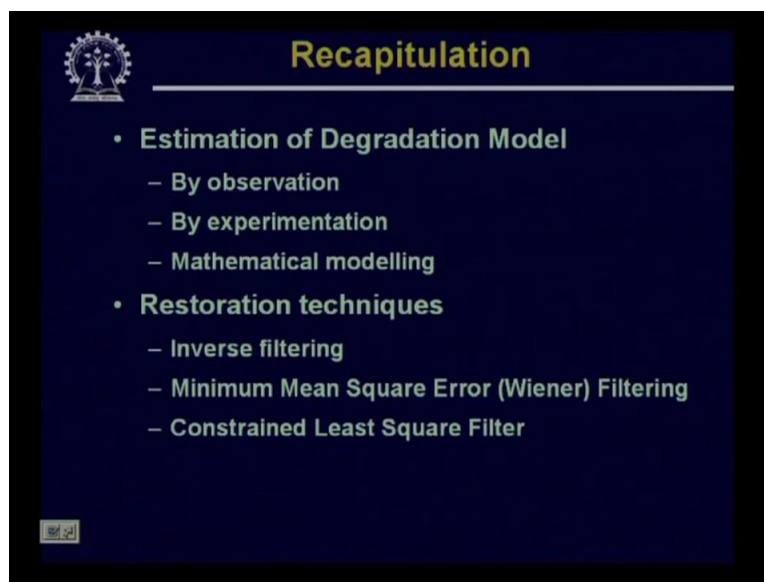


**Digital Image Processing**  
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**Indian Institute of Technology, Kharagpur**  
**Module 10 Lecture Number 46**  
**Other Restoration Techniques - 1**

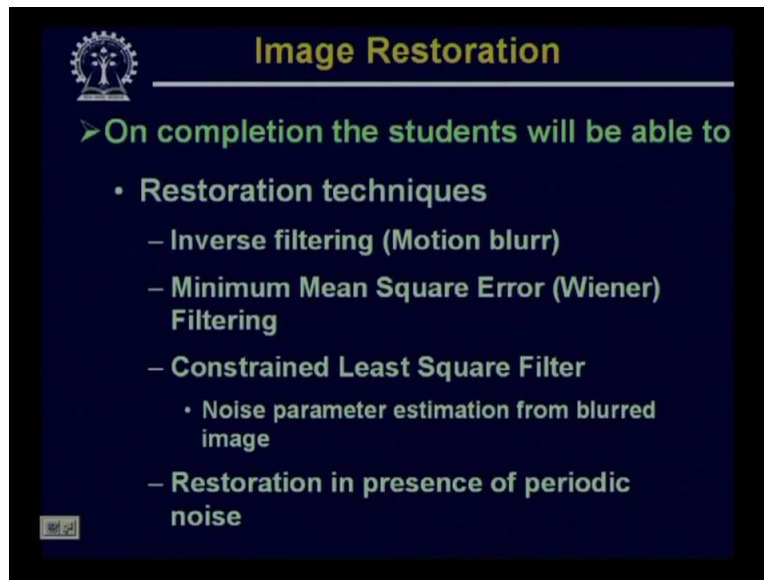
Hello, welcome to the video lecture series on Digital Image Processing. For last few classes we were discussing about restoration of blurred images. So what we have done in our last class is estimation of degradation models, we have seen that whatever restoration technique we use knowledge the restoration techniques mainly use the knowledge of the degradation model, which degrades the image. So estimation of the degradation model degrading an image is very, very important for the restoration operation.

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So in your last class we have seen three methods for estimation of the degradation model. The first, method we discussed is the estimation of the degradation model by observation, the second technique that we have discussed is estimation by experimentation. And the third technique that we have discussed is the mathematical modeling of degradation. Then we have also seen what should be the corresponding restoration technique and in our last class we have talked about the inverse filtering technique. And today we will talk about the other restoration technique. Which also make use of the estimated degradation model, of the estimated degradation function?

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The slide features a dark blue background with a white logo in the top left corner. The title 'Image Restoration' is centered at the top in a yellow font. Below the title, a green arrow points to the text 'On completion the students will be able to'. A bulleted list follows, detailing various restoration techniques in white text.

