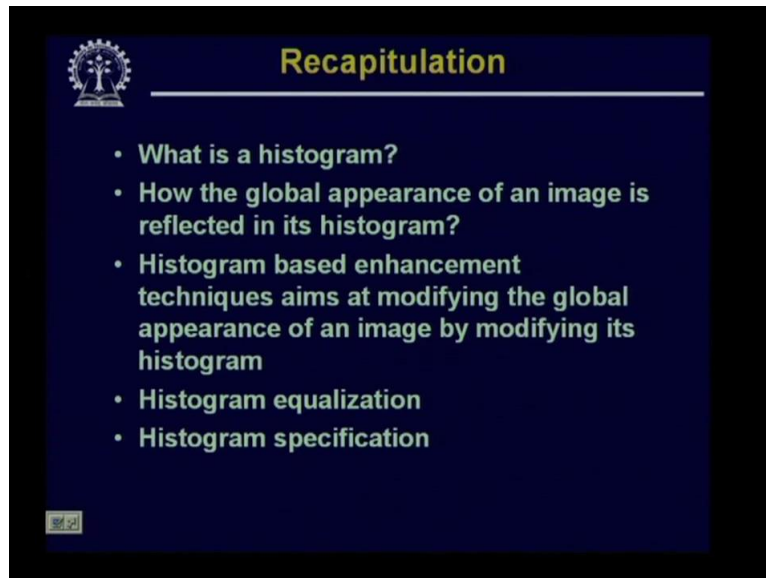


Digital Image Processing.
Professor P. K. Biswas.
Department of Electronics and Electrical Communication Engineering.
Indian Institute of Technology, Kharagpur.
Lecture-36.
Histogram Implementation-I.

(Refer Slide Time: 0:47)



Hello, welcome to the video lecture series on digital image processing. For last few classes, we have started our discussion on image enhancement techniques. So in the previous class, we have seen what is meant by histogram? We have seen how the global appearance of an image is reflected in its histogram? We have seen that histogram based enhancement techniques aims at modifying the global appearance of an image by modifying its histogram. Then we have started discussion on histogram equalization technique and histogram specification or histogram matching techniques.

(Refer Slide Time: 1:12)

