Digital Image Processing.
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Lecture-20.
Image Transformation-I.

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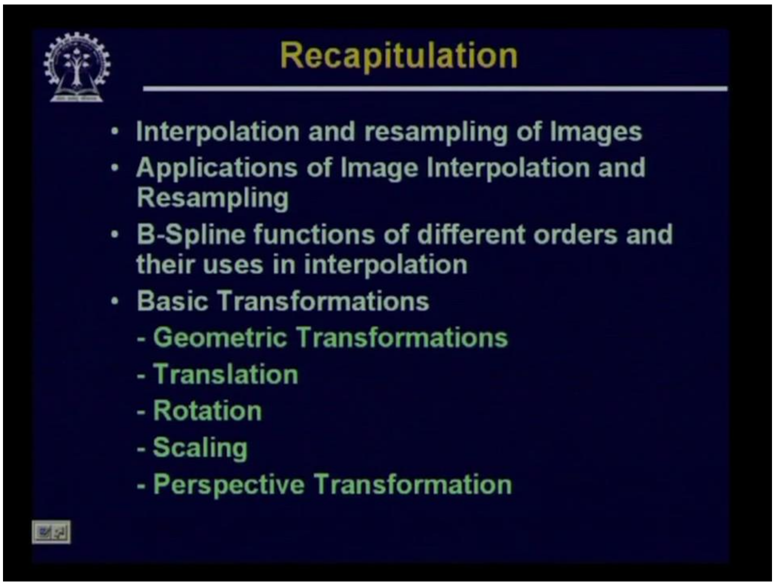

Welcome to the course on digital image processing. In the last class we seen the interpolation and resampling operation of images and we have seen different applications of the interpolation and resampling operations. So while we have talked about the interpolation and resampling, we have seen that it is the B-Spline functions or B-Spline interpolation functions of different orders which are mainly used for image interpolation purpose.

And before this interpolation we have also talked about the basic transformation operations and the transformation operations that we have discussed those were mainly in the class of geometric transformations. That is we have talked about the transformation like translation, we have talked about rotation, we have talked about scaling and we have seen that these are the kind of transformations which are mainly used for coordinate translation.

That is given a point in one coordinate system we can translate the point or we can represent the point in another coordinate system, where the second coordinate system may be a translated or rotated version of the first coordinate system. We have also talked about another type of transformation which is perspective transformation and this perspective transformation is mainly used to find out or to map a point in a three dimensional world coordinate system to a two dimensional plane where this two dimensional plane is the imaging plane.

So there our purpose was that given a point or the 3D coordinates of a point in a three dimensional coordinate system, what will be the coordinate of that point on the image plane when it is imaged by a camera. In todays lecture we will talk about another kind of transformation which we call as image transformation. So we will talk about or we will explain the different image transformation operations.

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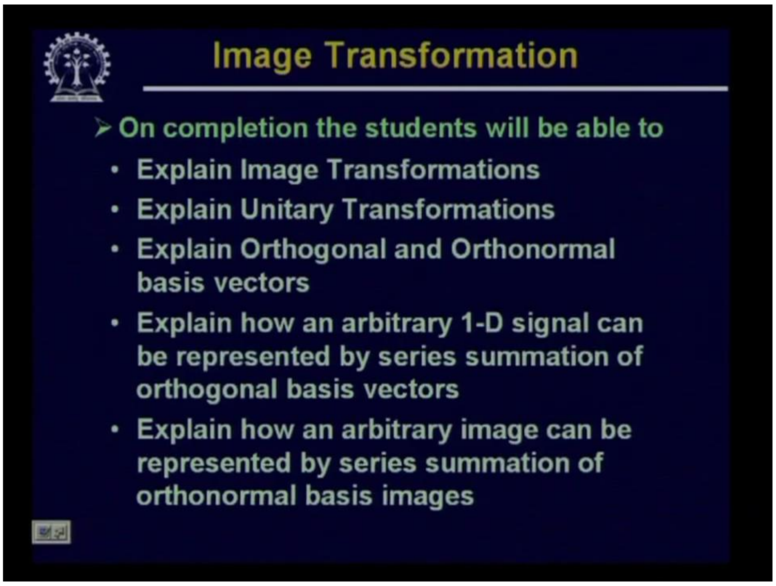

Now before coming to specific transformation operations like say fourier transform or discrete cosine transform or say discrete cosine transform. Before we come to such specific transformations. We will first talk about a unitary transformation which is a class of transformations or class or unified unitary transformations and all the different sort of transformations that is whether it is discrete fourier transform or discrete cosine transform or hadamard transform.

