

Design Principles of RF and Microwave Filters and Amplifiers
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Module # 3
Lecture # 11
Gain Definitions of Microwave Amplifiers

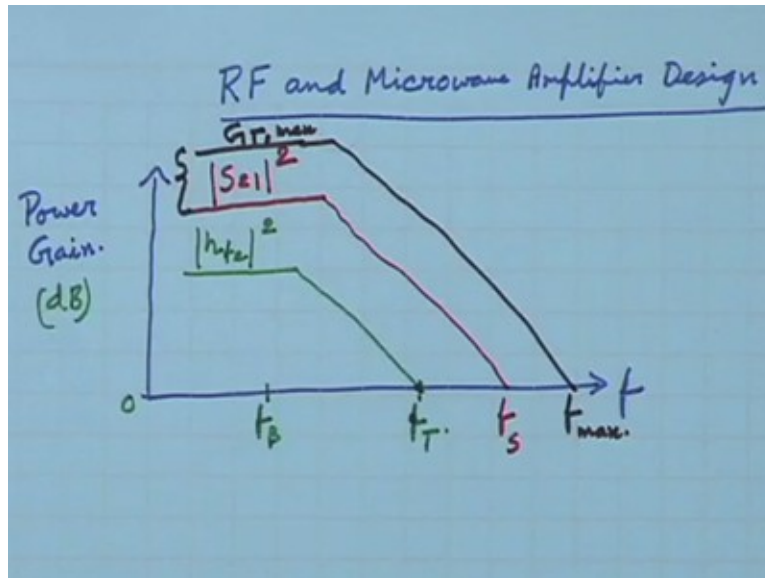
Welcome to this lecture series today will be starting the RF microwave and amplifier design. We have in the previous ten lecture we have seen the filter design microwave filter design. Today will start Amplifier Design. You know the Signal Amplification is one of the most basic needs of the Electronic Circuits. We are mainly concern with power amplification. We don't Generally not about the Voltage amplification as happens in low frequency or baseband cases.

Here we are bothered about the power amplification because, microwave power is at premium. It is very costly to produce microwave power. Also we generally wirelessly transmit that power. So we want sufficient power to receive get received at the receiver. That is why we take special care for microwave amplifier design. Also you know that after world War II when microwave Sources like Guyton, travelling wave tube, Mecatrons etc. were invented that time the microwave amplifier were based on tubes.

But with the advances in semiconductor technology now a days many transistor based amplifiers are used in Microwave region and already NPTEL courses, we have seen the microwave sources how those transistors, particularly the GaAs transistors gas fet then HBT HEMT etc., are used. What are the design principles please refer there?

Now in this course we will consider the transistor based amplifier as a block box. Again as a 2 port network and we would not see the inner mechanism or the device mechanism of a transistors. That we already seen earlier. Now we will see a S Parameter based description of the transistor. So we care 2 port S Parameterized transistor. We will try to design an amplifier and will try to get power amplification by that.

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Now if we compare the low frequency and high frequency power gains. We will see that at low frequency you know that low frequency we generally describe the transistor with a H parameter and most important parameter of that is h_{fe} . So the generally that power gain at low frequency something like this. This is generally h_{fe}^2 . So the this h_{fe} is the short circuit current gain. Now generally it power gain becomes if we express this in dB scale then at 0dB that means 1 value of 1 power gain so it is 10.

So this frequency at which the SC short circuit current gain h_{fe} that falls to 1 h_{fe}^2 that is called the beta cutoff the maximum usable frequency of the transistor and there is also relevant parameter called f_{β} which is the frequency at which DC short circuit current gain beta of the transistor falls to 1. f_T is much higher than f_{β} . But if we go higher we see that in microwave region. This H parameter description is not adequate there as I said that a whole transistor. Transistor we represent by S parameter with an characteristic impedance level Z_0 .

