

Design Principles of RF and Microwave Filters and Amplifiers
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Module No # 3
Lecture No # 04
Amplifier Design for maximizing transducer gain

Welcome to these 14 lectures of lecture series in pervious lecture we have seen that if we have a potentially unstable transistor device then how to choose my gamma L and Gamma S to make it conditionally stable. Now that time we raised the question can we judge from the S parameter can you determine whether need to draw the stability circles are not.

The answer is fortunately yes people have devised some analytic test based on S parameters to test whether the device is unconditionally stable or not if the device is unconditionally stable you go ahead with your actual amplifier design you need not worry about the stability. If the test fails then you need to locate or draw the stability circle on the smith chart and then determine the ranges from where you will have to choose you gamma S and gamma L value and then you proceed to amplifier design.

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Analytic Tests for Unconditional Stability

Rollet's Test
(K-Δ) test:
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 |S_{12} S_{21}|} > 1.$$

as well as:

$$|\Delta| = |S_{11} S_{22} - S_{12} S_{21}| < 1.$$

\Rightarrow device is unconditionally stable.

μ-test

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12} S_{21}|} > 1$$

\Rightarrow device is unconditionally stable.

So we will see that test, that test is called there are two test anyone you can do called Rollet's test or Rollet stability criteria. So Rollet you found out a parameter called K which is given by $1 - \frac{S_{11}^2 - S_{22}^2 + \Delta^2}{4 S_{12} S_{21}}$, delta is the determinant of the S matrix. Such a simple parameter he proved, it can be easily proved but that is you can see in books it is given.

But that is not so important for the design, so you know that if this K value, you see everything here depends on S parameters values. So you can find out K and if this K is greater than 1 as well as if delta which is nothing but if you see the S matrix it is $S_{11} S_{22} - S_{12} S_{21}$ this magnitude that means these are real number. If these is less than 1 then we say that this Rollet test then passes you and device transistor is unconditionally stable.

That means you can choose any passive load impedance source impedance and still you need not worry your device would not go into oscillation. This is called Rollet's test another test here are two conditions that should be simultaneously satisfied another test this is also sometimes called K delta test.

This is called Rollet's constant or Rollet's constant K another test is called new test it depends on here you have K and delta both you have to test but new test is an improvement of that it is recently people have found out and this test is that you test this parameter Mu is given by $1 - \frac{S_{11}^2 - S_{22}^2 - \Delta^2}{4 S_{12} S_{21}}$. So this mu is given by this here also you see this is a real number these is greater than 1.

Then we can again say that device is unconditionally stable so any one of this test you can do basically from this equations after manipulation can have this mu parameter. So this is nothing new but people have made it more convenient if you test this you can if not fails this test any of this test if it fails then you need to draw the stability circle. Ok let us see the example of this C in the rest part of this will see already we have seen stability criteria, unconditional stable devices.

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Outline

- **Stability Criterion**
- **Unconditionally Stable Devices**
- **Conditionally stable Devices**
- **Making conditionally stable Devices from potentially unstable Device**

Then we will see an example if we get a conditionally stable devices how that can be if we have an potentially unstable device how we can make conditionally stable amplifier design from that. So this is from Gonzalez book I already referred this book. So in page hundred of my edition this problem is given that suppose S parameter of transistor at 800 Megahertz is given like this are the value given first determine the stability.

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Example 1: Pp 100 Gonzalez

The s parameter of a transistor at 800 MHz is given

$$[S] = \begin{bmatrix} 0.65 \angle -95^\circ & 0.035 \angle 40^\circ \\ 5 \angle 115^\circ & 0.8 \angle -35^\circ \end{bmatrix}$$

1. Determine the stability ;
2. If potentially unstable then draw the stability circles;
3. Show how resistive loading can stabilize the transistor.

Then if potentially unstable then draw stability circles and then show how resistive loading can stabilize the transistor. We see this is the new thing we will discuss that with the example. So we will test Rollet's test so K we can calculate and delta we can calculate so it turn out that k is equal to 0.547 and delta is 0.504 and with the phase of this.

