Digital Image Processing
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Module 01 Lecture Number 03
Image Digitilization, Sampling Quantization and Display

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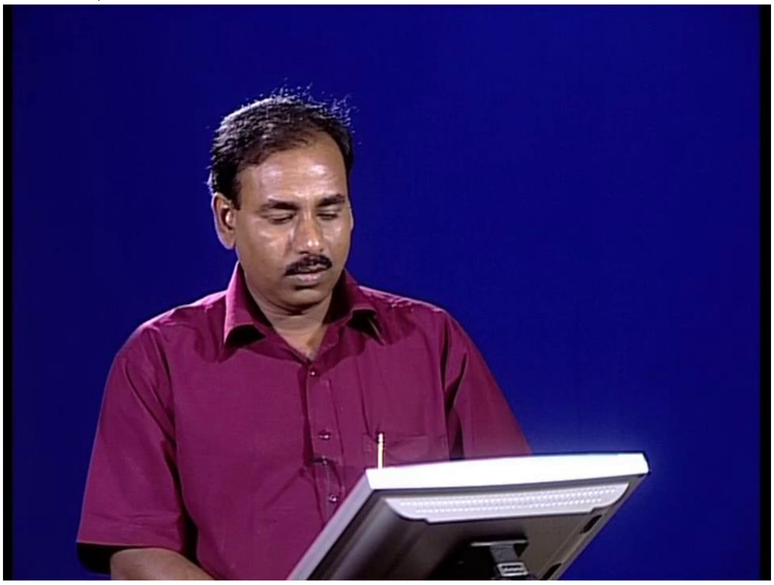

Welcome to the course on Digital Image Processing.

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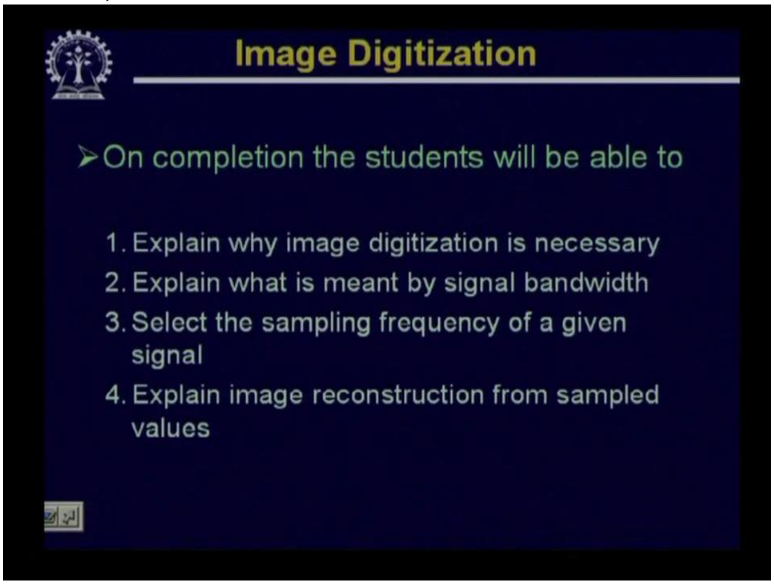

We will also talk about what is meant by signal bandwidth. We will talk about how to select the sampling frequency of a given signal, and we will also see the image reconstruction process from the sample values.

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So in today's lecture, we will try to find out the answers to 3 basic questions. The first question is why do we need digitization? Then we will try to find the answer to what is meant by digitization and thirdly we will go to how to digitize an image.

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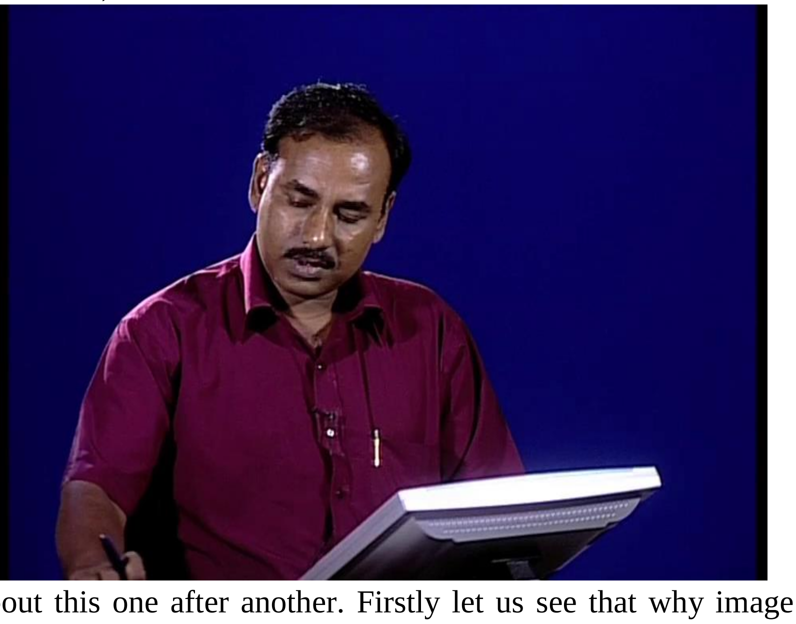

So let us talk about this one after another. Firstly let us see that why image digitization is necessary.

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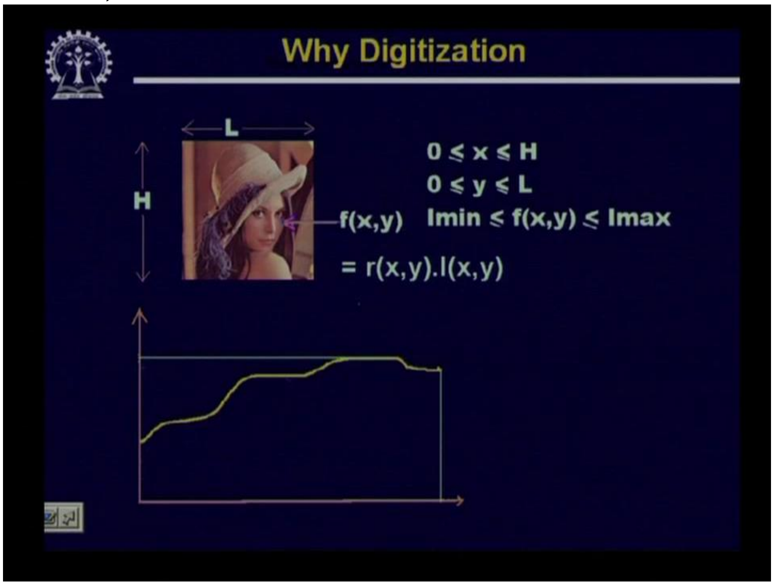

You will find that in this slide that we have shown an image, this is an image of a girl and as we have just indicated in our introductory lecture that an image can be viewed as a two dimensional function given in the form of f “x, y”. Now this image has certain length and certain height. The image that has been shown here has a length of L; this L will be in units of distance or units of length. Similarly the image has a height of H which is also in units of distance or units of length. Any point in this two dimensional space will be identify the image coordinates x and y. Now you will find that conventionally we have said that x axis is taken vertically downwards and y axis is taken as horizontal. So every coordinate in this two-dimensional space will have a limit like this. That value of x will vary from 0 to H and value of L, value of y will vary from 0 to L. Now if I consider any point “x, y” in this image, the point “x, y” or the intensity or the color value at point “x, y” which can be represented as function of x and y where “x, y” identifies a point in the inner space that will be actually a multiplication of two terms. One is r “x, y” and other one is i “x, y”. We have said during our introductory lecture that this r “x, y” represents the reflectance of the surface point of which this particular image point corresponds to. And i “x, y” represents the intensity of the light that is falling on the object surface. Theoretically this r “x, y” can vary from 0 to 1 and i “x, y” can vary from 0 to infinity. So a point f “x, y” in the image can have a value anything between 0 to infinity. But practically intensity at a particular point or the color at a particular point given by “x, y” that varies from certain minimum that is given by “i min” and certain maximum “i max”. So the intensity at this point “x, y” that is represented by “x, y” will vary from minimum intensity value to certain maximum intensity value.

Now you will find the second figure in this particular slide. It shows that if I take a horizontal line on this image space and if I plot intensity values along that line, the intensity profile will be something like this. It again shows that this is the minimum intensity value along that line and this is the maximum intensity value along the line. So the intensity at any point in the image or intensity along a line whether it is horizontal or vertical, can assume any value between the maximum and the minimum.

Now here lies the problem. When we consider a continuous image which can assume any value, an intensity can assume any value between certain minimum and certain maximum and the coordinate points x and y they can also some value between, x can vary from 0 to H, y can vary from 0 to L.

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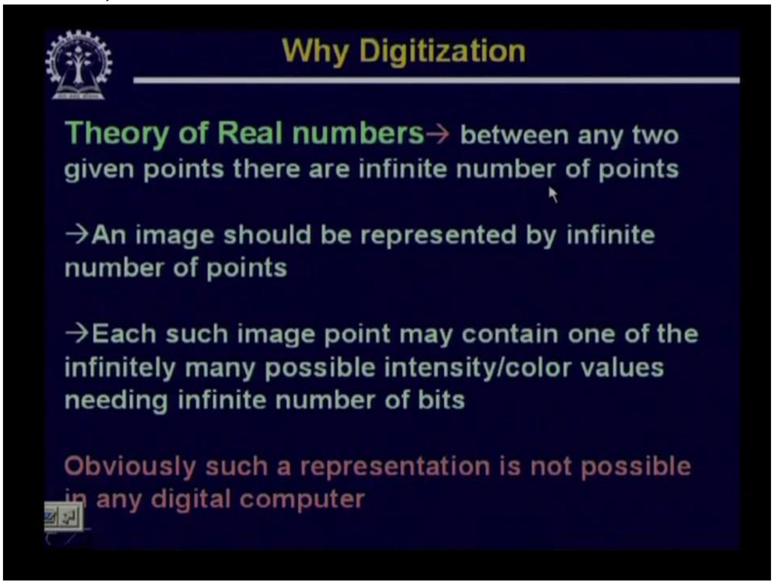

Now from the Theory of Real Numbers, you know that given any two points, that is, between any two points there are infinite numbers of points. So again when I come to

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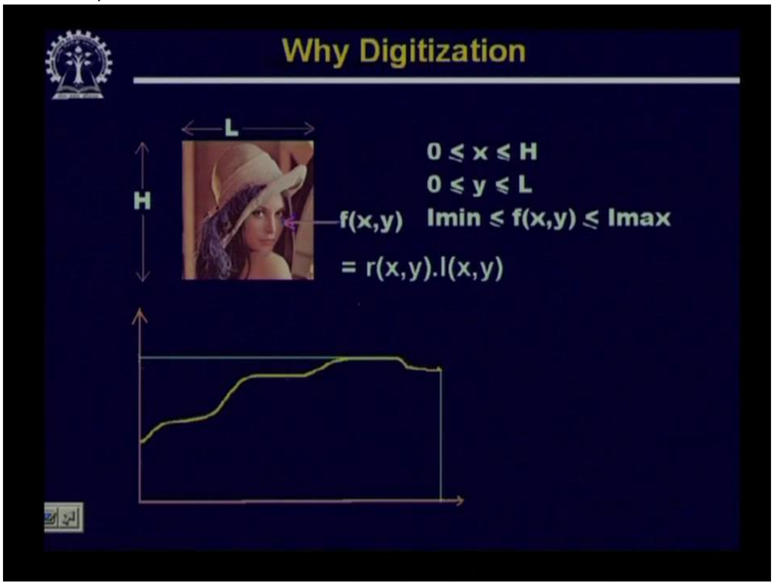

this image, as x varies from 0 to H, there can be infinite possible values of x between 0 and H. Similarly there can be infinite values of y between 0 and L. So effectively

