Digital Image Processing.
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Lecture-43.
Image Restoration Techniques-II.

Hello, welcome to the video lecture series on digital image processing. Now this formulation that we have done until now, this formulation is for the continuous case. And as we have said many times that in order to use this mathematical operation for our digital image processing techniques, we have to find out a discrete formulation of this mathematical model. So let us see that how we can have a equivalent discrete formulation of this particular degradation model.

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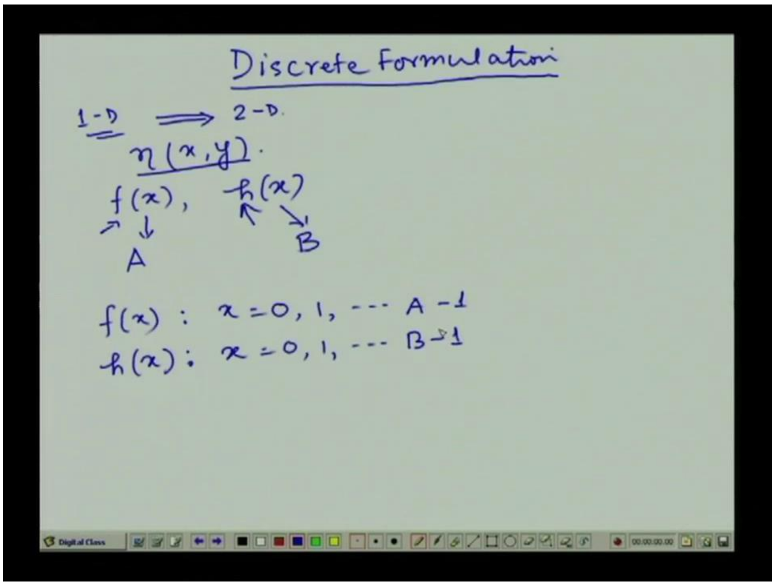

So to get a discrete formulation, firstly we will consider so we have to get a discrete formulation. So to obtain this discrete formulation for simplicity initially we will assume the cases in one dimension, and later on this we will extend to two dimensional cases for digital image processing operations. Again for simplicity, initially we will neglect the contribution of the noise term, that is eta(x,y). So in case of one dimension as we have done in case of in the continuous signal, we have two signals f(x), and another one is h(x), so we have said that f(x) is the input signal and h(x) tells us that what is the degradation function.

So f(x) is the input function and h is the, h(x) is the degradation function. For discretization of the same formulation, what we have to do is, we have to uniformly sample these two functions, f(x) and h(x). And we assume that f(x) is uniformly sampled to give an array of dimension A and h(x) is uniformly sampled to give an array of dimension B. that means, for f(x), in the discrete case x varies from 0, 1 to A – 1, and h(x), for h(x), x varies from 0, 1, to B – 1.

Then what we will do, we will add additional zeros to this a, f(x) and b(x) to make both of them of the same dimension and dimension equal to say capital M. So we make both of them to be of dimension capital M by adding additional number of zeros and we assume that both f(x) and h(x) after addition of these zero terms and making both of them to be of dimension m, they become periodic with a periodicity capital M.

So once we have done this, now the same convolution operation that we have done in case of our continuous case, now can also be written in case of discrete case. So in discrete case, the convolution operation we will write in this manner, so after converting both f(x) and h(x) in 2 arrays of dimension m, these new arrays that we get we represent it by fe(x) that is f extended x as we have extended it, and h we represent by he(x) that is the extended version of h(x).

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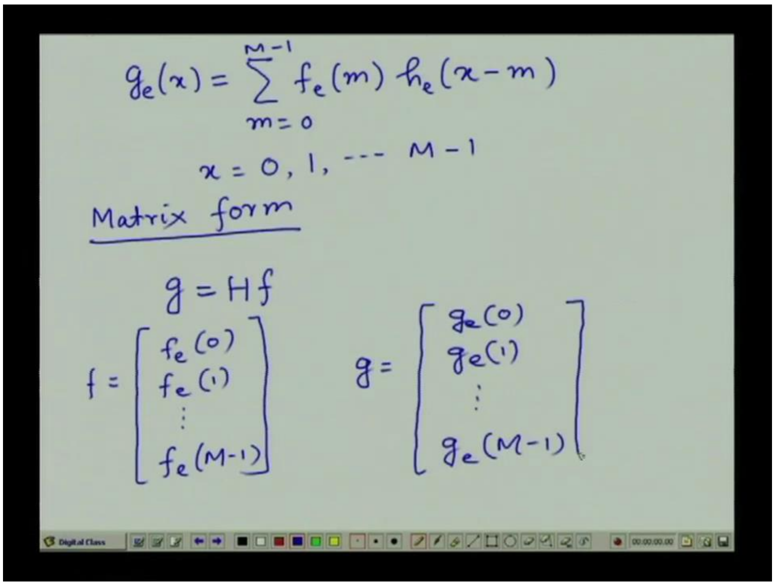