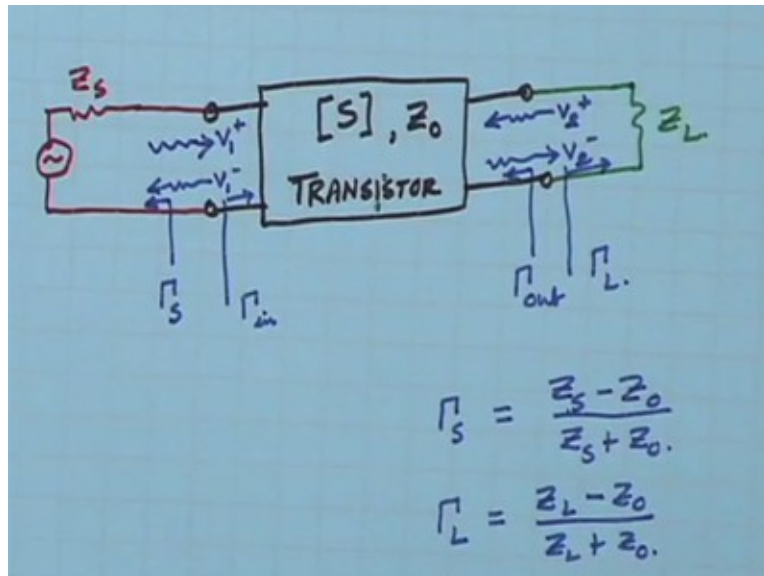
And if we see the typical power variation in the dB scale. That takes the form of this that generally this S_{21} . If we take the pure transistor, and if we try to find out that if we give some signal and without any loading or proper matching. Then this S_{21}^2 square that for certain region it is flat in dB scale and then after that it generally falls off generally with a roll off factor. And this frequency where the power gain becomes 1 that is called f_s .

The frequency at which this power gain of the transistor alone that falls to 1. However, by proper design we can have a improve this, We can add some extra gain and that actually will be the topic of this lecture series. That we can improve this part and make it go upto a more higher frequency F_{max} . Now this one we give a new name called GT_{max} . This GT is called transducer power gain and this maximum achievable thing is somewhat higher than is S_{21}^2 so this is the play that a microwave engineer or an RF engineer who carefully designs an amplifier.

He can get this much extra gain by proper designing and also he can extend the frequency a bit to F_{max} . So this difference between f_S and f_{max} that allows or RF engineer should know how to exploit this gap. And so he can extend or push the amplifier design to higher value. Obviously, below above this frequency no will be willing to use this. Because, the power gain that is falling to 1. So these transducer power gain is a new quantity or I will say that in microwave region we need to look carefully at various gain definitions.

I am saying from the beginning of this course and all NPTEL courses, that at high frequency or at microwave region we are very careful of the maximum power transfer and we are very careful of the impedance matching in one of the first NPTEL lectures for this microwave technology we have elaborately said about impedance matching. Now it when impedance matching is given its (()) (08:42) importance. The in a microwave based amplifier there are many as power gains that are possible.

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So, we will go into that now and for that we again assume that the transistor is represented by a 2 port network. This is a transistor block obviously, this is a microwave transistor. Generally, you know that HEMT or HBT this transistors we can use up to 100GHz people also trying to push it further. Now this transistor for our purposes will be represented by its 2 port is S parameter and also will assume that the character impedance level of this transistor is Z_0 .

Now obviously, this one will be connecting with the a source and a load. So, let us put SC source is here. And let us that source will have typically an source resistance. Similarly, we will be connecting in this side is to the load of load impedance is Z_L . Now, you see that there are what happens to the waves here so we can say that actually the terminal voltage here is B_1 .

Actually, we know that the at high frequency actually at all frequency but at high frequency we generally cannot neglect that actual signal propagation is in the form of waves. So, there will be you know for scattering parameter is define in terms of that voltage waves. So we have an incidence voltage wave going to here that will call V_{1+} .