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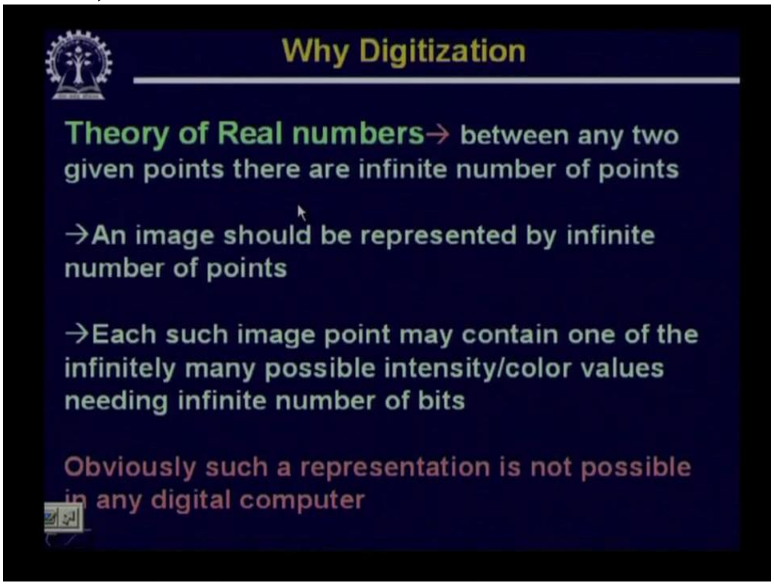

that means that if I want to represent

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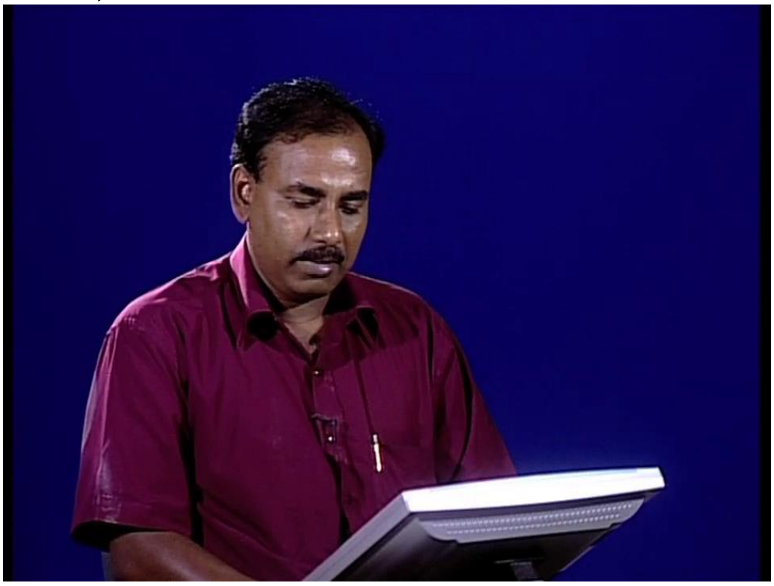

this image in a computer then this image has to be represent by infinite number of points.

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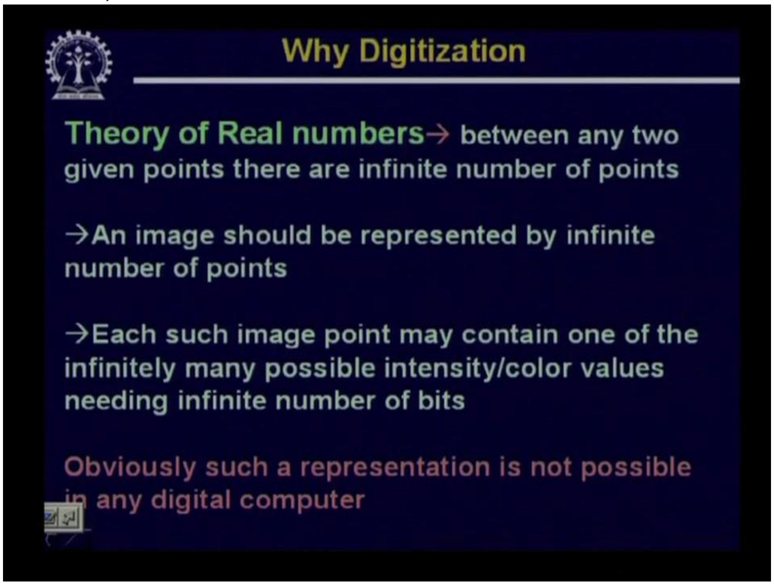

And secondly when I consider the intensity value at a particular point, we have seen

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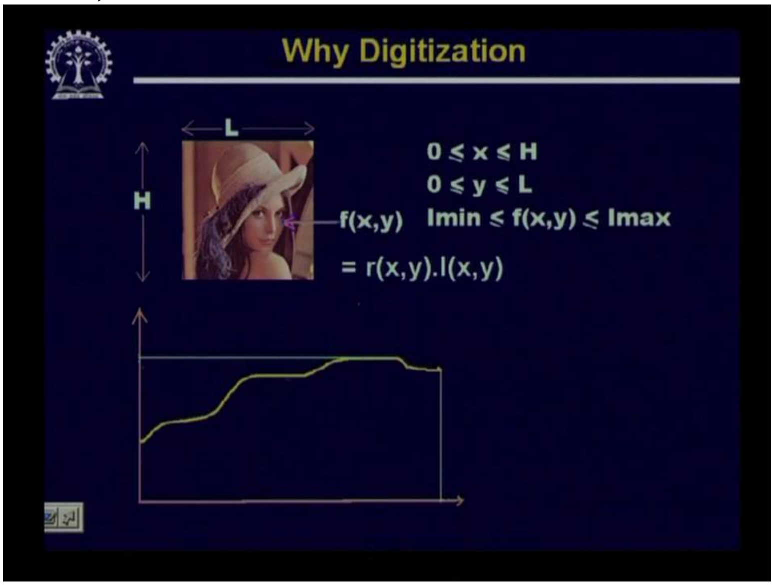

that intensity value f "x, y" it varies between certain minimum "i min" and certain maximum "i max". Again if I take these two "i min" and "i max" to be minimum and maximum intensity values possible

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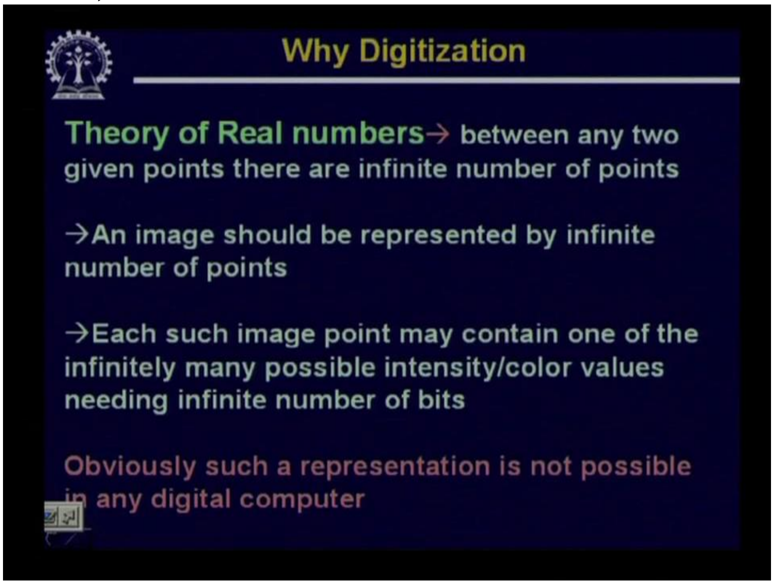

but here again the problem is the intensity values, the number of intensity values that can be between minimum and maximum is again infinite in number; so which again means that if I want to represent an intensity value in a digital computer then I have to have infinite number of bits to represent an intensity value. And obviously such a representation is not possible in any digital computer.

So naturally we have to find out a way out,

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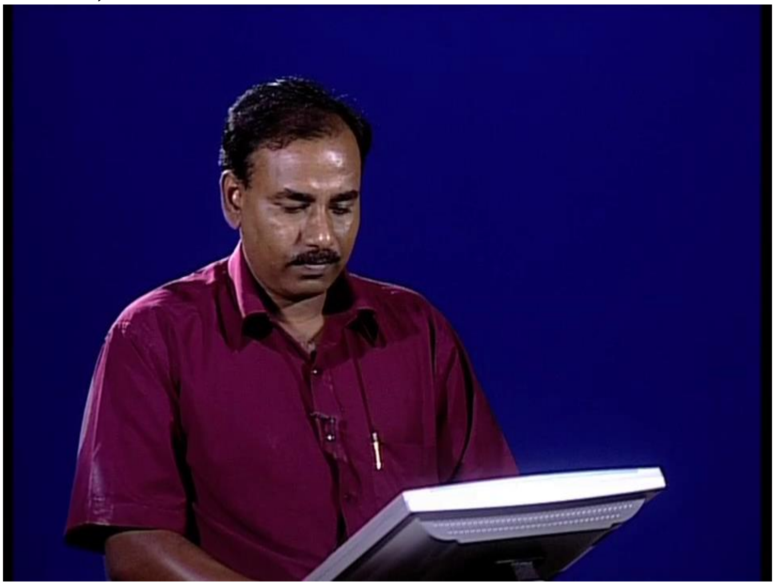

that is our requirement is we have to represent this image in a digital computer in a digital form. So what is the way out? In our introductory lecture, if you remember that we have said that instead of considering every possible point in the image space, we will take some discrete set of points and those discrete set of points are decided by grid. So if we have a uniform rectangular grid, then at each of the grid locations we can take a particular point and we will consider the intensity at that particular point. So this is a process which is known as sampling.

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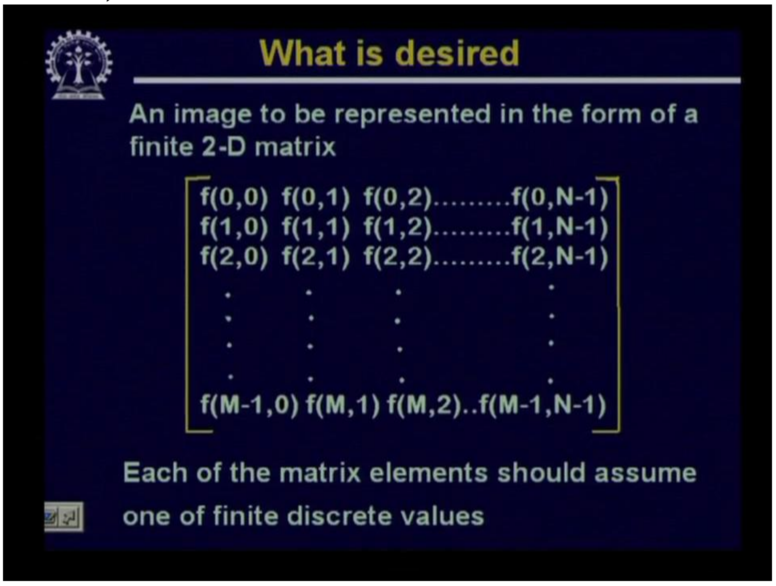

So what is desired is we want that an image should be represented in a form of finite two-dimensional matrix like this. So this is a matrix representation of an image and this matrix has got finite number of elements. So if you look at this matrix, you find that this matrix has got m number of rows varying from 0 to m minus 1 and the matrix has got n number of columns varying from 0 to n minus 1. Typically for image processing applications we have mentioned that the dimension is usually taken either as 256 by 256 or 512 by 512 or 1 K by 1 K and so on. But still whatever be the size, the matrix is still finite. We have finite number of rows and we have finite number of columns. So after sampling what we get is an image in the form of matrix like this.

