Digital Image Processing.

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Lecture-37.

Histogram Implementation-II.

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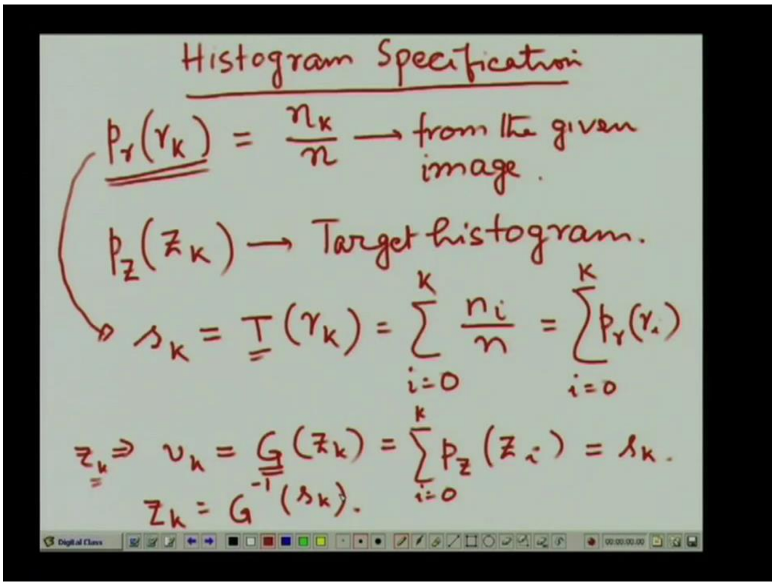

Hello, welcome to the video lecture series on digital image processing. Now let us come to the case of histogram specification or histogram modification as it is called. So, we will talk about histogram specification. So as we have told in our last class that histogram equalization is an automated process, so whatever output you get, whatever process image you get by using the histogram equalization technique that is fixed. And histogram equalization techniques is not suitable for interactive image manipulation, whereas interactive image manipulation or interactive enhancement can be done by histogram specification techniques.

So in histogram specification techniques, what we do is, we have the input image, we can find out what is the histogram of that particular image, then a target histogram is specified, and you have to process the input image in such a way that the histogram of the processed image will resemble, will be close to the target histogram which is specified. So here we have two different cases, firstly, we have Pr(rk) which as we have seen is nothing but nk divided by n, where nk is number of pixels in the given image with an intensity value equal to rk, and this we compute from the given image which is to be processed.

And we have a target histogram which is specified which is to be so that our processed image will have a histogram which is almost close to the target histogram, and the target histogram is specified in the form Pz(zk). So this is the target histogram which is specified, you note that we do not have the image corresponding to this particular histogram, so it is the histogram which is specified, okay. And we used the subscript, Pr(rk) and Pz(zk). So this subscripts r and z they are used to indicate that this two probability distribution, probability density functions Pr and Pz, they are different.

So in case of histogram specification, what we have to do is, the process is done in this manner, firstly using this Pr(rk), you find out the transformation function corresponding to equalization and that transformation function as we have seen earlier is given by sk equal to T(rk) which is nothing but sum of ni by n, where i varies from zero to k, and which is obviously equal to Pr(ri) i varying from zero to k. So this is a transformation function that is computed from the histogram Pr(rk) which is obtained from the given image.

And to obtain this histogram specification, the process is like this, you define a variable, say zk such that which we will follow this property we will have the transformation function say vk equal to G(zk) okay, and that will be equal to we can compute this from the specified histogram which is given in the form of Pz say (zi) where i varies from zero to k and we define this to be equal to say sk. So you find that this intermediate stage that is vk is equal to G(zk) where this transformation function G(zk) is given by Pz(zi) summation i from zero to k, this is an hypothetical case, because we really do not have the image corresponding to the specified histogram Pz Pz(zk), okay.

Now once I get this, to get the reconstructed image or the processed image, I have to take the inverse transformation, so here you find that as we have define that for this particular zk we have vk equal to G(zk) which is equal to sk and G(zk) is computed in this form, and from here to get the value zk, the intensity value zk in the processed image, we have to take the inverse transformation but in this case the inverse transformation is not taken with respect to T, but the inverse transformation has to be taken with respect to G.

So our Zk, in this case will be equal to G inverse of (sk). So what we have to do is, for the given image we have to find out the transformation function Tr, corresponding to the histogram of the given image, okay. And using this transformation function, every intensity value of the input image, which is rk has to be matched to and equalized intensity value which is equal to Sk, so that is the first step. The second step is, from the specified histogram Pz, we have to get a transformation function G, and then the sk that we have obtained in the previous step, that has to be inverse transformed using this transformation function G to give you the processed image intensity value in the processed image which is equal to zk.

Now, so far, we have concerned we have discussed that finding out T(rk) is very simple that is this forward process is very simple, but the difficulty comes for getting the inverse transformation that is G inverse. It may not always be possible to get analytical expressions for T and G, and similarly, it may not always be possible to get an analytical expression for G inverse. So the best possible way to solve this inverse process is to go for is to go for an iterative approach.