Also depending on the mismatch. We know there will be a V_{1-} here. And V_1 the sum of these at this port, port 1 V_{1+} and V_{1-} that will be calling V_1 . Also, we know that here the something will happen. That from this port there will be a incident voltage V_{2+} that will be entering. Because of this in general there will be mismatch here. So this V_{2+} will come and also

there will be the V_2^- . Again the port voltage is nothing but V_2^{++} and V_2^- . And also will see that there will be various reflections.

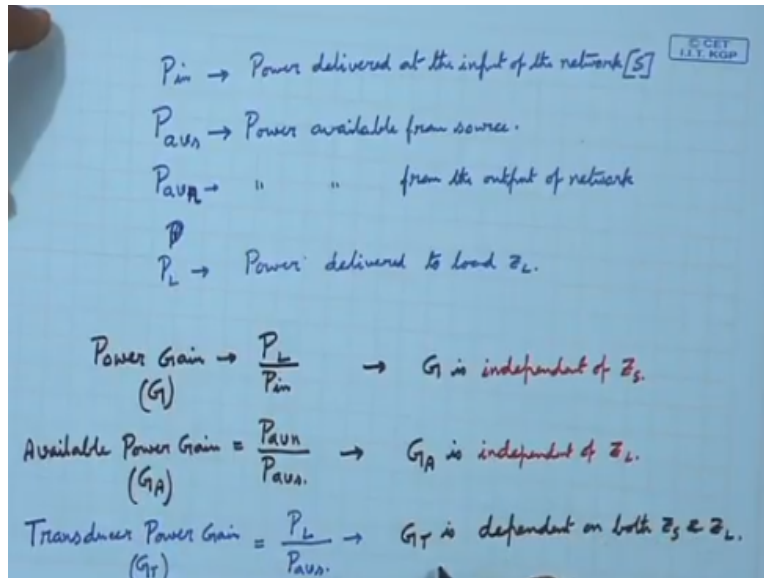
So if we look at the source side we know if we look at here we can see that there will be some source reflection. Similarly, if we look at here there will be some input reflection because input impedance will be seen here and depending on the impedance level there will be input reflections.

Similarly, from here if we look we can say that there will be a output reflection coefficient also from here if we look there will be a load reflection coefficient. Now, we know what are this value of this reflection coefficient. Let us see that this Γ_S the source reflection coefficient that we know that will be given by $Z_S - Z_0$ by $Z_S + Z_0$ and Z_S is a complex number. Z_0 is generally is a real number. So, Γ_S is a complex reflection coefficient because this is a complex number.

Similarly at the load side due to mismatch between Z_L and Z_0 . There will be $Z_L - Z_0$ by $Z_L + Z_0$. Now you see that I have certain power depending on what is the input impedance I am seeing here. This Z_S will be able to transfer a part of that power. We know that if Z_S is chosen as Z_{in} conjugate then maximum power can be transferred to this transistor. Again here this transistor which is has impedance level Z_0 . It is seen some output impedance so depending on that it will transfer a part of that power.

If we can have a conjugate match here. Then we know that it will able to deliver that power. And then again depending on the this output impedance and Z_L . We can have a maximum power transfer here. But also we won't be able to do maximum power transfer. So that time some amount of power transfer will be there. So we define three definitions of power.

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You see here with respect to this figure. We will say now three definitions of the power. One is that Power avs this is actually power available from source. That means the power that source can deliver that we are calling power available from source. Similarly, we have here actually we are calling this transistor it is a 2 port network. So it is in the symbols this will call as network.

So, Pavn this is power available from the output of network which actually for the amplifier is this transistor part. Then also, we have P or before this I will say Pin. Where Pin is power delivered at the input of the network S. Similarly I will have PL which is power delivered to load ZL. So, you see that I can now write four things. That means the power. Input power is what power is going from here.

