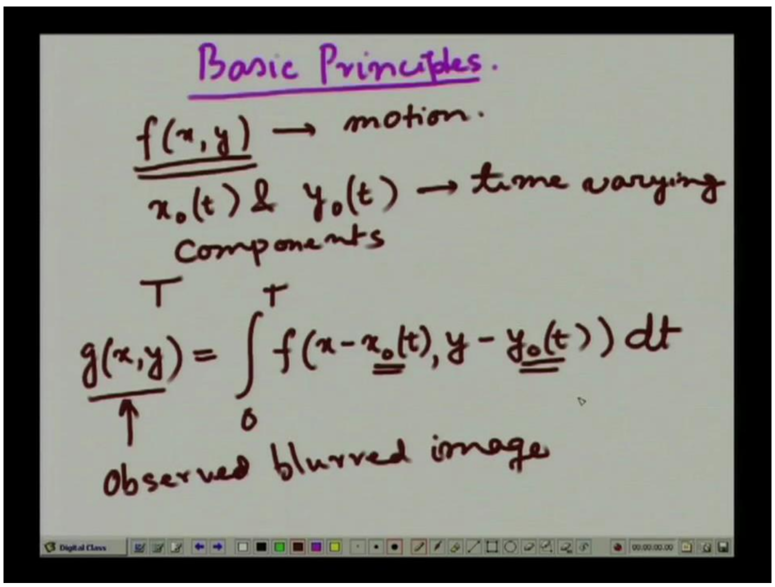
Digital Image Processing
Prof. P. K. Biswas
Department of Electronics and Electrical Communications Engineering
Indian Institute of Technology, Kharagpur
Module 09 Lecture Number 45
Estimation of Degradation Model and Restoration Techniques - 2

Hello, welcome to the video lecture Series on Digital Image Processing. Now there are other approaches of degradation mathematical model for to estimation, estimate the degradation which are obtained by fundamental principles. So from the basic principles also we can obtain what should be the degradation function?

(Refer Slide Time: 00:55)



So one such case so here we will discuss the basic principles the degradation model estimation from basic principles. And I try to find out what will be the degradation model, degradation function what the image is degraded by linear motion, and this is a very very common situation that if we try to estimate or if you time to image of fast moving object. In many cases we find that the image that you get is degraded. There is some short of blurring which is known as motion blurring. And this motion blurring occurs due to the fact that whenever we take the snap of the same the shutter of the camera is open for certain duration of time.

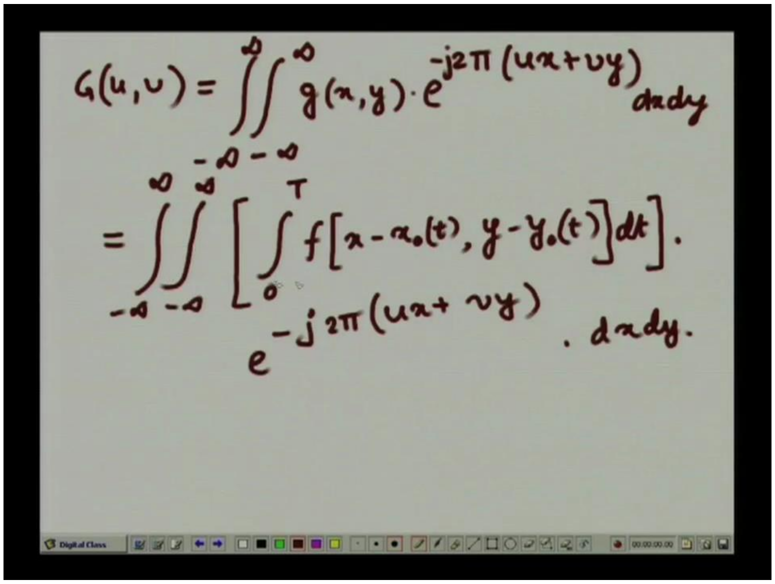
And during this period during which is the shutter is open. The object is not stationary the object is moving. So considering any particular point in the imaging plane here the light which arise from same doesn't not come from a single point. But the light you get at you particular point on the imaging sensor is the aggregation of the reflected light from various point in the sense. So that tells us that what should be the basic approach to model, to estimate the degradation model in case of motion of the same with respect to the camera.