All these different transform are different cases of this class of unitary transformations. Then when you talk about this unitary transformation we will also explain what is an orthogonal and orthonormal basis function. So we will see that what is known as an orthogonal basis function, what is also known as a orthonormal basis function. We will also explain how an arbitrary one dimensional signal can be represented by series summation of orthogonal basis vectors and we will also explain how an arbitrary image can be represented by a series summation of orthonormal basis images.

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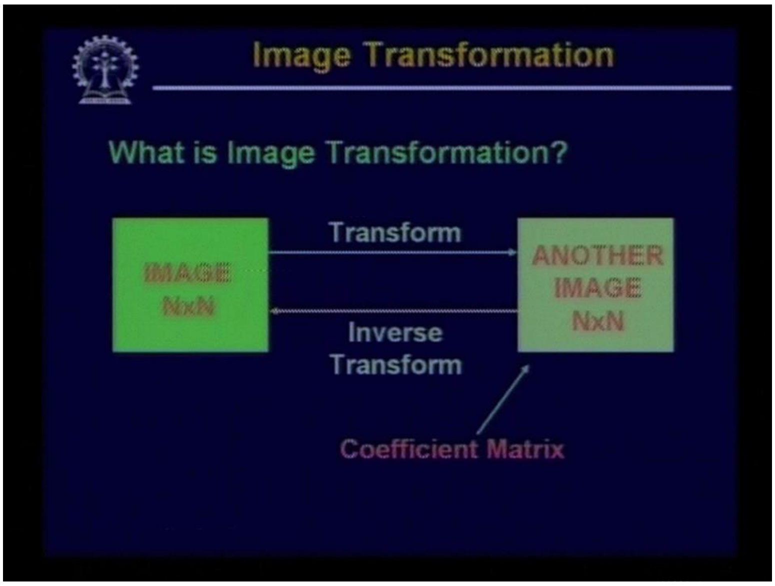

Now firstly let us see that what is the image transformation. You find that in this case we have shown a diagram, where the input is an image and after the image is transformed we get another image. So if the size of the input image is n by n, say it is having n number of rows and n number of columns. The transformed image is also of same size, that of size n by n. And given this transformed image, if we perform the inverse transformation we get back the original image.

That is image of size n by n. Now if given an image by applying transformation, we are transforming back to another image of same size and doing the inverse transformation operation we get back the original image then the question naturally comes that what is the use of this transformation. And here you find that after transformation the second image of same size n by n that we get that is called the transformed coefficient matrix.

So the natural question that arises in this case that if by transformation I am going to another image. And by using inverse transformation I get back the original image, then why do we go for this transformation at all. Now we will find and we will also see on in our subsequent lectures that this kind of transformation has got a number of very very important applications. One of the application is for preprocessing, in case of image preprocessing of the images.

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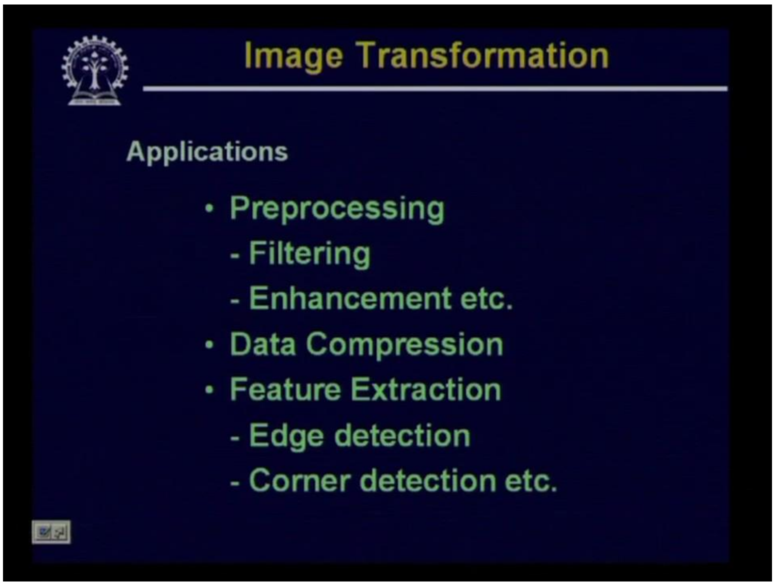