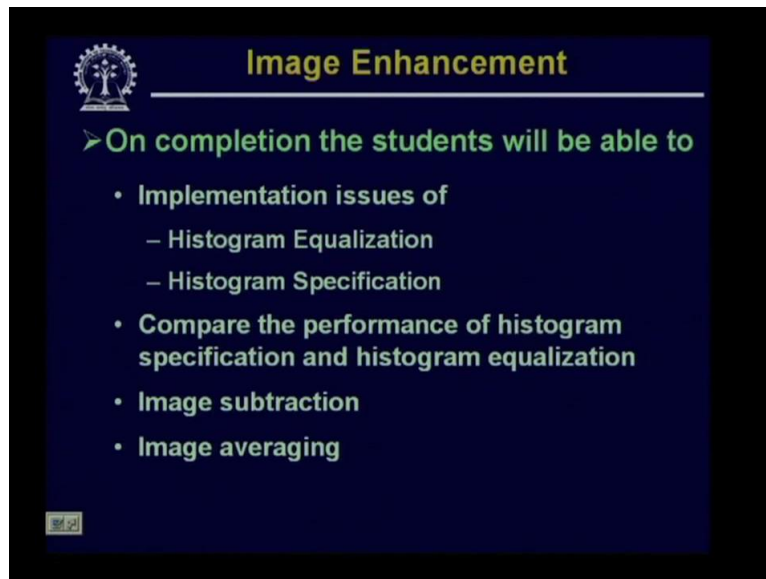


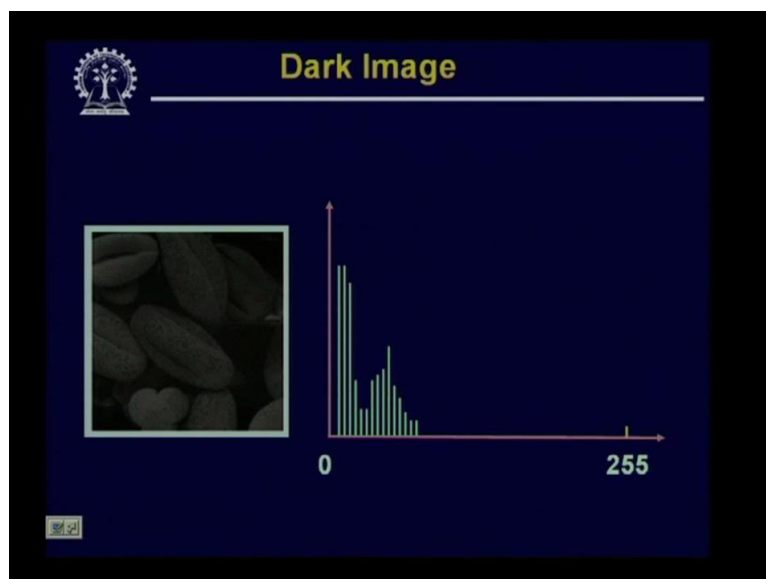
Image Enhancement

➤ On completion the students will be able to

- Implementation issues of
 - Histogram Equalization
 - Histogram Specification
- Compare the performance of histogram specification and histogram equalization
- Image subtraction
- Image averaging

So today's class, we will talk about some implementation issues of histogram equalization and histogram specification techniques, and we will talk about this implementation issues with respect to some examples, then we will also compare the performance of histogram specification and histogram equalization techniques with the help of some results obtained on some images. Then lastly we will talk about two more point processing techniques for histogram equalization, one of them is histogram subtraction and other one is histogram averaging techniques.

(Refer Slide Time: 2:05)



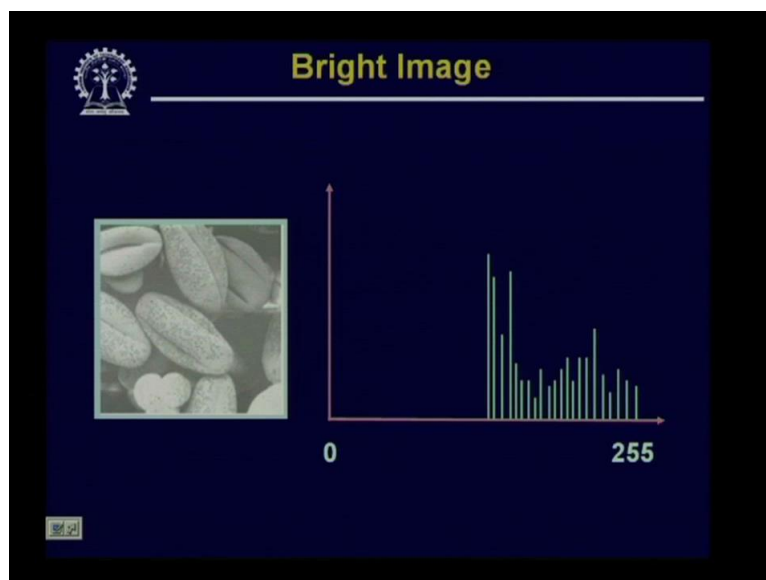
Dark Image

The slide displays a dark image of several dark, oval-shaped objects (possibly seeds or stones) on the left. To the right of the image is a histogram showing the distribution of pixel intensities. The x-axis is labeled from 0 to 255, and the y-axis represents frequency. The histogram shows a high concentration of pixels at low intensity values (near 0), with a long tail extending towards higher intensity values (up to 255).

So now, let us briefly recapitulate what we have done in the last class. As we have said, that histogram of an image that indicates what is the global appearance of an image, we have also seen these images in the last class, but just for a quick recapitulation you find that on the left hand side we have shown an image which is very dark, and we call this as the dark image. And on the right hand side we have shown the corresponding histogram and you find that this histogram shows that most of the pixels in this particular image, they are having an intensity value which is near about 0.