Then here it is taking P some of this power because all power depending on the mismatch all power cannot come. So P avs is the power that is available here. Then again P avn is the power that is available at the output. And again depending on the impedance level the power that gets delivered is PL.

So there are this four different power things. Which if everything is properly matched then I know that means if I do conjugate matching from maximum transfer theorem view point. Then I know whatever Pin it is supplying that maximum power will come here. That means generally

that is for resistive loads half of this power. Then again here due to this impedance level some of that will come here. From that again if I can do maximum power will be transferred here.

Now based on these I have 3 definitions of power gain. You see that there are three definitions of power gain the first one is Simple Power Gain. But is it it is P_L by P_{in} so P_L we know power to load and P_{in} is the power delivered at the input of the network S power delivered at the input of the network S that means this G if this generally called G .

So G I can say you see that G P_{in} power delivered at the input of network is so this is P_{in} this is my P_{avs} power available from the source this is P please correct it or again am writing that this my P_{avs} this is my P_{in} . Similarly this is P_{avn} power available from network this is P_L . So you see that when I am saying this that means I am basically taking the power ratio that how much power that I can deliver to the load. But am taking P_{in} here the input to the network that means that I am not talking Z_S level into consideration so I can say G is independent of Z_S

Similarly, I can have the next one is called available Power Gain. It is generally denoted as G_A and this is P_{avn} by P_{avs} . Where avn and avs are there that means what I am seeing here P_{avn} by P_{avs} here you see that I am considering power available here divided by this power that means here I have been incorporate Z_S . But, I have not incorporated Z_L this loading I have not considered. So I can say that G_A is independent of Z_L . So obviously this G is dependent on Z_L independent on Z_S .

This G_A is dependent on Z_S independent on Z_L . So in both this definition you can see I am keeping this open that in 1 case I am not keeping considering the effect of Z_S . In another case I am not considering the effect of Z_L . So obviously I need another because finally my job is how much power I have in the source and how much power I am delivering in the load.

There will be a final one. Which generally in low frequency we consider that but here this powers depending on the impedance level they are not all equal that why we are doing this. This final power calculation is called Transducer power gain G_T . The very important one and it is defined as P_L by P_{avs} . From the final P_L by P_{avs} I got this.

So this is the most important one and it is defined as you see I can write that GT is dependent on both ZS and ZL. So, when I complete loading is their both in the load side and also the source impedance is considered. Then what is my power gain that is called Transducer Power Gain. So G T is the most important factor but you know that out of this three obviously GT.

That will be the least because after taking everything into account this value will be reduced from any of this values this various parameter help us to have various separately consider various size , suppose if I want find out that what is the effect on the gain of GL then I use this what is the effect separately if I want what is the effect on gain of GS I use this when everything together the final output is this one.

Now Obviously if everywhere the power the conjugate matching is there that means the input is conjugate matched that means $Z_S = Z_{in}$ star also $Z_L = Z_{out}$ star. Then I know that maximum power transfer is taking place so whatever is available that I can delivery here has P_{in} and whatever P_{avn} is giving that I can delivery to P_L under that condition so we can write that. When input and output are conjugately matched all this definition are same $G = G_A = G_T$.

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input & output are conjugately matched

$$G = G_A = G_T$$

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = \frac{S_{11} + S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$V_1 = V_S \frac{Z_{in}}{Z_{in} + Z_S}$$

$$V_1^+ = \frac{V_S (1 - \Gamma_S)}{2(1 - \Gamma_{in} \Gamma_S)}$$

$$P_{in} = \frac{1}{2 Z_0} |V_1^+|^2 (1 - |\Gamma_{in}|^2)$$

$$= \frac{|V_S|^2}{8 Z_0} \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \Gamma_{in}|^2} (1 - |\Gamma_{in}|^2)$$

But otherwise not because for a G T is the function of Z S and Z L. So for particular selection of ZS and Z L only I get GT a particular selection of ZS and ZL only I get G and GA etc.. Now

these are in general different this power gains but under conjugate matching I know that this will be maximized that is called maximum power and that time if we look it G_T that is the actual G_T that you can get from a particular transistor power amplifier.

