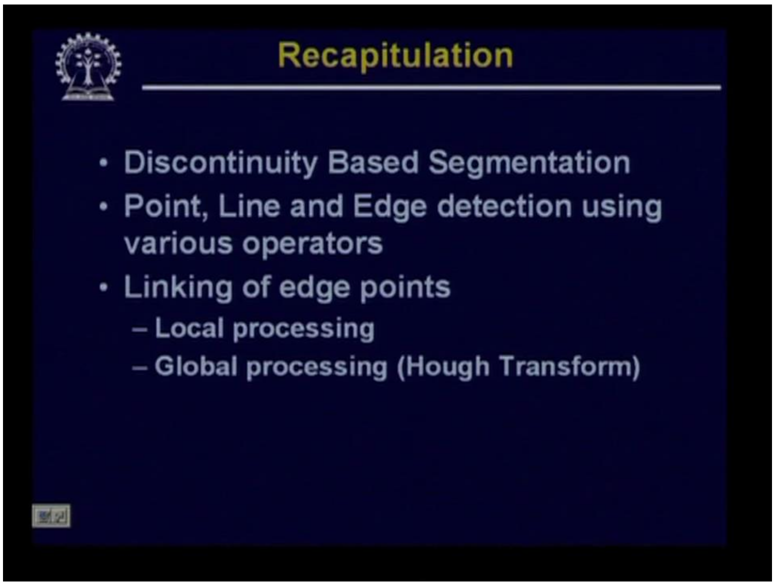
Digital Image Processing.
Professor P. K. Biswas.
Department of Electronics and Electrical Communication Engineering.
Indian Institute of Technology, Kharagpur.
Lecture-58.
Region Based Segmentation Operations. Thresholding Techniques.

(Refer Slide Time: 0:48)



Hello, welcome to the video lecture series on digital image processing. For last few lectures we were discussing about image segmentation operations and image analysis operations. So, in our last lecture we have talked about the discontinuity based image segmentation we have seen earlier or we have discussed earlier that are mainly two approaches of image segmentation. One is the discontinuity based segmentation and the other one is similarity based image segmentation.

For last two classes we have talked about the discontinuity based image segmentation and in discontinuity based image segmentation we have seen that the segmentation is done using the characteristics of variation of intensity values when there is a variation of intensity from say background to a foreground object. So, under this we have seen various point and the line and edge detection operations which are used in this segmentation process.

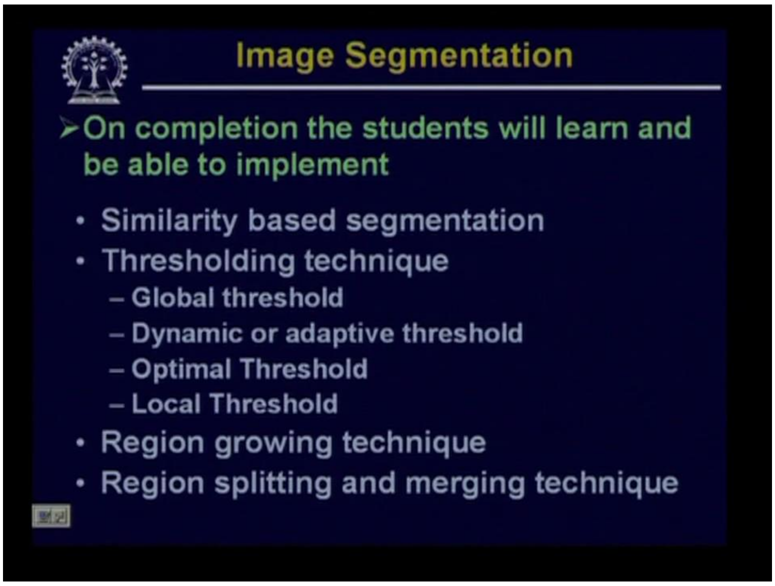
Here the basic purpose was that an object is to be described by it is boundary or it is imposing boundary which are to be obtained using one of this discontinuity based operations. And we have discussed that though we want that the object boundary should be continuous or it should have a complete definition but because of noise or may be because of non-uniform illumination after performing these different edge detection operations the edge points that we get they are not normally continuous.

So, to take care of this problem after this edge detection operation the edge points that we get they are to be linked. So, for that we have discussed about two different approaches. One is local linking operation where the edge points in the neighborhood are linked together if we find that those two edge points are similar in nature, and for that as similarity criteria we have taken the strength of the gradient operator or strength of the edge operator as well as the direction of the edge at these points.

So, if we find that within a neighborhood two edge points have the similar edge strength and also they have similar edge direction in that case those two points are linked together to be part of the same edge. Now, here again the problem is that if the points are not in the small neighborhood which is defined but the points are at a larger distance, in that case this local edge linking operation does not help.

So, in such cases what we have to go for is the global edge linking operation. So, we have discussed a technique that is hough transform. So, using hough transform we have been able to link the distant edge points and this is an operation called global edge linking operation or it is the global processing technique.

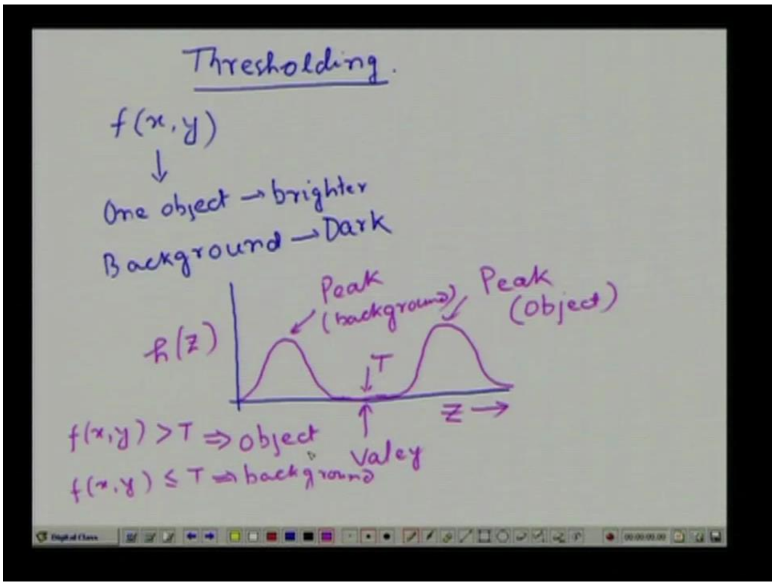
(Refer Slide Time: 4:09)



Now, today we will start our discussion on the other type of segmentation which is the similarity based segmentation. So, under similarity based segmentation there are mainly three approaches. One is called thresholding technique, the second approach is region growing technique and the third approach is region splitting and merging technique.