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Solution :

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}; \Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$K = 0.547 \text{ and } \Delta = 0.504 \angle 249.6^\circ; \Rightarrow \text{Since } K < 1$$

the transistor is potentially unstable at 800 MHz

[Unconditional stability requires: $K > 1$ and $|\Delta| < 1$]

Alternatively,

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|} = \frac{0.575}{0.5035 + 0.1750} = 0.8511 < 1$$

\Rightarrow **Potentially unstable.**



Now since k is equal to 1 so we say that it fails Rollet's test so transistor is potentially unstable at eight hundred megahertz because we know that unconditional stability K is greater than 1 and magnitude of delta is less than 1 so it as felt there. So though magnitude of delta is less than 1 but K is less than 1 so it is potentially unstable also in the μ test if you do that you see the value turns out to be 0.85 which is less than 1 but we know that μ needs greater than 1 for unconditionally stable.

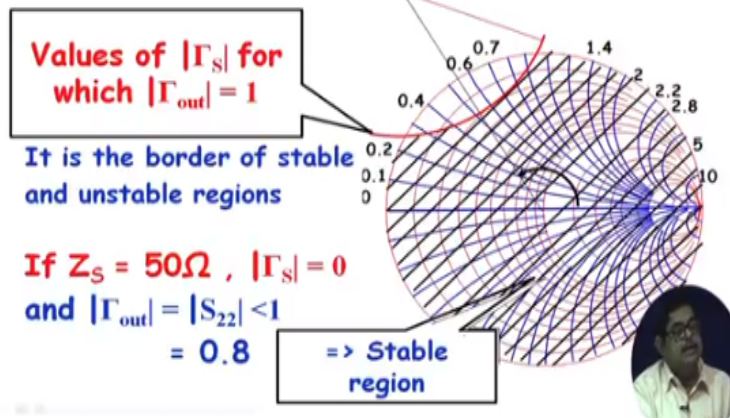
So it says that stability checking analytic stability indicate that this is a potential unstable device. So if we want to use this device we need to find out or plot stability circles and then determine what are the ranges of load impedance and source impedance, so that we can make it stable amplifier. So you see that first we have done input stability circle CS expression I have already given.

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I/P Source stability circle :

$$C_s = \frac{(S_{11} - \Delta S_{22}^*)}{|S_{11}|^2 - |\Delta|^2} = 1.79 \angle 122^\circ, \quad R_s = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \frac{(S_{11} - \Delta S_{22}^*)}{|S_{11}|^2 - |\Delta|^2} \right| = 1.04$$

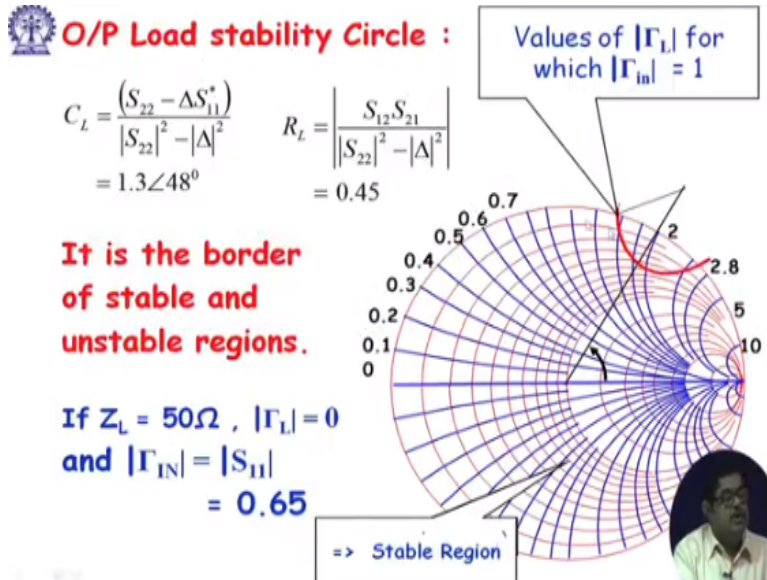


So those are all I can calculate that center will be 1.79 you know smith chart as a radius of 1. So the center falls outside of smith chart that you see the center here this is outside smith chart and 1.79. So these distance is 1.79 and this angle that it makes is hundred and twenty degree so it is here and RS is 1.04 so partially you see this is 1.79 the radius is 1.04 so it will partially intersect.

The smith chart intersect then we need to determine which one is stable zone so values of delta S for which gamma out = 1 this is the input stability circle it is the border of stable and unstable region. Then as I said assume that $Z_S = 50$ ohm so gamma S = 0, so gamma out magnitude turns out to be a S22 magnitude but what was S22 magnitude. You see S22 magnitude is 0.8 so it is less than 1 so S22 magnitude is 0.8.

