Digital Image Processing
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Module 10 Lecture Number 47
Other Restoration Techniques - 2

Hello, welcome to the video lecture series on digital image processing. So we have another operation another kind of filtering operation which is called constrained least square filtering.

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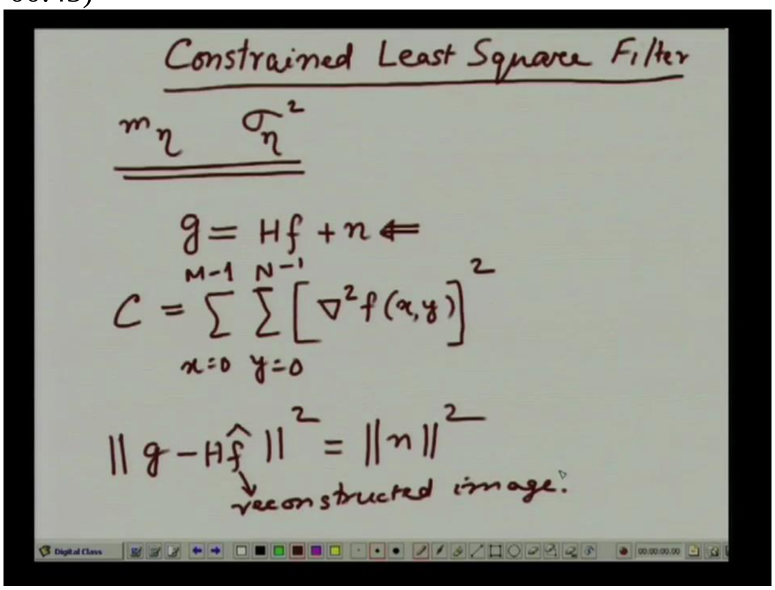

So now will considered a filtering operation which is called constrained least square filter. Now unlike in case of wiener filtering, what the performance of the weinering filtering depends upon the correct the estimation of the value of K that is the performance of wiener filtering depends upon, how correctly you can estimate, what is the power spectrum of the original undegraded image. In case this constant least square filter it does not make any assumption about the original undegraded image. It makes use of only the noise probability distribution function, probability density function, now is PDF.

And mainly it uses the mean of the noise which will write as say m eta and the variance of the noise which will write as sigma eta square. So will come will see that how the reconstruction using this constrained least square filter approach makes use of this noise parameters like mean of the noise and the variance of the noise. In case of this constrained least square filter, to obtain this constrained least square filter. We will start with the expression that we got in our first class that is g equal to Hf + n.

So you remember that this is an expression which we had derived in first class which tell us the, what is the degradation model for degrading the image, where H is the matrix which is derived from the impulse response Hx and n is the noise vector. Now, here you will notice that he the value H is very very sensitive to noise. So to take care of that what you do is, we define an optimality criteria and using that optimic optimality criteria the reconstruction has to be done.

And because this degradation function H of the degradation matrix H is noise dependent it is very very sensitive to noise. For so for reconstruction the optimality criteria that we will use is the image smoothness. So we know from earlier discussion that the second derivative operation or the Laplacian operator it tries to enhance the irregularities or discontinuity in the image.

So if we can minimize the Laplacian of the reconstructed image that will ensure that the image the reconstructed image will be smooth. So the our optimality criteria in this particular case is given by C is equal to double summation del square f (x,y) square, where y varies from 0 to N-1 and x varies from 0 to M-1. So our assumption is the image that we trying to reconstruct of that or blurred image that we have obtained that is of size M by N N.

So our optimality criterion is given by this where del square F(x,y) is nothing but the Laplacian operation. So this optimality criteria is Laplacian operator based. And our approach will be that will try to minimize this criteria subject to the constraint that g-H f hat square should be equal to n square, where this F hat, this is the reconstructed image. So will try to minimize this optimality criteria subject to the constraint that G- Hf hat square is equal to n square and that is why it is called constrained least square filtering.