So that is what we are trying to estimate here. So here we assume that the image f (x, y) this undergoes motion and when f (x, y) undergoes a motion then there will be some moving component. So assume two components x nought (t) and y nought (t) which are the moving components or the time varying components. So these are the time varying components along x direction and y direction respectively.

So once the object is moving then the intensity the total exposure at any point in the imaging plane can be obtained by aggregation operation or integration operation, where the integration has to be gone over the period during which the shutter remains open. So if I assume the shuttered remains open for a time duration given by capital T, in that case the total exposure at any point which is the observation at point (x, y) given by g (x, y) will be of this from f (x- x nought (t), y- y knot (t) dt and integration of this from 0 to T.

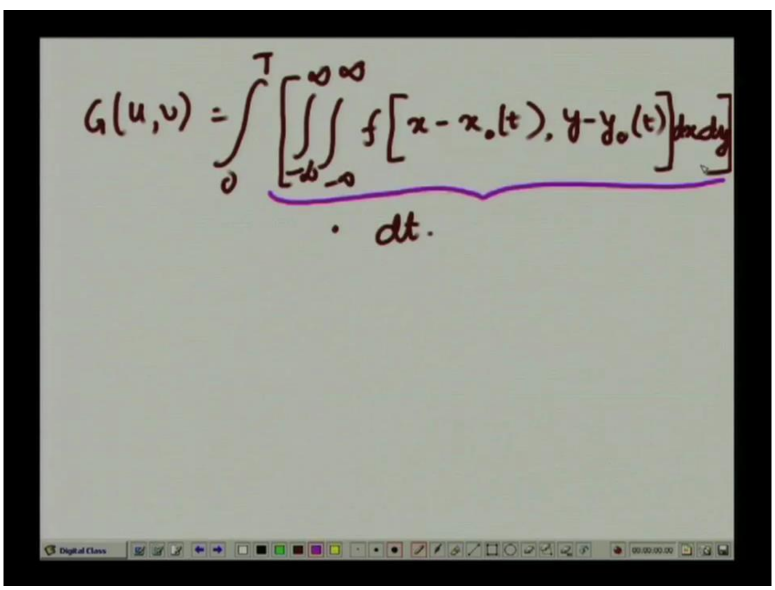
So here the capital T is the duration of time during which the shutter of the camera remains on and x nought (t) and y nought (t) these two terms, they are the time varying components along x direction and y direction respectively. And this g (x, y) gives us the observed blurred image. Now from this we have to estimate what is the degradation function or the blurring function. So once we get g (x, y) then our purpose is to get the Fourier transform of this that means we are interested in the Fourier transformation G (u, v) of g (x, y).

(Refer Slide Time: 05:15)



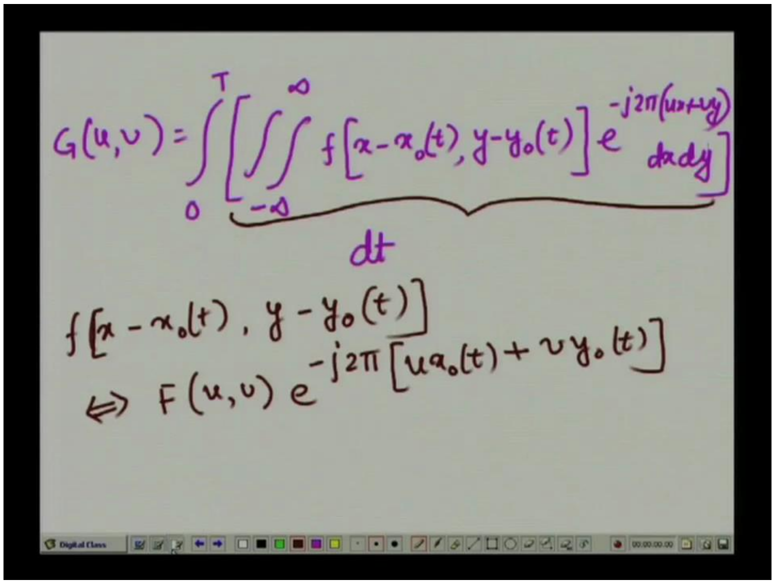
And this G (u, v) as we know from the Fourier Transformation equations is given by g (x, y) into e to the power –j2 pi (ux + vy) dxdy and take the integration, double integration from minus infinity to infinity over both x and y. So it is this expression using this Fourier Transformation expression we can find out what will be g (u, v), that is Fourier Transformation of the degraded image g (x, y). And if I derive this it will be of this form so minus infinity to infinity, again minus infinity to infinity.