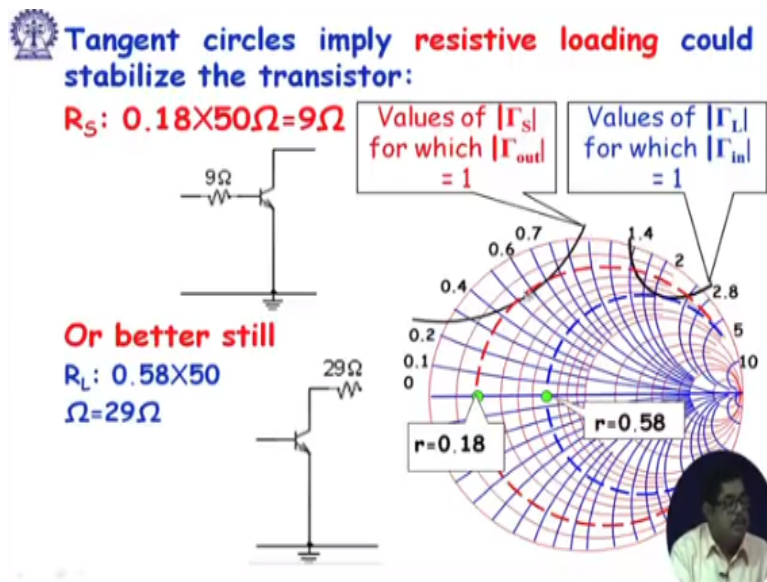
So I know this input the portion of smith chart that is containing center that is the stable region we have marked as stable region this black shading and this is the unstable region. So I will have to choose the gamma S from here similarly we draw output stability circle this is the center again outside the center is outside the smith chart and this time forty eight degree.

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So this angle is forty eight degree and you see the radius is 0.45 so $1.3 - .45$ there will be intersection this redline is that locus of the circle a part of the circle arch. And so again here we can check that when will simply match the load side gamma in become S11 magnitude and S11 magnitude in this case .65 you can again say that this input side is the stable one. So from this stable region we will have to choose out gamma L.

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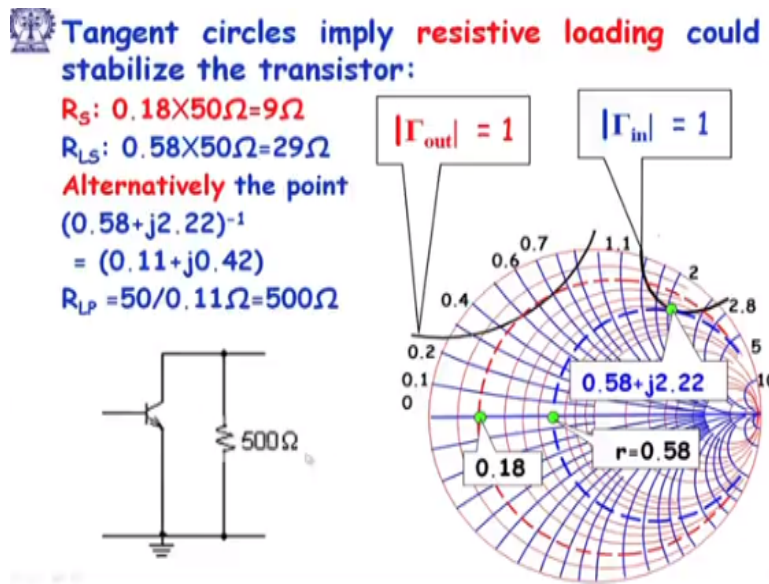


So how we can do that and also you see that if we want to give resistive loading suppose this is my input stability circle this is my output stability circle now I see all this constant resistance things so the one that is tangent to this input stability circle. So there I can choose the

corresponding this is the constant resistance circle of resistance value normalize resistant value 0.18. So I can out since this is input if I put this value you see I am further of here.

So I can add a 0.18 normalize resistance so with the 50 ohm characteristic impedance the value is here you see 9 ohm here I can add in the source side that means with the base of the transistor I can add a 9 ohm resistance similarly from the output stability circle I can find out tangent circle and find out that this is contact resistance circle this blue one of 0.58. So upon unnormalizing I see that 29 ohm so at the load side I can out 29 ohm series resistance and that will make the whole thing stable.

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So it will stabilize the transistor now sometimes at the output you want to have parallel loading so that we can see that this 0.58 if is just change it to the or this I can find out what is this point impedance and I change it that becomes .11 so from that point 11 you know for admittance conductance's I need to divide it so five hundred ohm.