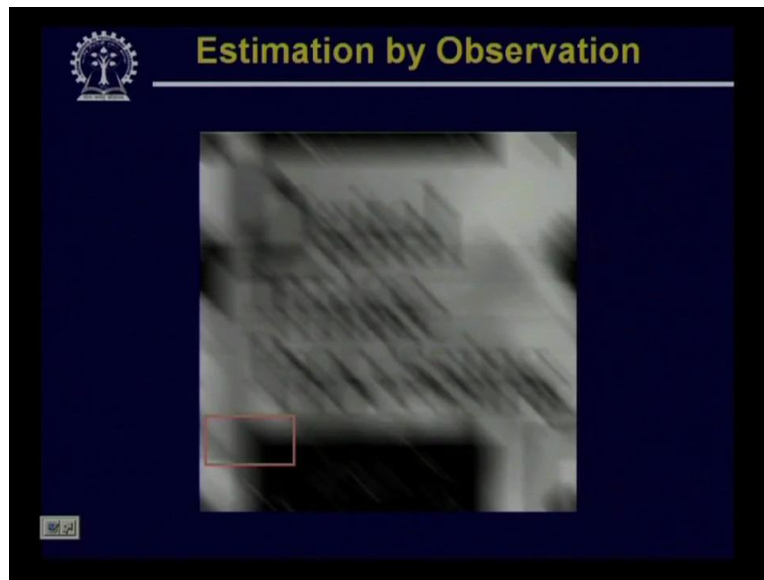
## Image Restoration

➤ On completion the students will be able to

- Restoration techniques
  - Inverse filtering (Motion blurr)
  - Minimum Mean Square Error (Wiener) Filtering
  - Constrained Least Square Filter
    - Noise parameter estimation from blurred image
  - Restoration in presence of periodic noise

So in today's lecture we will see the inverse filtering. The restoration of the motion blurred image using the inverse filtering technique, in our last class we have seen inverse filtering technique where the image blurs degraded by the turbulence atmospheric turbulence model. We will also talk about the minimum mean square error or wiener filtering approach for restoration of a degraded image. We will also talk about another technique called constrained least square filter, where the constrained least square filter mainly uses the mean and standard deviation of the noise which contaminates the image the degraded image. And then will also talk about the restoration technique where the noise present in the image is a periodic noise.

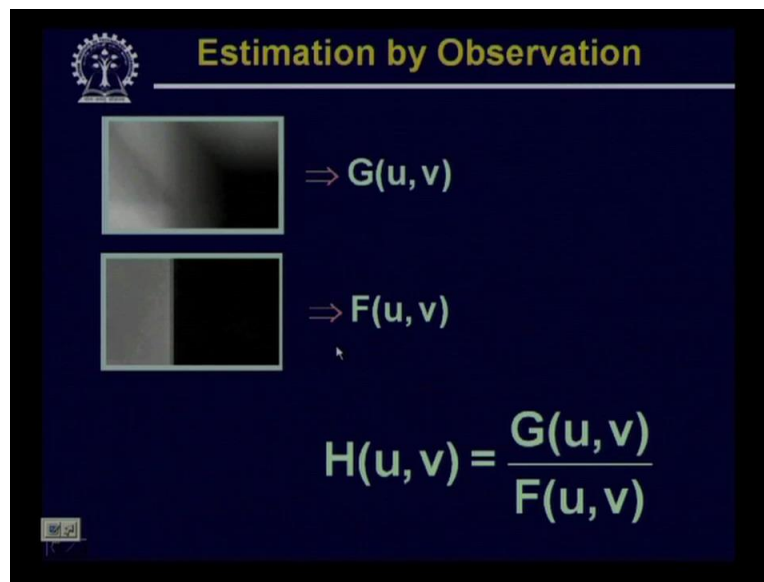
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So firstly let us quickly go through what we have done in our last class. So we are talking about the estimation of the degradation model because that is the very very basic requirement for the restoration operation. So the first one that we have said is the estimation of degradation model by observation so in this case what is given to us is the degraded model and by looking at the degraded model we have to estimate that what is the degradation function. From the degraded image we have to estimate what is the degradation function? So here we have shown one such a degraded image and we have said that once a degraded image is given we have to look for a part of the image, which contains some simple structure.

And at the same time the energy contain, the signal energy contained in that part in that sub image should be very high to reduce the effect of the noise. So if you look at this particular degraded picture you will find that this red rectangle it shows an image region in this degraded image which contain a simple structure and from this it appears that there is a rectangular figure present in this part of the image. And there are two distinct grey levels one is of the object which is towards the black and other one is the background which is a greyish background.

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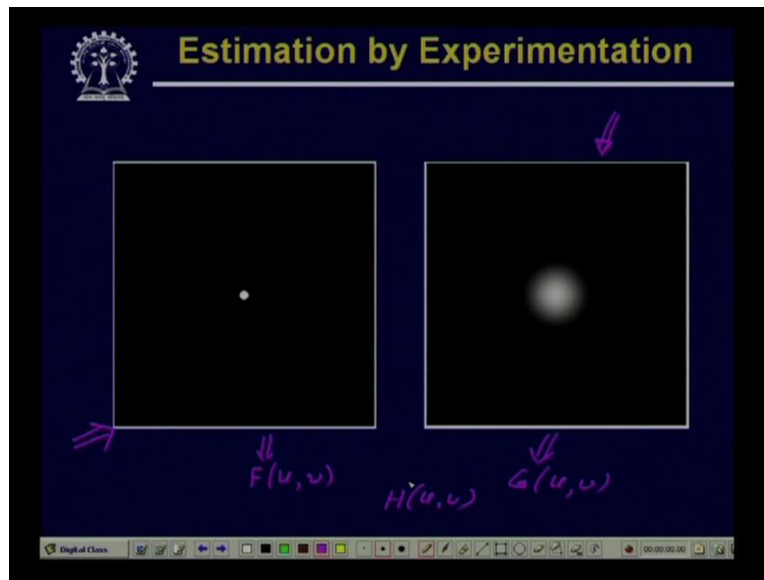


The slide, titled "Estimation by Observation", features a dark blue background with a white logo in the top left corner. It displays two rows of images. The top row shows a blurred, degraded image on the left, followed by an arrow pointing to the label  $G(u, v)$ . The bottom row shows a sharper, original image on the left, followed by an arrow pointing to the label  $F(u, v)$ . Below these images, the equation 
$$H(u, v) = \frac{G(u, v)}{F(u, v)}$$
 is presented in white text.