(Refer Slide Time: 07:05)



Now this g (x, y) is to be replaced by 0 to T the expression that you have got earlier, f [x- x nought (t), y- y nought (t)] dt into e to the power –j2 pi (ux+ vy) into dxdy. So if we just do some reorganisation of this particular integration equation, we can write G (u ,v) in the form, G (u, v) equal to integral 0 to T double integral minus infinity to infinity, minus infinity to infinity f [(x- x nought (t), y – y nought (t)] dxdy into dt.

(Refer Slide Time: 08:15)

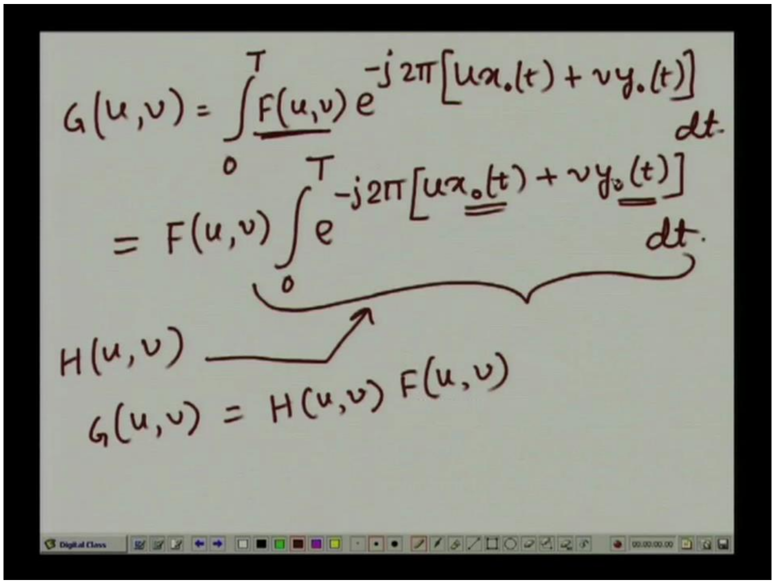


Now from this particular expression you find that the expression with in this bracket. Sorry there will be some more addition. So the final expression will be like this G (u, v) will be equal to 0 to T then within bracket we have to have this double integral varying from minus infinity to infinity, f [(x- x nought (t), y – y nought (t)] e to the power –j2 pi (ux+ vy) dt and then dxdy and then dt. So this will be the final expression.

Now in this if you look at this inner part this is nothing but the Fourier Transformation of shifted f (x, y) where the shift in the x direction is by x nought t and shift in the y direction is by y nought t. And from the properties of Fourier Transformation we know that the Fourier Transformation is shift in invariant in the sense that the Fourier Transform magnitude will remain the same only it will introduce some phase term.

So by doing that we can say that this f [x – x nought (t), y – y nought (t)], this will have a Fourier Transformation which is nothing but f (u, v) e to the power –j2 pi into [ux nought (t) +vy nought (t)]. So this is from the translaw translation invariant property of the Fourier Transformation.

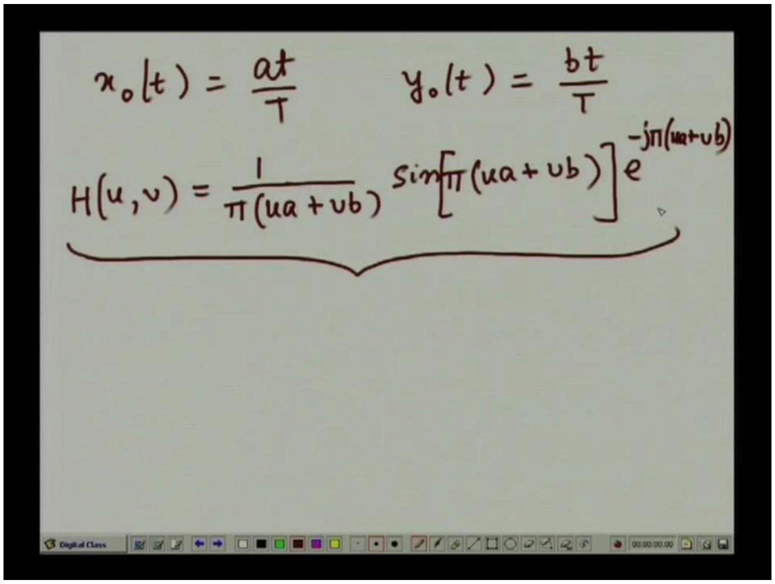
(Refer Slide Time: 10:45)