So instead of this previous case where I had you see that previous I have given a 29 ohm series resistance instead I can also give a five hundred ohm parallel resistance in the load side and that will also stabilize the transistor so by this resistive loading you can add but obviously this will change the overall S parameter or overall gain condition so we need to check that whether this

true so we can do this that the series resistance that we have had or shunt resistance that we have had we can find the ABCD parameter.

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 **S to ABCD Parameters** : Ref: Pozar Ch.4. p 211, 2nd Ed. :

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}} & B = Z_0 \frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}} \\ C = \frac{1}{Z_0} \frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}} & D = \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}} \end{bmatrix}$$

ABCD parameter for series N/W: $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

ABCD parameter for shunt N/W: $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$

ABCD to S Parameters :

$$[S] = \begin{bmatrix} \frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D} & \frac{2(AD-BC)}{A+B/Z_0+CZ_0+D} \\ \frac{2}{A+B/Z_0+CZ_0+D} & \frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D} \end{bmatrix}$$


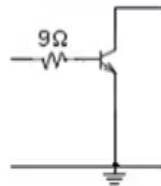
As you can see that this one is the shunt element so if I know that ABCD of this because I know that S parameter of this transistor so I can easily find out what is the ABCD parameter of this then I can find out this shunt element what is the ABCD parameter I can multiply this ABCD parameter to find the resistive loaded transistors ABCD parameter from there I can go to again S parameter back composite S parameter.

And from that I can again see whether this is resistive loading stabilize that this composite thing now needs to pass the rollet's stability criteria or MU test.

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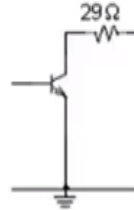
ABCD parameter for series N/W: $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

ABCD parameter for shunt N/W: $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$



Overall ABCD Parameters:

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & 9/50 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$



$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & 29/50 \\ 0 & 1 \end{bmatrix}$$

So that will be doing this are the formulas you can refer to so you can find out that when I have a series Z I know this is the ABCD parameter so overall ABCD parameter in that case will be this. Similarly, for the shunt one I will have these because these then and in shunt with that I have this so overall ABCD parameters comes out to be this.

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ABCD parameter for shunt N/W: $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$

Overall ABCD Parameters:

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} A_{tr} & B_{tr} \\ C_{tr} & D_{tr} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 50/500 & 1 \end{bmatrix}$$

Finally, overall S-parameters is obtained as

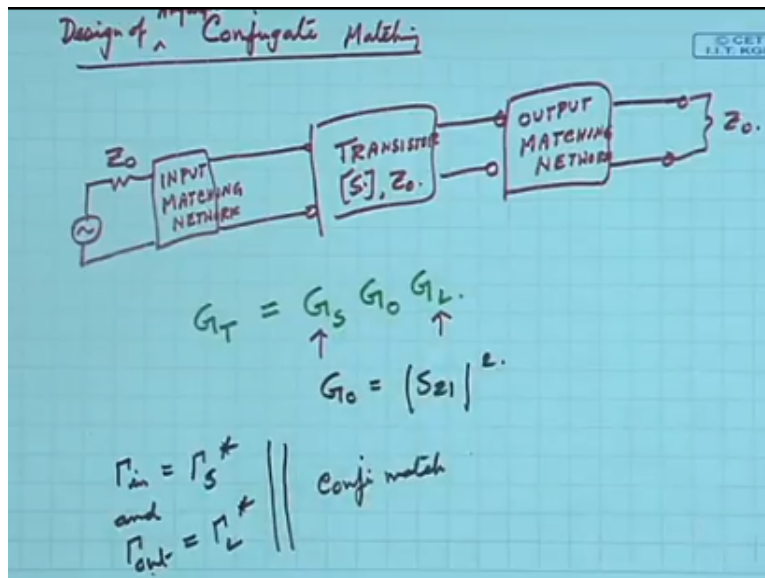
$$[S] = \begin{bmatrix} 0.65 \angle -94^\circ & 0.032 \angle 41.2^\circ \\ 4.62 \angle 116.2^\circ & 0.66 \angle -36^\circ \end{bmatrix}$$

Stability factor: $K=1.04$ and $\Delta = 0.409 \angle 250.13^\circ$
 => above N/W is unconditionally stable at 800 MHz

This are the transistor ABCD values and then I can reconvert after doing this multiplication I can reconvert it to overall S parameter is this you see stability factor so I can calculate the rollet's factor that turns out to be 1. So you see and beta magnitude that is 0.409 so I know that I have passed the test so above network is unconditionally stable at eight hundred megahertz.

