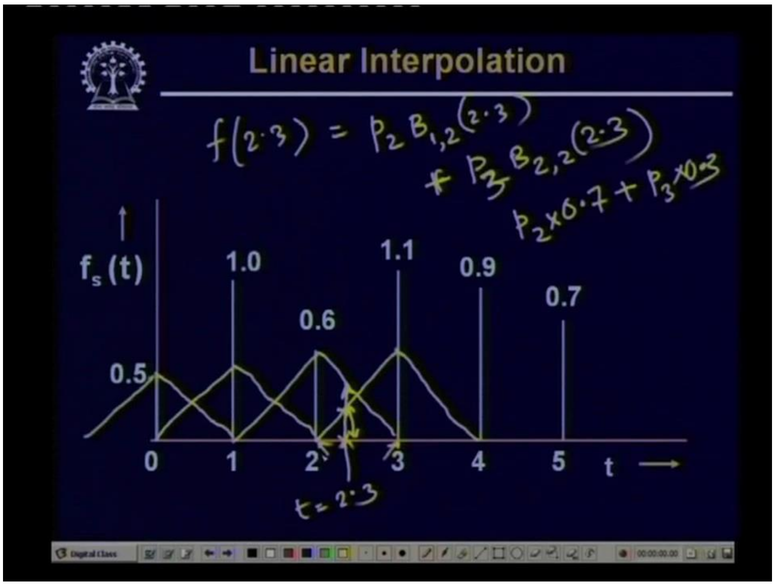

In case of linear interpolation, when I consider the value of p0. What I do is I consider B -1,2(t) and B -1,2(t) is something like this. Where from -1 to 0, this increases linearly attends a value of 1 at t equal to 0. Similarly if when I consider the contribution of p1, the corresponding Bezier interpolation function that I have to consider is B0,2(t) which is something like this. So now if I want to find out what is the value at the same point say 2.3.

You find that to find out the value at point 2.3, the contribution of p1 will be equal to 0. Because the value of B0,2(t) is equal to 0, beyond point t equal to t, t t equal to 2. And by this you will find the only contribution that you can get to this point t equal to 2.3 is from the point p2 and from the point p3, ok. And in this particular case I will have f(2.3) which will be nothing but p2 then B, because it is I have to make it I - 2 in I -1 in this particular case. So this will be equal to B1,2(2.3)+p2 into B sorry p3 into B2,2(2.3), just this.

So the contribution of this point to this point t equal to 2.3 will be given by this value and the contribution of point p3 will be given by this value. And here you find that because the function increases linearly from 0 to 1, between t equal to 2 and t equal to 3 and it decreases linearly from 3 to 0 when t varies from 3 to 4. So here you find that the value that will get will be nothing but p2 into the value of this function in this particular case will be equal to 0.7 plus it will be p3 into 0.3.

So if I simply replace the value of p2 which is equal to point 6 and p3 which is equal to 1.1, I can find out what is the value of f(t) at t equal to 2.3.

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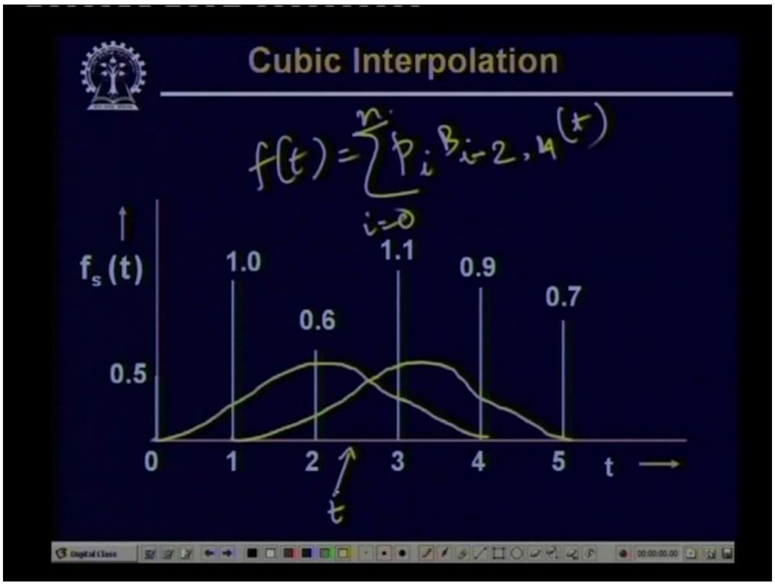

So similar is also the case for cubic interpolation and in case of cubic interpolation you find the your region of supports will be something like this, ok, sorry.