So using this expression, now the expression for G (u, v) can be written as G (u, v) will be equal to integral 0 to T f (u, v) e to the power –j2 pi (ux nought (t) + vy nought (t)) dt, and because this term f (u, v) is independent of t, so you can take this term f (u, v) outside the integration. So the final expression that you get is F (u, v) into integral 0 to T e to the power –j2 pi (ux nought (t) + vy nought (t)) dt. So from this you find that now if I defined my degradation function H (u,v) to be this particular integration. So if I define H (u,v) to be this then I get expression for G (u,v) is = H (u,v) into f (u, v).

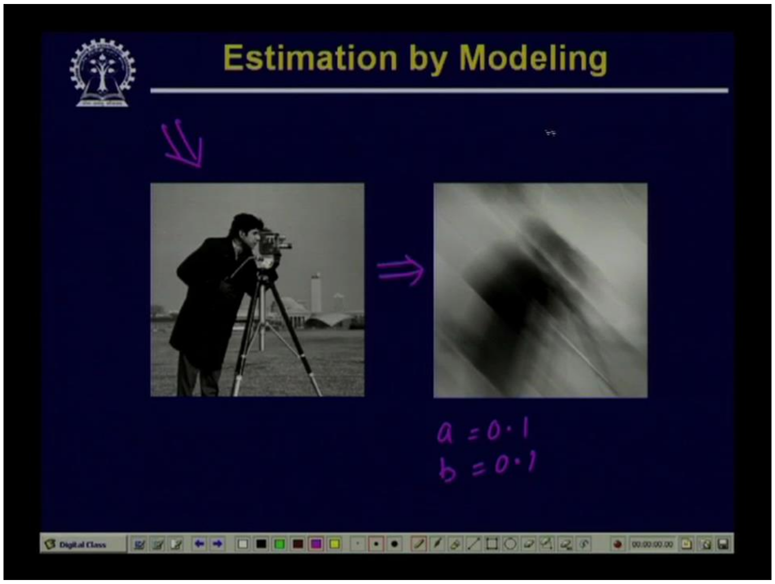
So here this motion term the Degradation function is given by integration of this particular expression and in this expression this x nought (t) and y nought (t) they are the motion variables, which are known. So if the motion variables are known then using those motion variables the values of the motion variables I can find out what will be the degradation function. And using that degradation function I can go for the degradation model. So I know H (u,v), I know G (u,v) and from that I can find out the restored image F (u,v).

(Refer Slide Time: 13:10)



Now in this particular case if assume that x nought (t) is equal to at upon T. And similarly y knot (t) I assume this also to be some constant bt upon T. That means over a period of T during which the camera shutter is open in the x direction the movement is by an amount A. And in the y direction the movement is by amount b. So by using by assuming this we can find that H (u,v) by using that integration by computing that integration will be given by 1 upon pi (ua+vb) into sin [pi (ua + vb)] into e to the power –j pi into (ua + vb). So this is what is the degradation function or the blurring function.

(Refer Slide Time: 14:35)