And there is practically no pixel having higher intensity values, and that is what gives this particular image a dark appearance.

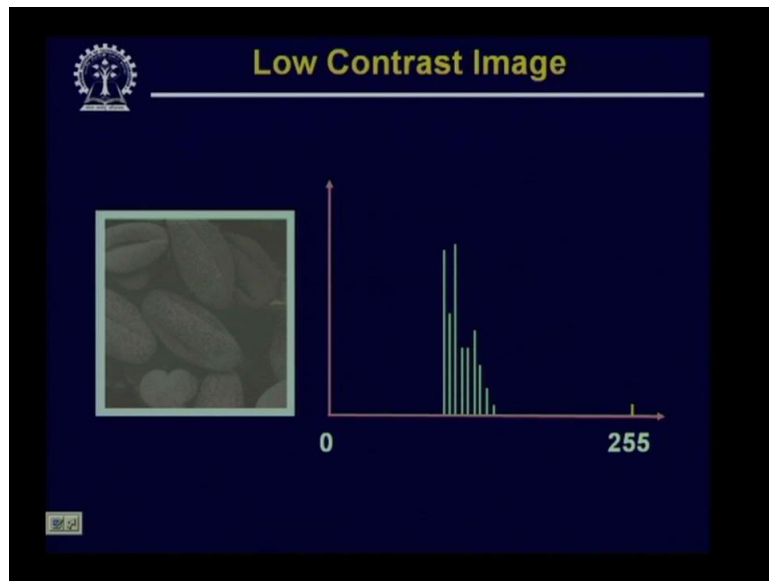
(Refer Slide Time: 3:05)



Then the second one that we have shown is a bright image or a light image, and again from this particular histogram you find that most of the pixels in this particular image have intensity values which are near to maximum values that is 255 in this particular case. And since we are talking about all the images in our application which are quantized or every pixel is quantized with eight bits so the intensity levels will vary from 0 to 255.

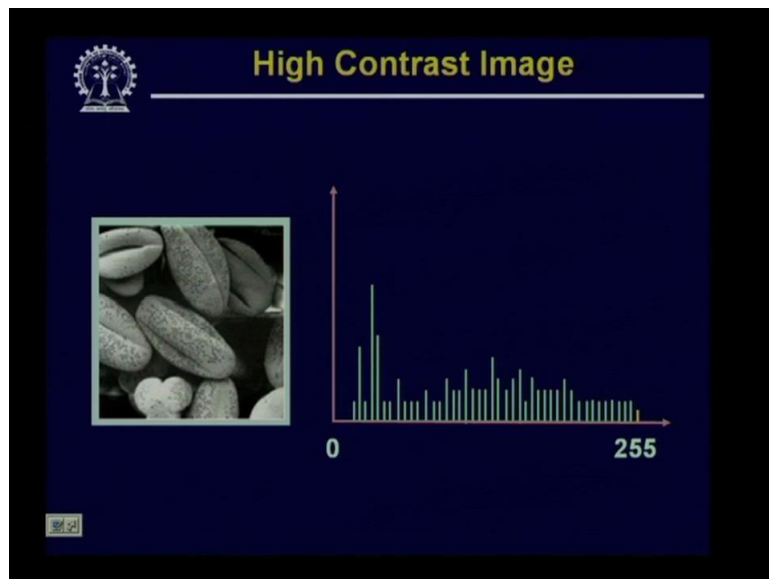
So in our case, the minimum intensity of a pixel will be zero, and the maximum intensity of a pixel will be 255. So in this particular example, you will find that the intensity of the images as this histogram shows that most of the pixels have intensity which are near 255, that is maximum value.