If the image contains noise, then you find or you know that contamination of noise gives rise to high frequency components in the image. So if by using some sort of unitary transformation we can find out what are the frequency components in the image. Then from these frequency coefficients, if we can suppress the high frequency components, then after suppressing the high frequency components, the modified coefficient matrix that you get if you take the inverse transform of that modified coefficient matrix. Then the original or the reconstructed image that we get that is a filtered image.

So filtering is very very important application where this image transformation techniques can be applied. The other kind of preprocessing techniques we will also see later on that it is also very very useful for image enhancement operation.

Say for example if we have an image which is very blurred, that is the contrast of the image is very very poor, then again in the transformation domain or using the transform coefficients, we can do certain operations by which we can enhance the contrast of the image so that is what is known as enhancement operation. We will also see that this image transformation operations are very very useful for data compression.

So if I have to transmit an image, or if I have to store the image on hard disk. Then you can easily think that if I have an image of size say 512by 512 pixels and if it is a black and white image. Every pixel contains 8 bits, if it is a color image contains normally 24 bits. So storing a an image color colored image of size 500 and 500, 512 by 512 pixel size takes huge amount of disk space.

So if by some operation I can compress the space or I can reduce the space required to store the same image then obviously on the on a limited disk space I can store more number of images. Similar is the case if I go for transmission of the image or transmission of image sequences or video.

In that case the bandwidth of the channel over which this image or the video has been transmitted is a bottle neck which forces us that we must employ some data compression techniques, so where the bandwidth requirement for the transmission of the image or the transmission of the video will be reduced. And we will also see later on that this image transformation techniques is the first step in most of the data compression or image or video compression techniques.

These transformation techniques are also very very useful for feature extraction operation. By features I mean that in the images if I am interested to find out the edges or I am interested to find out the corners of certain shapes. Then this transformation techniques or if I work in the transformation domain then finding out the edges or finding out the corners of certain objects that also becomes very very convenient.

So these are some of the applications where this image transformation techniques can be used so apparently we have seen that by image transformation I just transform an original image to another image. And by inverse transformation that transformed image can be retransformed to the original image. So the application of this image transformation operation can be like this and here I have selected only few of the applications we will see later that applications of these image transformations are much more than what I have listed here.

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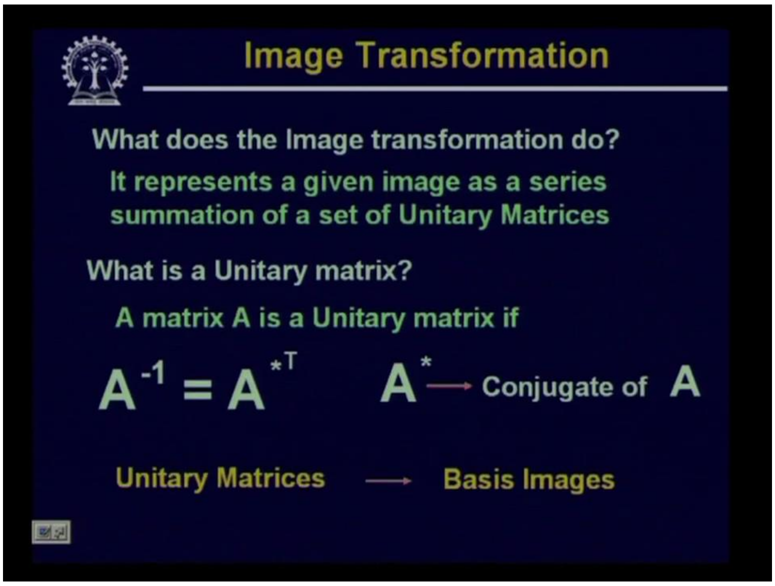

Now what is actually done by image transformation. By image transformation what we do is we try to represent a given image as a series summation of a set of unitary matrices. Now what is an unitary matrix, a matrix A is said to be an unitary matrix if A inverse or inverse of A is equal to A* transpose, where A* is the complex conjugate of A. So a matrix A will be called an unitary matrix if the inverse of the matrix is same as first you take the conjugate of the matrix A, then take its transpose.