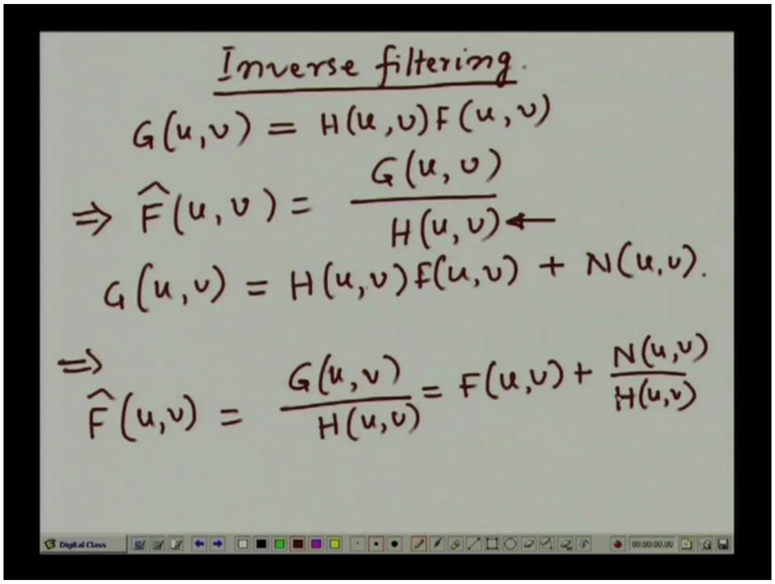


So now let us see that using this degradation function what is the kind of degradation that is actually obtained. So here again on the left hand side we have an original image and on the right hand side this is this corresponding blurred image or the blurring is introduced assuming uniform linear motion. And for obtaining this particular blurring here we assumed a is equal to 0.1 and b also equal to 0.1. So using this values of a and b. We have obtained this, we have obtained the blurring function or degradation function.

And using this degradation function we have obtained this type of degraded model. And you find that this is a quit a common sense whenever you take the image of a very first moving object. The kind of degradation that you obtained in the image is similar to this. Now the problem is we have obtained a degradation function, now once I obtained a degradation function or an estimated degradation function. Now given a blurred image how to restore the original image or how to recover the original image?

So as we have mentioned that there are different types of filtering techniques for obtaining or for restoring the original image from a degraded image.

(Refer Slide Time: 16:15)



The simplest kind of filtering technique is what is known as inverse filtering. Now the concept of inverse filtering is very simple. Our expression is G (u,v) that is the Fourier transform of the degraded image is given by H (u,v) into F (u,v) where H (u,v) is the degradation function in the frequency domain and F (u,v) is the Fourier transform of the original image, G (u,v) is the Fourier transform of the degraded image.

Now because this H (u,v) into F(u,v) this is a point by point multiplication that is for every value of u and v the corresponding f component and the corresponding H component will be multiply together to give you the final matrix which is again in the frequency domain. Now from this expression it is quite obvious that I can have f (u,v) which is given by G (u,v) upon H (u,v).

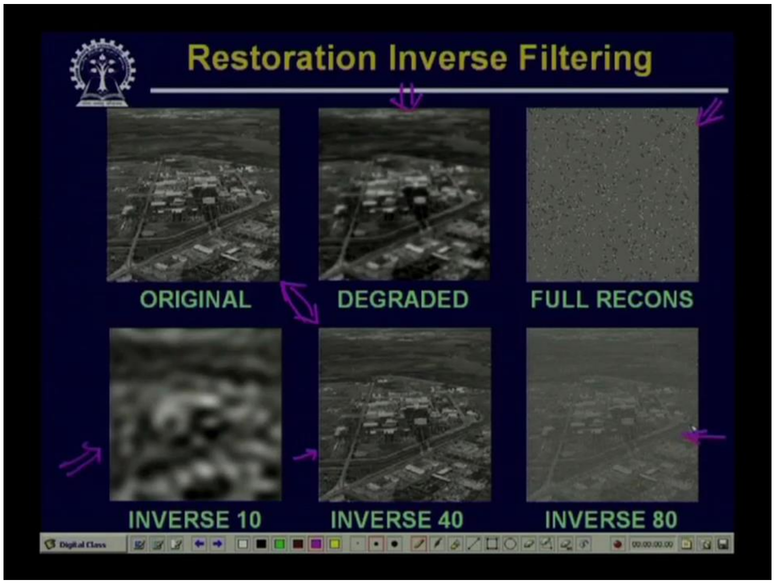
So have this H (u,v) is our degradation function in the frequency domain and G (u,v) I can always compare by taking the Fourier transformation of the degraded image that is obtained. So if I divide the Fourier transformation of the degraded image by the degradation function in frequency domain what I get is the Fourier transformation of the original image. And as I said that when I compute this H (u,v) this is just an estimated H(u,v). It will never be exact.

So the reconstruction of the recovered image that we get is not the actual image but it is an approximate image, approximate original image which we represent by H (u,v) F hat (u,v). Now here as we have already said that G (u,v) if I consider the noise term is given by H (u,v) into F (u,v) plus the noise term N (u,v). Now from here if I compute the Fourier transform of the reconstructed image that will be F hat (u,v) which is equal to G (u,v) upon H (u,v).