Under thresholding technique again we have four different types of thresholding, one is called global threshold, the other type of thresholding is called dynamic or adaptive thresholding there, is something called optimal thresholding and there is also a thresholding operation which is called local thresholding. So, we will discuss about these different region based segmentation operations either thresholding or region growing and the region splitting and merging techniques one after another.

(Refer Slide Time: 5:21)



Now, let us first start our discussion with the thresholding technique. So, first we will discuss about the thresholding technique for segmentation. Now, thresholding is one of the simplest approach of segmentation. Suppose, we have an image and as we have said earlier that image is described by a represented by a 2 dimensional function f(x, y) and let us assume that this image contains dark object against a light background.

so, in such cases if there is a dark object against a light background or even if it is the reverse that we have a light object against a background then you will find that the intensity values they are mainly concentrated near two regions or we call them two modes. One of the regions will be towards the darker side or towards the lower intensity values and other one other mode will be towards the brighter side or towards the higher intensity values.

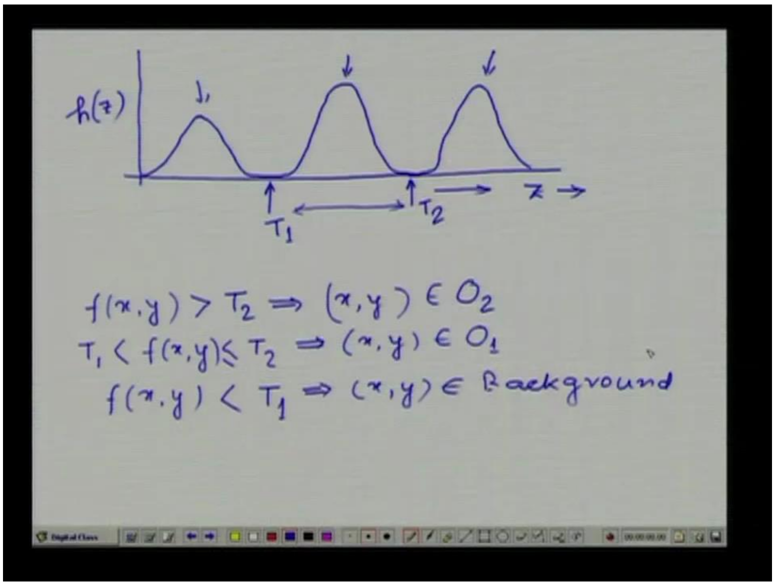
So, if you plot the histogram of such a image, so here we assuming that we have one object and let us assume that the object is brighter and the background, the background is dark. So, if we plot the histogram of such an image the histogram will appear something like this. So, on this side we put say intensity value z and this side say is our histogram of z. So, as we said that because we having one object and we are assuming that the object is bright which is placed against dark background.

So, the intensity values will tried to be accumulated will the histogram will give rise to a bimodal histogram where the intensities will be concentrated on dark side as well as on the brighter side. So, for such a bimodal histogram you find that there are two peaks. One peak here and the other peak here and these two modes or these two peaks are separated by a deep valley. So, this is the valley and this is one peak and this is the other peak and as we have assumed that our object is bright and the background is dark so all these pixels which are grouped in the lower intensity region. These pixels belong to background and the other group of pixels they belong to the object.

Now, the simples form of segmentation is if we can choose a threshold value say t in this valley region and we take a decision that if a pixel at location (x, y) or the have the intensity value f(x, y) which is say greater than t, then we say this pixel belongs to object, whereas if f(x, y) is less than or equal to the threshold t then this pixel belongs to the background. So, this is our simple decision rule which is to be used for thresholding purpose. So, what we have to do is we have to choose a threshold in the valley region and then check the image.

The segmentation is simply testing each and every pixel to check whether it is intensity value less than the threshold or the intensity value is greater than the threshold. So, if the intensity value is greater than the threshold then we will say that it belongs to the pixel belongs to an object, whereas if the intensity value is less than or equal to threshold we say that the pixel belongs to the background.

(Refer Slide Time: 10:37)



Now, the situation can be even more general that is instead of having bimodal histogram we can have multimodal histograms that is a histogram can even be of this form like this. So, this is our pixel intensity set and of this side is the histogram. So, here you find that the histogram has three different modes which are separated by two different values. So, now what we can do is we can choose one threshold say t1 in the first valley region and the other threshold t2 in say second valley region.

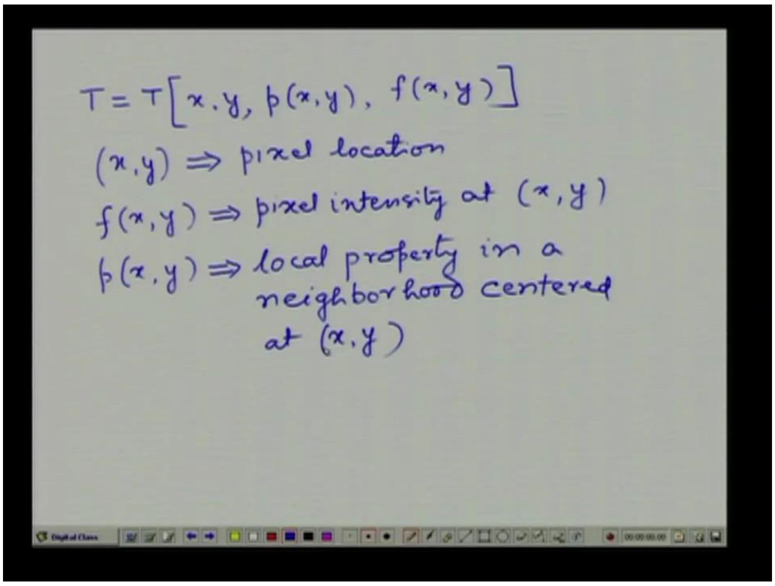
So, what this histogram indicates is that there are three different regions or three different intensity regions which are separated by some other intensity band. Okay and those three different intensity regions are represented or gives rise to these three different peaks in the histogram.

So, here our decision rule can be something like this that if we find that the intensity value f(x, y) at a pixel location (x, y) is greater than threshold t2 then we say that the point (x, y) belongs to say objects o2. So, all the intensity values, all the pixels having intensity values greater than t2, these pixels we will say that they belong to the object o2. In the other case if a pixel has an intensity value in this region that is greater than t2 greater than t1 and the less than t2 then we will say that this particular pixel belongs to object o1.

So, our decision rule will be that t1 less than f(x, y) less than or equal to t2 then this indicates that the corresponding pixel (x, y) it belongs to object o1 and obviously, the third condition will be that if f(x, y) the intensity value at a location (x, y) is less then threshold t1 in that case we say that the corresponding pixel (x, y) it belongs to the background.