So A inverse will be equal to A* transpose, where A* is the complex conjugate of the matrix A. That is complex conjugate of each and every element of matrix A. And these unitary matrices will call as the basis images. So the purpose of this image transformation operation is to represent any arbitrary image as a series summation of such unitary matrices, or series summation of such basis images.

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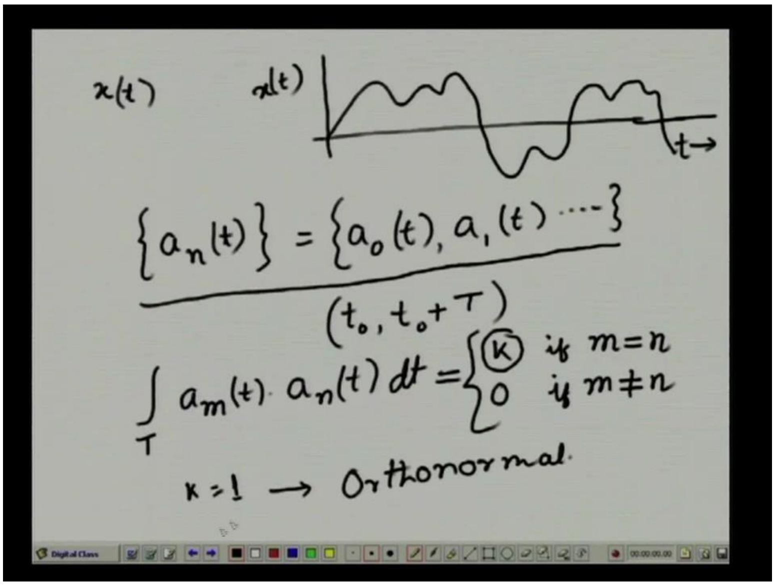

Now to start with I will first try to explain with the help of one dimensional signal. So let us take an arbitrary one dimensional signal say I take a signal say x(t). So I take an arbitrary signal x(t) and you see that this is a function of t. So this x(t) the nature of x(t) can be anything say let us take that I have a signal like this, x(t) which is a function of t. Now this arbitrary signal x(t), can be represented as a series summation of a set of orthogonal basis function.

So I am just taking this as an example in for one dimensional signal and later on we will extend to two dimension that is in for the images. So this arbitrary signal this one dimensional signal x(t), we can represent by the series summation of a set of orthogonal basis functions. Now the question is what is orthogonal, by orthogonal I mean that if I consider a set of real valued continuous functions.

So I consider a set of real valued continuous functions say an(t) which is equal to set say a0(t),a1(t) and so on, ok. So this is a set of real valued continuous functions and this set of real valued continuous functions is said to be orthogonal over an interval say t0 to t0+T. So I define that this set of continuous real valued functions will be orthogonal over an interval t0 to t0+ capital T, if I take the integration of function say am(t) into an(t)dt and take the integration of this over the interval capital T, then this integral will be equal to some constant k, if m is equal to n. And this will be equal to 0, if m is not equal to n.

So I take two functions am(t) and an(t), take the product and integrate the product over interval capital T. So if this integration is equal to some constant say k, when m is equal to n and this is equal to 0 whenever m is not equal to n. So if this is true for this set of real valued continuous functions, then this set of real valued continuous functions form an orthogonal set of basis functions.

And if the value of this constant k is equal to 1, so if the value of this constant k is equal to 1 than we say that the set is orthonormal, ok. So an orthogonal basis function as we have defined this non 0 constant k if this is equal to 1, then we say that it is a orthonormal set of basis functions. Let us just take an example that what do you mean by this. Suppose we take a set like this.