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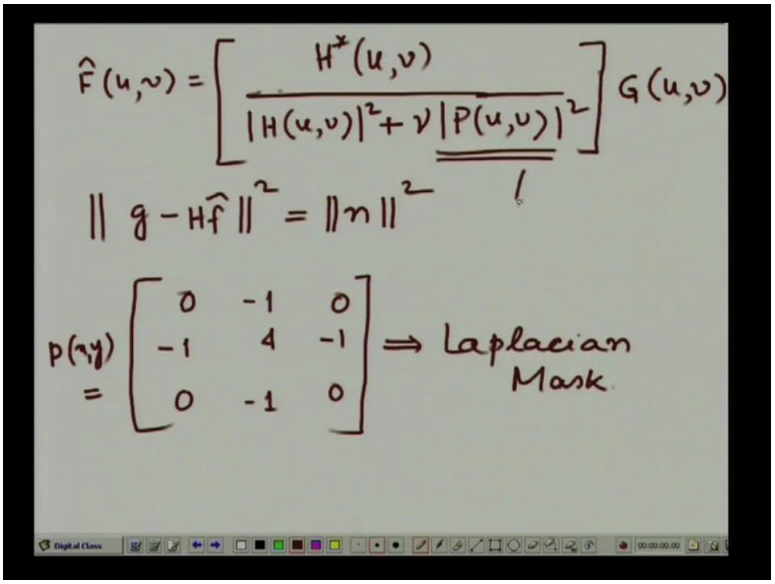

Again without going into the details of mathematical derivation we will simply give the frequency domain solution of this particular constrained least square estimation. But the frequency domain solution now is given by f hat (u, v) is equal to H star (u,v) upon H (u,v) square plus a constant gamma times P (u,v) square this times G (u,v). Again as before this H star indicate that it is the complex conjugate of H.

Here again we have a constant term given as gamma but the gamma is to be adjusted so that the specified constant that is g - Hf hat square is equal to n square, this constant is meet. So here is gamma is a scalar quantity scalar constant whose value is to be adjusted to for , so that this particular constraint is maintained. And this quantity P (u,v) the term P (u,v) it is the Fourier spectrum or Fourier transform of the mask given by 1 ,0,-1,0,-1,4,-1,0,-1,0.

So this is my P (x,y) and this P (u,v) is nothing but the Fourier spectrum of or the Fourier transform of this P (x,y). And you can easily identify that this is nothing but the Laplacian operator mask, of the Laplacian mask that we have already discussed in our earlier discussion. Now here for implementation of this for computation of this we have to keep in mind that our image is of size M by N.

So before we compute Fourier transformation of P (x,y) which is given in the form of a 3 by 3 mask, we have to paired appropriate number of zeros so that this P(x,y) also becomes a function of dimension N by M or an array of dimension M by N. And after only converting this two an array of dimension of M by N we can compute P (u,v) and that P (u,v) has to be use in this particular expression.

So as we said that this gamma has to be adjusted manually for obtaining the optimum result and the purpose is that this adjusted value of gamma, the gamma is adjusted so that the specified constant is maintained. However it is also possible to automatically estimate the value of gamma by an iterative approach.

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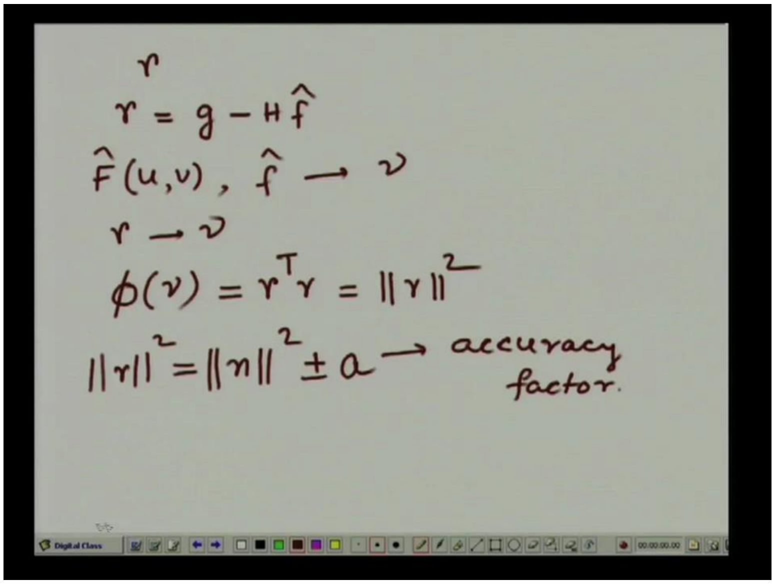