So, even in cases we can have histograms which are even which will have even more number of peaks, more than three peaks. Such cases also similar such classification is possible but what we have to do for this thresholding based segmentation technique is that we have to choose proper threshold values.

(Refer Slide Time: 14:17)

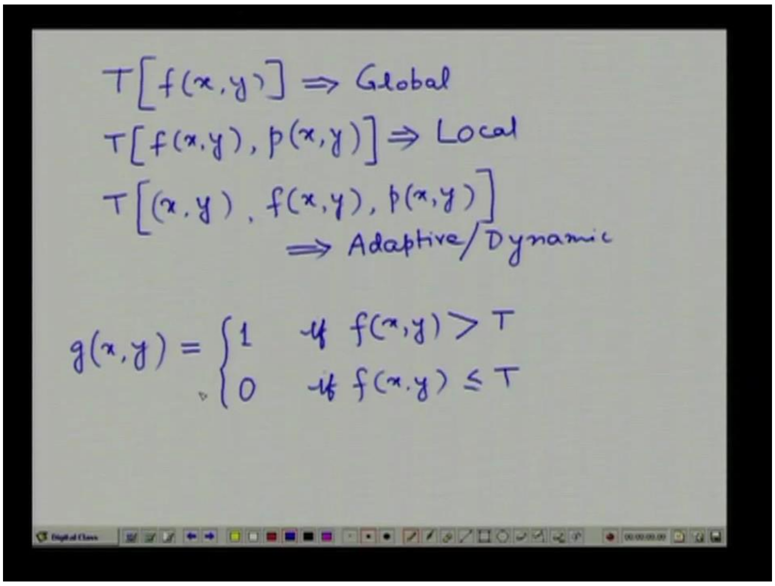


Now, this threshold value or the thresholding operation can be considered as an operation that involves testing against a function t where this function t is of the form t is equal to (x, y), p(x, y) and f(x, y). So, this thresholding operation what we are doing is we are considering or this can be viewed as an operation to test the image pixels against a function t where this function t is of this form, this function t is a function of (x, y) which is nothing but the pixel location in the image, f(x, y) which is nothing but the intensity value at location (x, y).

So, this is pixel intensity at location (x, y) and p(x, y) it is some local neighborhood property, some local property in a neighborhood centered at (x, y). So, in general this threshold t is a function, can be a function of pixel location the pixel value as well as the local property within a neighborhood around a pixel location (x, y). So, any combination of these three that is pixel location, pixel value and neighbor property, neighborhood property. This neighborhood property can even be the average intensity values within a neighborhood around pixel (x, y).

So, any combination of this t can be a function of any combination of these three terms and depending upon this combination this t can be either a global threshold or a local threshold or it can even be an adaptive threshold.

(Refer Slide Time: 16:57)



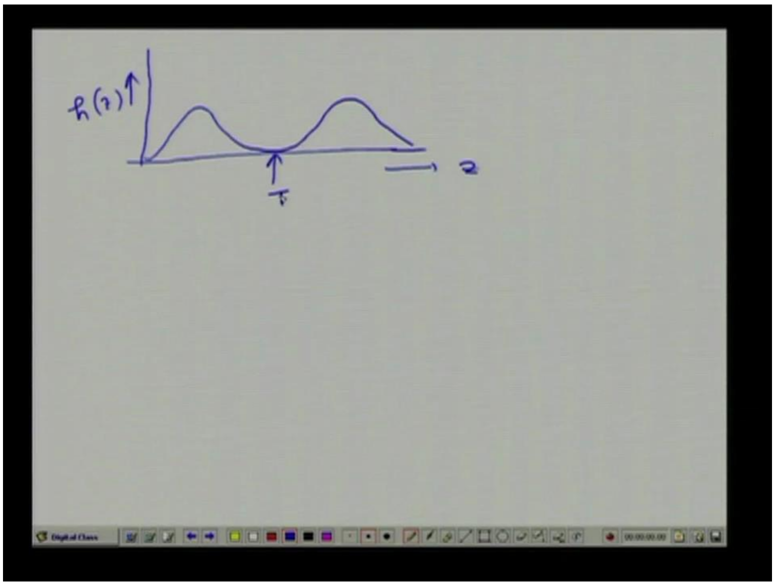
So, in case the t is the threshold t is only a function of f(x, y) we say that the threshold is a global threshold whereas if t is a function of f(x, y) and the local property that is p(x, y) then we say that the threshold t is a local threshold. And if in the addition to all these t is also a function of the location of the pixel that is in the more general case if t is the function of (x, y), f(x, y) as well as p(x, y) then we say that this threshold t is an adaptive or dynamic threshold.

Now, whichever the nature of the threshold of the t is whether it is local or global or adaptive our thresholding operation is by using this threshold we want to create a thresholded image say g(x, y) from our input image f(x, y) and we set the value of g(x, y) is equal to 1 if the corresponding function or the intensity of the image at that location that is f(x, y) is greater than the threshold t.

Now, this threshold t can be either global or local or adaptive and we set g(x, y) is equal to zero if f(x, y) is less than or equal to the chosen threshold t. So, you find that the basic aim of this thresholding operation is we want to create a thresholded image g(x, y) which will be a binary image containing pixel values either zero or one and this value will be set to zero one depending up on whether the intensity f(x, y) at location (x, y) is greater than t or it is less than or equal to t.

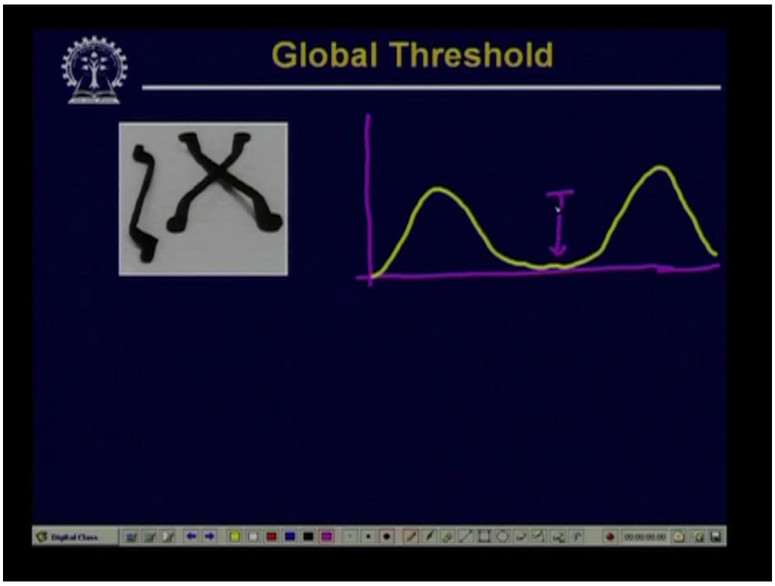
so, if we have a bright object against a dark background, in that case g(x, y) equal to one this indicates that the corresponding pixel is an object pixel, whereas g(x, y) equal to zero this will indicate the corresponding pixel is a background pixel. On the contrary if we have dark objects against bright background in that case what we will do is we will set g(x, y) equal to one if f(x, y) is less than or equal to t again indicating that in the thresholded image, a pixel location having an intensity value of one, that indicates the corresponding pixel belongs to the object. And in such case we will put g(x, y) equal to zero if f(x, y) is greater than t again indicating that the pixel in the thresholded image g(x, y), if it is equal to zero, the corresponding pixel is a background pixel.