(Refer Slide Time: 3:55)



Then the next image shows, that here the image has got in pixels having intensity value in the middle range, but the range of intensity values is very narrow. So as a result, the image is neither very bright nor very dark, but at the same time because the dynamic range of the intensity values is very low, the image contrast is very poor.

(Refer Slide Time: 4:23)



So as the next slide shows, which we call a high contrast image, where you find that most of the details of the objects present in the image are visible, and by looking at the corresponding histogram, we find that the pixels in this particular image have wide range of intensity values starting from very low value which is near about 0 to the maximum value which is near about

55. So this says that we will say that image, a particular image has a high contrast if its intensity values, the pixel intensity values have a wide range of values starting from a very low value to a very high value.

So all this 4 examples, tell us that how the global appearance of an image is reflected in its corresponding histogram, and that is why all the histogram based enhancement techniques they try to adjust the global appearance of the image by modifying the histogram of the corresponding image. So the first technique of this histogram based enhancement that we have discussed in the last class is called histogram equalization. So let us quickly review what we mean by histogram equalization?

(Refer Slide Time: 6:06)

The image shows a handwritten derivation on a digital class screen. At the top, the probability mass function is given as $p_r(r_k) = \frac{n_i}{n}$. A bracket under this equation is labeled "Histogram Equalization". To the right, a note explains that $n_k \rightarrow$ no. of pixels with intensity value $= r_k$. Below this, the transformation function is derived as $s_k = T(r_k) = \sum_{i=0}^k \frac{n_i}{n}$. This is further simplified to $= \sum_{i=0}^k p_r(r_i)$ with the constraint $0 \leq s_k \leq 1$.

So here, in case of histogram equalization, if I consider a discrete case, in a discrete case we have seen that the histogram of an image is given in the form like this, that is $Pr(r_k)$ where r_k is an intensity level present in the image, and which is given by summation of n_i by n , i varying from zero to k where n_k , is the number of pixels with intensity value is equal to r_k , sorry this is not summation Pr_k is given by n_i by n .

So as this expression suggests, that this particular expression tells us what is the probability of a pixel having a value r_k present in the image. And the plot of all these Pr_k values for different values of r_k , defines what is the histogram of this particular image. Now when we talk about the histogram equalization, the histogram equalization technique makes use of this histogram to find out the transformation function between a intensity level in the original image, to an intensity level in the processed image.

And that transformation function is given by, s_k is equal to say transformation function we represent by $T(r_k)$ which is given by summation of n_i by n , where i varies from zero to k , and which is nothing but summation of $P_r r_i$ where i varies from zero to k . So this is the transformation function that we get which is to be used for histogram equalization purpose. Now we find, that in this particular case, because the histogram which is defined, that is $P_r r_k$ equal to n_i by n , it is a normalized histogram. So every value of $P_r r_k$ will be within the range zero to one, and similarly this transformation function, that is $T(r_k)$ when it gives us a value s_k corresponding to an intensity level in the input image which is equal to r_k , the maximum value of s_k also in this particular case will vary from zero to one.

(Refer Slide Time: 9:43)

Handwritten mathematical equations on a digital whiteboard:

$$0 \quad r_{L-1} \rightarrow \underline{255}$$

$$s_k = T(r_k)$$

$$s' = \text{Int} \left[\frac{s - s_{\min}}{1 - s_{\min}} \times (L-1) + 0.5 \right]$$

So the minimum value of the intensity as suggested by this particular expression will be zero and the maximum value of the intensity will be equal to one. But we know that when we are talking about the digital images, the minimum intensity of an image can be a value zero and the maximum intensity can have a value r_{L-1} minus one, as k varies from zero to L minus one, and in our discrete case, this r_{L-1} minus one is equal to 255, because in our case the intensity values of different images are quantized with 8 bits, so we can have an intensity varying from 0 to 255.