Now the second requirement is, if I don't do any other processing on these matrix elements, now what this matrix element represents? Every matrix element represents an intensity value in the corresponding image location. And we have said that these intensity values or the number of intensity values can again be infinite between certain minimum and maximum which is again not possible to be represented in a digital computer. So here what we want is each of the matrix elements should also assume one of finite discrete values. So when I do both of these

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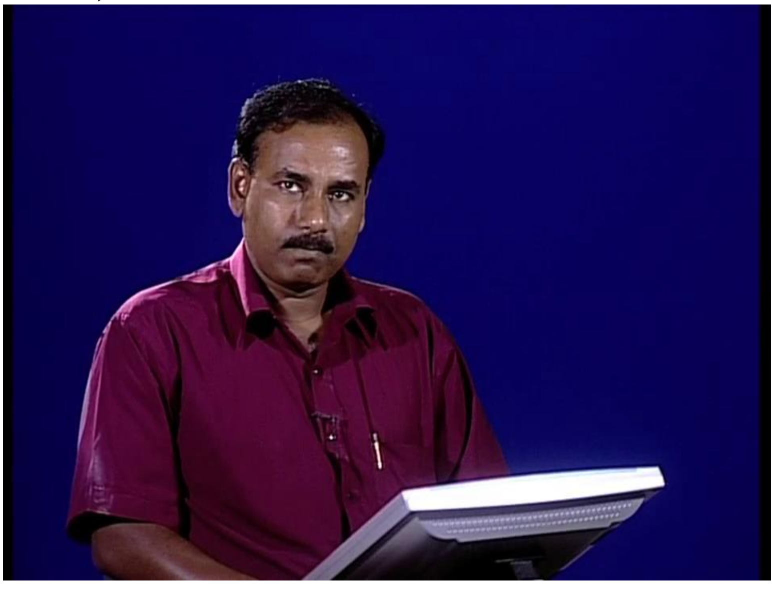

that is first operation is sampling to represent the image in the form of a finite two-dimensional matrix and each of the matrix elements again has to be digitized so that the intensity value at a particular element or a particular element in a matrix can assume only values from a finite set of discrete values. These two together completes the image digitization process.

Now here is an example.

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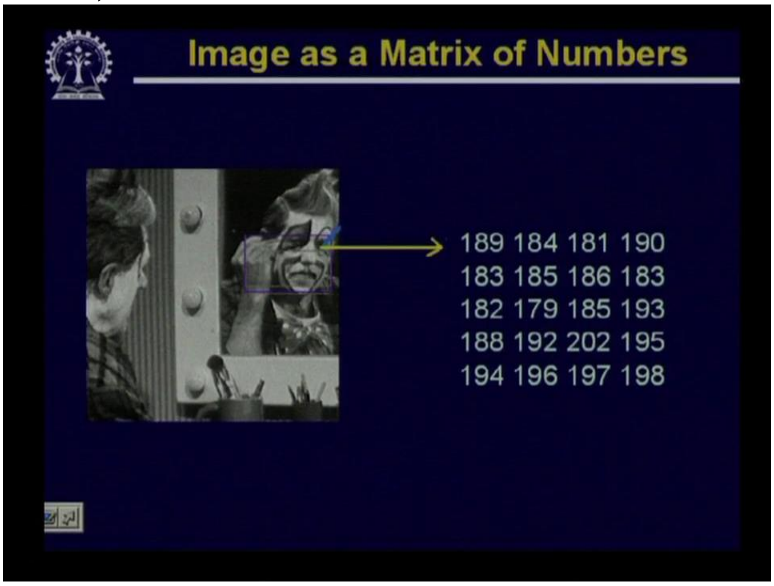

You find that we have shown an image on the left hand side and if I take a small rectangle in this image and try to find out what are the values in that small rectangle, you will find that these values are in the form of a finite matrix and every element in this rectangular, in this small rectangle or in this small matrix assumes an integer value. So an image when it is digitized will be represented in the form of a matrix like this .

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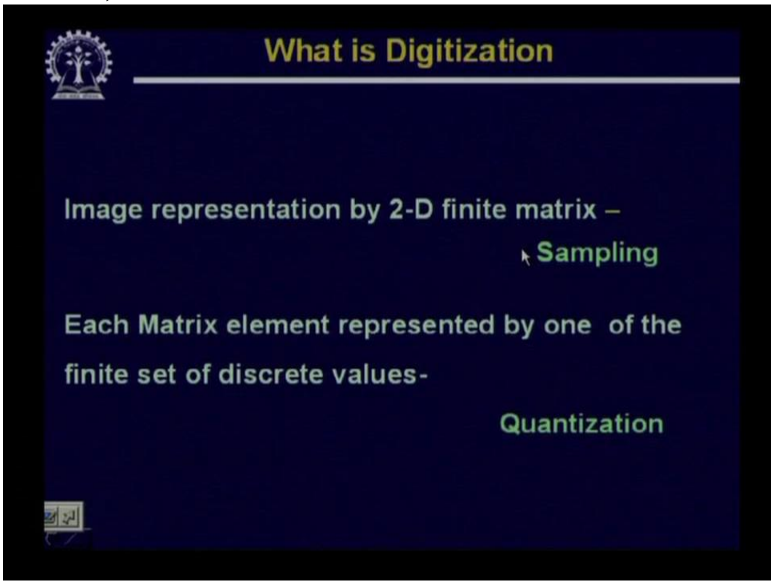

So typically what we have said till now, it indicates that by digitization what we mean is an image representation by 2D two-dimensional finite matrix, the process known as sampling and the second operation is each matrix element must be represented by one of the finite set of discrete values and this is an operation which is called quantization. In today's lecture

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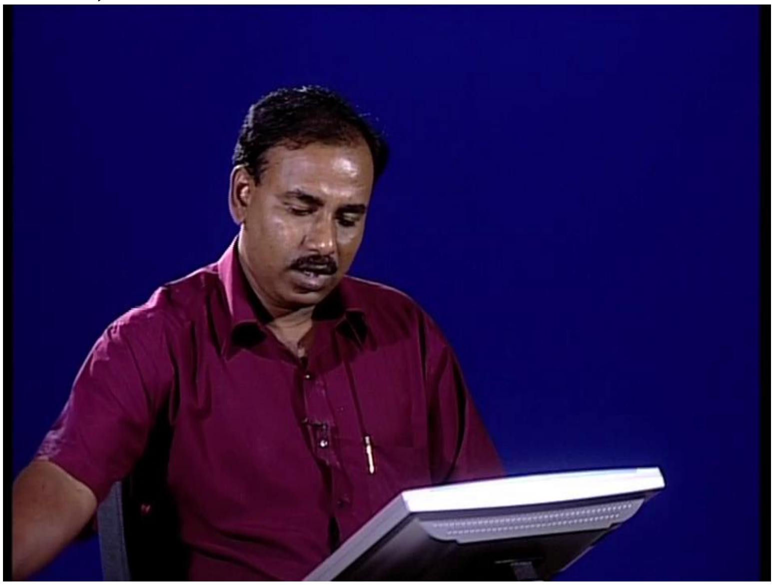

we will mainly concentrate on the sampling; and quantization we will talk about later.

Now let us see that how,

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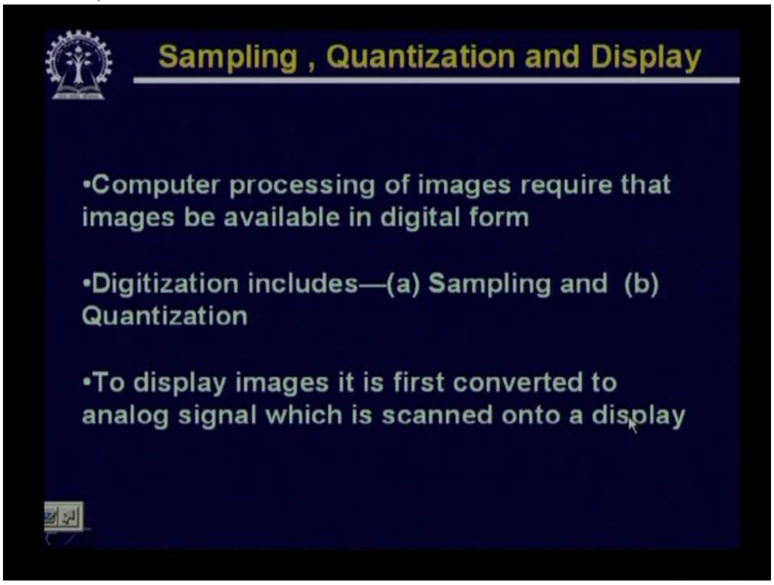

what should be the different blocks in an image processing system. Firstly we have seen that computer processing of images require that images be available in digital form and so we have to digitize the image. And the digitization process is a two-step process. The first step is sampling and the second step is quantization. Then finally when we digitize an image processed by computer, then obviously our final aim will be that we want to see

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that what is the processed output

So we have to display the image on a display device. Now when then image has been processed, the image in the digital form but when we want to have the display, we must have the display in the form of analog. So whatever process we have done during digitization, during visualization or during display, we must do the reverse process. So for displaying the images, it is first, it has to be first converted into the analog signal which is then displayed on a normal display. So if we just look in the form of a block diagram

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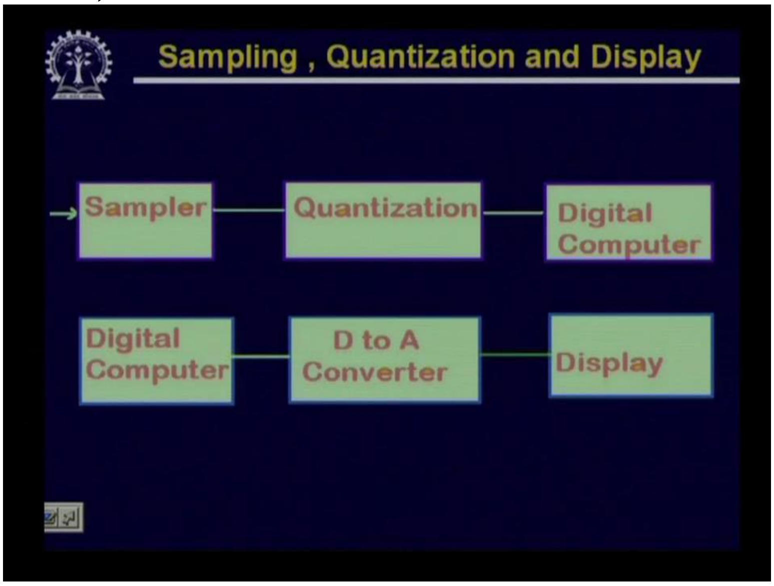

it appears something like this.