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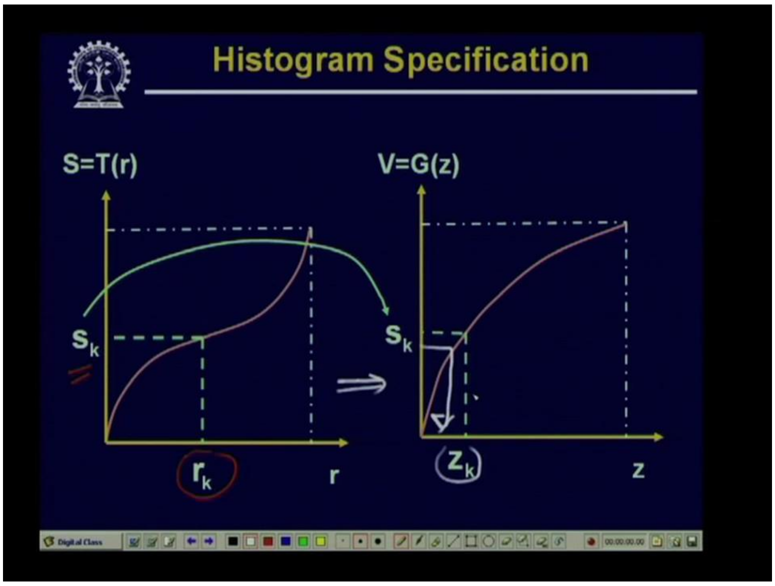

Now let us see that what does this entire expression mean. So here we have shown the same formulations graphically, on the left hand side we have the transformation function S equal to T(r) and this transformation function is obtained from the histogram of the given image, and on the right hand side we have given the transformation function V equal to G(z) and this transformation function has to be obtained from the target histogram that is specified.

Now once we have these two transformation functions, you find that these two transformation functions, it may tell you that given an rk I can find out what is the value of Sk, given a zk I can find out what is the corresponding value vk. But the problem is zk is unknown, we do not know what is the value of zk, this is the one that we have to find out by using the inverse transform zee inverse. So the process as per our definitions since we have seen that vk equal to Sk, so what we do is for the given rk, we find out the corresponding value Sk by using this transformation S equal to T(r) and once we get this, then we set this vk equal to Sk that is from the first transformation function we come to the second transformation function.

So we set vk equal to Sk, and then find out zk in the reverse direction, so now our direction of the transformation is reverse and we find out zk from this value of Sk using this transformation called Gz. But as we have mentioned that it may not always be possible to find out the analytical expressions for r and G. So though the method appears to be very simple, but its implementation is not that simple. But in the discrete domain, the matter, this particular case, can be simplified. It can be simplified in the sense that both this transformation functions, that is S equal to T(r) and V equal to G(z), they can actually be implemented in the form arrays.

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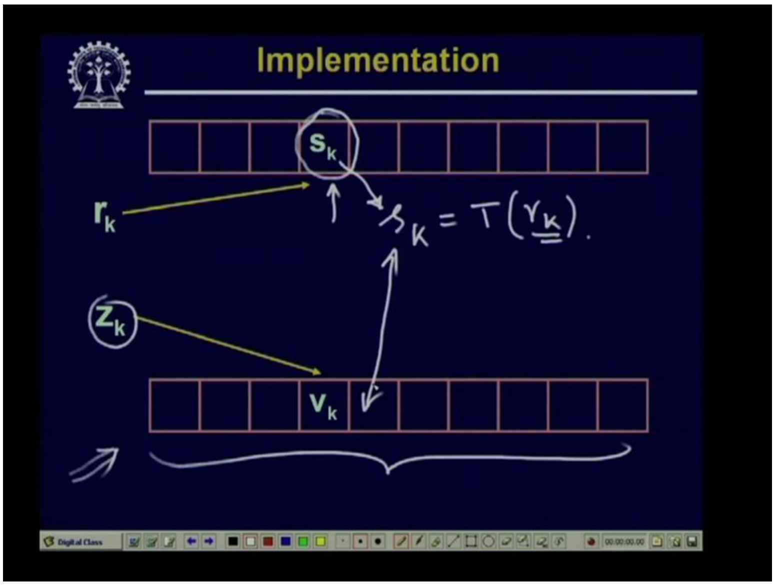

So the arrays are like this, so I have an array r, as shown in the top, where the intensity value of the input image rk is to be taken as an index to this particular array. And the content of this array element, that is s is the value, the corresponding value Sk. Similarly the second transformation function Gz, that can also be implemented with the help of an array, where zk is an index, is an index to this array, and the corresponding element in the array gives us the value vk.

Now find, that using this arrays the forward transformation is very simple when we want to find out Sk is equal to T(rk), what we do is using this rk, you simply come to the corresponding element in this particular array, find out what is the value stored in that particular location and that value gives you the value of Sk. But the matter is not so simple when we go for the inverse transformation, so for inverse transformation, what you have to do is, we have to find out a location in this second array, that is we have to find out the value of zk, where the element is equal to Sk.

So this is what we have to do, so we find that as the forward transformation that is Sk equal to T(rk) was very simple, but the reverse transformation is not that simple. So to do this reverse transformation, what we have to do is, we have to go for an iterative procedure.