Whereas this particular transformation function, that is s_k equal to $T(r_k)$ this gives us a maximum intensity value s_k in the processed image which is equal to one. So for practical implementation, we have to do some sort of post processing, so that all this s_k values that you get in the range of zero to one, can now be mapped to the maximum dynamic range of the

image that is from 0 to 255. And the kind of mapping function that we have to use is given by, say I can write it as, s dash is equal to say Integer value because we will be getting only integer values into s minus s minimum divided by one minus s minimum into L minus one, where L minus one is the maximum intensity level plus you give a DC shift of 0.5.

So whatever value of s we get by this transformation s_k equal to $T(r_k)$, that value of s has to be scaled by this function to give us an intensity level in the processed image which varies from zero to maximum level, that is zero to capital L minus one, and in our case this capital L minus one will be equal to a value 255.

(Refer Slide Time: 12:04)

$$r \rightarrow 0, 1, 2, \dots, 7$$

$$s \rightarrow 0, 1, 2, \dots, 7$$

$$p_r(0) = 0, \quad p_r(1) = p_r(2) = 0.1$$

$$p_r(3) = 0.3, \quad p_r(4) = p_r(5) = 0$$

$$p_r(6) = 0.4, \quad p_r(7) = 0.1$$

Now let us take an example to illustrate this, suppose we have an input image having eight discrete intensity values, that is r varies from 0, 1, 2 upto say 7, so we have eight discrete intensity values.

Similarly, the processed image that you want to generate that will also have eight discrete intensity values varying from zero to seven. Now suppose the probability density functions or the histogram of the input image is specified like this, so it given that $Pr(1)$ that is the probability that an intensity value will be equal to one, sorry $Pr(0)$ the probability that an intensity value will be equal to zero is equal to 0, $Pr(1)$ that is probability that intensity value will be equal to one, is same as $Pr(2)$ which is given as say 0.1 $Pr(3)$ that is given as 0.3, $Pr(4)$ is equal to $Pr(5)$, which is given equal to zero, $Pr(6)$ is given as 0.4 and $Pr(7)$ is given as 0.1. Now, our aim is that given this histogram of the input image, we want to find out the

transformation function T_r , which will map such an input image to the corresponding output image and the output image will be equalized.

(Refer Slide Time: 14:33)

r	$P_r(r)$	$T(r) = \sum_{i=0}^r P_r(i)$	s'
0	0	0	0
1	0.1	0.1	1
2	0.1	0.2	2
3	0.3	0.5	4
4	0	0.5	4
5	0	0.5	4
6	0.4	0.9	7
7	0.1	1.0	7

$$s' = \text{Int} \left[\frac{s - s_{\min} \times r + 0.5}{1 - s_{\min}} \right]$$

So to do this, what we have to do is, we have to find out the mapping function T_r . So this mapping function, we can generate in this form, let us have all this values in the form of a table, so I have this r , r varies from zero to seven. The corresponding P_r , the probability values are given by 0, 0.1, 0.1, 0.3, 0, 0, 0.4 and 0.1. Then obviously, from this probability density function, we can compute the transformation function T_r , which is nothing but summation of, say $P_r(i)$ where i varies from zero to r .

So if we compute the transformation function, you will find that the transformation function comes out to be like this, this is 0, this is 0.1, here it is 0.2, here it is 0.5, because the next two probability density function values as zero, so it will remain as 0.5, this will also remain as 0.5, then this will be 0.9, and here I will get 1.0. So this is the transformation function that we have. So this means that, if my input intensity is zero, this transformation function will give me a value s , this is nothing but the value say s_k which will be equal to zero. If the intensity is one, input intensity is one the output s_k will be equal to point one, if the input intensity is two, output s_k will be equal to point two.