That while digitization, first we have to sample the image by a unit which is known as sampler, then every sampled values we have to digitize, the process known as quantization and after quantization

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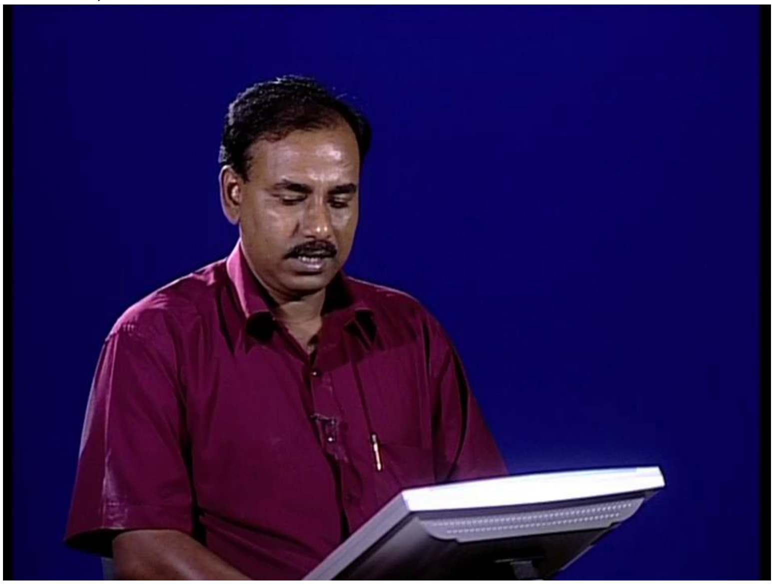

we get a digital image which is processed by the digital computer.

And when we want to see the processed image that is how does the image look like after the processing is complete then for that operation it is the digital computer which gives the digital output.

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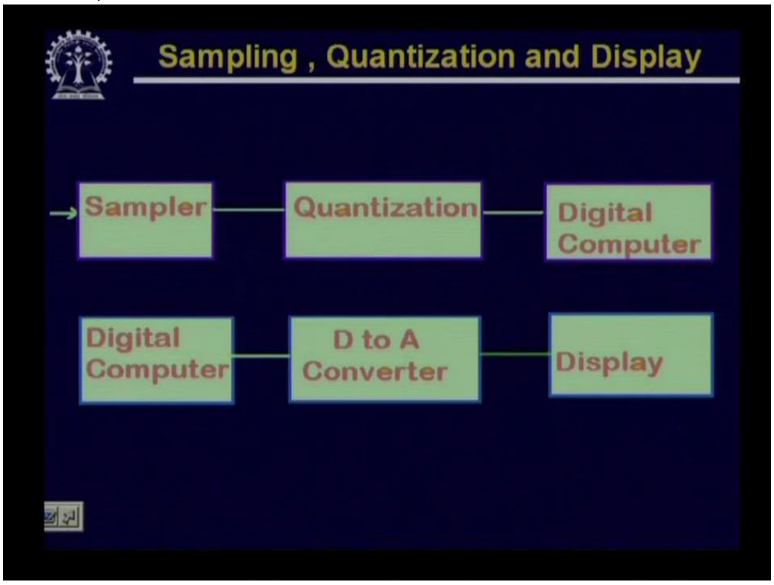

This digital output goes to D to A converter and finally the digital to analog converter output is fed to the display and on the display we can see that how the processed image looks like.

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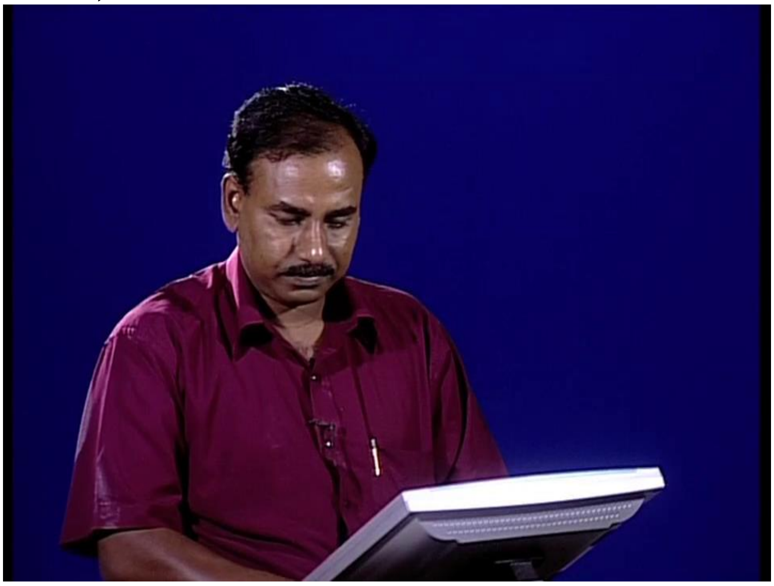

Now

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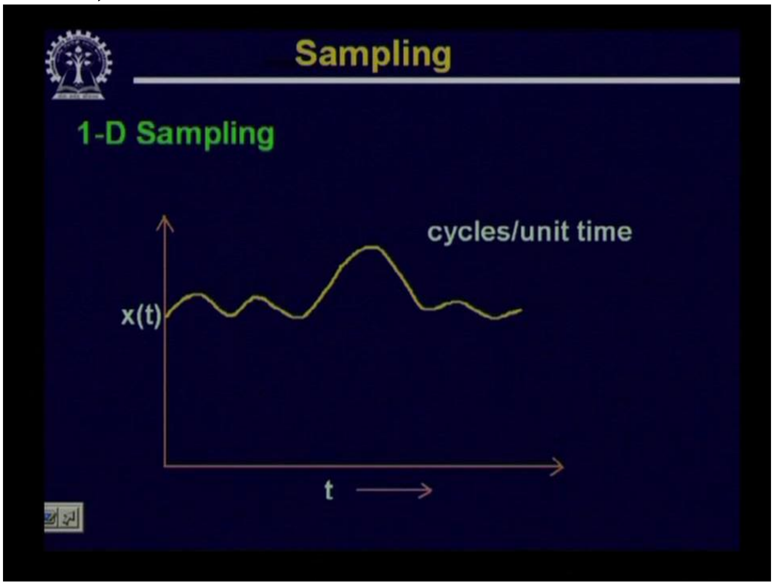

let us come to the first step of the digitization process that is sampling. To understand sampling, before going to the two-dimensional image let us take an example from one dimension. That is, let us assume that we have a one-dimensional signal x t which is a function of t. Here we assume this t to be time and you know that whenever some signal is represented as a function of time, whatever is the frequency content of the signal that is represented in the form of Hertz and this Hertz means it is cycles per unit time. So here again, when you look at this particular signal x t you find that this is an analog signal that is t can assume any value. t is not discretized. Similarly the functional value x t can also assume any value between certain maximum and minimum. So obviously this is an analog signal and we have seen that an analog signal cannot be represented in a computer. So what is the first step that we have to do? As we said, for the digitization process, the first operation that you have to do is the sampling operation.

So for sampling what we do is

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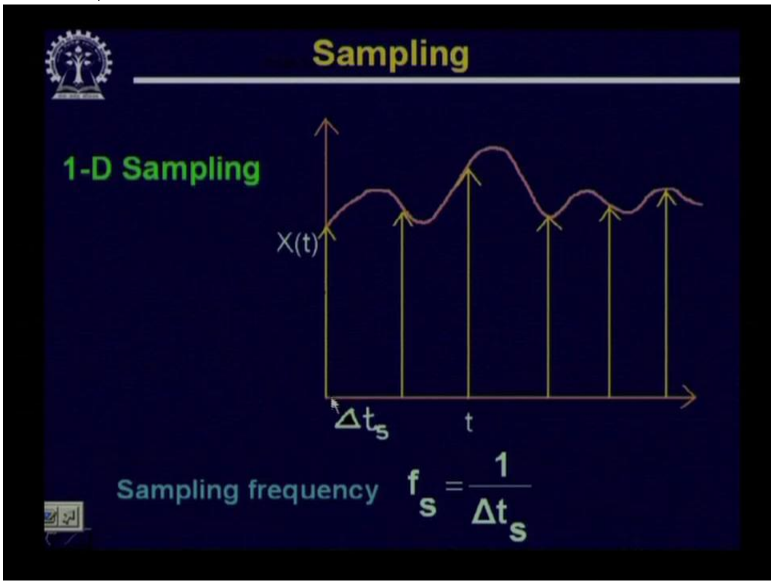

instead of taking, considering the signal values at every possible value of t, what we do is, we consider signal values at certain discrete values of t. So here in this figure, it is shown that we assume the value of the signal x t at t equal to 0, we also consider the value of the signal x t at t equal to 2 delta "t s". Assume the value of signal x t at t equal to delta 2 "t s", t equal to delta 3 "t s" and so on. So instead of considering signal values at every possible instant, we are considering the signal values at some discrete instants of time. So this is a process known as sampling. And here when we are considering the signal values at an interval of delta "t s", so we can find out what is the sampling frequency. So delta "t s" is the sampling interval and corresponding sampling frequency, if I represent it by "f s" it becomes 1 upon delta "t s".

Now when you sample the signal like this, you will find that there are many informations which are being missed. So for example, here we have a local minimum, here we have a local maximum, here again we have a local minimum, local maximum, here again we have a local maximum and when we sample at an interval of delta "t s", these are the informations that cannot be captured by these samples. So what is the alternative?

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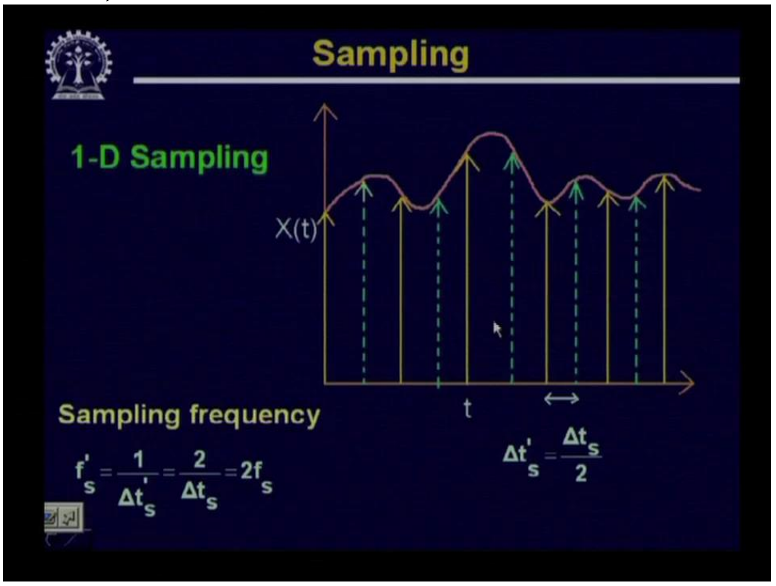

The alternative is let us increase the sampling frequency or let us decreasing the sampling interval. So if I do that you will find that these bold lines, bold golden lines