(Refer Slide Time: 21:11)



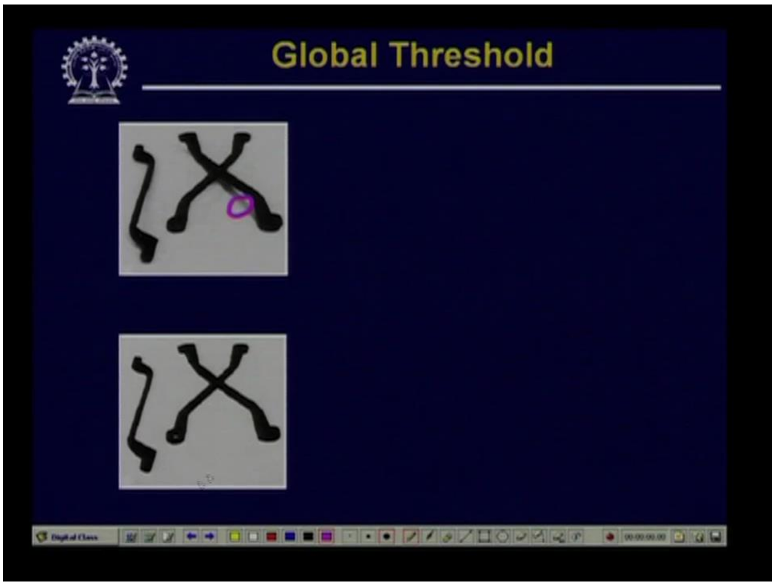
Now, the question is how to choose this threshold value? Now, for that let us come to the case again considering the histogram we have said that if my histogram is a bimodal histogram of this form then what I can do is by looking at the histogram. So, this is our intensity value z and on this we have h (z). By inspecting this histogram we can choose a threshold in this deep valley region and using this threshold I can go for the segmentation operation.

(Refer Slide Time: 21:47)



Now, by doing this I will show you one particular result say, for example, in this particular case. Here, you find that we have an image where the objects are dark whereas the background is bright. So, naturally in this case I will have a histogram where the histogram will be a bimodal histogram. So, the nature of the histogram will be like this. So, this will be a bimodal histogram.

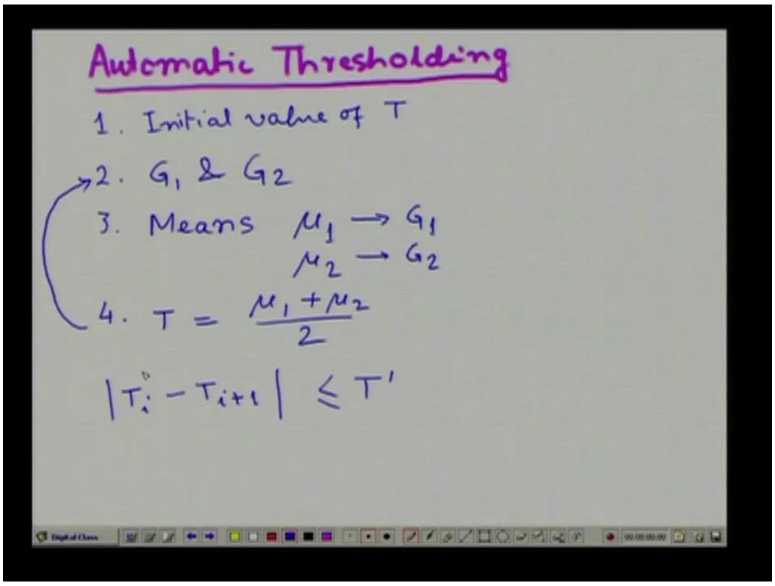
(Refer Slide Time: 22:54)



So, here if I choose a threshold t in this region and using this threshold I segment this image then the then the kind of segmentation that we get is as given here. So, here you find in this second image in the segmented image that your background and object regions have been clearly separated, even the shadow which is present in the original image that has been removed in the segmented image.

So, this segmentation though it is a very simple operation if you choose the threshold in the valley region between the two modes in a bimodal histogram then this segmentation, the simple segmentation operation can clearly take out the object regions from the background. But here what we have done is we have chosen the histogram to choose the threshold that is you inspect the histogram and then from inspection of the histogram you have to choose the threshold value. But is it possible to automate this process? That is instead of finding the histogram by instead of finding the threshold value by looking at the histogram, can we automatically determine what is the threshold value which should be used for segmenting an image?

(Refer Slide Time: 24:11)



So, this operation can be done by using an iterative procedure, so automatic threshold. So, here again for detecting this threshold automatically what we can do is, we can first choose an initial value of threshold. So, arbitrarily or by or somehow we can choose an initial value of threshold and using this initial value of threshold what we can do is, we can have a segmentation of the image.

So, when we segment the image using this initial value of threshold the segmentation operation basically will partition your histogram into two partitions or the image will be divided into two groups of pixels. So, we can say that one group of pixels we termed them as group g1 and the other group of pixels we term them as group g2. So, the pixel intensity values in group g1 will be similar and the pixel intensity values in group g2 will also be similar but these two groups will be different.

Now, once I separate a partition the image intensities into these groups g1 and g2 the next step that what we will do is, you compute the means or the average intensity values mu1 for group g1 and the average intensity value mu2 for group of pixels g2. So, once I get this mu1 and mu2 that is the average intensity value in the group of pixels g1 and also the average intensity value for the groups of pixels g2 then in the fourth step what I do is, I choose a new threshold t which is equal to mu1 plus mu2 divided by 2. And after doing this you go back to step to two and perform the operation thresholding operation once again.