So the region the nature of the region of support will be something like this and again by using the same formulation that is f(t) is equal to pi Bi -2 in this particular case into k value of k is equal to 4(t) I can find out I am taking the summation for i equal to 0 to n, I can find out what will be the value of f(t) for any particular time instant t, or any arbitrary time instant t.

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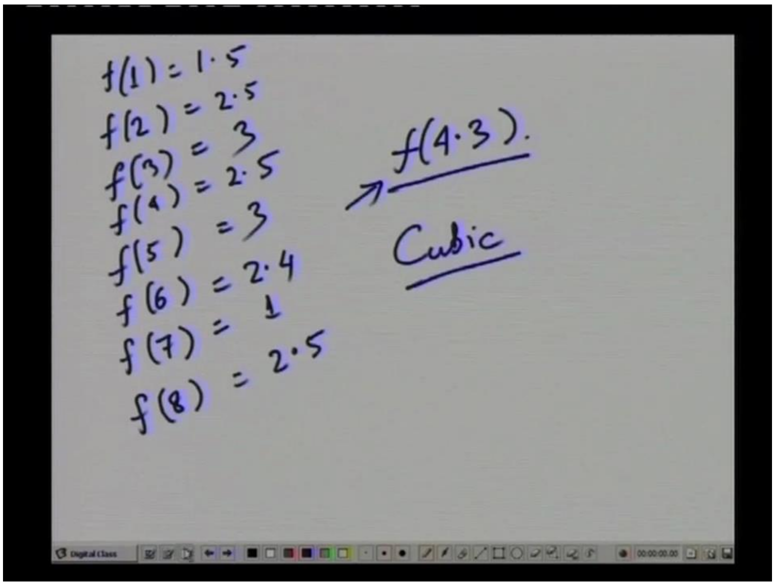

So by modifying this interpolation operations we can go for all this different types of interpolation. Now to explain this let us take a particular example so I take an example like this I take a function say f the function values are like this that f of (0). So I take f of (1) is equal to say 1.5, I take f of (2) is equal to 2.5, I take say f of (3) is equal to say 3, I take f of (4) is equal to something like say 2.5, I take f of (5) is equal to again 3, f of (6) may be something like 2.4, I can take f of (7) to be something say 1, I can take f of (8) to be something like 2.5.

And I want to find out the approximate value of this function at say t equal to 4.3, so given this sample values. I want to find out what is the value of this function at t equal to 4.3, so I want to find out f(4.3) given these sample values. And suppose the kind of interpolation that I will use is a cubic interpolation. So I use cubic interpolation and using this samples I want to find out the value of f(4.3).

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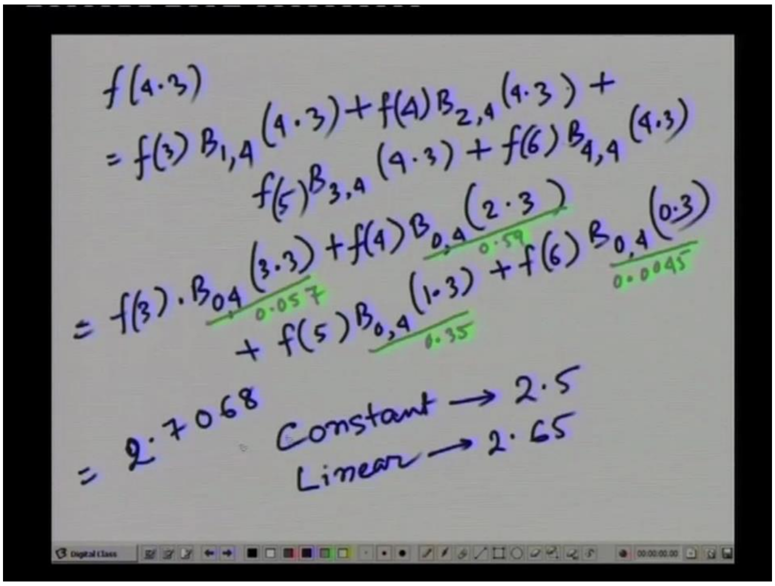