So by having a small sub image from this portion what I do is I try to manually estimate that what should be the corresponding original image. So as shown in this slide the top part is that degraded image which is cut out from the image that we have just shown at the bottom part is the estimated original image. Now what you do is from this if you take Fourier transform of the top one, what I get is the  $G(u, v)$  as we said the it is a Fourier transformation of the degraded image.

And the lower one we are assuming this to be original so if I take the Fourier Transform of this what I get is  $F(u, v)$  that is the Fourier Transform of the original image. And obviously here the degradation model of that degradation function in the Fourier domain is given by  $H(u, v)$  which is equal to  $G(u, v)$  by  $F(u, v)$ . And you remember that in this case division operation has to be done point by point. So this is how the degradation function can be estimated by observation when only the degraded images are available.

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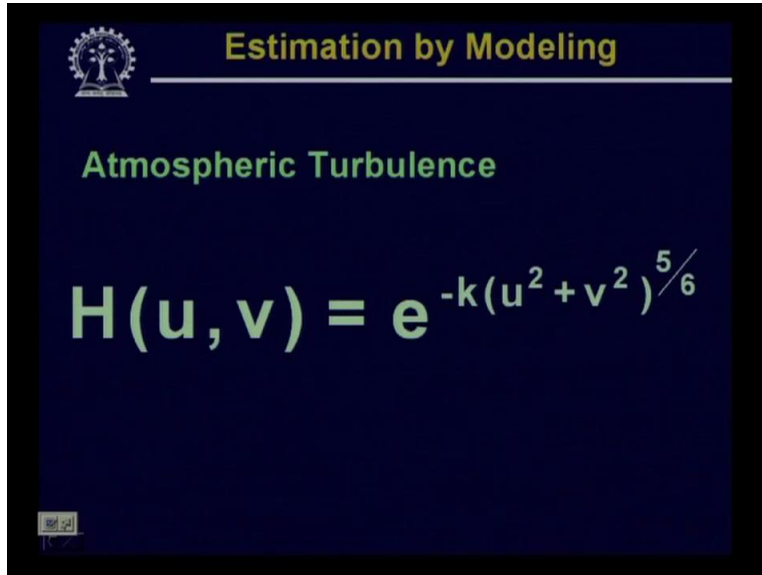
The other approach for estimation the degradation function. We have said is the estimation by experimentation. So our requirement is that whichever imaging device or imaging setup that has been used for getting a degraded image which has been used to record a degraded image in our laboratory for experimental experimentation purpose. We will have a similar such imaging set up. And then we try to find out that what is the impulse response of that imaging set up.

As we have already discussed that it is the impulse response which completely characterises any system. So if you know what is the impulse response of the system? We can identify we can always calculate, what is the response of the system to any type of input signal. So by experimentation what we have done is we have taken a similar imaging set up and then you simulate an impulse by using a very narrow strong beam of light. So as has been shown in this particular diagram.

From the left hand side what is shown is one such simulated impulse in this particular diagram. So the left hand side this one shows such simulated impulse. And of the right hand side what we have is the response of this impulse as recorded by the imaging device. So now if I take the Fourier transform of this, this is going to give me  $F(u,v)$ . And if I take the Fourier transform of this, this is going to give me  $G(u,v)$ . And you see that because of the input, the original is an impulse.

The Fourier transform of an impulse is a constant. So if I simply take the Fourier transform of the response which is the impulse response or in this case it is the point spread function then this divided by the corresponding constant will give me the degradation function which is  $H(u,v)$ . So this is how we estimate the degradation function of or the degradation model of the imaging set up through experiment.