And now in discrete domain, the convolution function can be written as ge(x) = summation fe(m) he(x - m) where this m will vary from 0 to capital M – 1. And x we will assume values from 0 to capital M – 1. So this is the discrete formulation of the convolution equation that we have obtained in case of continuous signal cases. Now if you analyze this convolution expression, you find that this convolution expression can be written in the form of a matrix, matrix operation, so we can have the matrix form. In matrix form, these equations will be like this, g = some matrix H times f, where the function f or array f will be simply fe(0), fe(1), this way up to fe(capital M – 1) and function g, similarly will be ge(0), ge(1), to like this it will be ge(M, capital M – 1).

So you recollect, you just recollect that fe and ge, these are the names which are given to the sampled versions of the functions f(x) and g(x) after extending the functions by addition of addition by adding additional number of zeros to make them of dimension of capital M.

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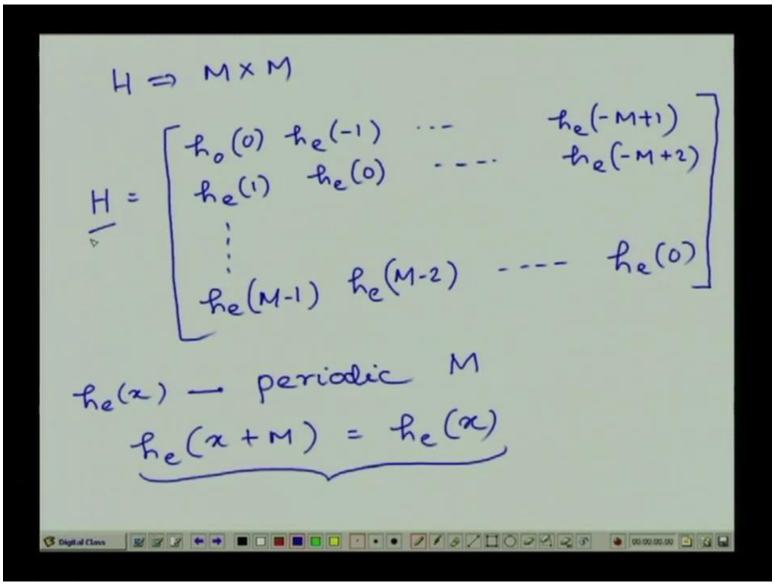

And in this particular case, the matrix H will have the matrix H will be of dimension capital M by capital M, where the elements of H will be like this, he(0), he(-1) continue like this…it will be he(-M+1), here it will be he(1), he(0),… It will be he(-capital M+2) and if we continue like this, it will be he(capital M -1), he(capital M -2), ... like this it will be he(0). So this is the form of the matrix capital H which is the degradation matrix in this particular case.

And here you find that the elements of this degradation matrix capital H, are actually generated from the degradation function he(x). Now remember, that we have assumed that our he(x), this function is actually periodic, this is which we have assumed with periodicity of capital M.

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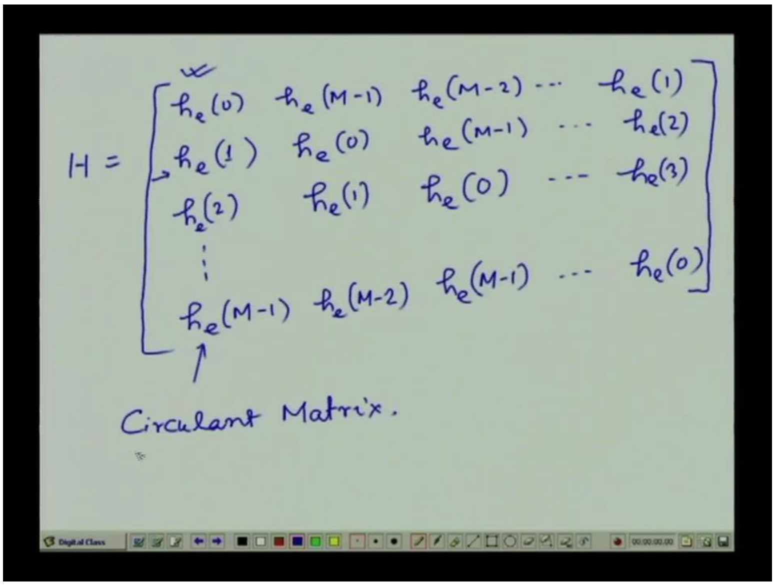