So for that iterative approach what we do is? We use defined a residual vector say r, but this residual vector is nothing but r equal to g- H f hat. So remember that this g was, is obtained from the degraded image, the matrix degradation matrix H is obtained from the degradation function Hx, and f hat is actually the estimated restored image. Now, here since f hat. We have seen earlier that f hat sorry, we seen have earlier that f hat (u,v) and so this f hat in the special domain they are functions of in the special domain, they are functions of gamma so obviously r which is function of f hat, so this r will also be the function of gamma.

Now if I define a function say phi of gamma which is nothing but r transpose r or which is nothing but the Euclidean form of r. It can be shown that this function is monotonically increasing function of gamma that means whenever gamma increases this Euclidean norm of r are also increases if gamma decreases the Euclidean norm of r also decreases.

And by making use of this property it is possible to find out what is the optimum value of gamma within some specified accuracy. So approach in this case or aim is that we wants to estimate the value of gamma such that the Euclidean norm of r, that is r square will be equal to n square plus minus some constant A, but this A is nothing but what is the specified accuracy factor, of this gives the tolerance of reconstruction.

Now obviously you find that if r square is equal to n square then the specified constant is exactly made. However it is very very difficult so we specify some tolerance by giving the accuracy factor A. And we want that the gamma should be such that r will be the Euclidean norm of r will be within this range.

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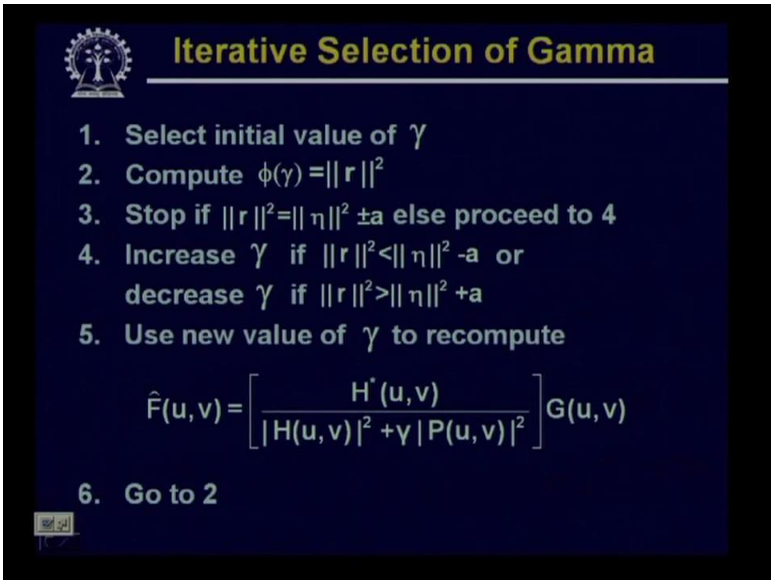

Now given this background an iterative approach, iterative algorithm for estimation of the value of gamma can be put like this. So you select an initial value of gamma then compute y phi gamma which is nothing but Euclidean norm of r, then you terminate the algorithm if r square is equal to n square, here it has been written as eta square plus minus A.

If this is not the case then you processed to step number four where you increase the value of gamma. And if r square is less than eta increase the value of gamma, if r square is less than eta square minus A, or you decrease the value of gamma, if r square is greater than eta square plus A.

Now, using whatever new value of gamma that you get you recompute the image and for that the image reconstruction function as we have said in frequency domain is given by this particular expression. And with this reconstructed value of f you go back to step number two and you do this iteration until and unless this termination condition that is r square is equal to n square plus minus A, this condition is met.