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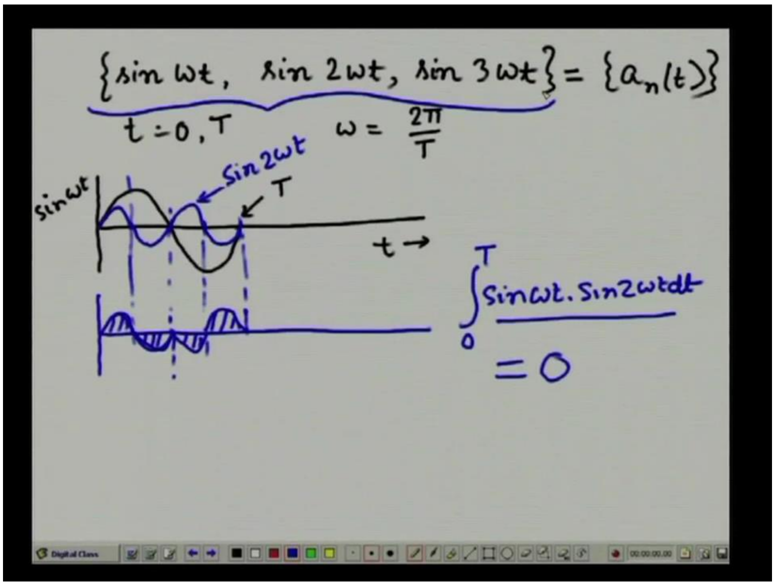

So sin omega t, sin twice omega t and sin three omega t. So this is my set of functions an(t), ok. Now if I plot sin omega t over interval t equal to 0 to capital T, ok. So this will be like this and where omega is equal to 2pi by capital T. So capital t is the period of this sinusoidal wave. Then if I plot this sign omega t, you will find that sign omega t in the period 0 to capital T is something like this.

So this is t, this is sign omega t and this is the time period capital T. If I plot twice omega t over this same diagram sin of twice omega t will be something like this, ok. So this is sin of sorry this is sin of twice omega t, now if I take the product of sin omega t and sin twice omega t in the interval 0 to capital t, the product will appear something like this. So we find that in this particular region but sin twice omega t and sin omega t they are positive.

So the product will be of this form, in this region sign omega t is positive but sin twice omega t is negative, so the product will be of this form. In this particular region sin twice omega t is positive, whereas sin omega t is negative. So the product is going to be like this, this will be of this form. And in this particular region both sin omega t and sin (ome) twice omega t they are negative so the product is going to be positive, so it will be of this form.

Now if I integrate this, so if I integrate sin of omega t into sin of twice omega tdt over the interval 0 to capital T. This integral is nothing but the area covered by this curve. And if you take this area you will find that the positive half will be cancelled by the negative half and this product will come out to be 0. This integration will come out to be 0.

Similar is the case if I multiply sin omega t with sin thrice omega t and take the integration. Similar will also be the case if I multiply sin twice omega t will with sin three omega t and take the integration. So this particular set that is sin omega t, sin twice omega t and sin three omega t, this particular set is the set of orthogonal basis functions.

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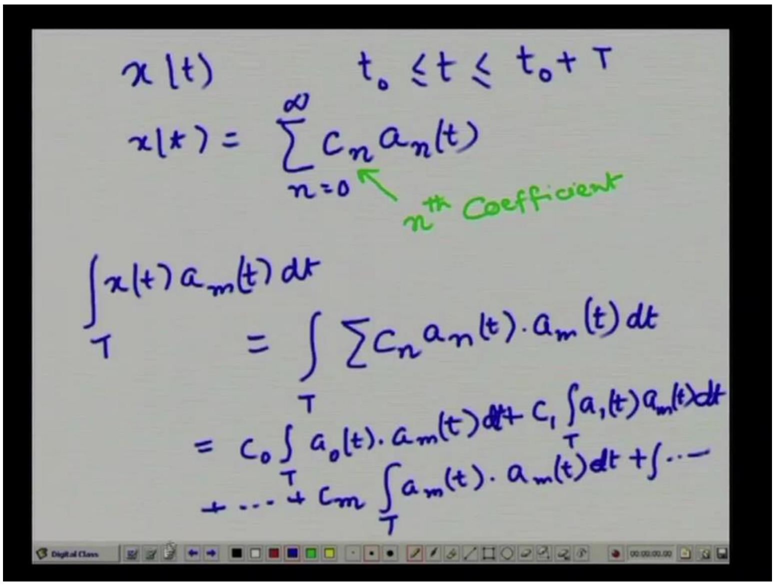