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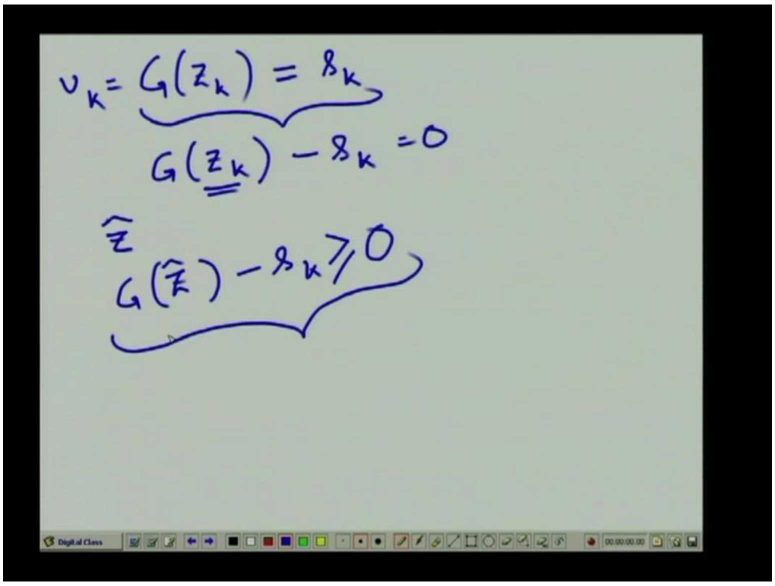

The iterative procedure is something like this, so we do the iterative procedure following this. You find that as per our definition, we have said that G(zk) this is vk, vk is equal to G(zk) is equal to sk, so if this is equal to two, then we must have G(zk) minus sk which is equal to zero, the solution would have been very simple if zk was known, but here we are trying to find out the value of zk.

So to find out the value of zk, we take we take help of an iterative procedure, so what we do is, we initialize the value of zk to some value say z hat, and try to iterate on this z hat until and unless you come to a condition, until and unless a condition like G(z hat) minus sk is greater than or equal to zero. So until and unless this condition is satisfied, you go on iterating the value of z hat, incrementing the value of z hat by one at every iteration. And for this what you have to do is, we have start with a minimum value of z hat, then increment the value of z hat by steps of one in every iteration, until and unless we come to a condition like this, that is G(z hat) minus sk equal to zero.

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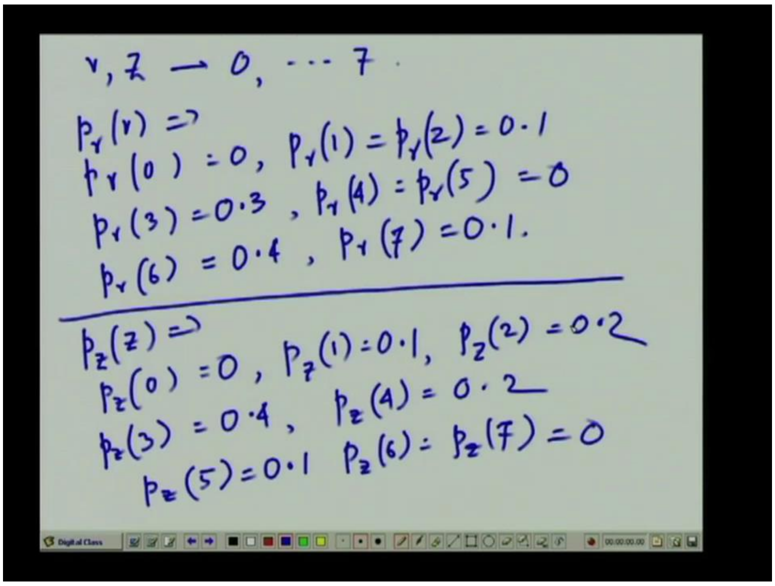

And the minimum value of z hat for which this condition is satisfied, that gives you the corresponding value of zk. So this is a simple iterative procedure. So again as before, we can illustrate this with the help of an example, here again we take Pr we assume that both r and z, they varies from, they assumes value from zero to seven and we take the probability density functions of Pr(r), like this, that is Pr (0) is equal to 0, Pr(1) is equal to Pr(2) is equal to 0.1, Pr(3) is equal to 0.3, Pr(4) is equal to Pr(5) is equal to 0, Pr(6) is equal to 0.4, and Pr(7) is equal to 0.1.

So this what we assume that it is obtain from the given image. And similarly, the target histogram is given in the form Pz(z) but the values are Pz(0) is equal to 0, Pz(1) is equal to 0.1, Pz(2) is equal to 0.2, Pz(3) is equal to 0.4, Pz(4) is equal to 0.2, Pz(5) is equal to 0.1, Pz(6) is equal to Pz(7) this is equal to 0. So this is the target histogram that has been specified. Now our aim is to find out the transformation function or the mapping function from r to z.