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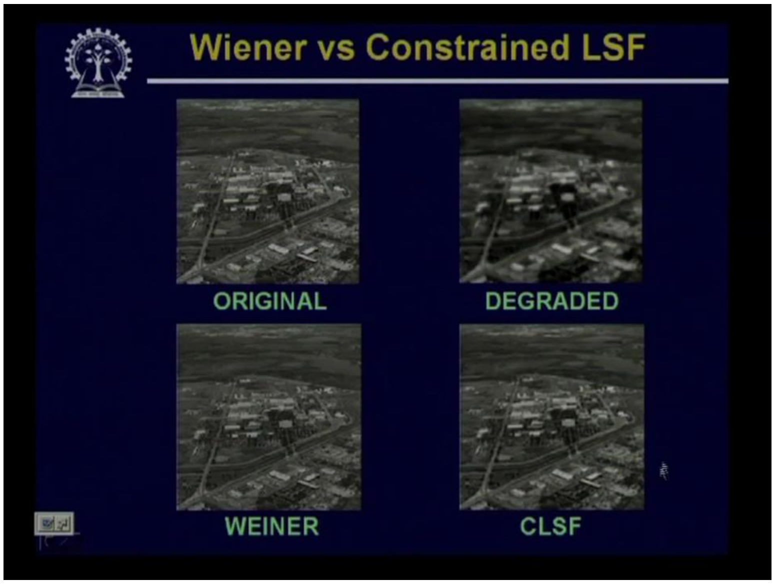

Now using this kind of approach, what we have obtained is we have got some reconstructed image. So here you find that it is the same original image this is the degraded version of that image. On the bottom row the left hand side gives you the reconstructed image using the wiener fettering. And again on the bottom row on the right hand side this gives you the reconstructed image which is obtained using the constrained least square filter.

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The same for the motion degraded image. Here we have also considered some additive noise so again on the top row on the left this is the image which is obtained by direct inverse filtering. And you find the prominence noise in this particular case. The right one is the one which has been obtained by wiener filtering, here also you find that the amount of noise has been reduced but the still the image is noisy and the bottom one is the one that has been obtained by using the constrained least square filtering.

Now if you look at these three images you find that the amount of noise is greatly reduced in the bottom one that is this particular image which has been obtained by this restored image, which has been obtained by constrained least square filtering approach. And as we said the constrained least square filtering approach makes you use of the estimation of mean of the noise at the standard deviation of the noise. So it is quite expected that the noise perforce of the least square constrained least square filter will be quit satisfactory.

And that is what is observed here that this image which is obtained using this constrained least square filter, the image has been the noise has been removed to a great extent whereas the other reconstructed image cannot remove the noise component to that extent however if you look at this reconstructed images. The reconstruction quality of this image is not that clearly good.

So that clearly says that using the optimality criteria the reconstructed image that you get. The optimum reconstructed image may not always be visually the best, so to obtain a visually best image the best approach is you manually adjust that particular constant gamma.

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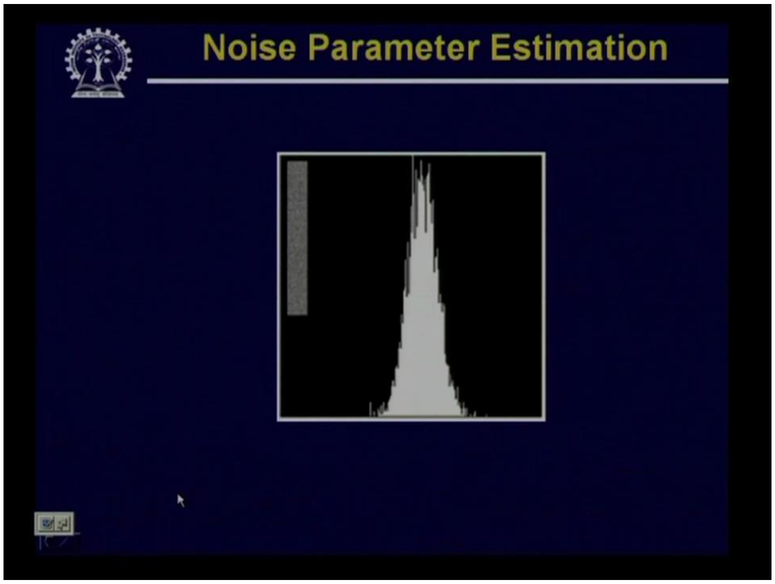

Now, as I said that this constrained least square filtering approach makes use of the noise parameters that is the mean of the noise and the variance of the noise. Now, how can we estimate the mean and variance of the noise from the degraded image Xf.