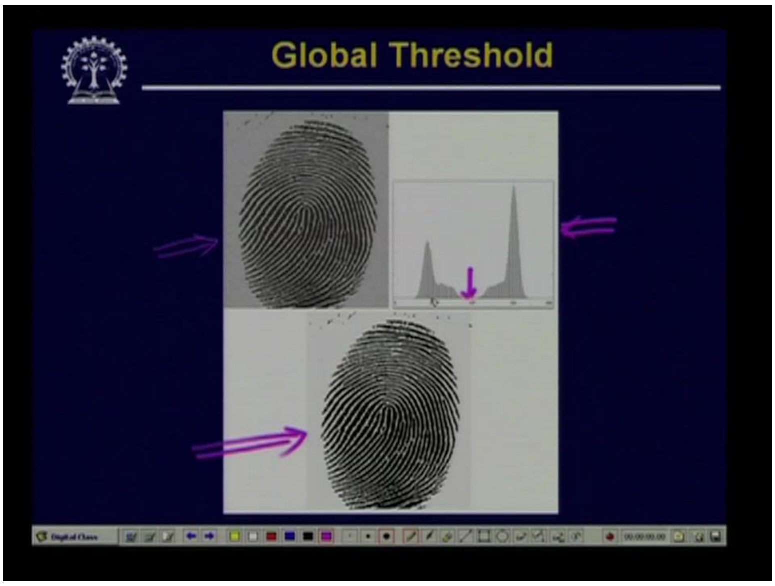
So, what we are doing is, we are choosing an initial value of threshold using that initial value of threshold we are thresholding the image. By thresholding what we are doing is, we are separating intensity values into two groups g1 and g2. For group g1 I find out the average intensity value mu1 for group g2 I also find the average intensity value g2 then I find out a new threshold which is the mean of these two averages that is mu1 plus mu2 by 2 and using this new threshold I threshold the image again.

So, thereby these groups g1 and g2 will be modified and I repeat this process that is thresholding to grouping then finding out the intensity averages in the two different groups two separate group. Recalculating the threshold this entire process will be repeated until and unless I find that the variation in two successive iterations in the computed value of t is less than some pre-specified value.

So, this operation has to continue until you find that in a one at iteration ti and the next iteration ti plus 1 the threshold value in the ith iteration ti and in the I plus, first iteration ti plus 1. The difference between these two is less than or equal to some pre-specified value say t prime. So, when I attempt this condition I stop my thresholding operation. So, here you find that we do not have to go to the histogram to choose the threshold rather what we do is we choose some initial value of threshold.

Then go on modifying this threshold value iteratively finally you converge, you come to a situation where you find that in two subsequent iterations the value of the threshold does not change much and at that position whatever the thresholded image that you have got that is your final thresholded value.

(Refer Slide Time: 29:18)

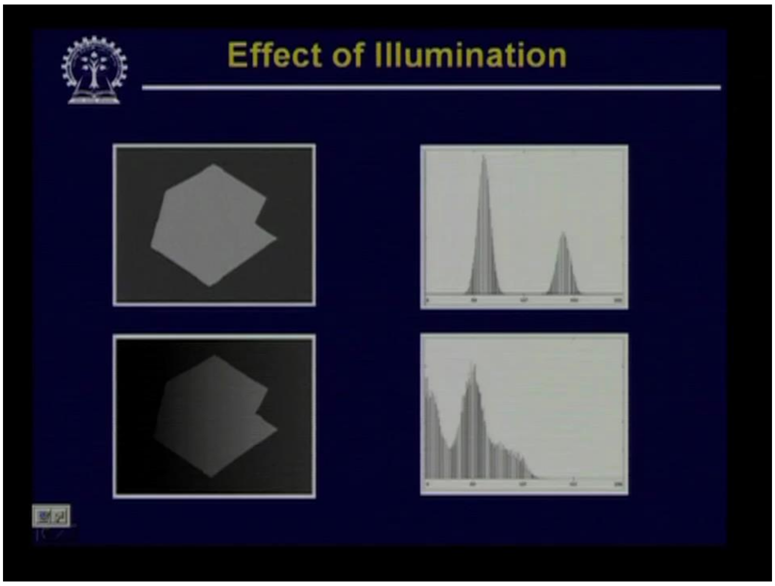


So, using this kind of adaptive threshold the kind of result that can be obtained is something like this. So, here you find that this is one input image and you can identify this that this is fingerprint image. this is the histogram of that particular image. So, obviously in this from this histogram also I can choose a threshold here. But this thresholded output that has been obtained is not by choosing a threshold from the histogram but this is by automatic threshold selection process that is by doing this iterative process.

And it can be observed that from this histogram whatever threshold you chose by this automatic process the threshold will be similar to that, and here you find that since the threshold that you have chosen this does not consider the pixel location or the local neighborhood of the pixel intensity values, here the threshold is a global one that is for the entire image you choose one particular threshold and using that threshold you go for segmenting the image.

So, the kind of thresholding operation that we have done in this particular case this is called a global thresholding operation. Now, you find that in this particular case this global thresholding will give you very good result if the intensity or the illumination of the scene is uniform. But there may be cases where the scene illumination is non-uniform and in case of such non-uniform illumination getting a global threshold which will be applicable over the entire image is difficult.

(Refer Slide Time: 31:11)

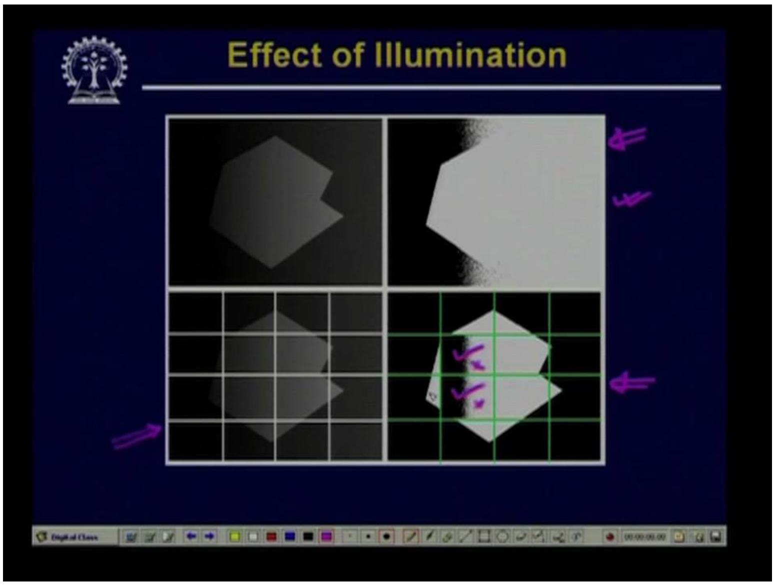


So, let us take one particular examples in this particular case. On the top we have an image and you can easily find out that for this image if I plot the histogram. The histogram will be as shown on the right hand side. Clearly, this histogram is a bimodal histogram and there is a valley in between the two modes. So, these two modes a separated by a deep valley. So, obviously for such a kind of histogram I can always choose a threshold inside the valley and segment this image successfully.