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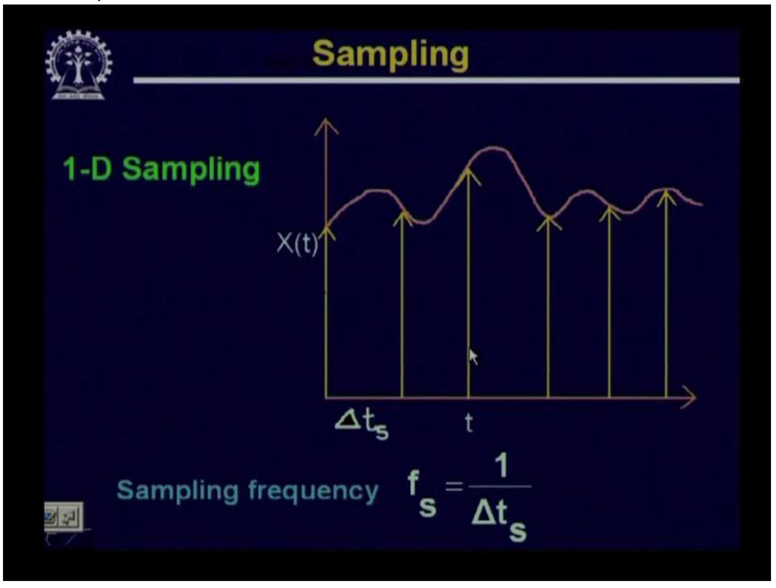

they represent the earlier samples that we had, like this

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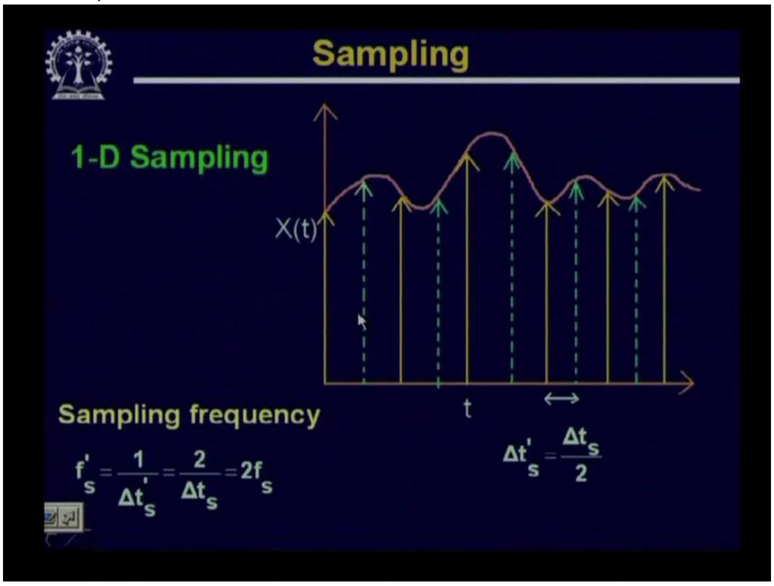

whereas these dotted green lines, they represent the new samples that we want to take. And when we take the new samples, what we do is we reduce the sampling interval by half. That is our earlier sampling interval was delta "t s", now I make the new sampling interval which I represent as delta "t s dash" which is equal to delta "t s" by two; and obviously in this case, the sampling frequency which is "f s dash" equal to 1 upon delta "t s dash" now it becomes twice of "f s". That is, earlier we had the sampling frequency of "f s", now we have the sampling frequency of delta 2 "f s", twice "f s". And when you increase the sampling frequency, you find that with the earlier samples represented by this solid lines you find that this particular information that is tip in between these two solid lines were missed. Now when I introduce a sample in between, then some information of this minimum or of this local maximum can be retained. Similarly here, some information of this local minimum can also be retained. So obviously it says that when I increase the sampling frequency or I reducing the sampling interval

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then the information that I can maintain in the sampled signal will be more than when the sampling frequency is less. Now the question comes, whether there is a theoretical background by which we can decide that what is the sampling frequency, proper sampling frequency for certain signals, that we can decide. We will come to that a bit later.

Now let us see that what does this sampling actually mean?

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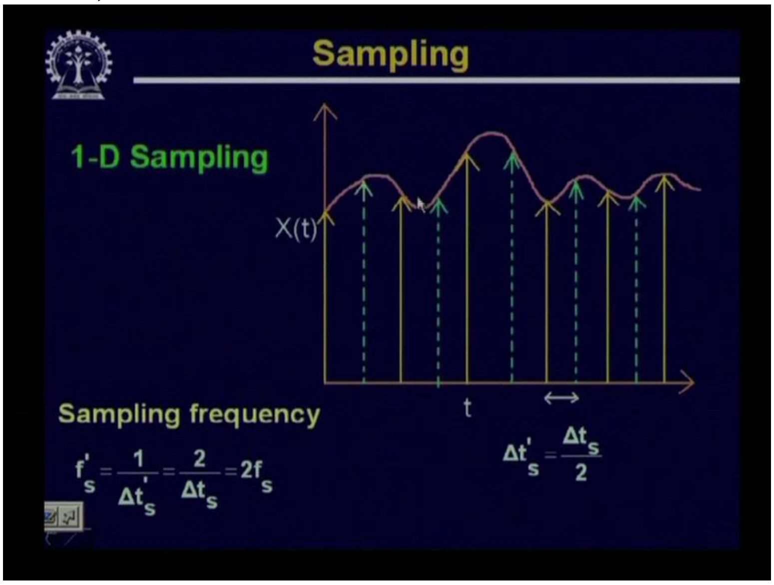

We have seen that we have a continuous signal x t and for digitization instead of considering the signal values at every possible value of t, we have considered the signal values at some discrete instants of time t, Ok. Now this particular sampling process can be represented

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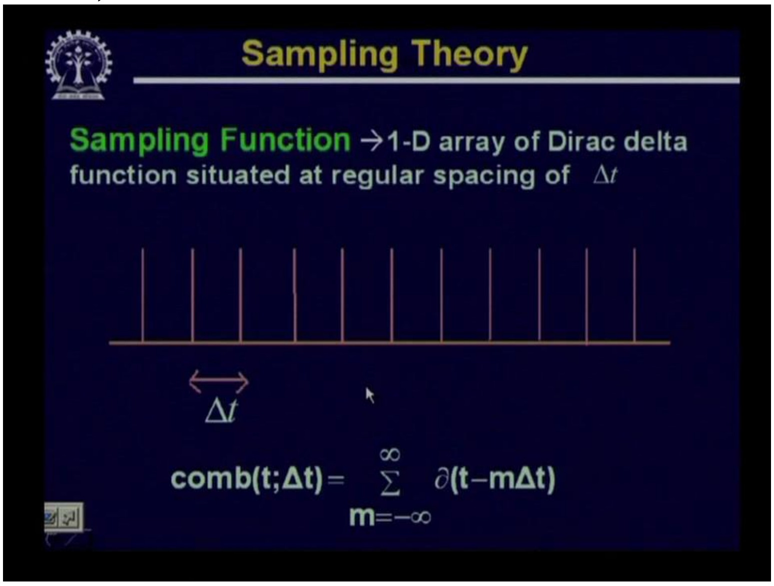

mathematically in the form that if I have, if I consider that I have a sampling function and this sampling function is one-dimensional array of Dirac delta functions which are situated at a regular spacing of delta t. So this sequence of Dirac delta functions can be represented in this form. So you find that each of these are sequence of Dirac delta functions and the spacing between two delta functions is delta t. In short, these kind of function is represented by comb function, a comb function t at an interval of delta t and mathematically this comb function can be represented as delta t minus m into delta t where m varies from minus infinity to infinity.

Now this is the Dirac delta function. The Dirac delta function says that if I have a Dirac delta function delta t then the functional value will be 1 whenever t equal to 0 and the functional value will be 0 for all other values of t. In this case when I have delta t minus m of delta t then this functional value will be 1 only when this quantity, that is, t minus m delta t within the parenthesis becomes equal to zero. That means this functional value will assume a value 1 whenever t is equal to m times delta t for different values of m varying from minus infinity to infinity.

So effectively this mathematical expression gives rise to a series of Dirac delta functions in this form where at an interval of delta t, I get a value of 1. For all other values of t, I get values of 0. Now this sampling

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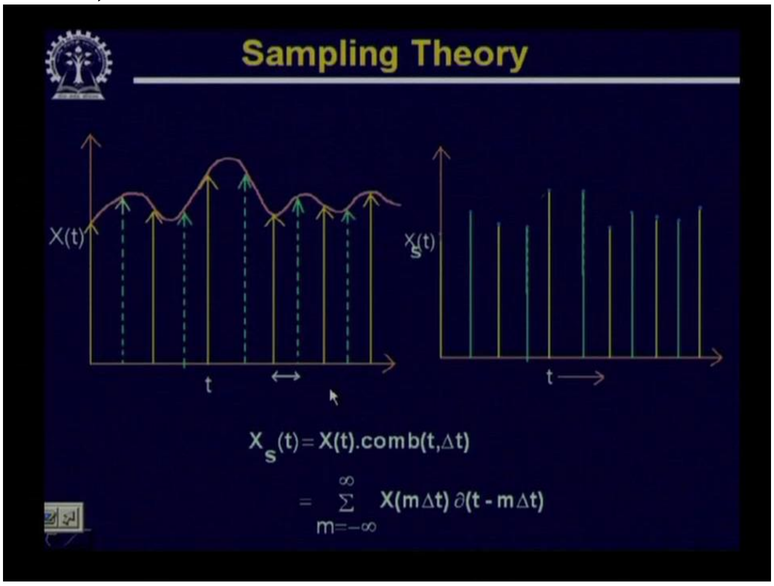

as you find that we have represented the same figure here, we had this continuous signal x t, original signal. After sampling we get a number of samples like this. Now here, these samples can now be represented by multiplication of x t with the series of Dirac delta functions that you have seen, that is, comb of t delta t. So if I multiply this, whenever this comb function gives me a value 1, only the corresponding value of t will be retained in the product and whenever this comb function gives you a value 0, the corresponding points, the corresponding values of x t will be set to 0. So effectively, this particular sampling, when from this analog signal, this continuous signal, we have gone to this discrete signal, this discretization process can be represented mathematically as "“x s”" t equal to x t into comb of t delta t and if I expand this comb function and consider only the values of t where this comb function has a value 1, then this mathematical expression is translated to x of m delta t into delta t minus m delta t where m varies from minus infinity to infinity, right.

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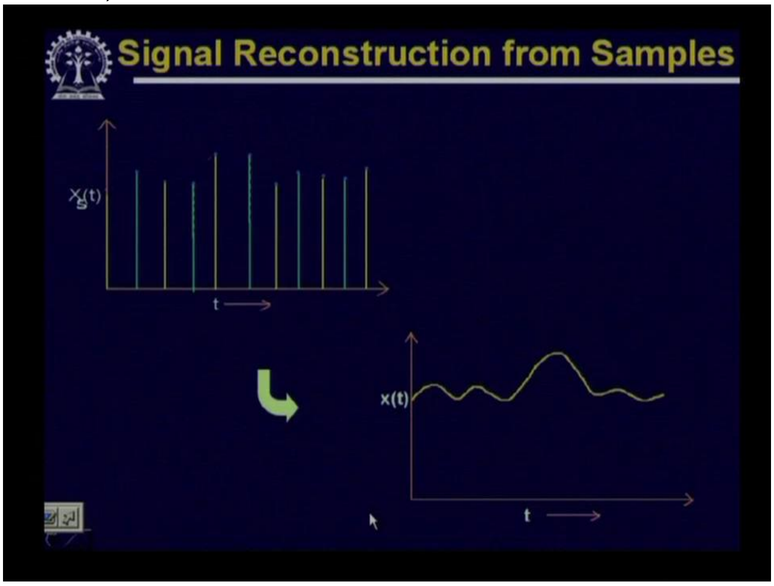

So after sampling what you have got is, from a continuous signal we have got the sampled signal represented by “x s” t where the sample values exist at discrete instants of time.

Sampling, what we get is a sequence of samples as shown in this figure where “x s” t has got the signal values at discrete time instants and during the other time intervals, the value of the signal is set to 0. Now this sampling will be proper if we are able to reconstruct the original, continuous signal x t from these sample values. And you will find out that while sampling we have to maintain certain conditions so that the reconstruction of the analog signal x t is possible.