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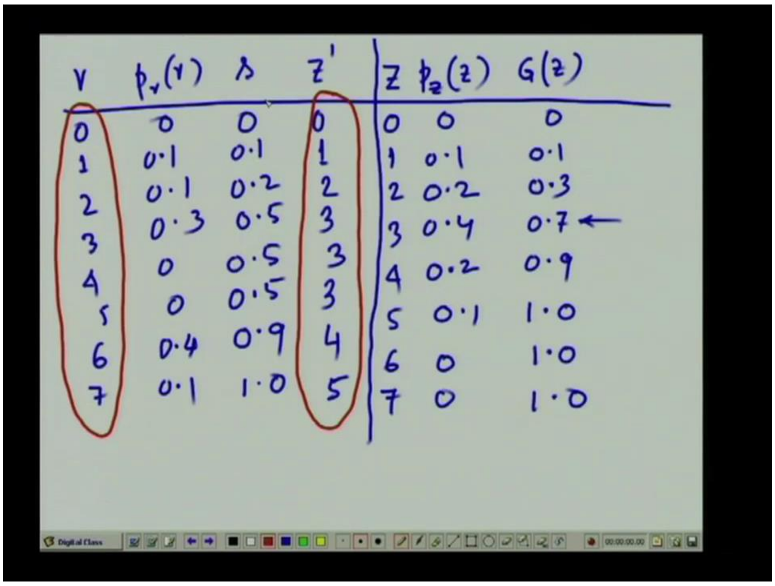

So for doing this, we follow the similar procedure as we have done in case of histogram equalization, and in case of histogram equalization we have found that for different values of r, we get Pr(r) like this, for r is equal to 0 we have Pr(r) is equal to 0, for r equal to 1, pr(r) is equal to 0.1, for r equal to 2 this is also 0.1, 3, this is 0.3, for 4, this is 0, for 5 this is also 0, for 6, this is 0.4, for 7, this is 0.1.

And from this, we can find out what is the corresponding value of s, and the corresponding values of s will be given by 0, 0.1, 0.2, 0.5, 0.5, 0.5, then here you get, 0.9 and here you get 1.0. Similarly, from the target histogram we get for different values of z, 0, 1, 2, 3, 4, 5, 6, 7 the corresponding histogram is given by Pz(z) which is 0, 0.1, 0.2, 0.4, 0.2, 0.1, 0, 0. And the corresponding G(z) is given by, 0, 0.1, 0.3, 0.7, 0.9, then 1.0, 1.0, and 1.0.

Now if I follow the same procedure, that to map from r to r to z, first I had to map from r to s, then I have to map from s to z. And for that, I have to find out the minimum value of z, for which G z minus s is greater than or equal to zero. So for this when I come to value of s is equal to 0, so here I put the corresponding values of z, let me put it as z prime. So when s is equal to 0, the minimum value of z for which zee z minus s is greater than equal to zero is zero. For s is equal to 0.1, again I start with z is equal to 0, I find that the minimum value of z for which G z minus s will be greater than or equal to equal to zero is equal to one.

Here, the minimum value of z, for which G(z) minus s will be greater than or equal to zero is equal to two. When I come here, that is r is equal to 3, the corresponding value of s is equal to 0.5, again I do the same thing, you find that the minimum value of z for which the condition will be equal to two, that is equal to three. When I come to r is equal to 4, you find that here the value of s is equal to 0.5 and when I compute G(z) minus s, the minimum value of z, for which G(z) minus s will be equal to greater than or equal to zero, that is equal to three, because for three, G(z) equal to 0.7 so this is also be equal to three.

And if I follow the similar procedure, I will find that the corresponding functions will be like this. So for r is equal to 1, the corresponding processed image will have an intensity value r is equal to 0, the corresponding processed image will have an intensity value equal to 0, for r is equal to 1, it will be equal to 1, r is equal to 2 processed image will be equal to 2, r is equal to 3, processed image will also be equal to three.

But r is equal to 4 and 5, the processed image will have intensity values which is a, which are equal to three. For r is equal to 6, the processed image will have an intensity value which is equal to 4, r is equal to 7, the processed image will have an intensity value which is equal to 5. So, these two columns, the first column of r, and the column of z prime, this two columns gives us that mapping between an intensity level and the corresponding processed image intensity level when I go for this histogram equalization sorry, histogram matching.

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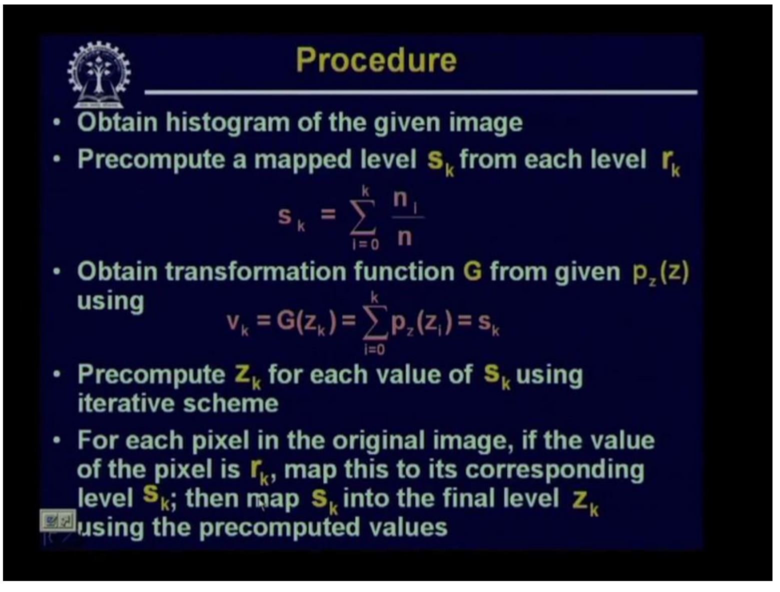

So, you find that our iterative procedure will be something like this, that first you obtain the histogram of the given image, then precompute a mapped level Sk from for each level rk giving this using this particular relation, then from the target histogram, you obtain the mapping function G and for that, this is the corresponding expression, then precompute values of zk for each value of Sk using the iterative scheme. And once these two are over, I have a precomputed transformation function which is in the form of a table which maps an input intensity value to the corresponding output intensity value, and once that is done, then for final enhancement of the images.