And from this expression this is nothing but F (u,v) plus N (u,v) upon H (u,v). So this expression says that even if H (u,v) known as exactly the perfect reconstruction may not be possible. Because we have seen earlier that in most of the cases the Fourier transformation coefficient are very very small when the value of u and v is very large. So that means for those cases N (u,v) by H (u,v) this term will be very high that means the reconstructed image will be dominated by noise.

And that is what is obtained practically also. So to have to avoid this problem what will be, what we have to do is for reconstruction purpose instead of considering the inter frequency plane we have to restrict or reconstruction to a component of the frequencies in the frequency plane which are nearer to zero. Since, if I do that kind of reconstruction that limited reconstruction in that case the dominance of noise can be avoided. So now let us see that what kind of result we can obtain using this inverse filtering.

(Refer Slide Time: 20:40)



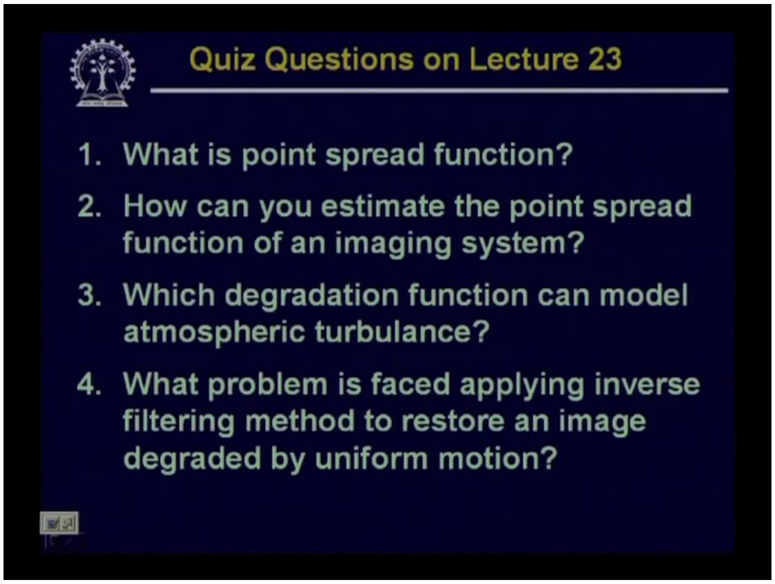
So this shows an inverse filtering result. Here we have the original image, in the middle we have the degraded image. So this degraded image we had already shown. So this is the degraded image. And you find that on the right hand side this is a reconstructed image using the inverse filtering, going for reconstruction all of the frequency coefficient are considered. And as we have said that as you go away from zero zero component in the frequency domain that is as you go away from the origin H (u,v) term become very very negligible so it the noise term which tends to dominate.

So you find that in this reconstructed image nothing is available. Whereas if we go for restricted reconstruction that is we consider only few frequency component near the origin as has been shown here that you considered only those frequency terms within a radius of ten from the origin. So this is the reconstructed image and as it is obvious, because our domain of reconstruction the frequency component that we have considered very very limited so the image becomes reconstructed image becomes very blurred, and that is the property of the low pass transform.

This is nothing but a low pass filter and that is the property of low pass filter. If the cut of frequency is very low then the reconstructed image has to be very very blurred. In the middle of the bottom row again we have shown the reconstructed image, but in this case we have the increase the cut of frequency. The cut of frequency instead of using ten, now we have used cut of frequency for the 40. And here you find that if you compare the original image with this reconstructed image, you find that the reconstruction is quit accurate.

If increase the cut of frequency further as we said that the it is the noise term which is going to dominate so from the right most here we have increase the cut of frequency to 40. So here you find that we can observe the reconstructed image but as if the object are behind a curtain of noise. That means it is the noise term which is going to dominate as we increase the cut of frequency of the filter.

(Refer Slide Time: 23:10)



So with this we complete our today's discussion now let us come to the questions on today's lecture. So the first one is what is point spread function? The second one is how can you estimate the point spread function of an imaging system? Third question which degradation function can model atmospheric turbulence? And the fourth question what problem is faced applying inverse filtering method to restore an image degraded by uniform motion? Thank you.