So this is the way by which I have an potentially unstable device still with proper loading finding from Smith chart I can make it conditionally stable. Ok so after that we will try to do that the actuals so this is stability checking after that we will see the conjugate matching. So design of amplifier with conjugate matching because that is out actual M so we will do that design with conjugate matching and you see that.

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I have now made that stability analysis I have chosen the gamma S and gamma L ranges etc. if we refer to our original diagram in previous class we have said that finally under conjugate matching I will have a source then I will have a conjugate (()) (17.09) then an input matching network. This is conjugate impedance matching this will be connected to the transistor and then I will have output matching network.

And then it will be finally looking at Z_0 so I know that actual Z_L is different but with output matching it will appear as Z_0 . Similarly actual source impedance is Z_S but with input matching network I can make it and that day we have also seen that the transducer power gain that comprises of these that you can say this generally we call Z_S then this the transistor side that we call Z_0 and this is Z_L and this Z_0 is typically not in our hand because transistor manufacturer he has made the transistor that transistor is giving me some S_{21} values.

So we know that typically this Z_0 is given by S_{21} square now and micro wave designer and amplifier designer is job is to now design input matching network and output matching network. So that I will choose my proper Γ_L and Γ_S . So that Z_S that will appear as Z_0 here to the source similarly the Z_L here was Z_L but with this output matching network it will as Z_0 then it will be conjugately matched. Now what is conjugate matching that time we have seen the Γ_{in} is nothing but Γ_S^* .

And simultaneously because in general for any transistor unless and until we do unilateral assumption so for bilateral transistor S_{12} and S_{21} are different values S_{21} . So in that case when I have a bilateral transistor this Γ_{in} that is a function of Γ_L and Γ_{out} is a function of Γ_S . So together simultaneously the input and output should be matched for conjugate matching and conjugate matching says that you choose Γ_{in} is equal to Γ_S^* and Γ_{out} is equal to Γ_L^* .

So that you can become a conjugate match and you get maximum power transfer. So this is already given by the device manufacturer so our job is to now design for this Z_S and this input matching network and output matching network so I can get maximum of this Z_S and this Z_L . Now last time we found have out the expression for this Z_S so that expression I am writing again that or before that this G_T we have also found out the expression for G_T in terms of S parameters of Γ_S and Γ_L .

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$$G_{T_{max}} = \frac{1}{(1-|\Gamma_S|^2)} |S_{21}|^2 \frac{(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2}$$

\uparrow
 $G_{S_{max}}$
 \uparrow
 $G_{L_{max}}$

Now under conjugate match under GT will be maximized transducer power gain is maximized and that equation obviously we are assuming that this input matching network and output matching network are lossless. We have seen in the basic building blocks of microwave engineering NPTEL lectures that how to design lossless input matching network and output matching network those are not lump elements.

L section lumped elements they are used for lower frequencies is but at high microwave frequencies you need to have transmission lines and transmission lines stubs to design lossless input matching network. So that GT max will become $1 - |\Gamma_S|^2$ this expression we have already found last time $|S_{21}|^2$ and $1 - |\Gamma_L|^2$ by $|1 - S_{22}\Gamma_L|^2$. So with our this nomenclature $G_{S_{max}}$ Z_0 $G_{L_{max}}$ basically this I can say is my $G_{S_{max}}$ maximum this is my $G_{L_{max}}$ maximum.

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$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$$

Conj. match

$$\Gamma_S^* = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$$

2 eq. 2 unknowns
 Γ_S & Γ_L

So now we have enforced this condition so again start from this basic conditions that I have $\Gamma_{in} = S_{11} + S_{12} S_{21} \Gamma_L$ by $1 - S_{22} \Gamma_L$ $\Gamma_{out} = S_{22} + S_{12} S_{21} \Gamma_S$ by $1 - S_{11} \Gamma_S$ and you see as I am always saying that Γ_{in} is a function of Γ_L and also S parameters of the device Γ_{out} is a function of Γ_S and S parameters of the device. Now for conjugate matching I know that this Γ_N .