Similarly if the input intensity is six, the output intensity value will be point nine. But naturally, because the output intensity is has to vary from zero to the maximum value which is equal to seven, so we have to scale this particular function, this particular s values to cover this entire range of intensities. And for that, we use the same expression, the same mapping

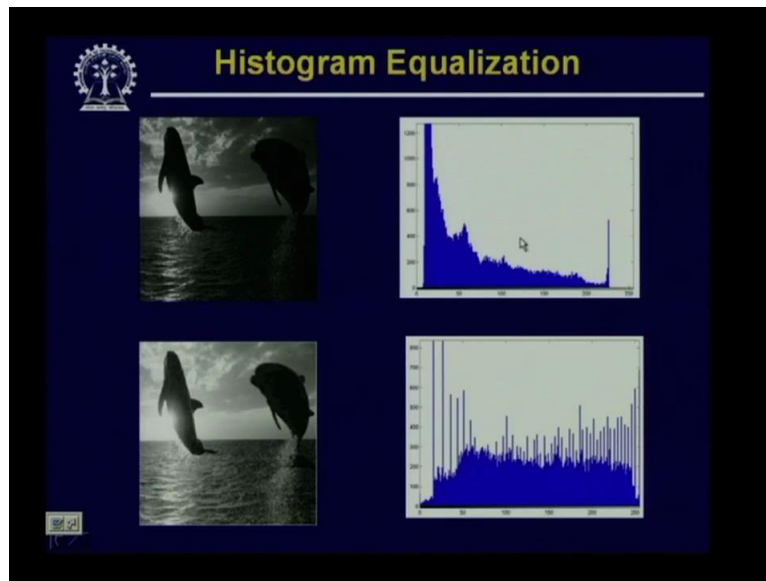
function as we have said that $s_{\text{dash}} = \text{integer of } s_{\text{min}} \text{ divided by } (1 - s_{\text{min}})$, and in this particular case $L - 1 = 7 + 0.5$.

So doing this calculation and taking the nearest integer value whatever we get, that will be my reconstructed intensity level. So if I do this, then you will find that for all these different values of s , the reconstructed s_{dash} will be, for $r = 0$, the reconstructed s_{dash} will be equal to zero, for $r = 1$, the reconstructed s_{dash} will also be equal to one, for $r = 2$, the reconstructed s_{dash} will also be equal to two, but for $r = 3$, so in this case $r = 3$, I get $s = 0.5$ and minimum s is zero. So this becomes 0.5 and denominator is also equal to one, so 0.5 into seven that gives you three point five plus point five which is equal to four.

So when my input intensity is three, the corresponding output intensity will be equal to four, similarly for $r = 4$ the output intensity will also be equal to four, for $r = 5$, the output intensity will also be equal to four, for $r = 6$, now the output intensity if you calculate following the same relation it will come out to be seven, for $r = 7$, the output intensity will also be equal to seven. So this first column that is for different values of r , and the last column that is the different values of s .

So this first column, and the last column, this gives us the corresponding mapping between the given intensity value to the corresponding output intensity value, and this is the image which is the processed image of the enhanced image which is to be displayed. So this is how the histogram equalization operations have to be done, and we have seen in the last class that using such histogram equalization operations, we have got the results which are given like this.

(Refer Slide Time: 19:27)



So here we have shown an image which is a very very dark image, and on the right hand side we have the corresponding histogram, and once we do histogram equalization, then what we get is an equalized image or the processed image, and on the bottom row you find that we have a brighter image which is the histogram equalized image, and on the right hand side, we have the corresponding histogram.

As we have mentioned in our last class that whenever we are going for histogram equalization, then the probability density function of the intensity values of the equalized image, they are ideally normal ideally uniform distribution. In this particular case you will find that this histogram of this equalized image that we have got, this is not absolutely uniform. However this is near to uniform, so that theoretical derivation which shows us that the distribution value, intensity distribution will be uniform that is a theoretical one. In practical cases, in discrete situations in most of the cases we do not get a uniform probability distribution, the uniform intensity distribution.

The reason being that in discrete cases, there may be a situation that many of the allowed pixel values will not be present in the image, and because of this the histogram that you get or the intensity distribution that you get in most of the cases they will not be uniform. So this all shows us the cases of histogram equalization. Thank you.