I take the input intensity value and map to the corresponding output intensity value using the mapping functions, so that will be our final step, and if this is done for each and every pixel location in the input image, the final output image that will be an enhanced image, where the enhanced image whose intensity levels will have a distribution which is close to the distribution that is specified.

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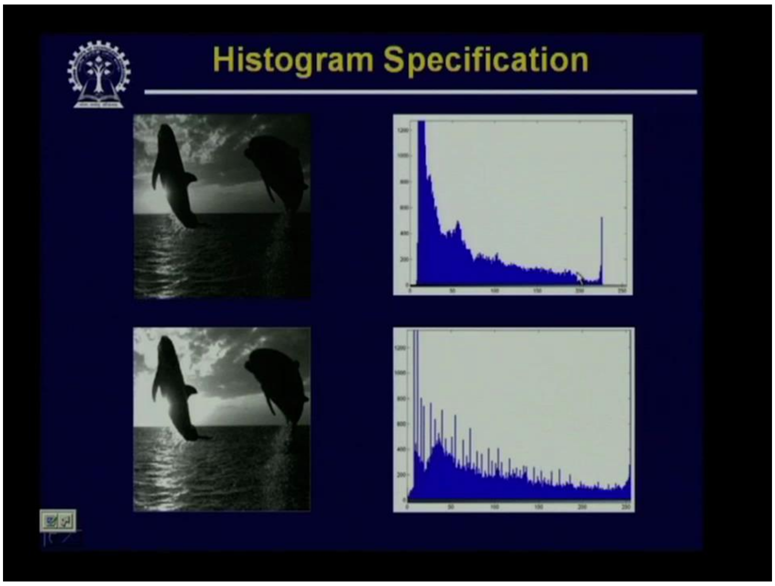

So using this histograms equalization technique, you find that what are the results that you get. Again, I take the same image, that is dolphin image, on the top is the original image, and on the right hand side I have the corresponding histogram.

On the bottom, I have an equal histogram matched imaged, and on the right hand side, I have the corresponding histogram. So you find that, these two histograms are quite different and I will come a bit later that what is the corresponding target histogram that you have taken. Now to compare the result of histogram equalization with histogram specification, you find that on the top I have shown the same histogram equalized image as we have done that we have shown earlier. And on the bottom row, we have shown this histogram matched image.

And at this point, this histogram specification, target histogram which was specified was the histogram which is obtained using this equalization process. So this particular histogram was our target histogram, so this histogram was the target histogram and using this target histogram, when we did this histogram specification operation, then this is the processed image that we get, and this is the corresponding histogram.

Now if you compare this two, the histogram equalized image, and the histogram matched image, you find you can note a number of differences. For example, you find that this background is contrast of the background is much more than the contrast of the histogram equalized image. And also, the details on this water front, the water surface is more prominent in the histogram specified image, than in the case of histogram equalized image. And similar such differences can be obtained by specifying other histograms.

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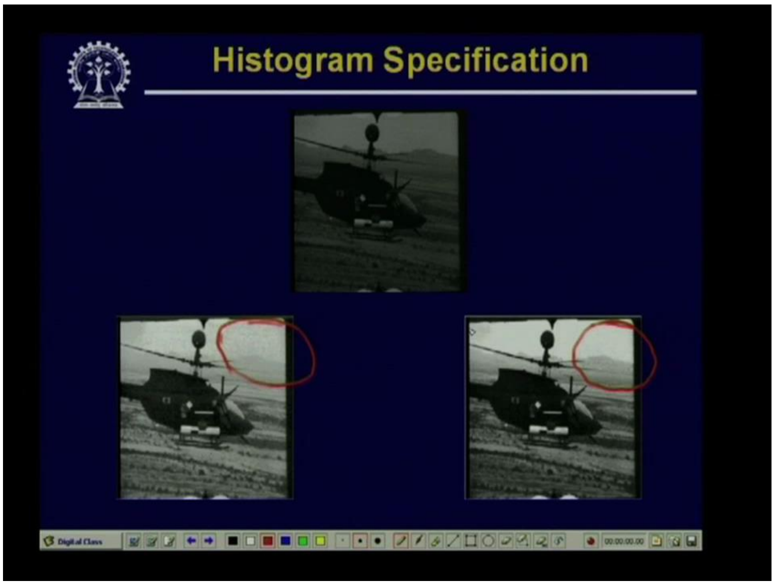

So this is our histogram specification operation. So this shows another result with histogram specification, on the top we have the dark image, on the left bottom we have the histogram equalized image, on the right bottom we have the histogram specified image.

Here again you find, that the background in case of histogram equalized image is almost washed out, but the background is highlighted in case of histogram specified image. So this is the kind of difference that we can get between an histogram equalized image and an histogram specified image. So this is what is meant by histogram specification and histogram equalization. Now as we have said, that I will discuss about two more techniques, one is image differencing technique and the other one is image averaging technique for image enhancement.