So if this function is periodic with periodicity capital M, that means he(x + capital M) that will be same as = he(x). So by using this periodicity assumption, now this particular degradation matrix H can be written in a different form where this matrix H will now be represented as he(0), he(capital M – 1), he(capital M – 2) up to… he(1). The second row will be he(1), he(0), he(capital M – 1) …. And this will be he(2). Third row will be, he(2), he(1), he(0) like this it will be … He(3). And the last row, continue in the same manner will be he(capital M – 1), he(capital M – 2), he(capital M - 3)… and the last term will be equal to he(0).

Now if you analyze this particular matrix, you find that this degradation matrix capital H has a very very interesting property. That means the first property is different rows of this matrix are actually generated by rotation to the right of the previous row. So here if you look at, the second row you find that this second row is actually generated by rotating the first row to the right. Similarly third row is generated by rotating the second row to right by one.

So this is so in this particular matrix, the different rows are actually generated by rotating the previous row to the right. So this is called a circulant matrix because different rows are generated by a circular rotation. And the circularity in this particular matrix is also complete in the sense that if I rotate this last row to right what I get is the first row of the matrix. So this kind of matrix is known as a circulant matrix. So here I find, we find that in case of discrete formulation, the discrete formulation is also a convolution operation.

And here in the matrix equation of the degradation model the degradation matrix H that we obtained that is actually a circulant matrix. Now let us extend the concept of this discrete formulation from one dimension to two dimension. So let us see what we get in case of two dimensional functions, that is in case of two dimensional images. So in case of two dimension, we have the input function or the image function which is given by f(x,y) and we have the degradation function which is given by h(x,y).

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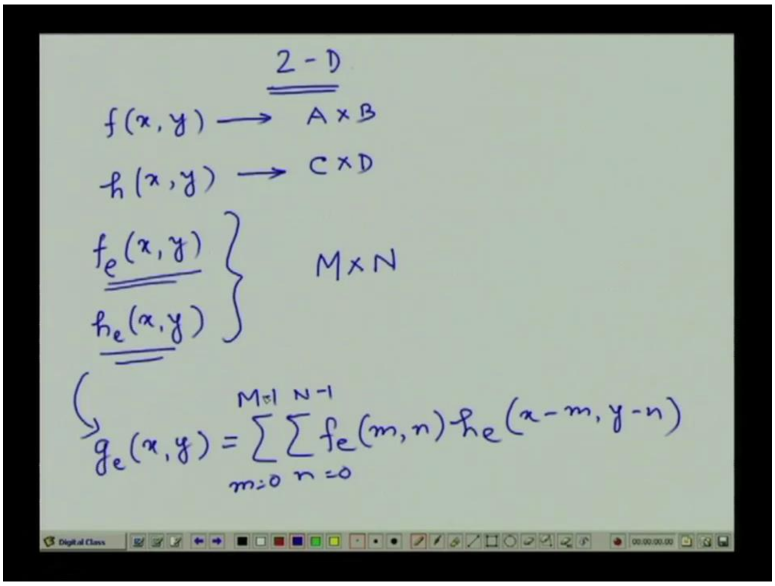

And we assume, that this f(x,y) is sampled to an array of dimension capital A by capital B and say h(x,y) is sampled to an array of dimensions say capital C by capital D. Now as we have done in one dimensional case, that is the functions f(x,y) and h(x) are actually extended by using by putting additional number of zeros to make both of them of same size say capital M. In the same manner, here we add additional number of zeros to both these f(x,y) and h(x,y) to get the extended functions fe(x,y) and he(x,y) to make both of them of dimensions say capital M by capital N.

And we also assume that this fe(x,y) and he(x,y), they are periodic and in x dimension, the periodicity will be of period capital M, and in y dimension the periodicity will be of period capital N. Now following similar procedure, we can obtain a convolution expression in two dimension which is given by ge(x,y) which is nothing but fe(m,n) he(x -m, y -n), where n varies from zero to capital N – 1 and m varies from zero to capital M – 1.