So this is general now conjugate matching that time will make this choice that this Γ_{in} will choose as Γ_S^* . So Γ_S^* is $S_{11} + S_{12} S_{21} \Gamma_L$ by $1 - S_{22} \Gamma_L$ and Γ_L^* that will be $S_{22} + S_{12} S_{21} \Gamma_S$ by $1 - S_{11} \Gamma_S$. So you see that by conjugate matching I have replaced Γ_{in} and Γ_{out} with their appropriate Γ_S and Γ_L values. So you see this is two equations here all S parameters are known I have two unknown two equation so I can write two equations two unknowns.

What are the unknown Γ_S and Γ_L so I can solve for that let us solve first for Γ_S . From this equation if I solve for Γ_S then Γ_S become something like this you see just simple manipulation but obviously complex number manipulation. So that part you need to take care but otherwise it is trivial and this becomes everyone do it write take it pen and paper and do it.

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number so B1 is also a real number so this and this is also a real number. If this is greater than 0 then only this will have a solution so people have found out that ok this it can be shown.

So first this will have a solution and once that solutions is there we can easily because all this are known quantities you see there are no unknowns no gamma L is involved in this solution. In this only I need to know the S parameter values delta is also dependent on net parameter so I can immediately solve for this the route will exist. Route will have a physical existence if this is satisfied.

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Solving for Γ_L

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$B_2 = 1 + (S_{22})^2 - (S_{11})^2 - |\Delta|^2$$

$$C_2 = S_{22} - \Delta S_{11}^*$$

$$B_2^2 - 4|C_2|^2 > 0.$$

$$K > 1 \text{ and } |\Delta| < 1.$$

$$G_{T, \text{max}} = \frac{|S_{21}|}{|S_{12}|} (K - \sqrt{K^2 - 1}).$$

conj. matched gain

Similarly if I solve for gamma L solving for gamma L we get another quantitative equation and that solution is we call it 2 or B2 plus minus root over B 2 square – 4C2 square by 2C2 where B2 is given 1 + S22 square – S11 Square – delta square and C2 is S22 – delta S11 star. Now it will have solution if B2 square – 4C2 square that is greater than 0. So I got two such conditions B1 square – 4C1 square greater than 0 and B2 square – 4C2 square were is 0 where beyond B1 B2 C1 C2 are this.

People have shown that this conditions enforcing this two simultaneously that B1 square – 4C1 square greater than 0 and B2 square – 4 magnitude C2 square greater than 0. This is equivalent to that criteria that K which was rollet's stability factor k greater than 1 and delta less than 1. So

that mean what is this mean that if I have made a conditionally stable amplifier then this things are automatically satisfied and so gamma S and gamma L are values.

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denom $|K| < 1,$

$K = 1.$

$G_{max} = \frac{|S_{21}|}{|S_{12}|}$
max. stable gain

Unilateral

$S_{12} \approx 0$

$\Gamma_S = S_{11}^*$

$\Gamma_L = S_{22}^*$

$G_{TU_{max}} = \frac{1}{(1 - |S_{11}|^2)} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2}$

So I can always find the gamma S and gamma L value and when the roots for chosen like this then we can maximize GT max and that is given by S21 by S12 K – root over K square – 1. So this gain that maximum gain for conjugate matching that sometimes is also called matched gain. Actually, more scientifically it should be call conjugately (()) (31:55) matched gain there is also a simple match case.

No if we have a devices for whom if we have devices for whom k is less than 1 then conjugate matching is not possible then we have but what we can do we can enforce $K = 1$ that is the maximum possible transducer gain. So that is called G maximum stable gain this name is maximum stable gain and that is given by simply S21 by S12. So if you have if you have failed you device have failed the rollets test then also you can make proper choice from these and that time you will get a gain of S21 by S12.

Usually in any practical transistor device S21 is a quite high value S12 is low value. So you get good about of gain even from a device without this. Now this becomes further simplified if we do it for unilateral case that time the input and output their coupling that means gamma in depends on gamma L that is not there for unilateral case.

You know unilateral means S_{12} is 0 or approximately 0 under that case we know that Γ_S under becomes that means S_{11}^* and Γ_L becomes S_{22}^* so you see with this formula we can write the GT unilateral maximum gain possible that will be given by simply $1 - |S_{11}|^2$ instead of $1 - |S_{21}|^2$ and $1 - |S_{22}|^2$. So this for unilateral case which many times we will see that we can assume this and that time we can achieve a transducer gain maximum like this.

For unilateral case design is much simpler so I have talked out all the possibilities we will see since this lecture does not have any more time in the next lecture we will see a conjugate match design as an example. Thank you