So let us see that how we can do it. Here you find that f(4.3) can be written as using considering the region of support this can be written as f(3) into B1,4(4.3), so we find that since this f(3) is nothing but p3 in our case. So this Bi becomes Bi -2, so 3 -2 is equal to 1 so I am considering B1,4 and at location t equal to (4.3). So this will be +f(4) which is nothing but p4 into B2,4(4.3) + it will be f(5) into B3,4 at location t equal to 4.3 + it will be f(6) into B4,4 at location t equal to (4.3), ok.

Now as we said that we have told that Bi,k(t) is nothing but B0,k(t-i). So just by using this particular property of the B-Spline functions, I can now rewrite this equation in the form that this will be equal to f(3) or p3 into B0,4 and this will be now t -i, i is equal to 1, so this will be B(3.3)+f(4) into B 0,4(2.3)+f(5) into B0,4(1.3)+f(6) into B0,4 and this will be equal to 0.3.

Now you can compute the values of this B0,4(3.3), you can compute the values of B0,4(2.3), you can compute the values of B0,4(1.3), and you can also compute the value of zee B0,4(0.3) using the approximate analytical formula of B0,4(t) that you have given and you have seen that this is nothing but a cubic formula of variable t.

So if I do this you will find that and using the sample values this B0,4(3.3) this gets a value of 0. 0.57, B2,4 B0,4(2.3) this gets a value of 0.59, B0,4(1.3) this gets a value of 0.35, and B0,4(0.3) gets a value of 0.0045 and you can verify this by using the computation the analytical formula that we have given.

And by using the values of f(3), f(4), f(5) and f(6) if I compute this equation then I get the final interpolated value to be 0.7068, sorry 2.7068. Now if do the same computation using constant interpolation and as I said that the constant interpolation is nothing but a nearest newer interpolation. So when I try to find out the value at f(4.3), the point t equal to 4.3 is nearest to point t equal to 4 at which I have a sample value.

So using nearest neighbour or constant interpolation f(4.3) will be simply equal to f(4) and in which in our case is equal to 2.5, whereas if I go for linear interpolation again you can compute this using the linear equations that we have said, that using linear interpolation the value of f(4.3) will be equal to 2.65. So we find that there is slight difference of the interpolated value whenever we go for constant interpolation or we go for linear interpolation or we go for cubic interpolation.

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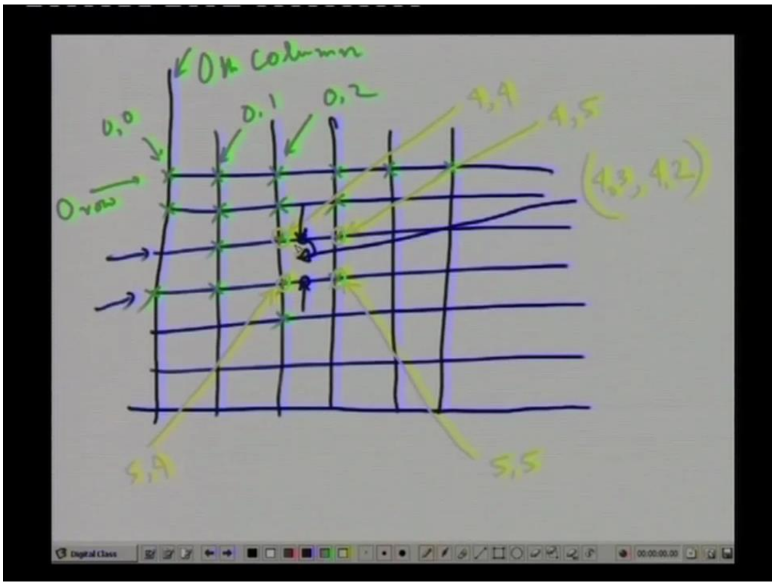