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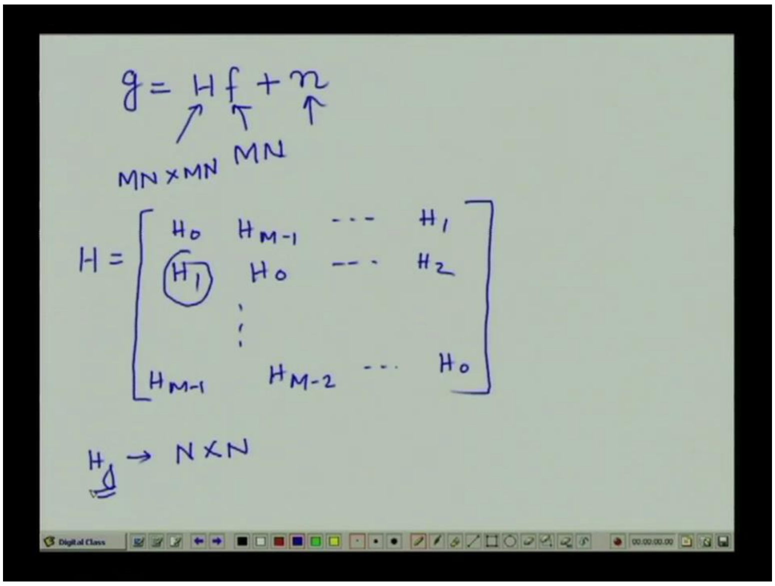

And if I write this convolution expression in the form of a matrix and incorporating the noise term eta(x,y), I will get a matrix equation which is of the form g = H f + n. Where this matrix where this vector f is a vector of dimension capital M into N, which is obtained by concatenating different rows of the two dimensional function f(x,y), that is the first n number of elements of this vector f will be the elements of the first row of matrix f(x,y). Similarly we also obtain this particular vector n by concatenation of rows of the matrix eta(x,y). And this particular degradation matrix h(x) in this case will be of dimension M into N by M into N.

And this matrix H will have a very very interesting form, this matrix H can now be represented as H0, HM -1, like this … Up to H1, the second row can be H1, H0, up to …H2, and the last row is HM -1, HM -2, like this ….We have H0, where each of this terms Hj is a matrix, so each of this Hj is actually a matrix of dimension N by N, where this Hj is generated from the jth row of the degradation function h(x,y).

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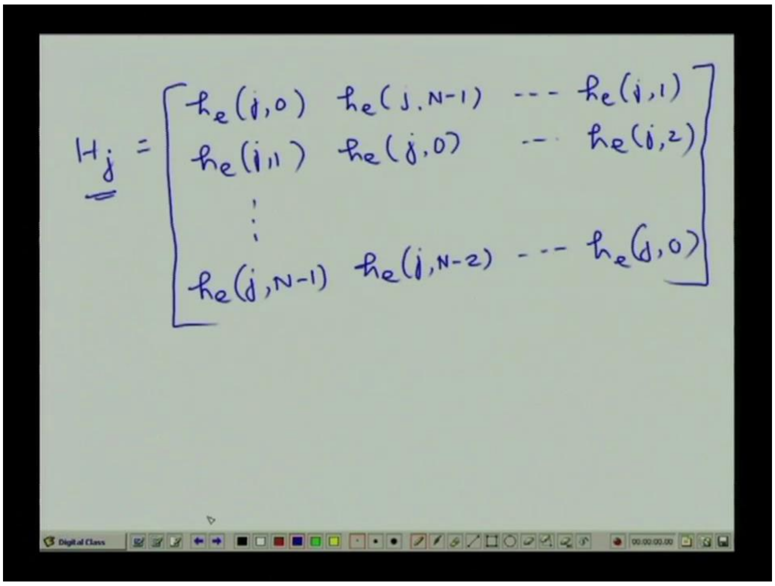

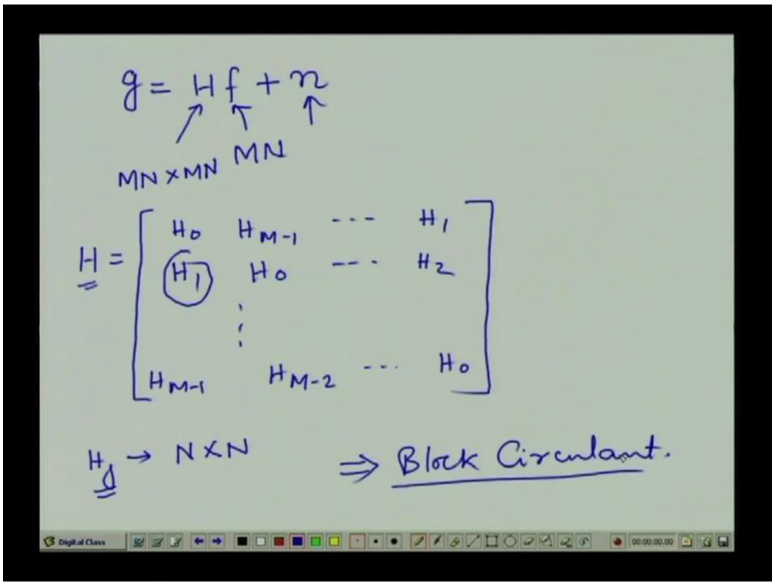

That is, this Hj, we can write this matrix Hj in the form he(j0), he(j, N -1) like this up to … he(j,1), second row will be he(j,1), he(j,0) this way …he(j,2) and if I continue like this, the last row will be he(j, N -1), he(j,N -2) like this if I continue the last element will be … he(j,0).