It is possible that if you look at a more or less uniform intensity region in the image if you take a sub image of the degraded image where the intensity is more or less uniform, and if you take the histogram of that the nature of the histogram is same as the probably density function PDF of the noise which is contaminated with that image.

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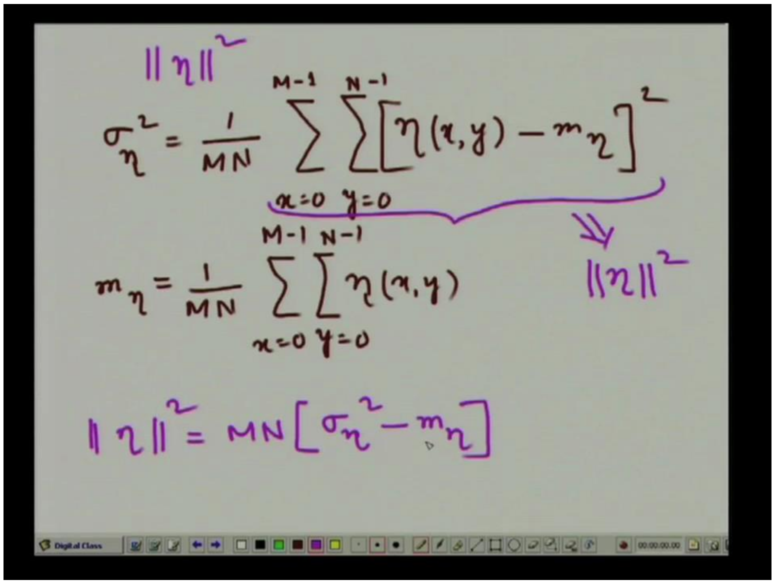

So we can obtained the noise estimate or we can compute the noise term that is eta square in our expression which is used for this constrained least square filtering in this way. We have the noise variance which is given by sigma n square sigma eta square which is nothing but M into N into double summation eta (x,y) minus M eta. Which is the mean of the noise square where y varies from 0 to N-1 and x varies from 0 to M-1.

And the noise means that is m eta is given by the expression 1 upon M into N then double summation eta (x,y) where again where y varies from 0 to N-1 and x varies from 0 to M-1. Now, from this we find that this particular term. This particular term this is nothing but what is our eta square. So while making use of this and making use of the mean of the noise we get that eta square this noise term is nothing but M into N where M into N, M and N the dimension of image into sigma n square, sigma eta square minus m eta.

And as in our constrained that we have specified it is eta square which is use in constrained term, and which is only dependent upon sigma eta at M eta. So this clearly says that this optimally reconstruction is possible if I have the information of the noise standard deviation of the noise variance and the noise mean. Now the estimation of noise variance and noise mean is very very important. If I have the only the degraded image what I will do is, I will look at some uniform available region within the degraded image.

Find out the probability; find out the histogram of that particular region and the nature of the histogram is same as the probability density function of the noise.

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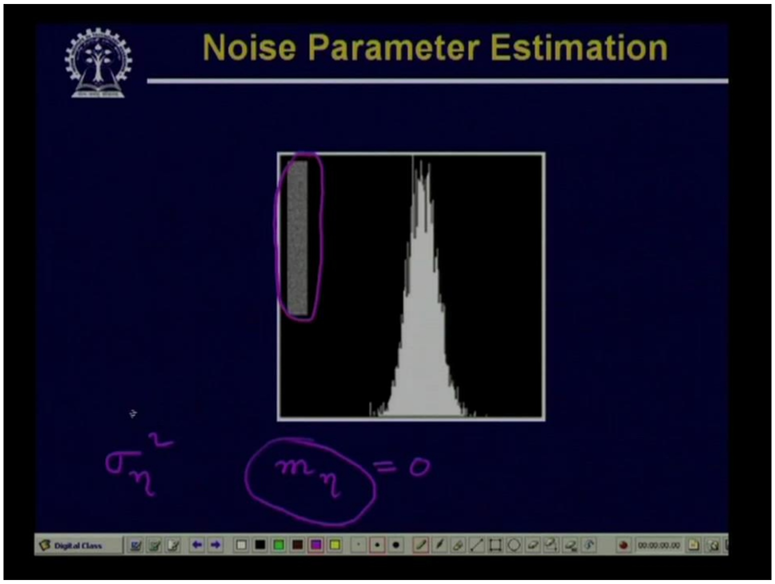