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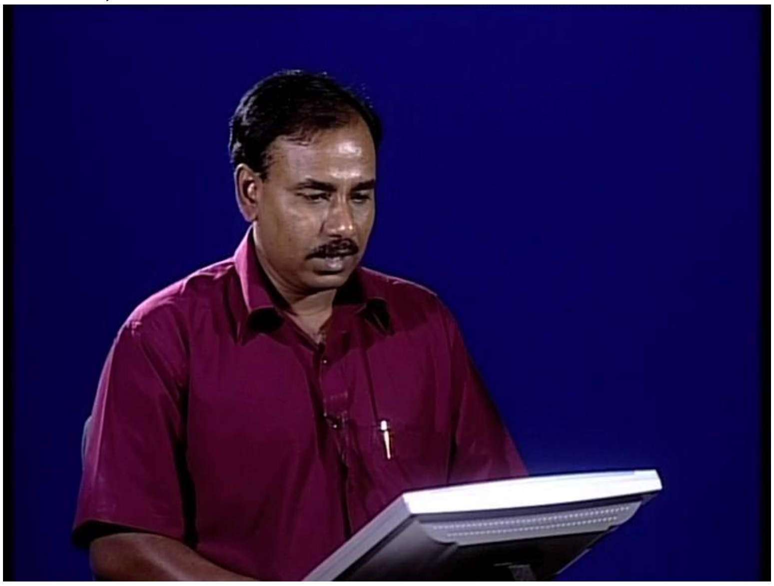

Now let us look at some mathematical background which will help us to find out the conditions which we have to impose for this kind of reconstruction. So here you find that if we have a continuous signal in time which is represented by x t

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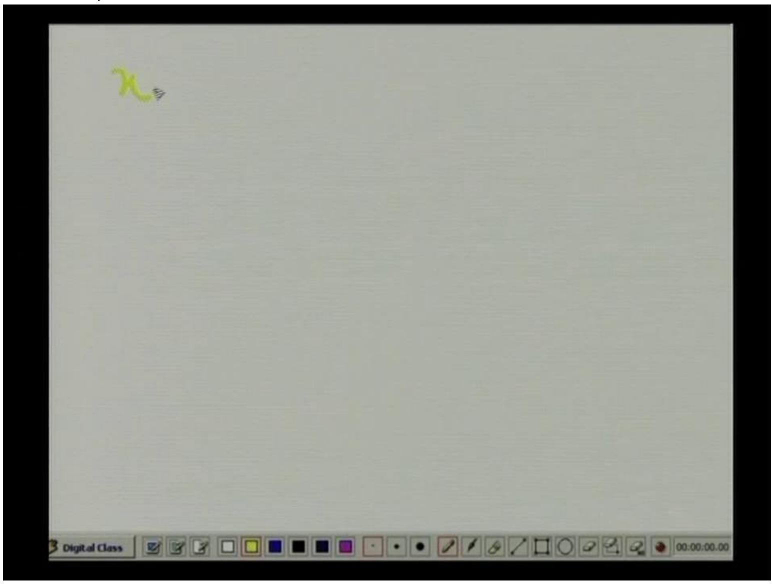

then we know that the frequency components of this signal x t can be obtained by taking the Fourier transform of this x t. So if I take the Fourier transform of x t which is represented by f of x t which is also represented in the form of capital x of omega where omega is the frequency component and mathematically this will be represented as x t e "to the power minus j omega t" d t and we have to take the integration of this from minus infinity to infinity. So this mathematical expression gives us the frequency components which is obtained by the

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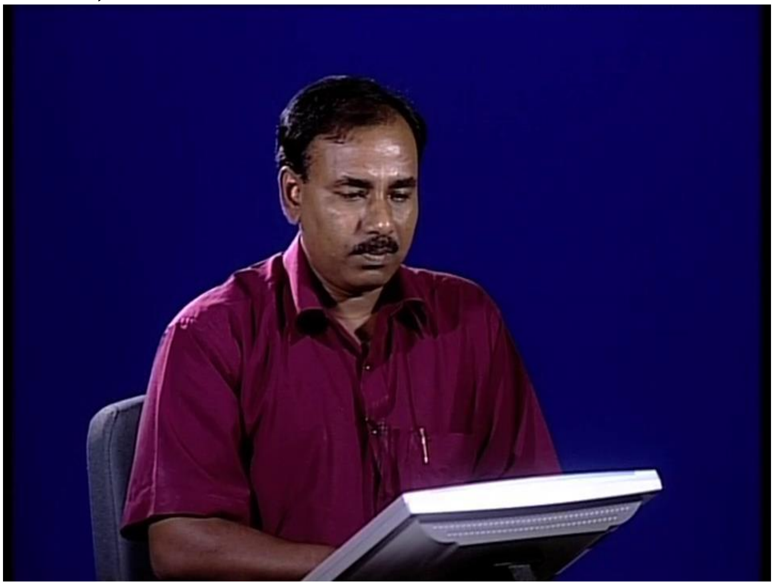

Fourier transform of the signal x t Now this is possible if the signal x t is aperiodic. But when the signal x t is periodic, in that case the instead of taking the Fourier transform, we have to go for Fourier series expansion. And the Fourier series expansion of a periodic signal say v t

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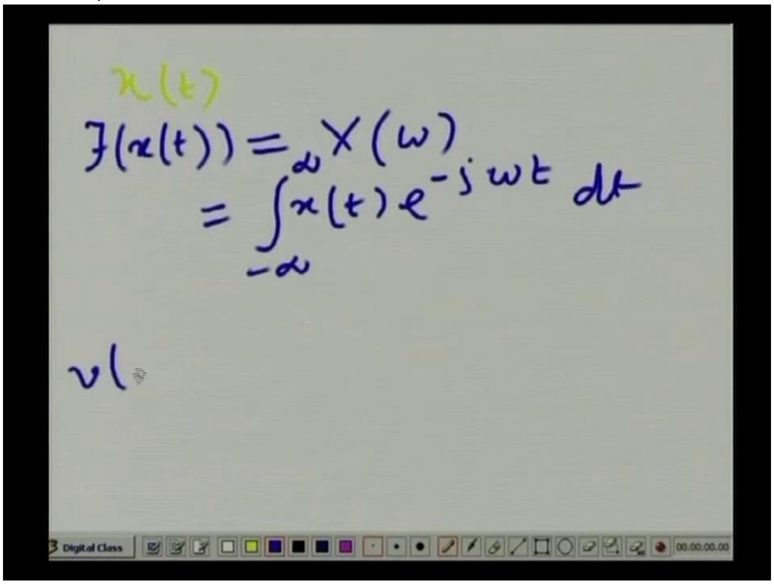

where we assume that v t is a periodic signal, is given by this expression

where omega 0 is the fundamental frequency of this signal v t

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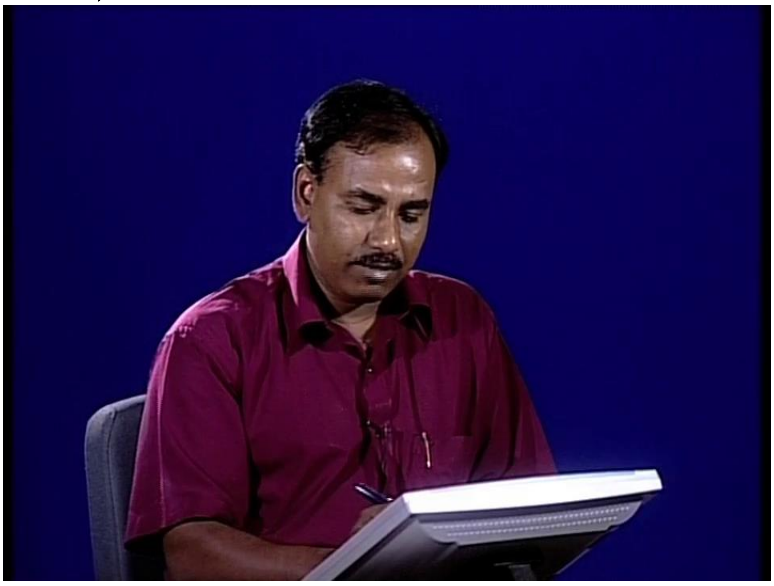

and we have to take the summation from

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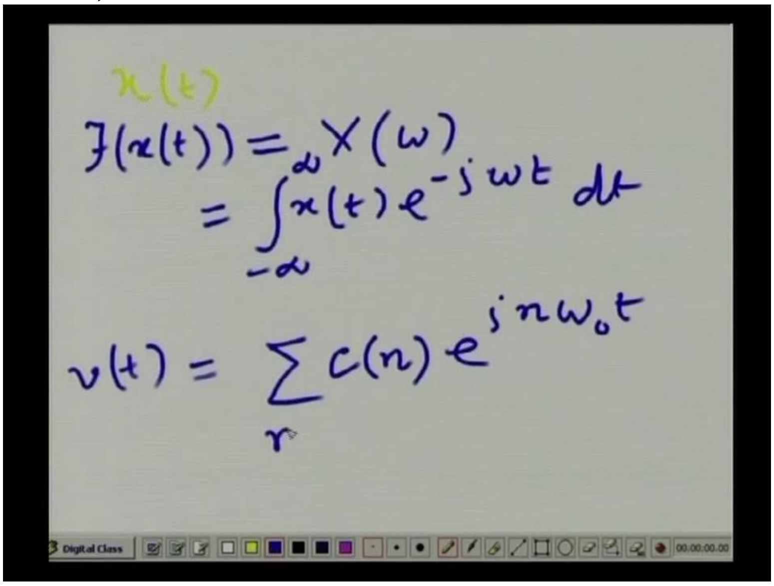

n equal to minus infinity to infinity

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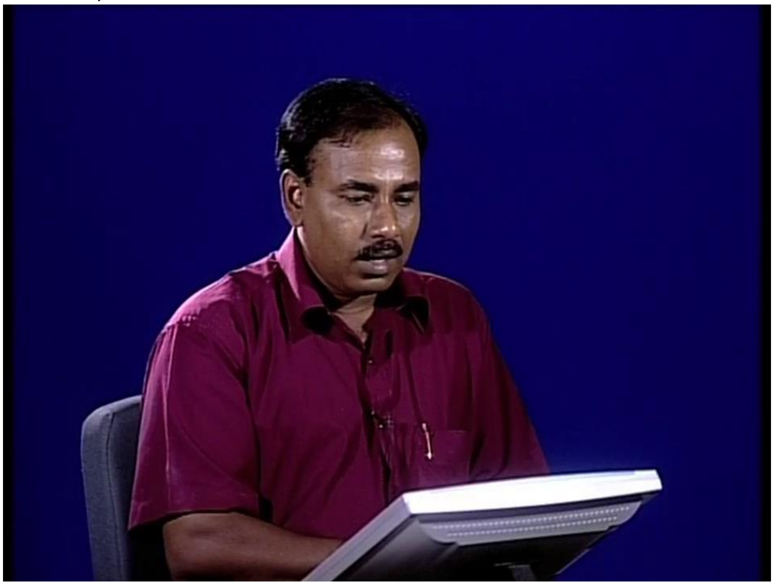

Now in this case the c n is known as Fourier coefficient. So nth Fourier coefficient and the value of c n is obtained as

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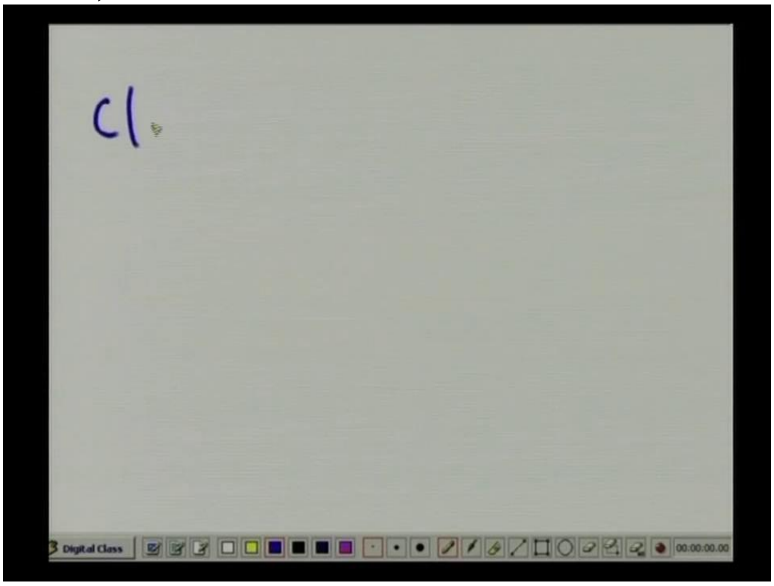

c n is equal to 1 upon “t naught” v t e “to the power minus j n “omega naught” t” d t and this integration has to be taken over a period that is “t naught”. Now in our case when we have this v t in the form of series of Dirac delta functions, in that case we know