But what happens if the illumination is not proper? If the background illumination is not uniform, then this image because of this non-uniform illumination may turn out to be an image like this. And whenever I have such an image with poor illumination and you find that the histogram of this image appears as given on the right hand side, and here you find that though the histogram appears to be a bimodal one but the valley is not well defined.

So, this simple kind of thresholding operation or the global thresholding operation is likely to fail in this particular case. So, what should we do for segmenting this kind of images using a thresholding operation? Now, one approach is you sub-divide this image into a number of smaller sub images assuming that in each of this sub image the intensity will be more or less uniform or the illumination is more or less uniform then for each of the sub image we can find out a threshold value. And using this threshold value you can threshold the sub images and then the combination of all of them or the union of all of them will give you the final thresholded output.

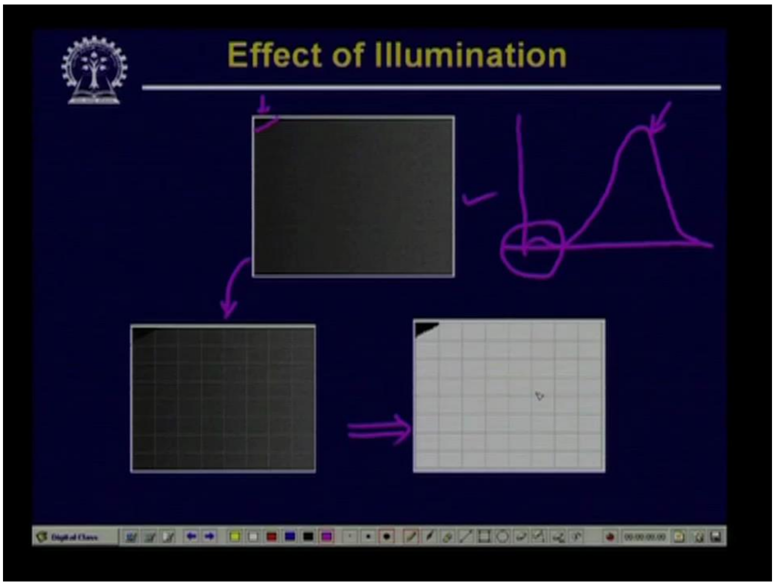
(Refer Slide Time: 33:09)



So, let us see what you get in this case. As we said that for this kind of images where the illumination is non-uniform if I apply a single global threshold then the kind of output the thresholded output that we are going to get is something like this. So, here you find that the thresholding has failed miserably. Whereas if I sub divide this image into a number of sub images as given on this left hand bottom and then for each of these sub images I identify the threshold and using that thresholding you go for segmenting that particular sub image and the thresholded output that you get is given on this right hand side.

Here, you find that excepting these two rest of the sub images have been thresholded property. So, at least your result is better than what you get with a global threshold operation. So, now because we are going for a threshold selection of a threshold which is position dependent because every sub image has a particular position. So, now because this threshold selection is position dependent it becomes an adaptive thresholding operation. Now, let us try to analyze that why this adaptive threshold has not been successful for these two sub regions.

(Refer Slide Time: 34:43)



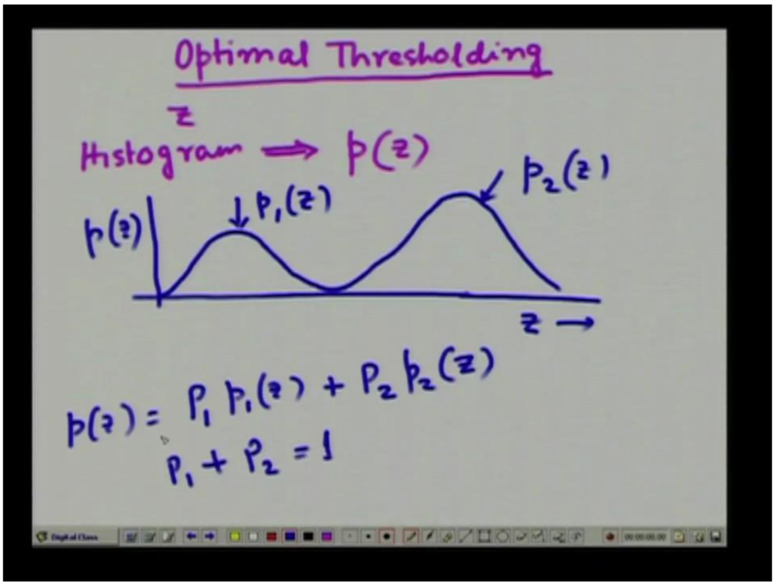
So, if I look at the nature of the image here if you look at this top image you find that in this top image here is a boundary. Okay where this small portion belongs to the background and this large portion of the image belongs to the object. Now, if I plot the histogram of this the histogram will be something like this, because the number of pixels in the background is very small so the contribution of those pixels to the histogram that is within this region is almost negligible. So, instead of becoming a bimodal histogram the histogram is dominated by a single peak, and that is the reason why this threshold operation has not has not given good result for this particular sub region.

So, how to solve this problem? Again our solution approach is same you sub divide this image into smaller sub division. So, you go for sub dividing further and for each of these smaller sub divisions now you try to find out the threshold and segment each of the sub divisions with each of the sub sub divisions using this particular threshold. So, if I do that you find that the kind of result that we get is here and here in the segmentation output is quite satisfactory.

So, if the scene illumination is non-uniform then a global threshold is not going to give us a good result. So, what we have to do is? We have to subdivide the image into a number of sub regions and find out the threshold value for each of the sub regions and segment that sub regions using this estimated threshold value. And here because your threshold value is position dependent, it depends up on the location of the sub region, so the kind of thresholding that we are applying in this case is an adaptive thresholding.

Now, in all these thresholding whether it is global thresholding or adaptive thresholding that we have discussed so far none of this cases we have talked about the accuracy of the thresholding or how accurate or what is the error that has been involved that is by this thresholding process.

(Refer Slide Time: 37:33)



So, we can go for a kind of thresholding by making use of some stastical property of the image where the mean error of the thresholding operation will be minimum. So, that is a kind of thresholding operation which is called optimal thresholding. So, what is this optimal thresholding? Again let us assume that the image contains two principle gray levels intensity regions.

One intensity region corresponding to the object and the other intensity region corresponding to the background. And we use a variable and we assume that this intensity variables can be modeled as a random variable and this random variable is represented by a variable say z.