So as has been shown here in this particular diagram. Here you find that this bar that has been shown, this is taken from one such noise image and this is the histogram of this particular region. And this histogram tells you that what is the PDF of the noise which is contaminated with this image.

So once I have this probability density function, from this I can compute what is the standard or the variance sigma eta square, and I can also compute what is the mean that is m eta. And in most of the cases for the noise term the noise is assumed to be zero mean. So this m eta equal to zero, so what is important is for us is only this sigma eta square and using this sigma eta square we can go for optimum reconstruction of the degraded image.

Now, in some situation in many cases it is also possible that the image is contaminated with period image. So how do we remove the periodic noise present in the image? We find that if you take the Fourier transformation of the periodic noise and display that Fourier transformation, in that case because the noise is clearly the corresponding dots the corresponding, at the corresponding u,v location in the Fourier transformation plane you will get very very bright dots.

And that dot indicates that what is the frequency of the periodic noise, present in the image then we can go for a very simple approach that once I know the frequency component. I can go for band pass filtering. Just to remove that part of the coefficient on the Fourier transform. And whatever is the remainder Fourier coefficient we have. If you go for inverse Fourier transformation of that we will get the reconstructed image.

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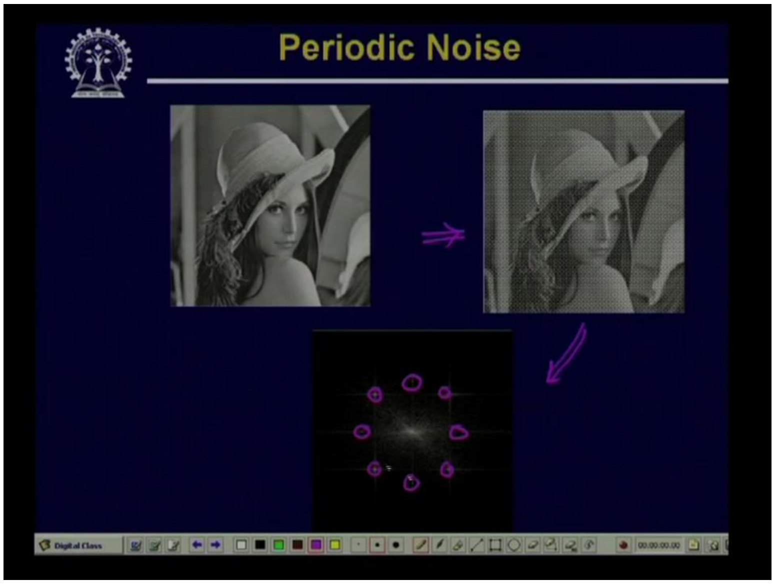

So as has been shown here so here you find that we are taking the same image which we are taking number of time earlier. And if you look at the right most image, and if you look closely to this right most image you find that this is contaminated with periodic noise. And if I take the Fourier transform of this here you find that in Fourier transform. There are few bright dots, one bright dot here, one bright dot here, one bright dot here, one bright dot here, one bright dot here, one bright dot here. One is at this location and one is that this location.

So all this bright dot tell us that what is the frequency of the periodic noise which contaminates the image. So once I have this information I can go for an appropriate band pass filtering to filter out that region from the Fourier Transformation or that part of the Fourier coefficient. So that is what has been shown next so this is what is a band reject filter.