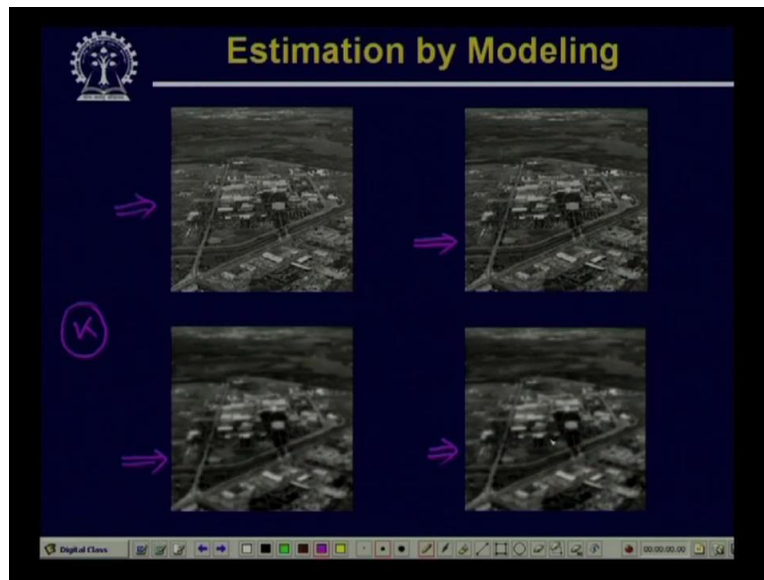
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The slide features a dark blue background with a white logo in the top left corner. The title "Estimation by Modeling" is written in yellow at the top. Below it, "Atmospheric Turbulence" is written in green. The main equation is displayed in white: 
$$H(u, v) = e^{-k(u^2 + v^2)^{5/6}}$$

The third approach for obtaining the degradation model is by mathematical modelling. So in our last class we have considered two such mathematical models. The first mathematical model that you considered which try to give the degradation function corresponding to atmospheric turbulence. And the function is given the degradation function in the frequency domain is given likes this that  $H(u,v)$  is equal to  $e$  to the power  $-k(u^2 + v^2)$  to the power five by six. And using this degradation model we have shown that how our degraded image will look like.

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So in this particular case we have shown four images. Here the top image, the top left image this is the original one. And the other three are the degraded image which have been obtained by using the degradation model that we have just said. Now in that degradation model when I said that  $e$  to the power  $-k(u^2 + v^2)$  to the power five by six. It is the constant  $K$  which tells you that what is the degree of the degradation or what is the intensity of the disturbance.

So low value of  $K$  indicates the disturbance is very low. Similarly higher value of  $K$  indicates that disturbance is very high. So here this image has been obtained with a very low value of  $K$ . This image has been obtained with a very high value of  $K$ . And this image as has been obtained with a medium value of  $k$ . So you find that depending upon the value of  $K$  how the degradation of the original images changes?

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**Estimation by Modeling**

**Motion Blurr**

$$H(u, v) = \int_0^T e^{-j2\pi[ux_0(t) + vy_0(t)]} dt$$
$$\Rightarrow \frac{T}{\pi(ua + vb)} \sin[\pi(ua + vb)] e^{-j\pi(ua + vb)}$$

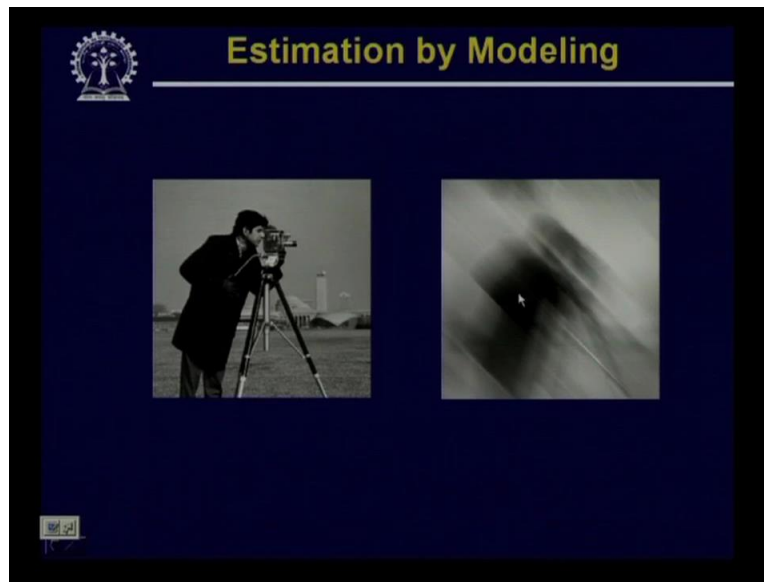
The second mathematical approach that or the second model that we have considered is motion blurred or the blurring which is introduced due to motion. And we have said that because the camera shutter is kept open for a finite duration of time, the intensity which is obtained at any point on the imaging sensor is not really coming from a single point of the same. But the intensity at any point of the sensor is actually the integration of the intensities or light intensities which are falling to that particular point from different points of the moving object.

And this integration has to be taken over the duration during which the camera shutter remain on. So using that concept what we have got is a motion a mathematical model for motion blur which in our last class we have derived that it is given by  $H(u, v)$  equal to integration 0 to T  $e$  to the power  $-j2\pi[ux \text{ nought}(t) + vy \text{ nought}(t)] dt$ .

Where this  $x \text{ nought}$  is the movement this  $x \text{ nought}(t)$  indicates movements along  $x$  direction. And  $y \text{ nought}(t)$  indicates movement along the  $y$  direction. So if I assume that  $x \text{ nought}(t)$  equal to  $a$ , and  $y \text{ nought}(t)$  equal to  $b$ . Then this particular integral can be computed and we get our final degradation function or degradation model as given in the lower equation.