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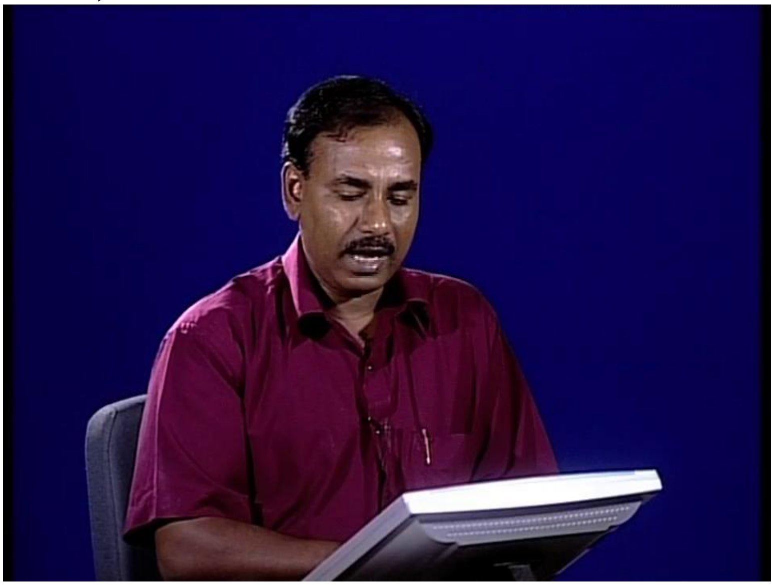

we know that value of v t will be equal to 1 when t equal to 0 and value of v t is equal to 0 for any other value of t within a single period.

So in our case

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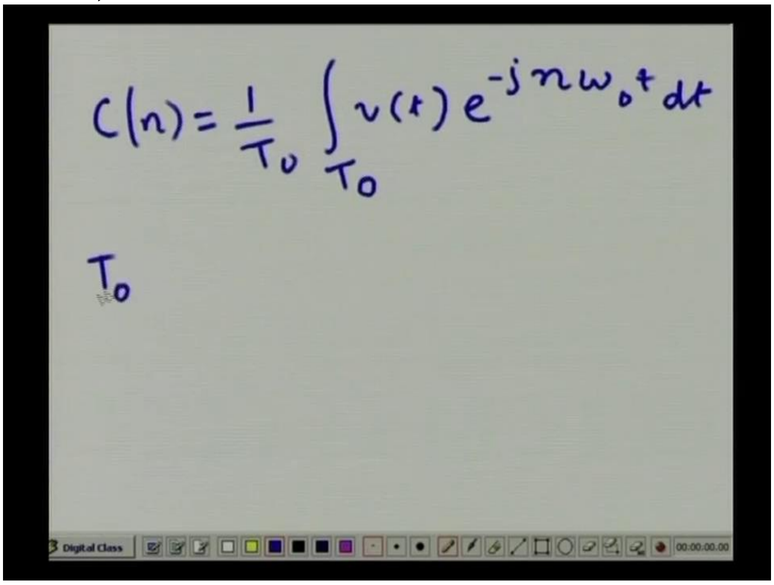

"T naught", that is the period of this periodic signal is equal to delta "T s" because every delta function appears at an interval of delta "T s". And we have v t is equal to 1 for t is equal to 0 and v t is equal to 0, otherwise, Ok.

Now if I impose this condition

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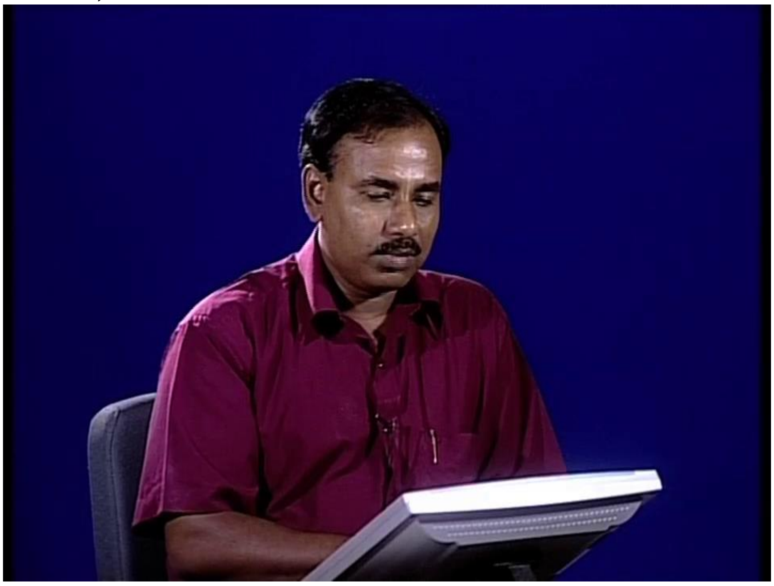

to calculate the value of c n, in that case you will find

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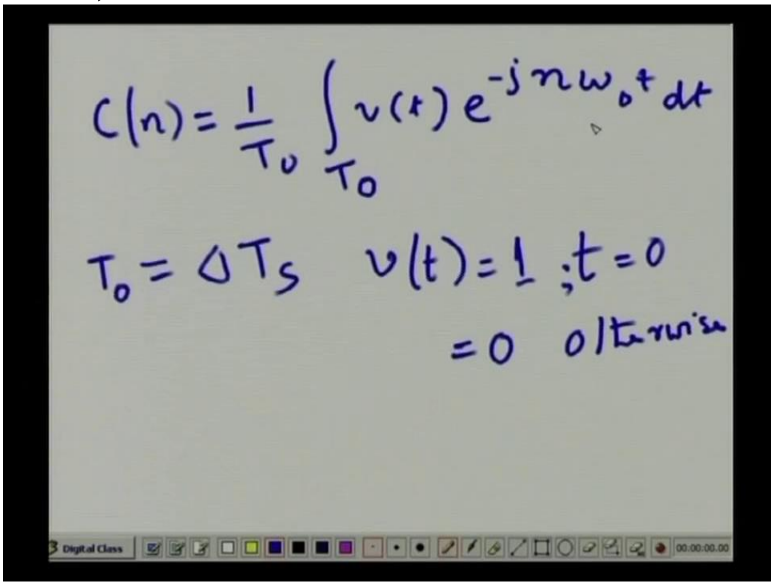

that the value of this integral will exist only at t equal to 0 and it will be 0 for

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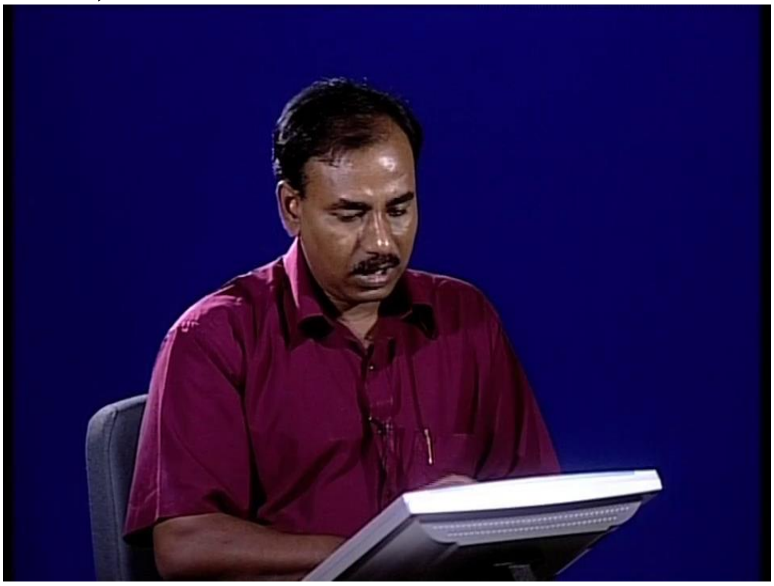

any other value of t So by this we find that c n now

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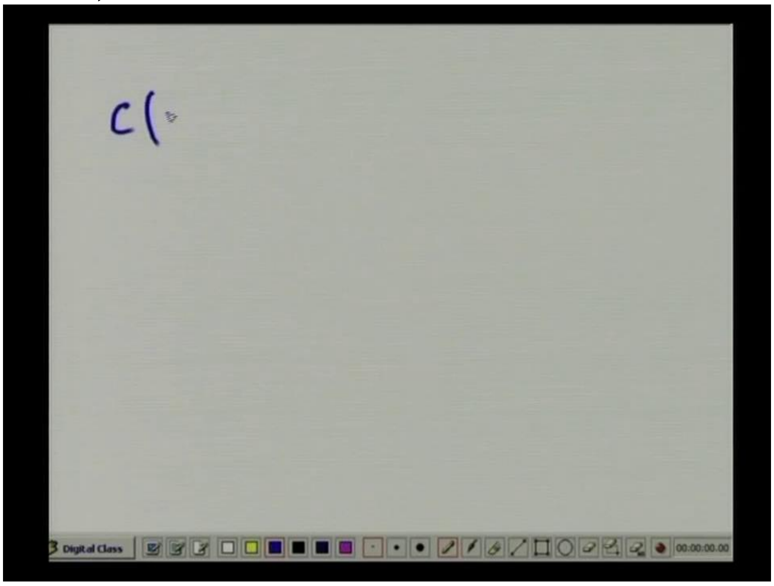

becomes equal to 1 upon delta “T s” and this 1 upon delta “T s” is nothing but the sampling frequency we put at, say omega s. So this is the frequency of the sampling signal.