Now, we want to find out what are the powers because we have written this expression but we need to make it u know gain etc. finally we need to convert to the devices parameters. So here I can have any excitation V_S . But I need to find out an expression of this G , G_A , G_T in terms of the S parameters of the transistor and also the load coordination and source coordination it should be independent of the excitation of that I give because any volt source that I can change voltage source gain etc. they should not get changed.

So we will now need to manipulate and get some expression of powers in terms of the source voltage V_S , so that I will do So before that first letter write what is our input impedance input reflection coefficient I have input reflection coefficient here simply this reflection coefficient is nothing but V_1^- by V_1^+ so I can write V_1^- / V_1^+ but B_1 this side is loaded so this V_1^+ is entering going from here as V_2^- then coming back as V_2^+ again coming here as V_1^- .

So all this affects that so already in the first NPTEL lecture basic building blocks or basic tools of microwave engineering we have seen signal flow graph and that time. I have established that if I have two port network and This reflection coefficient in the input that can be easily expressed in terms of the S parameters of the 2 port network that becomes S_{11} please refer their that how it is it can be very simply from signal flow graph it can be derived.

Also, please remember this generally students beginners they make mistakes that they think that input reflection coefficient that is always S_{11} no its S_{11} we have this term goes to zero that means Γ_L is zero when Γ_L is zero Γ_L zero means out is match so if port 2 is match then only I can say that input reflection coefficient = S_{11} . Otherwise let I will have to consider that what is mismatched here what is the reflection coefficient also S_{12} and S_{21} and S_{22} they matter.

So this we have earlier derived and we know that this what is gamma in from impedance perspective this is nothing but $Z_{in} - Z_0$ by $Z_{in} + Z_0$. Now also we know what is relation because ultimately will have to relate to source voltage V_S so what is the relation between V_1 and V_S . V_1 is the terminal voltage so I can write it that V_1 is V_S into Z_{in} by $Z_{in} + Z_S$ simple voltage division.

So from this you just since our actual quantities are $V_1 +$ and $V_1 -$ you express this V_1 as $V_1 +$ $V_1 -$ and will do simple manipulation that I can find out expressions, $V_1 +$ in terms of V_S as this also incorporate what is gamma S. I have already written gamma S here. So in terms of V_S and gamma S we can do very simple manipulation. So if I have these then what is the input power you see what is my P_{in} P_{in} is whatever $V_1 +$.

This voltage have getting how much power. average power if we assume the P_i I can write as half $Z_0 V_1 +$ square into $1 - \gamma$ I_n square this we have earlier seen how to do this P_{in} is this we know Impedance level is Z_0 and there is a reflection here so if we subtract that value reflected power we know how much power is being inputed to this network.

So P_{in} is this now with V_1 finally we want to because $V_1 +$ it will change from various loading condition but V_S it is the source voltage that is independence of all let us relate it there. And if we do that means if you put that expression of $V_1 +$ value. We get expression of V_S square by $8 Z_0 1 - \gamma$ S square by $1 - \gamma$ S gamma I_n square to $1 - \gamma$ in square

So this is an important relation P_{in} is this finally we got it that this is an expression on P_{in} . Also I need to find the other 2 expressions. That what is P_{avs} ? What is P_{avn} ? What is P_L . So I have done P_{in} .