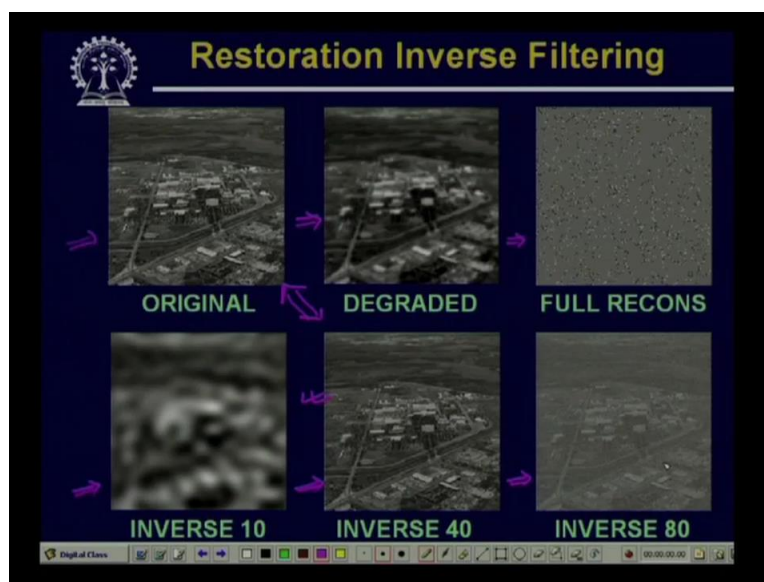


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And after this in our last class what we have done is we have used the inverse filtering. And using this motion blurring model this is the kind of blurring that we obtained. This is the original image on the right hand side we have shown the motion blurred image. Then we have seen that once you have the the model for the blurring operation then you can employ inverse filtering to restore a blurred image. So in your last class we have used the inverse filtering to restore the images which are blurred by atmospheric turbulence.

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And here again on the left hand side of the top row we have shown the original image so this is the original image this is the degraded image. And we have said in the last class because by inverse filtering, the function, the blurring function  $H(u,v)$  comes in the denominator. So if the value of  $H(u,v)$  is very low then  $G(u,v)$  upon  $H(u,v)$  that particular term very becomes very high. So if I considered all the frequency components or all values of  $(u,v)$  in  $H(u,v)$  for inverse filtering, the result may not be always good.

And that is what has been demonstrated here that this image was reconstructed considering all values of  $u$  and  $v$  and you find that this fully reconstructed image does not contained the information that you want. So what you have to do is along this inverse filtering we have to employ some sort of low pass filtering operation so that the higher frequency components of the higher values of  $u$  and  $v$  will not be considered for the reconstruction purpose.

So here on the bottom row the left most image this shows the reconstructed image where we have considered only the values of  $u$  and  $v$  which are within a radius obtained from the centre of the frequency plane. And because this is a low pass filtering operation where the cut of frequency was very very low so the reconstructed image is again very very blurred.

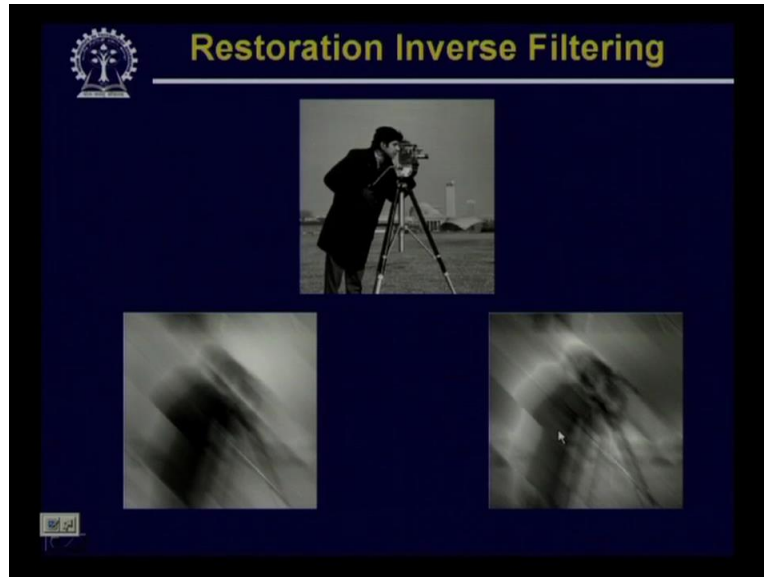
Because many of the low frequency components along with the high frequency components have also been cut out. The middle image shows to some extent very good image why there is a distance within which  $(u,v)$  values were considered for the construction where taken to be forty. So here you find that this reconstructed image this contain most of the information which was contain in the original image.

So this is a fairly good restoration. Now, if I increase the distance function, the value of the distance. If I go to 80 that means many of the high frequency components also we are going to incorporate while restoration. And you find that the right most image on the bottom row that is this one where the value of the distance was equal to 80. This is also reconstructed image but it appears, that the image is behind a curtain of noise.