So using this type of formulation we can go for interpolation of the one dimensional sampled functions. Now when I go for the interpolation of image functions, you find that images consists of a number of rows and a number of columns. Now the interpolation that you have discussed so far these interpolations are mainly valid for one dimension. Now the question is how do we extend this one dimensional interpolation operation into two dimension so that it can be applied for interpolation images as well.

So in case of image what we will do is as the image is nothing but a set of pixels arranged in a set of rows and in a set of columns. So let us consider a two dimensional grid to consider the image pixels. So I have the grid points like this, so in case of an image I have the image points or image pixels located at this location, located at this location, so these are the grid points for I have the image pixels.

So it is something like this, so I say this is location 0,0, this is location say 0,1, this is location 0,2 and so on. So I have this as the 0th row, this as the 0th column. And similarly I have row number 1, row number 2, row number 3, row number 4 and column number 1, column number 2, column number3, column number4 and so on. Now given this particular situation, so I have the sample values present at all these grid points.

Now giving this particular pixel array which is again in the form of two dimensional matrix. If I have to find out what will be the pixel value at this particular location. Suppose this is say location 4,4, let us assume. this is at pixel location say 4,5, fourth row, fifth column. This may be a pixel location say 5,4, that is fifth row, fourth column. And this may be the pixel location say 5,5, that is fifth row and fifth column.

So at these different pixel locations I have the intensity values or pixel values. And using this I want to interpolate what will be the pixel value at location say 4.3 and say 4.2. So I want to compute what will be the pixel value at this particular location 4.3 and 4.2. So now we find that the earlier discussion what we had for interpolation in case of one dimension. Now that has to be extended for two dimension to get the image interpolation.

Now the job is not very complicated, it is again a very simple job what you have to do is I have to take the rows one after another, I also have to take the columns one after another. So first what you do is you go for interpolation along the rows and then you try for interpolation along the column. And for this interpolation again I can go for either constant interpolation, or I can go for linear interpolation, or I can go for cubic interpolation.

But now because our interpolation will be in two dimension, so the kind of interpolation if it is a linear interpolation it will be called a bilinear interpolation. For cubic interpolation it will be called a bicubic interpolation. So let us see that what will be the nature of interpolation if I go for a constant interpolation. So effectively what we will do is, so in this particular case, we will interpolate along the row 4, will also interpolate along the row 5.

So we will try to find out what is the value pixel value at location 4,4.2, will also try to find out what is the pixel location at point 5,4.2. So once I have the pixel values interpolated values at this location and this location, that is 4,4.2 and 5,4.2 using these two sample values, I will try to interpolate the value at the this location 4.3, 4.2.

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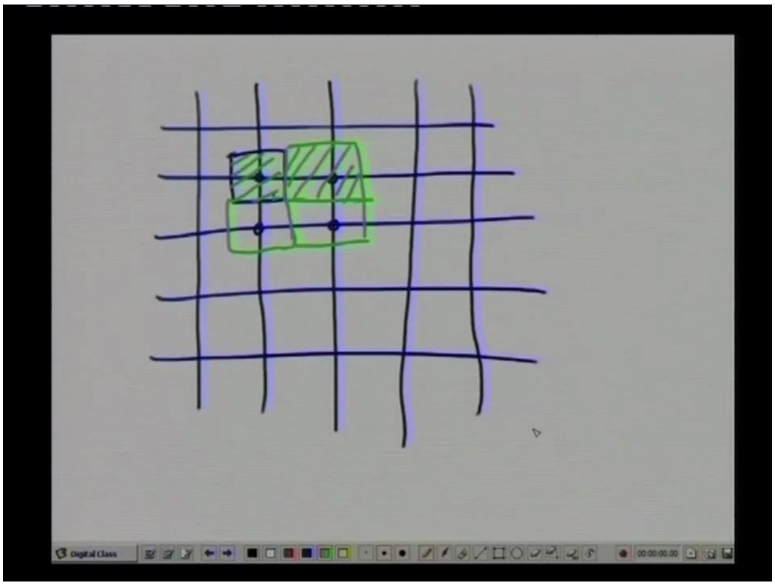