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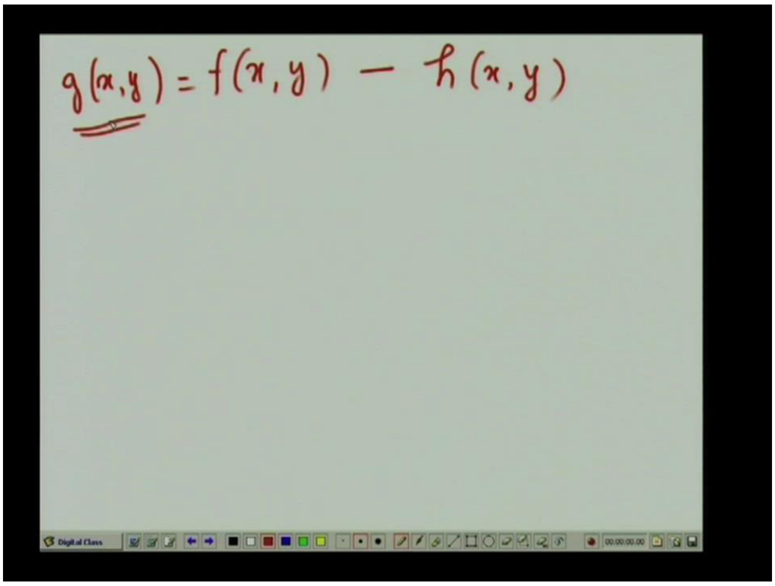

Now as the name suggests, when I, whenever we go for image differencing, that means we have to take the difference of pixel values between two images. So given two images, say one image is say f(x,y) and the other image say h(x,y), so the difference of this two images is given by, g(x,y) is equal to f(x,y) minus h(x,y). Now as this operation suggests, that g(x,y) and g(x,y), all those pixel locations will be highlighted wherever there is a difference between corresponding locations in f(x,y), and the corresponding location in h(x,y). Wherever f(x,y) and g(x,y) are same, the correspond f(x,y) and h(x,y) are same, the corresponding pixel in g(x,y) will have a value which is near to zero.

So this kind of operation, image differencing operation mainly highlights the difference between two images or the locations where the two image contents are different. Such a kind of image difference operations is very very useful in particularly in medical image processing. So in case of medical image processing, there is a operation which is called say mask mode radiography. In mask mode radiography what is done is, you take the image of certain body part of a patient an x-ray image which is captured with the help of a TV camera, where the camera is normally placed opposite to a x-ray source.

And then what is done is, you inject a contrast media into the blood stream of the patient, and after injecting this contrast media, again you take a series of images using the same TV camera of the same anatomical portion of the patient body. Now once you do this, the first one, the image which is taken before injection of this contrast media that is called a mask and that is why the name is mask mode radiography.

So if you take the difference of all the frames that you obtain after the injection of the contrast media, take the difference of those images from the mask image, then you will find, that all the regions were the contrast media that flows to the artery, those will be highlighted in the difference image.

And this kind of application and this kind of processing is very very useful to find out how the contrast media flows through artery of the patient, and that is very very helpful to find out if there is any arterial disease of the patient, so for example if there is any blockage in the artery or similar such diseases. So this mask mode radiography makes use of this difference image operation to highlight the regions in the patient body or the arterial regions in the patient body and which is useful to detect any arterial disease or arterial disorder.

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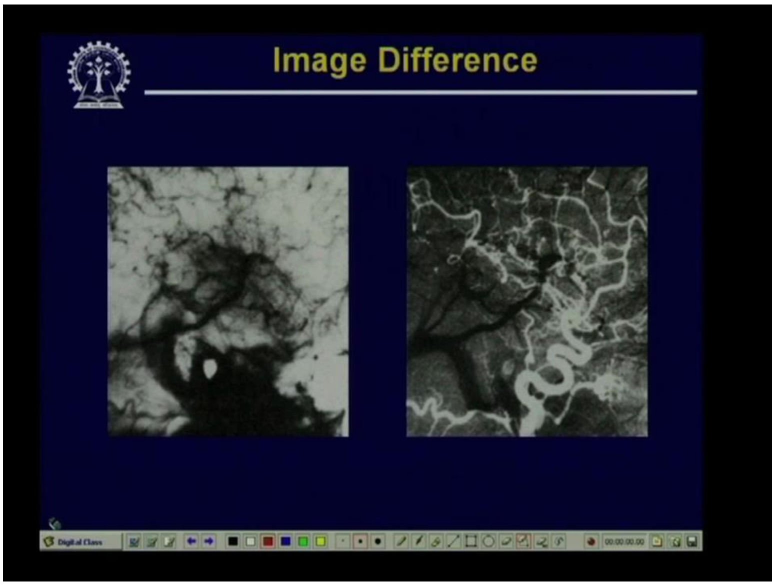

Now to show a result, here this particular case, you find that on the left hand side whatever shown that is the mask which is obtained, and on the right hand side this is the difference image, this difference image is the difference of the images taken after injection of the contrast media with the mask. And here you find that all this arteries through which the contrast media is flowing those are clearly visible.

And because of this, it is very easy to find out if there is any disorder in the artery of the patient. Now the other kind of image processing applications, as we said, so this is our difference image processing. So as difference image processing can be used to enhance the contents of certain regions within the within image wherever there is a difference between two images.