Now suppose we have an arbitrary real valued function x(t) and this function x(t) is we consider within the region t0 less than or equal to t, less than or equal to t0+ capital T. Now this function x(t) can be represented by as a series summation. So we can write x(t) as summation Cn a n(t), so you remember an(t) is the set of orthogonal basis functions. So represent x(t) as a series summation so x(t) equal to sum of Cn an(t), where n varies from 0 to infinity.

Then, this term Cn is called the nth coefficient of expansion. This is called nth coefficient of expansion. Now the purpose is the problem is how do we find out or how do we calculate the value of Cn. To calculate the value of Cn what we can do is we can multiply both the left hand side and the right hand side by another function from the set of orthogonal basis function. So multiply both sides by function say an(t) and take the integration from t equal to 0 to capital t or take the integration over the interval capital T.

So what we get is, we get an integration of this form x(t)am(t)dt integral over capital T this will be equal to again integral over capital T and this integral of Cn an(t) into am(t) because we are multiplying both the left hand side and the right side by the function am(t)dt. And you take the integral over the interval capital T.

Now if I expand this you find that if I expand this, this will be of the form C0 integration over capital T a0(t) into am(t)dt+C1 integration over the same interval capital T a1(t) into am(t)dt+ it will continue like this will have one term say Cm integral over T am(t) into am(t)dt+ some more integration terms.

Now as per the definition of the orthogonality that we have said, that a integral of an(t) into am(t) into dt that will be equal to some constant k, if and only if m is equal to n. And this integral will vanish for all the cases wherever m is not equal to n. So by using that formula of orthogonality what we get in this case is we simply get integral x(t) into am(t)dt this integral over capital T.

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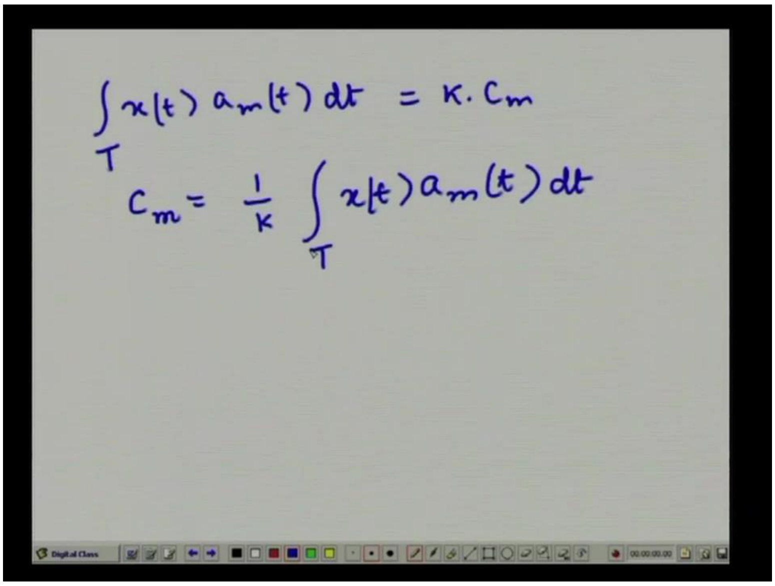

This will be simply equal to constant k times Cn because the right hand side of this integration that we have said this right hand side all these terms will be equal to 0 only for this term am(t) into an(t) dt the value will be equal to k. So what we get here is integration x(t)am(t)dt is equal to the constant k times Cn. So from this we can easily calculate that the mth coefficient Cm will be given by 1 upon k integration x(t)am(t) into dt where you take the integration over the interval capital T.

And obviously we can find out that if the set is an orthonormal set not an orthogonal set, in that case the value of k is equal to 1. So we can easily get the mth coefficient Cm to be x(t) c am(t)dt integrate this over the interval T. So the value of term k will be equal to 1. So this is how we can get the mth coefficient of expansion of any arbitrary function x(t), right and this computation can be done if the set of basis functions that we are taking that is the set am(t) is an orthogonal basis function.