So simply extending the concept of 1D interpolation to 2D interpolation. Now what will be the nature of this interpolation if I go for constant interpolation. So as we said the constant interpolation simply takes the nearest neighbour and copies to the particular the arbitrary location which does not occur on the regular rate. So at this location I have a pixel value, at this location I have a pixel value, at this location I have a pixel value, at this location I have a pixel value.

Now if I want to find out what will be the value of the pixel at this particular location. What I do is, I simply try to find out which is the nearest neighbour of this particular point. And here you find that the nearest neighbor of this particular point is this point. So all the points which are nearest to this particular point so within this square all the pixel values will get the value of this particular pixel. So it will be something like this.

Similarly all the pixels, all the points lying within this region will get the pixel values of this particular region. Similar is the case here and similar is the case here. When you go for bilinear interpolation, when I try to interpolate the pixel at any particular location like this.

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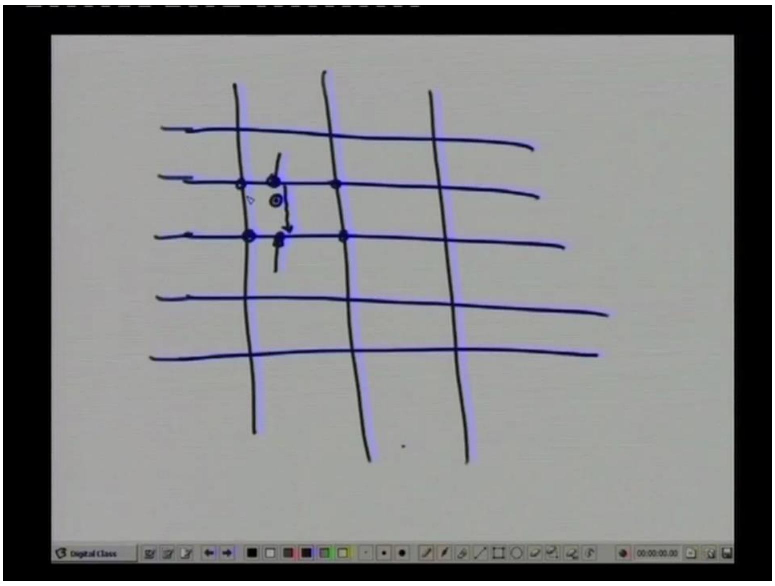

So again here in case of bilinear interpolation if I want to find out the pixel value at any particular any arbitrary grid location say something like this. Say somewhere here, what I have to do is I have to consider this pixel and this pixel, do the bilinear interpolation to find out the pixel at this location. I consider this pixel and this pixel, do bilinear interpolation to find out the pixel value at this location. Then using this and this, doing bilinear interpolation along the column I can find out what is the pixel value at this particular location.

And the same concept can also be extended for bicubic interpolation. So by this we explain how to do interpolation either constant interpolation which we have said is also the nearest neighbour interpolation, we can go for linear interpolation, in case of image it is bilinear interpolation or the cubic interpolation in case of image it is bicubic interpolation. So where you have to do interpolation both along the rows and after the rows you can do along the columns.

It can also be reversed, first you can do the interpolation along column then using the interpolated value along two or more columns, I can find out the interpolated value on any row location which does not fall on any regular grid point, ok. So now let us see that what are the results, this results we had already shown in the last class.