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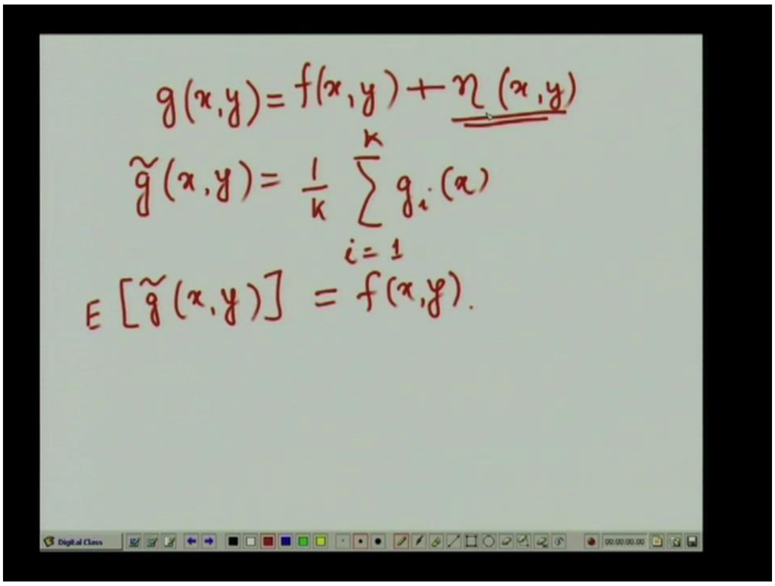

Similarly, if we take the average of a number of images of the same scene, then it is possible to reduce the noise of the image, and that noise reduction is possible because of the fact that normally if I have a pure image, say f(x,y), then the image that will capture, if I call it as g(x,y), that is the captured image, this captured image is normally the purest image f(x,y), and on that we have a contaminated noise, say eta(x,y). Now if this noise eta(x,y) is additive and zero mean, then by averaging a large number of such noisy frames, it is possible to reduce the noise.

Because, simply because, if I take the average of k number of frames of this, then g tilde(x,y), which is the average of k number of frames, is given by 1 over k, then summation of gi(x,y), where i varies from 1 to k, and if I take the expectation value or the average value of this **t** g tilde(x,y), then this average value is nothing but f(x,y), and our condition is the noise must be zero mean additive noise and because it is zero mean I assume that at a, at empty pixel location, the noise is uncorrelated and the mean is zero.

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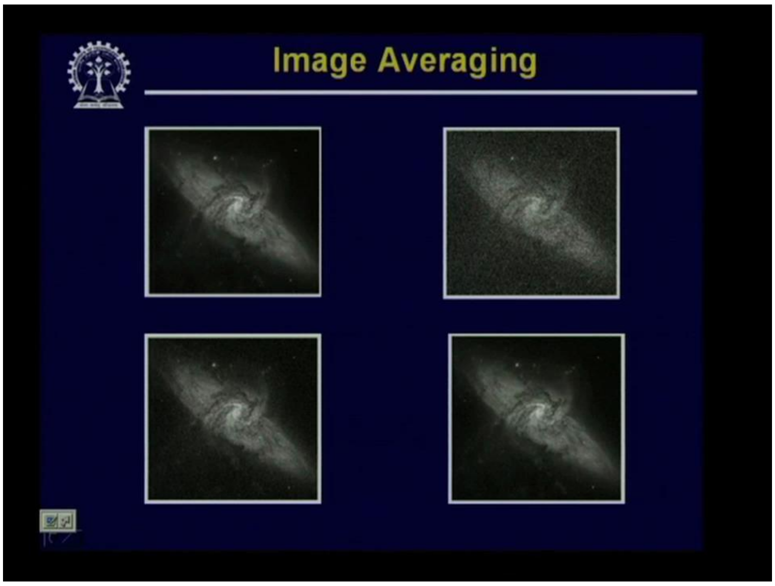

So that is why if you take the average of a large number of frames, the image, the noise is going to get cancelled out. And this kind of operation is very very useful for astronomical cases, because in case of astronomy, normally the objects which are imaged, the intensity of the images are very very low. So the image that you capture is likely to be dominated by the presence of noise. So here, this is the image of a galaxy, on the top left, on the top right, we have the corresponding noisy image, and on the bottom, we have the images which are averaged over a number of frames.

So, the last one is an average, is an average image, by the average is taken over one twenty eight number of frames and here the number of frames is less, and as it is quite obvious from this that as you increase the number of frames, the amount of noise that you have in the processed image is less and less. So with this we come to an end to our discussion on point processing techniques for image enhancement operations. Now let us discuss the questions that we have placed in the last class.