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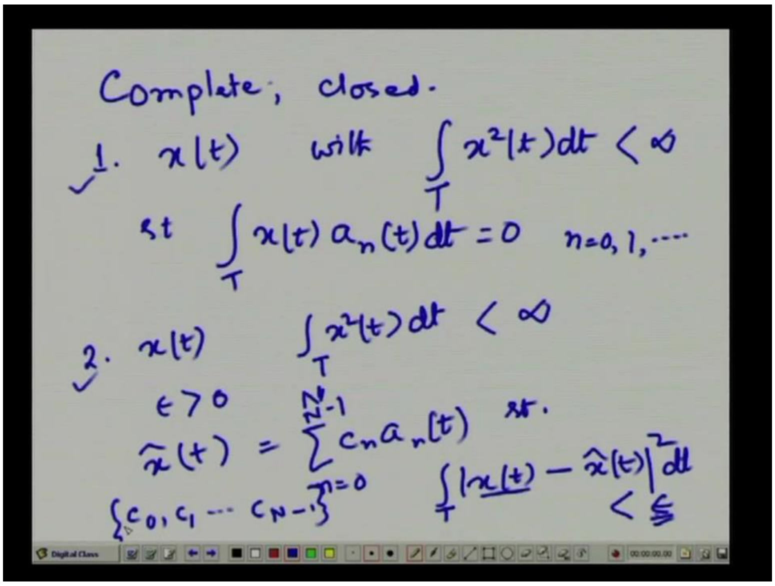

Now the set the orthogonal basis the set of orthogonal basis functions an(t) is said to be complete we say that this orthogonal basis function is complete, if this is complete or closed if one of the two conditions hold. The first condition is there is no signal say x(t) with integral x square (t)dt over the interval capital T less than infinity. So this means the signal with finite energy.

So that does not exist any signal x(t) with x square (t)dt less than infinity such that integral x(t) an(t)dt is equal to 0 this integration has to be taken over the interval capital T or n equal to 0,1 and so on. And the second condition is that for any piecewise continuous signal x(t), so x(t) is piecewise continuous and with the same condition of finite energy that is x square (t)dt, integral over capital T must be less than infinity and if there exist and epsilon greater than 0, however small this epsilon is there exists an N and a finite (**expen**) expansion such that x hat (t) is equal to Cn an(t), now n varies from 0 to capital N-1, such that integral x(t)-x hat (t) square dt taken over the same interval capital T must be less than epsilon.

So this is that form a piecewise continuous function x(t) having finite energy, there must be an epsilon which is greater than 0 but very small and there must be some constant capital N such that if we can have an expansion that x hat (t) is equal to summation of Cn an(t), now this n varies from 0 to capital N-1, for which this term x(t) – x hat (t) square dt (**ove**) integral over capital T, this is less than epsilon.

So we find that this x(t) is the original signal x(t) and x hat (t) earlier case we have seen that if we go for infinite expansion then then this x(t) can be represented exactly. Now what we are doing is we are going for a truncated expansion, we are not going to take all the infinite number of terms but we are going to take only capital N number of terms. So obviously this x(t) it is not being represented exactly but we are we are going to have its approximate expansion.

And if x(t) is of finite energy, that is integral of x square (t)dt integration over capital T, is less than infinite then we can say that there must be a finite N capital N the number of terms, for which the error of the reconstructed signal. So this x(t) – x hat (t) square dt, this is nothing but the energy of the error signal, or the error that is introduced because of this truncation, which must be limited, it must be less than or equal to epsilon, where epsilon is a very very positive small value.

So we say that the set of orthogonal basis functions an(t) is complete or close if one of this conditions hold, atleast one of this conditions hold, that is the first condition or the second condition. So this says that when we have a complete orthogonal function then this complete orthogonal function expansion enables representation of x(t) by a finite set of coefficients, where the finite set of coefficients are C0,C1 like this upto CN-1.

So this is the finite set of coefficients, so if we have I complete orthogonal function set of orthogonal functions then using this complete set of orthogonal functions, thank you.