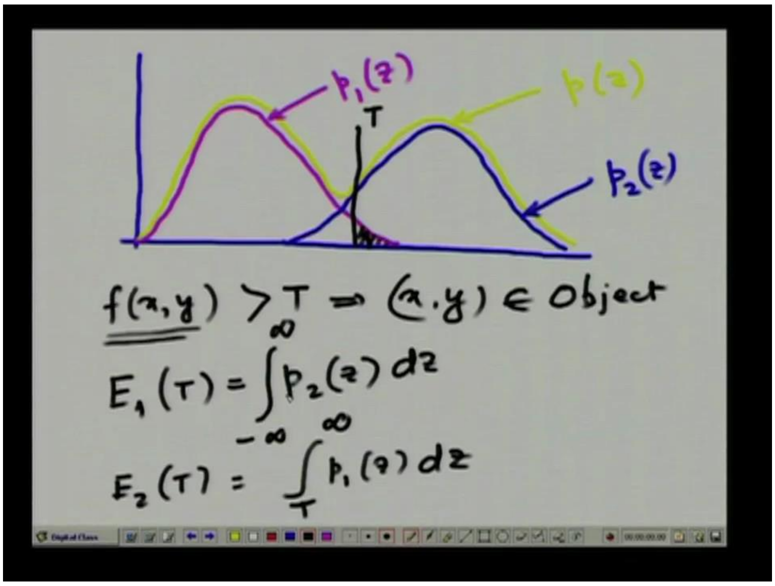
Now, once we represent the random variable by this z then the histogram of this particular image or the normalized histogram can be viewed as a probability density function of this random variable z. So, the normalized histogram can be viewed as a probability density function p(z) of this random variable z. Now, as we have assumed that the image contains two major intensity regions two dominants intensity values.

So, our histogram is likely to be a bimodal histogram. So, the kind of histogram that we will get for this image is a bimodal histogram. So, it will be something like this or, and ya as we said that threshold histogram we are assuming to be to be a density function of the intensity variable z. So, this bimodal histogram can be considered as a combination of two probability density functions or combination of two pdfs. Okay. So, one of them is say probability distribution function p1 (z) the other one is probability density function say p2 (z).

So, p1(z) indicates the probability distribution function the probability density function of the intensities of pixels which belong to say background and p2(z) is the probability density function of the pixel intensity values which belong to say object. Now, this overall histogram that is p (z) can now be represented as the combination of p1 (z) and p2 (z). So, this overall p (z) we can write as p1 into p1 (z) plus p2 into p2 (z), where this p1 indicates the probability that a pixel will belong to the background and p2 indicates that indicates the probability that a pixel belongs to an object.

So, obviously this p1 plus p2 this will be is equal to 1. So, these are the pixel probabilities which belong to either foreground or the background. So, here our assumption is that we have a bright pixel against a dark background because we are saying that p1 sorry, ya the p1 it is the probability that a pixel belongs to the background and capital p2 is the probability that a pixel belongs to the foreground or the object.

(Refer Slide Time: 42:01)



Now, what is our aim in this particular case? Our aim is that we want to determine a threshold t which will minimize the average segmentation error. Now, we find that since this overall probability is modeled as a combination of two different probabilities, so, it is something like this. I can say that I have one probability distribution function which is given by this and the other probability distribution function is say given by this, so that my overall probability distribution function is of this form.

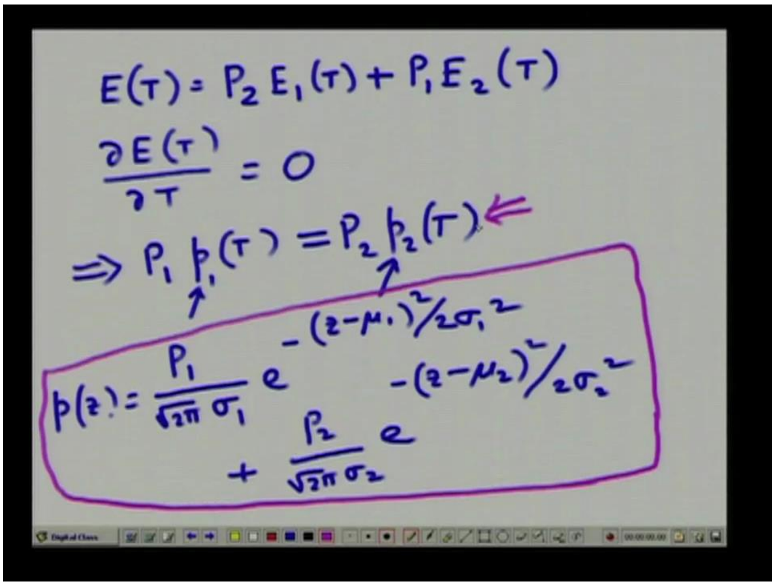
This is my overall probability distribution function. So, this blue color this indicates p2 (z) and the pink color this indicates p1 (z) and the yellow color indicates my overall probability density function that is p (z). Okay. So, in this particular case if I choose a threshold t somewhere here. So, this is my threshold t and I say that if f(x, y) is greater than t then (x, y) belongs to object. Okay.

Now, here you find that though we are taking a hard decision that if f(x, y) is greater than t then (x, y) belongs to object but the pixel with intensity value f(x, y) also has a finite probability say given by this that it may belong to the background. So, while taking this decision we are incorporating some error. The error is the area given by this probability curve for the region intensity value greater than t.

So, the probability of considering a background point as an object point of the error leads to an error. Okay, that is a background point may be classified as an object point. So, the error that you encounter in that particular case is given by say e1 (t) because this error is threshold t dependent. So, write this as e1 (t) is equal to say p2 (z) dz, take the integral of this minus infinity to infinity. So, what is this? This is the probability that this is the error incorporated that an object pixel may be classified as a background pixel.

Similarly, if a background pixel is classified as an object pixel then the corresponding error will be given by e2 (t) is equal to integral p1 (z) dz where the integral has to be taken from t to infinity. So, this gives you the two error values one of them gives the error that you encounter if you classify a background pixel as an object pixel and the other one if you segment object pixel as a background pixel.