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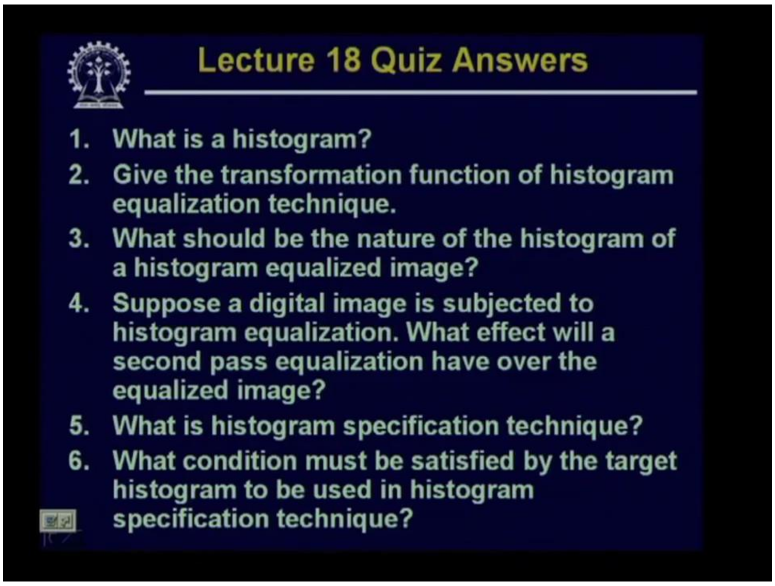

The first one is what is an image histogram? So you find that few of the questions are very obvious, so we are not going to discuss about them. Now the fourth one is very interesting that suppose a digital image is subjected to histogram equalization. What is the effect, what effect will a second pass equalization have over the equalized image? So as we have already mentioned that once a image is histogram equalized, the histogram of the processed image will be a uniform histogram, that means it will have an uniform probability density function.

And if I want to equalize this equalized image, then you find the corresponding transformation function will be a linear one, where the straight line will be inclined at an angle of 45 degree with the x axis. So that clearly indicates that whatever kind of equalization we do over an already equalized image that is not going to have any further effect on the processed image. So this is ideal case, but practically we have seen that after equalization the histogram that you get is not really uniform, so there will be some effect in the second pass, the effect but the effect may be negligible.

Six one is again a tricky one, what condition must be satisfied by the target histogram to be used in histogram specification technique? You find that in case of histogram specification technique the target histogram is used for inverse transformation that is G inverse. So it must be that the it must be true that the transformation function G has to be monotonically increasing, and that is only possible if you have the value of Pz(z) non zero for every possible value of z. So that is the condition that must be satisfied by the target histogram.

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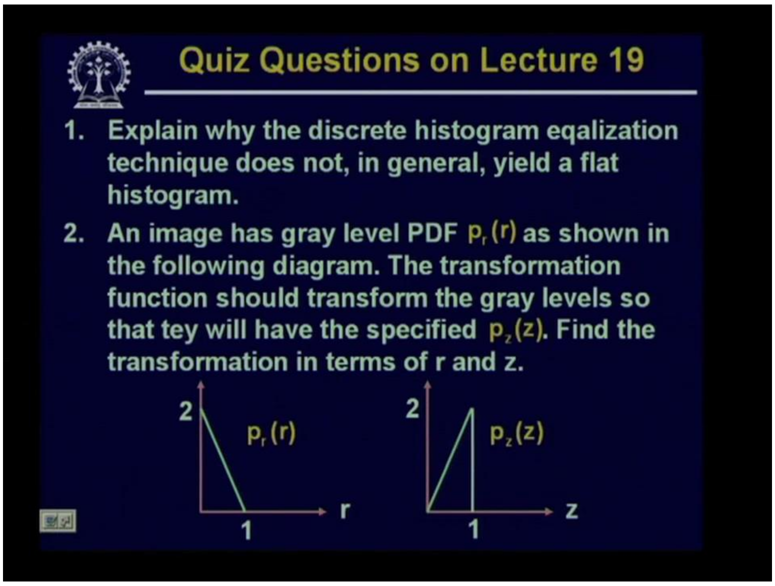

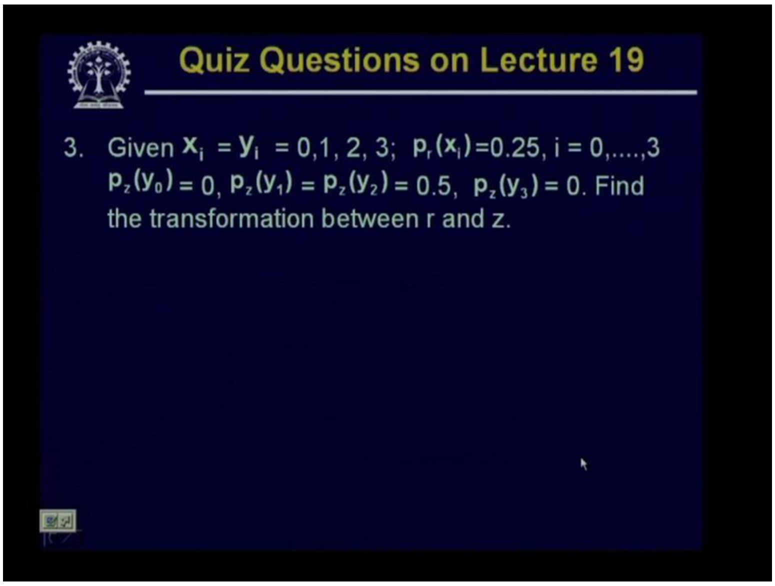

Now coming to today's questions, first one is, explain why the discrete histogram equalization technique does not, in general, yield a flat histogram. The second, an image has gray level PDF Pr(r) as shown here and the target histogram as shown on the right, we have to find out the transformation in terms of r and z that is what is the mapping from r to z. The third question we have given the probability density functions, 2 probability density functions, again you have to find out the transformation between r and z. Thank you.