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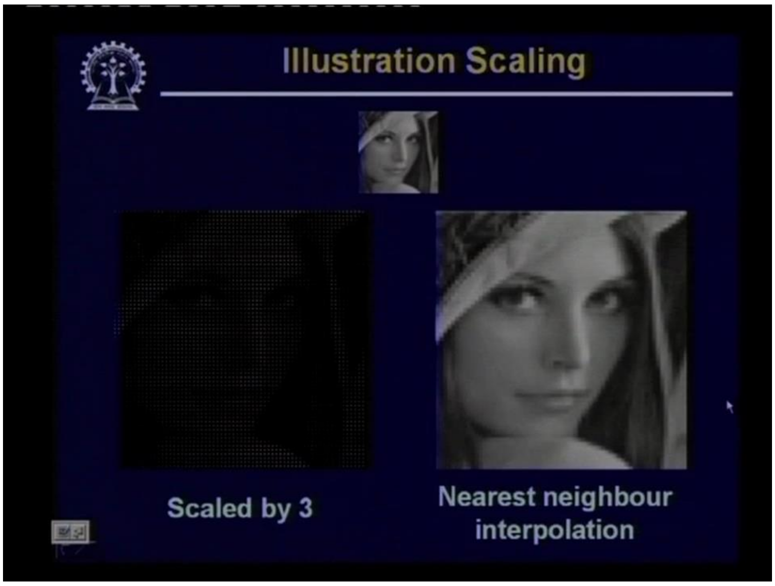

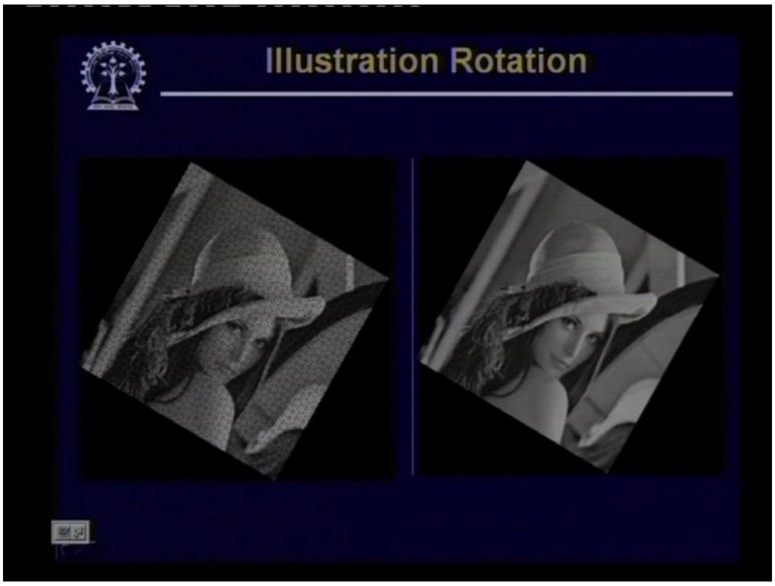

So we find that the first one is interpolated using nearest interpolation and as we have explained that because the value of the nearest pixel is copied to all the arbitrary location. So this is likely to a blocking art effect. And in this nearest neighbor interpolated image you also find that those blocking art effects are quite prominent. We have also seen the output with other interpolation operations.

We have shown the output with linear B-Spline interpolation. We have also shown the output with cubic B-Spline Interpolation, ok. So this is the case with rotation again when you rotate if you do not interpolate you get a number of patches black patches as this as is shown in the left image. If you go for interpolation all those black patches will be removed then you get a continuous image.

As it is shown in the right image, now this interpolation operation is useful not only for this translation or rotation kind of operations, you find that in many other application for example in case of satellite imagery when the image of the Earth surface is taken with the help of a satellite. Now because of Earth's rotation the image which is obtained from the satellite the pixels always does not fall on regular grids.

So in such cases what we have to go for is to rectify the distortion or to correct the distortion, which appears in the satellite images and this distortion is mainly due to the rotation of the Earth's Surface. So for correction of those distortions the similar type of interpolation is also used, thank you.