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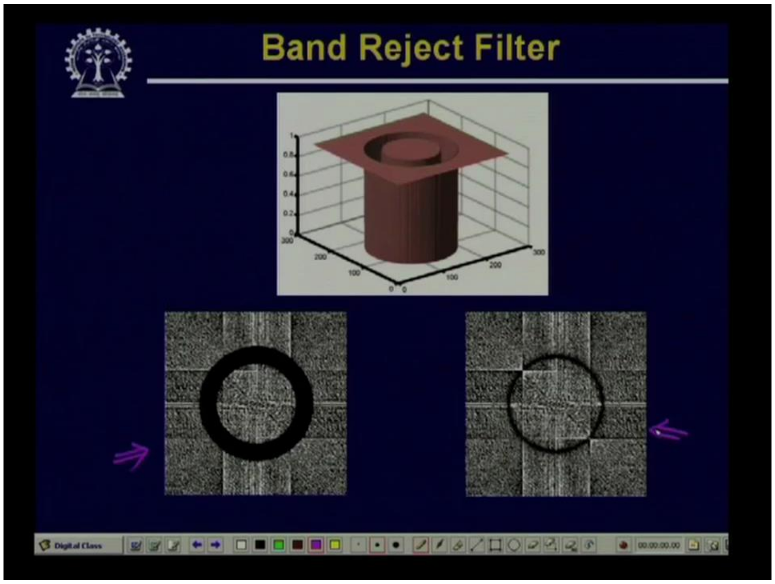

So this is the perspective plot of an ideal band reject filter and here the band reject filter are shown super imposed on the frequency plane. So on the left what I have is an ideal band reject filter and on the right what I have is the corresponding Butterworth band reject filter. So by using this band reject filter we are removing a band of frequencies from the Fourier coefficient corresponding to the frequency of the noise.

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So after removal of this this frequency component, if I go for inverse Fourier Transform then I'm going to get back my reconstructed image, and that is what we get in this particular case. So here you find that on the left top this is again the original image, on the right top it is the noise image, contaminated with periodic noise. If I go for ideal band reject filter and then reconstruct and this is the image I get which is on the bottom left and.

If I go for the Butterworth band reject filter then the image reconstructed image that I get is in the bottom right. So we have talked about the reconstruction of the restoration images using the various operations. And the last one that we have discussed with is, if we have an image contaminated with periodic noise then we can make use of band reject filter, do frequency operation improve a band reject filter to move those frequency component and then go for inverse filtering to reconstruct the image.

And her you find the that the quality of the reconstructed images a quit good earlier used this band reject filter the frequency domain. So with this we complete our discussion on image restoration. Now, let us have some question on today's lecture.

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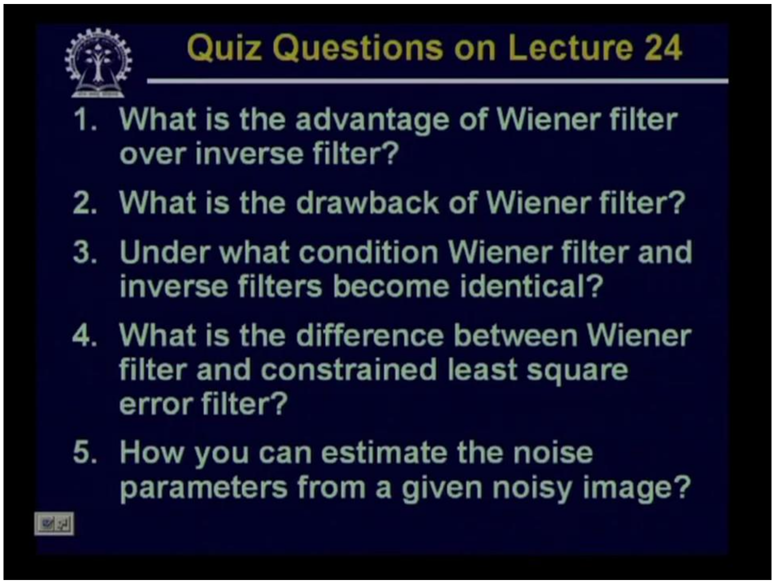

So our first question is what is the advantage of wiener filter over inverse filter? The second question what is the drawback of wiener filter? Third one under what condition wiener filter and inverse filters become identical? Fourth one what is the difference between wiener filter and constrained least square error filter? And the last question how can you estimate the noise parameters from a given noisy image or from the given blurred image? Thank you.