So you find, that this matrix Hj which is actually a component of the degradation matrix capital H is a circulant matrix that we have defined earlier. And using this block matrix the degradation matrix H is also have been subscripted in the form of a circulant matrix. So this matrix H in this particular case is what is known as a block circulant matrix. So this is what is called a block circulant matrix. So in case of a two dimensional function, that is in case of a digital image we have seen that the degradation model can simply be represented by this expression g = H into f + N, where this vector f is a vector of dimension M into N.

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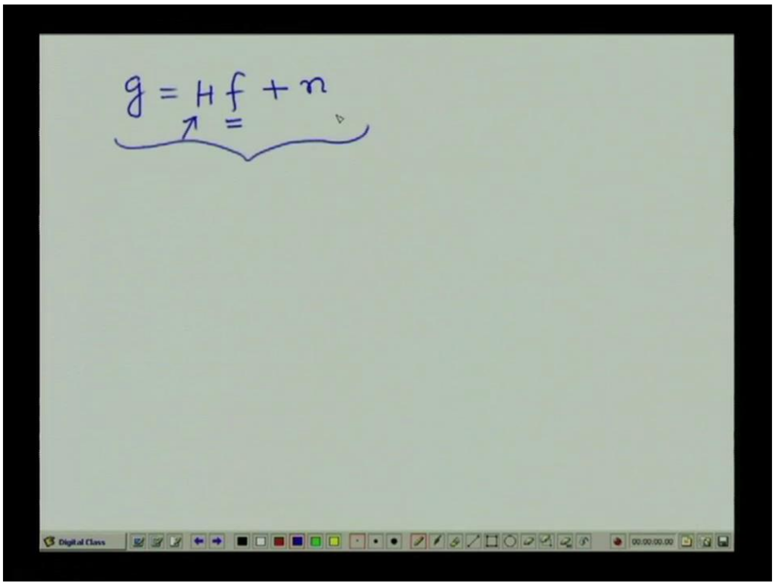

And the degradation matrix H, which is of dimension M into N by m into n is actually a block circulant matrix, where for each block the matrix is obtained from the jth row of the degradation function h(x,y). So now in our next lecture we will see what will be the applications of this particular degradation model for to restore an image from its degraded version. So now let us see some of the questions of this particular lecture.

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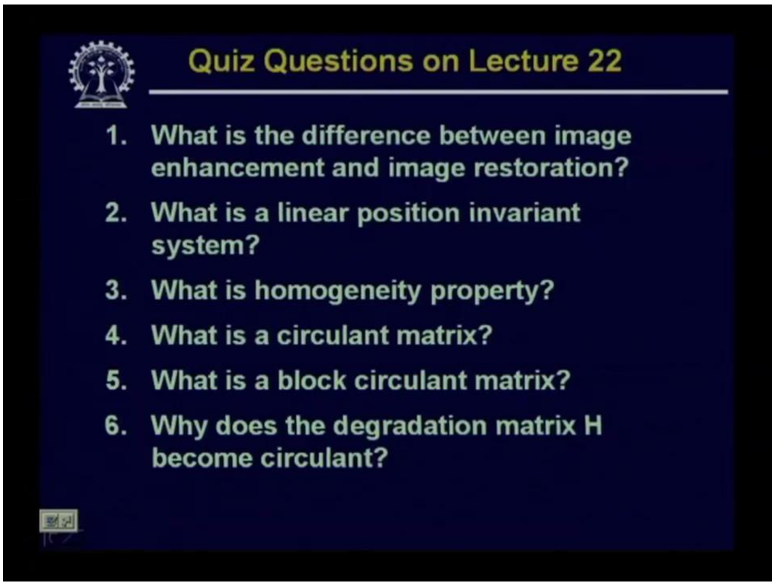

So the first question is what is the difference between image enhancement and image restoration? Second question is, what is a linear position invariant system, third question, what is homogeneity property? Fourth, what is a circulant matrix? What is a block circulant matrix? Why does the degradation matrix H become circulant? Thank you.