(Refer Slide Time: 46:01)



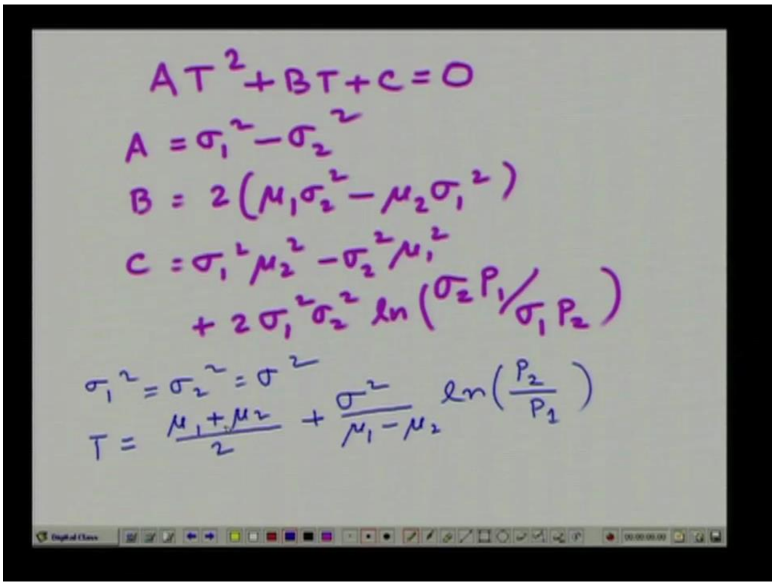
So, from these two error expressions the overall error probability can now be represented as e (t) is equal to capital p2 into e1(t) plus capital p1 into e2(t). So, you find that this e2(t) was the probability it was the error of classifying a background pixel as a foreground pixel and e1(t) was the of classifying an object pixel as a background pixel, and p1 is the probability that a pixel belongs to background and p2 is the probability that a pixel belongs to the object. So, the overall probability of error will be given by this expression capital p2 into e1(t) plus capital p1 into e2(t).

Now, for minimization of this error what we have to do is we have to take the derivative del e (t) delt and equate this to zero. So, whatever is the value of the t that you get that what is going to give you minimum error. So, if I put this restriction then this above expression we are not going to the details of mathematical derivation, I will just give you the final result this can be given by capital p1 into p(t) p1(t) plus capital p2 into p2(t). Sorry, this is not plus this is equal.

So, we are going to get an expression of this form. And the solution of this equation gives the values of t. So, if we try to solve this you find that what I need is the knowledge of this probability density functions p1(t) and p2(t). So, as we know that in most of the cases we normally assume the gaussian probability density function. So, if I assume that gaussian probability density function, in that case the overall probability p(z) is represented by capital p1 divided by square root of 2 pie sigma1 e to the power minus z minus mu1 square by 2 sigma1 square plus capital p2 by square root of 2pie sigma2 e to the power minus z minus mu2 square by 2 sigma2 square.

Where mu1 is the average intensity value of the background region and mu2 is the average intensity value of the object region and sigma1 and sigma2 they are the standard deviations of the intensity values in the background region and the intensity values in the object region.

(Refer Slide Time: 49:56)

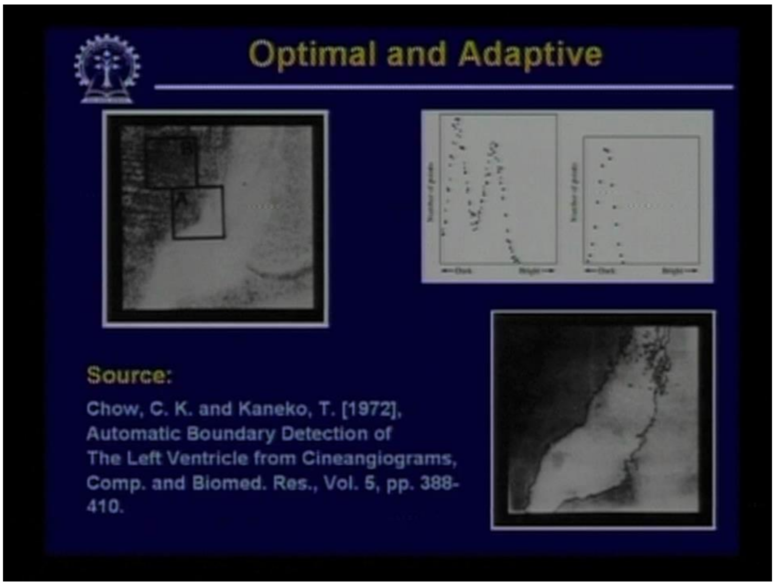


So, by assuming this gaussian probability density function we get the overall probability density function as given by this expression. And by assuming this and then from this particular expression the value of t can now be found out as the solution for t is given by, solution of this particular equation at square plus bt plus c is equal to zero, where this a is equal to sigma1 square minus sigma2 square, b is equal to 2 into mu1 sigma2 square minus mu2 sigma1 square and c is given by sigma 1 square mu2 square minus sigma 2 square mu1 square plus 2 sigma1 square sigma2 square ln sigma2 capital p1 by sigma1 capital p2.

And here if we assume that sigma1 square is equal to sigma2 square is equal to say sigma square then the value of the threshold t comes out to be t is equal to mu1 plus mu2 divided by 2plus sigma square up on mu2 mu1 minus mu2 ln capital p2 divided by capital p1. So, this is a simple expression for the value of threshold that you can obtain in this optimal thresholding operation. And this is optimal in the sense that these value of the threshold gives you minimum average error. And here again you find that if the probability the capital p1 and capital p2 they are same, in that case the value of t, t will be simply mu1 plus mu2 by 2 that gives the mean of the average intensities of the foreground region and the background region. Okay.

So, as we said that by estimating a threshold by this process if we segment the image then the average error of segmentation will be minimum that is minimum number of foreground pixels will be classified as object pixels and minimum number of object pixels will be classified as foreground pixels. Now, let us see an example that why this optimal thresholding can give us good results.

(Refer Slide Time: 52:55)



Let us take a very complicated case like this. This is the cardiogram angiography which in which the purpose is to detect the ventricle boundaries. Okay. You find that the image that is given here is very very complex and though we can somehow figure out that there is a boundary somewhere here but it is not very clear. So, the approach that was taken is this image was divided into a number of sub images, for every sub image the threshold was estimated, the optimal threshold was estimated, and then the thresholding was done.

So, for this optimal thresholding what was done is, for each of the sub image say, for example, this was divided into a number of sub images like this. For each of the sub image what was computed is the histogram and the threshold was computed for those sub images which shows a bimodal histogram like this. Whereas you find that if I take a sub image here this normally shows a unilevel histogram which is given here.

This for these sub images no threshold was detected the threshold was detected only for those sub images which showed bimodal histogram. And the threshold for other sub images where estimated by interpolation of the threshold of the regions having bimodal histogram. And then a second level of interpolation was done, iteration was done to estimate the threshold value at each of the pixel locations and after doing that for each pixel location using that particular threshold the decision was taken whether the corresponding value should be equal to zero or the in the thresholded image the corresponding value should be equal to 1.

So, using this the thresholded image was obtained and the boundary of such thresholded image when superimposed on this particular image you find that this one shows the boundary of the thresholded image. So, as was estimated that this was the estimated boundary the boundary points are quite well estimated in this particular case. So, with this we stop this particular lecture on thresholding operations. Thank you.