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$$P_L = \frac{|V_L|^2}{2Z_0} (1 - |\Gamma_L|^2)$$

$$= \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 (1 - |\Gamma_L|^2) (1 - |\Gamma_S|^2)}{|1 - S_{22}\Gamma_L|^2 |1 - \Gamma_S\Gamma_{in}|^2}$$

P_{avs} is power P_{in} when $Z_{in} = Z_S^*$
 $\Gamma_{in} = \Gamma_S^*$

$$P_{avs} = \frac{|V_S|^2 |1 - \Gamma_S|^2}{8Z_0 (1 - |\Gamma_S|^2)}$$

P_{avn} is power P_L when $Z_L = Z_{out}^* \Rightarrow \Gamma_{out} = \Gamma_L^*$

$$P_{avn} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 (1 - |\Gamma_{out}|^2) (1 - |\Gamma_S|^2)}{|1 - S_{22}\Gamma_{out}^*|^2 |1 - \Gamma_S\Gamma_{in}|^2}$$

Now also what will be PL let us see PL so PL can I write from this. What will be PL You see what V_2 – square by $2 Z_0 1 - \gamma_L$ square and then I can V_2 - here I have done for V_1 - V_2 – also we can relate with this gamma S and gamma L. So this finally if we do we can write that this will also turn out to be BS square by $8 Z_0 S_{21}$ square into $1 - \gamma_L$ square into $1 - \gamma_S$ square divided by $1 - S_{22} \gamma_L$ square $1 - \gamma_S \gamma_{in}$ square.

So we got PL in terms of VS square and all these are S parameters and gamma S and gamma L and gamma in.

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Similarly, now let us see the other two available things, what is P available S? that is the power that the source can make available. so this is source can make available maximum power under when this input side is conjugate match. That means, P avs is power is that power so I can say Pavs is power Pin when $Z_{in} = Z_S^*$. So already have P in expressions.

I should enforced this condition $Z_{in} = Z_S^*$. It means $\Gamma_{in} = \Gamma_S^*$. Put it here. So I have the expression of the P avs. Similarly, P avn is the maximum power transferred to the load. So, available from the network and that can be transferred to the load when this side is conjugately match. I have P L expressions. So I can say, P avn when Z_{in} is chosen as Z_S^* . So I already have Pin expression you see already have Pin expression there I should enforce this condition that $Z_{in} = Z_S^*$.

$Z_{in} = Z_S^*$ means $\Gamma_{in} = \Gamma_S^*$ so if I put it here I get the expression of Pavs that is $V_S^2 \frac{1 - \Gamma_S^2}{8 Z_0 (1 - \Gamma_S^2)}$ similarly what is P avn pavn is the maximum power that can be transferred to the load. So available from that network and that can be transferred to the load when this side when this side is conjugately matched so again I have PL expression there I will have to put the conjugate matching.

So I can say Pavn is power PL when $Z_L = Z_{out}$ start that means this implies that $\Gamma_{out} = \Gamma_L^*$. So I can if I do that then I get the expression of Pavn and that becomes $V_S^2 \frac{1 - \Gamma_S^2}{8 Z_0 S_{21}^2 (1 - \Gamma_{out}^2)} \frac{1 - \Gamma_S^2}{1 - S_{22}^2 \Gamma_{out}^2}$.

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P_{avs} is power P_{in} when $Z_{in} = Z_S^*$
 $\Gamma_{in} = \Gamma_S^*$

$$P_{avs} = \frac{|V_s|^2 |1 - \Gamma_S|^2}{8 Z_0 (1 - |\Gamma_S|^2)}$$

P_{avn} is power P_L when $Z_L = Z_{out}^* \Rightarrow \Gamma_{out} = \Gamma_L^*$

$$P_{avn} = \frac{|V_s|^2}{8 Z_0} \frac{|S_{21}|^2 (1 - |\Gamma_{out}|^2) (1 - |\Gamma_S|^2)^2}{|1 - S_{22} \Gamma_{out}^*|^2 |1 - \Gamma_S \Gamma_{in}|^2}$$

So, I have these expressions. So all 4 power quantities we have expressed in terms of source voltage, the impedance level of the active device that means transistor then S parameter of transistor and various reflection coefficients depending on the load and source conditions. So we are in a position to find out analytical expressions for a various gain quantity that we will see in a next lecture.