So this clearly indicates that if I go on increasing or if I take more and more  $u$  and  $v$  values. The frequency components for restoration using inverse filtering then the restored image quality is going to be degraded. It is likely to be dominated by the noise components present in the image.

Now, though this inverse filtering operation. What's fine for this turbulence kind of blurring the blurring introduce by the atmospheric turbulence.

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But this direct inverse filtering does not give good result in case of motion blur. So you find that here we have showed the result of direct inverse filtering in case of motion blur. So on the top most one this is the original image. On the left we have shown the degraded image. And the right most one on the bottom is the image, restored image obtained using the inverse direct inverse direct filtering.

And the blurring which you have considered in this particular case is the motion blur. Now let us say why this direct inverse filtering does not give satisfactory result in case of motion blur. The reason is if you look at the degradation function.

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**Estimation by Modeling**

**Motion Blurr**

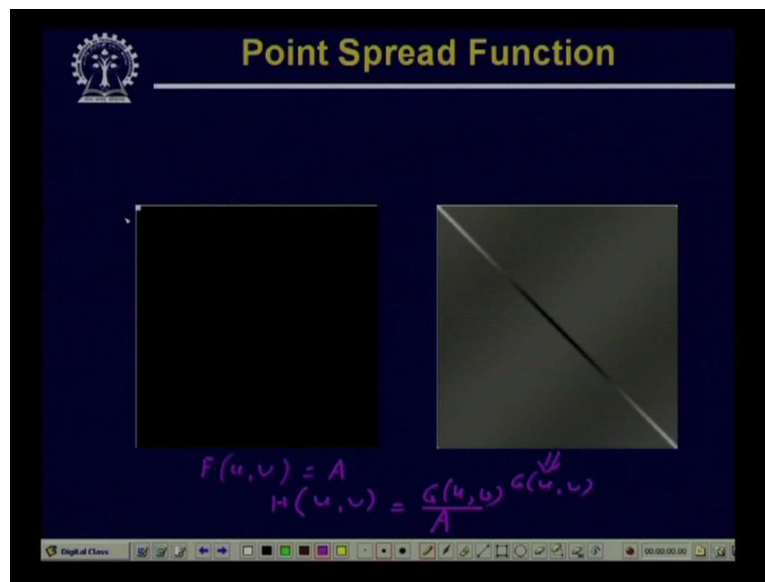
$$H(u, v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$
$$\Rightarrow \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)] e^{-j\pi(ua+vb)}$$

The motion degradation function, Say for example in this particular case you find that the degradation function  $H(u, v)$  is given by this expression in the frequency domain. Now this term will be equal to zero whenever this component  $ua + vb$  this is going to be integer. So for any integer value of  $ua + vb$  the  $H(u, v)$  the corresponding component  $H(u, v)$  will be equal to zero.

And for nearly integer values of this  $ua + vb$  term  $H(u, v)$  is going to be very very low. So about direct inverse filtering when we go for dividing  $G(u, v)$  by  $H(u, v)$  to have the Fourier transformation of the reconstructed image wherever  $H(u, v)$  is very low near about zero. The corresponding  $F(u, v)$  term will be, will be abnormally high.

And when you take inverse Fourier transform of this that very very high value is deflected in the reconstructed image and that is what gives to a reconstructed image as shown in this form. So what is the way out, Can't we use the inverse filtering for restoration of motion blurred image. So we have attempted round about approach what you have done is again we have taken impulse. Try to find out what will be the point spread function of. If I employ this kind of motion blur.

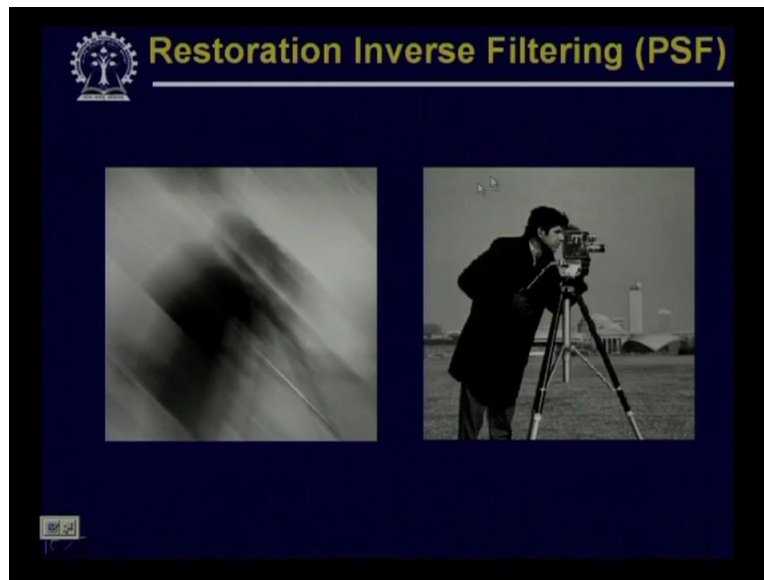
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So by using the some motion blur function or the motion blur model you blur this impulse and what I get is an impulse response like this which is the point spread function in this particular case. Now once I have this point spread function then as before. What I do is? I take the Fourier transform of this. And this Fourier transformation now gives me  $G(u,v)$  and because my input was an impulse where so for this impulse  $F(u,v)$  is equal to constant which is a something like  $A$ .