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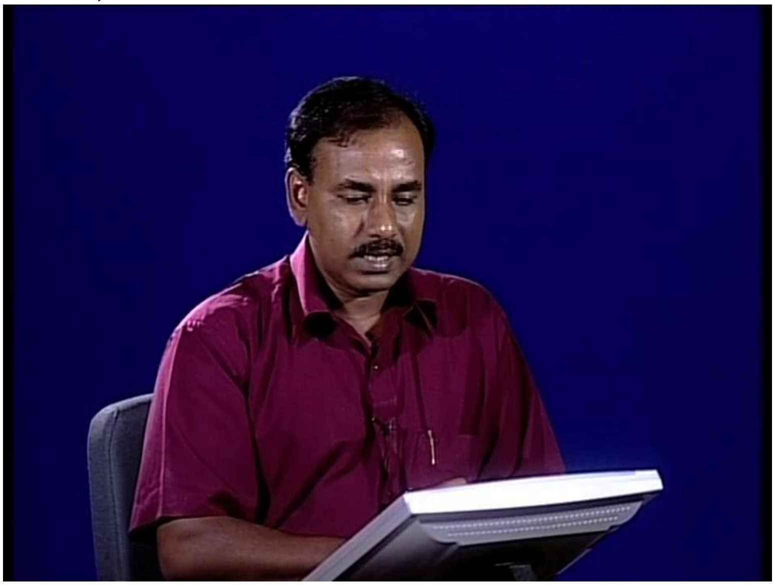

Now with this value of c n, now the periodic signal

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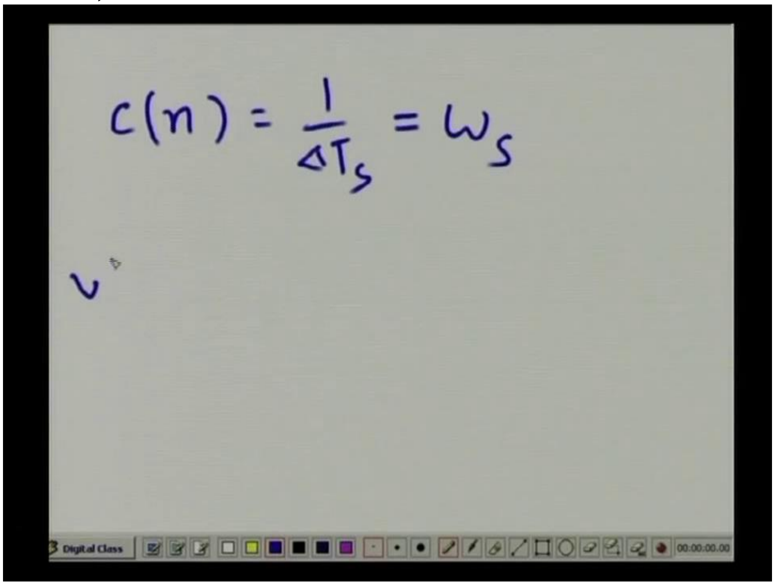

v t can be represented as 1 upon delta “T s”, summation of e “to the power j n “omega naught” t” for n equal to minus infinity to infinity. So what does it mean? This means that if I take the Fourier series expansion of our periodic signal which is in our case Dirac delta function, this will have frequency components

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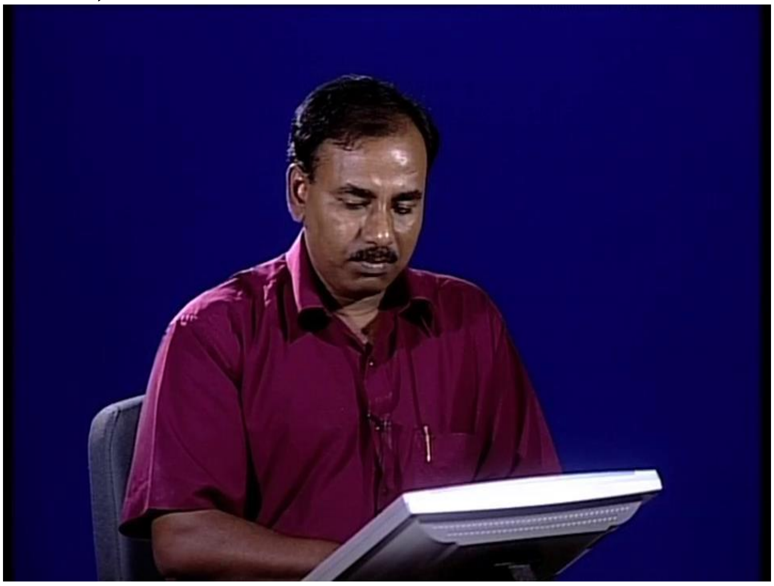

various frequency components for the fundamental component of frequency is "omega naught" and it will have other frequency components of twice "omega naught", thrice "omega naught", four times "omega naught" and so on.

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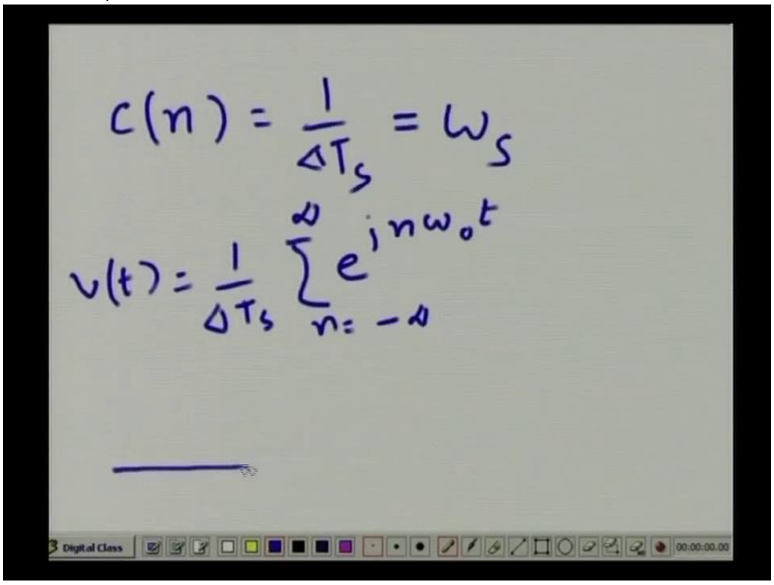

So if I plot those frequencies or frequency spectrum we find that will have the fundamental frequency "omega naught" or in this case, this "omega naught" is nothing but same as the sampling frequency "omega s". We will also have a frequency component of twice "omega s", we will also have a frequency component of thrice "omega s" and this continues like this. So you will find that the comb function as the sampling function

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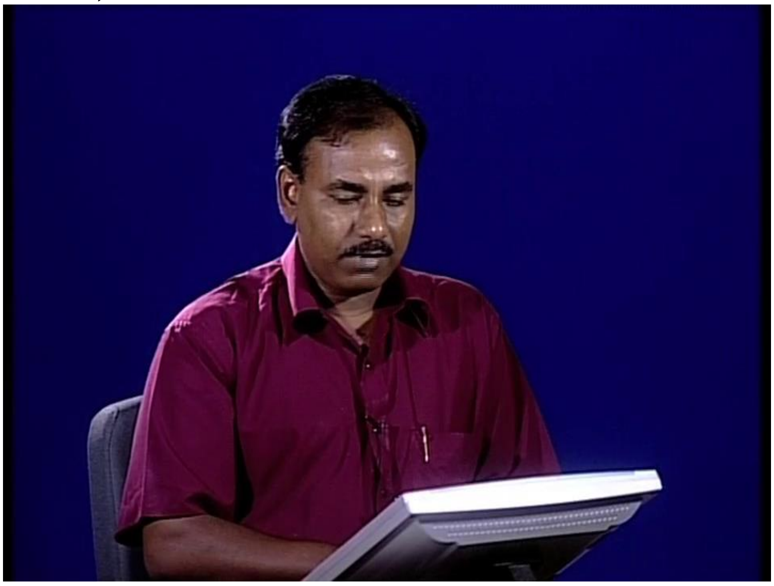

that we have taken, the Fourier series expansion of that is again a comb function. Now this is about the continuous domain.

When you go to discrete domain, in that case,

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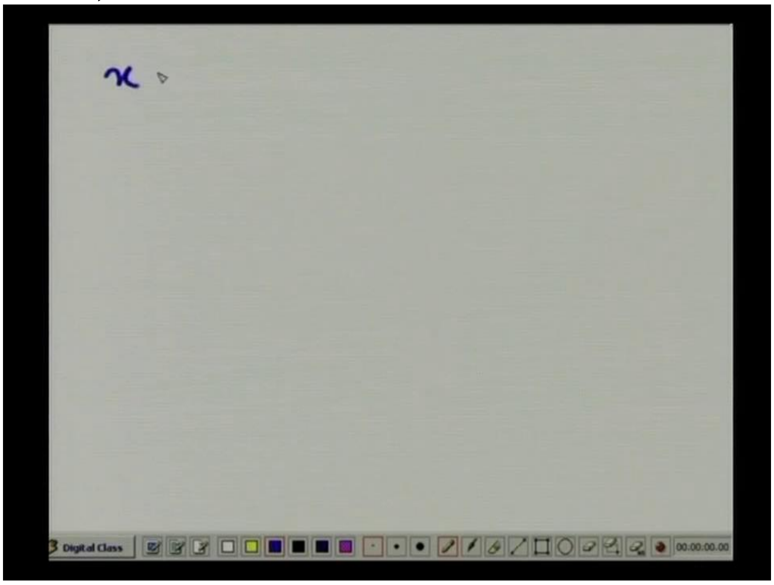

for a discrete time signal say x n where n is the nth sample of the signal x, the Fourier Transform of this is given by x k is equal to sum of x n e “to the power minus j “2 pi by N” into n k”, where value of n varies from 0 to N minus 1 where this capital n indicates number of samples that we have for which we are taking the Fourier transform. And given this Fourier Transform, we can find out the original sampled signal by the inverse Fourier transformation which is obtained as x n is equal to sum of x k e “to the power j “2 pi by N” n k" and this time, the summation has to be taken over k for k equal to 0 to N minus 1. So you find that we get a Fourier Transform pair, in one case from the discrete time signal we get the frequency component, discrete frequency components by the forward Fourier transform and in the second case, from the frequency components, we get the discrete time signal by the inverse Fourier transform. And these two equations taken together form a Fourier Transform pair.

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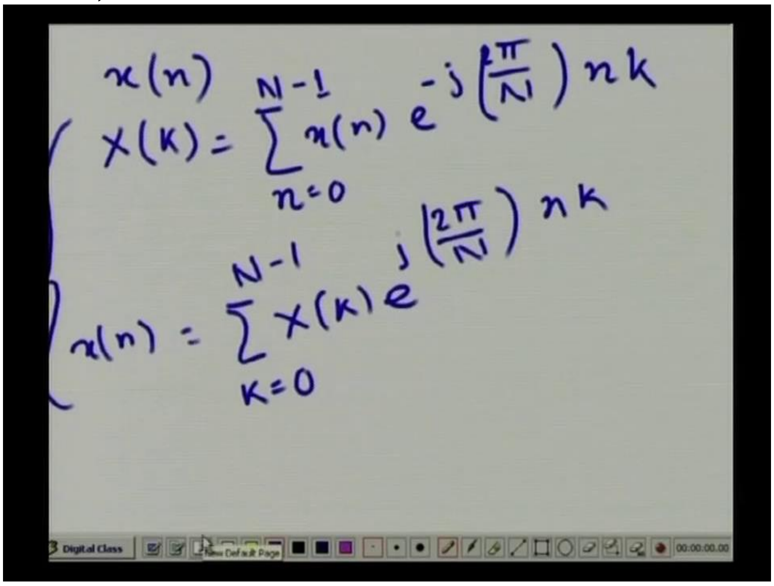

Now let us go to another concept.

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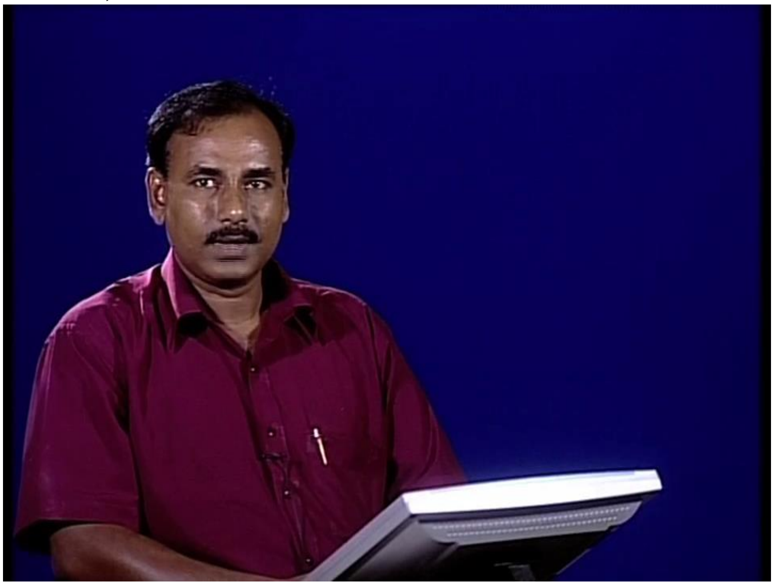

Thank you.