Now from this two I can compute recomputed  $H(u,v)$  which is given by  $G(u,v)$  divided by this constant term  $A$ . Obviously the value of constant is same as what is the intensity of this impulse so. If it an unit impulse, if I take an unit impulse then the value of constant  $A$  will be equal to one and in that case the Fourier Transformation of the point spread function directly gives me the degradation function  $H(u,v)$ .

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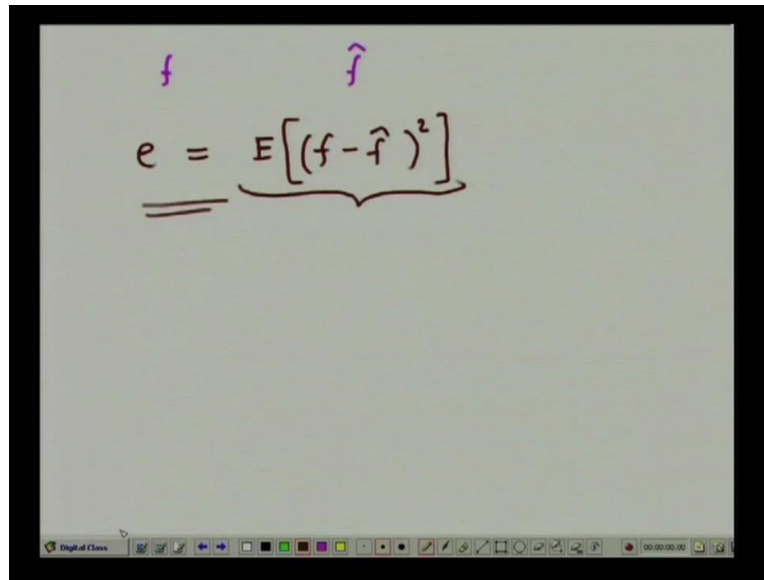
Now if I perform the inverse filtering using this recomputed degradation function then we find that the reconstruction result is very very good. So this was the blurred image. This is the reconstructed image during by using the inverse filtering direct inverse filtering, but here the degradation model was recomputed from the point, from the Fourier transformation of the point spread function.

So though using the direct inverse transform of the mathematical model of the motion blur does not give me good result, but recomputation of that degradation function gives a satisfactory result. But again with this Inverse filtering approach the major problem is as we said that we have to considered the  $u, v$  values for reconstruction which is within a limited domain.

Now how do u say that up to what extent of  $u, v$  value we should go that is again image depended. So it is not very easy to decide that to what extent of frequency component we should consider for the reconstruction of the original image if I go for direct inverse filtering. So there is another approach which is the minimum mean square error approach or it is also called the wiener flirting approach.

In case of wiener flirting approach, the wiener flirting tries to reconstruct the degraded image by minimizing an error function.

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$$\underline{e} = E[(f - \hat{f})^2]$$

So it is something like this. So if my original image is  $f$  and my reconstructed image is  $\hat{f}$  then the wiener filtering tries to minimize is the error function which is given by expectation value of  $f - \hat{f}$  square. So the error value  $e$  which is given by the expectation value of  $f - \hat{f}$  square where  $f$  is the original degraded image and  $\hat{f}$  is the restored image from the degraded image.

So  $f - \hat{f}$  square this gives you the square error and this wiener filtering tries to minimize the expectation value of this error. Now here our assumption is that the image intensity and the noise intensity are uncorrelated, and using that particular assumption this wiener filtering works. So here will not go into the mathematical details of the derivation.

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$$\hat{F}(u,v) = \left[ \frac{H^*(u,v) S_f(u,v)}{S_f(u,v) |H(u,v)|^2 + S_\eta(u,v)} \right] G(u,v)$$

Power spectrum of Original Image

Noise power spectrum

But it can be shown that a frequency domain solution of this I means whenever this error function is minimum. The corresponding  $f(u,v)$  in frequency domain is given by  $\hat{f}(u,v)$ , this will be equal to  $H^*(u,v)$  into  $S_f(u,v)$  divided by  $S_f(u,v)$  into  $|H(u,v)|^2$  plus  $S_\eta(u,v)$ . This into  $G(u,v)$ . Where this  $H^*$  indicates it is the complex conjugate of  $H(u,v)$  and  $G(u,v)$  as before it is the Fourier transform of the degraded image. And  $\hat{F}(u,v)$  is the Fourier transform of the reconstructed image and in this particular case this term  $S_f(u,v)$ , this is the power spectrum, power spectrum of original image, undegraded image, and  $S_\eta(u,v)$  is the noise power spectrum.



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The image shows a handwritten derivation of the Wiener filter equation. It starts with the general form of the filter:

$$\hat{F}(u,v) = \left[ \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}} \right] G(u,v)$$

Then, it defines the ratio of noise power spectrum to signal power spectrum as a constant:

$$\frac{S_\eta(u,v)}{S_f(u,v)} = \kappa.$$

Finally, it substitutes this constant into the filter equation to get a simplified form:

$$\hat{F}(u,v) = \left[ \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \kappa} \right] G(u,v)$$

Now, if I simplify this particular expression, I get an expression of this form, that  $\hat{f}(u,v)$  is equal to  $\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}} \cdot G(u,v)$ . So this is the expression of the Fourier transform of the reconstructed image when I use Wiener filtering.

Now in this case you might notice that if the image does not contain any noise then obviously  $S_\eta(u,v)$  which is the power spectrum of the noise will be equal to zero. And in that case this Wiener filter becomes identical with the inverse filter. But if the noise contains additive, if the degraded image also contains additive noise in addition to blurring in that case Wiener filter and the inverse filter is different.

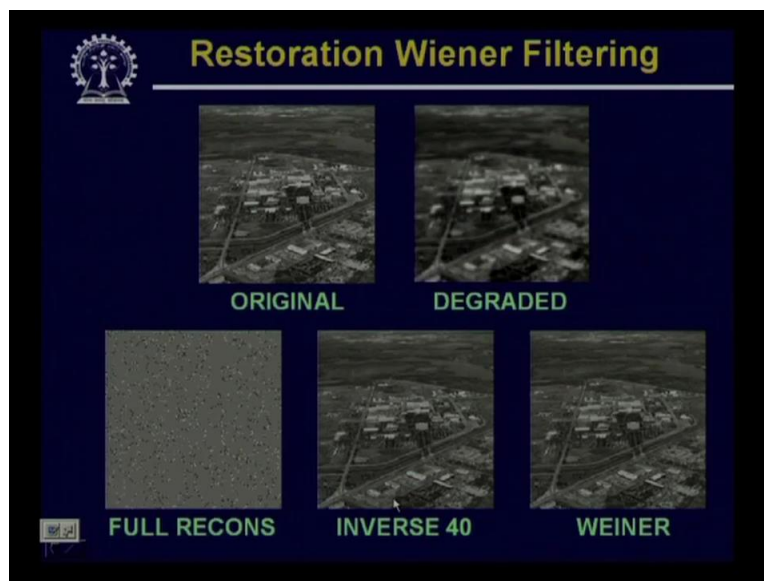
Now here you find that this Wiener filter considers the ratio of the power spectrum of the noise power of the noise power and the power of the power spectrum of the original undegraded image. Now even if I assume that the additive noise which is contained in the degraded image is a wide noise for which the noise power spectrum will be constant.

But it is not possible to find out. What is the power spectrum of the original undegraded image. So for that purpose what is done is, normally this ratio that is  $\frac{S_\eta(u,v)}{S_f(u,v)}$  that is the ratio of power spectrum of the noise to the power spectrum of the original undegraded image is

taken to be a constant  $K$ . So if I do this in that case the expression for  $\hat{F}(u,v)$  comes out to be one upon  $H(u,v)$  into  $H(u,v)$  square upon  $H(u,v)$  square plus  $k$  into  $G(u,v)$ .

So where this  $K$ , the term  $K$  is a constant which has to be adjusted manually for the optimum reconstruction of the, for the reconstructed image which appear to be visually best. So using this expression let us see what kind of image that or what kind of reconstruction image that we get.

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So here we have shown the image restoration of the degraded image using wiener filter. Again on the left hand side of the top flow it is the original image. Right hand side of the top two gives you the degraded image . Left hand, left most image of the bottom row that shows you the full reconstructed image or the reconstructed using the inverse filtering where all the frequency where considered. The middle one shows the reconstructed image using inverse filtering where only the frequency component within a distance forty from the center of the frequency plane has been considered for reconstruction.

And the rightmost one is the one which is obtained using wiener filtering and for obtaining this particular reconstructed image, the value of  $k$  was manually adjusted for best appearance. Now if you compare these two, that is the inverse filtered image with the distance equal to forty with the wiener filtered image, you will find that the reconstructed images or more or less same. But if

you look very closely it may be found that the wiener filtered image is slightly better than the inverse filtered image.

However visually they appeared to be, they appear to be almost same. The advantage in case of wiener filter is that I don't have to decide that what extent of frequency component I have to consider for reconstruction or for restoration of the undegraded image. But still the wiener filter has got a disadvantage that is the manual adjustment of the value of  $K$ .

As we have said that the value of  $K$  has been used for simplification of the expression where this constant  $K$  is nothing but a ratio of the power spectrum of the noise to the power spectrum of the undegraded image. And in all the cases have you taking this ratio to be constant may not be justified approach. Thank you.