

Design Principles of RF and Microwave Filters and Amplifiers
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Module No # 4
Lecture No # 19

Quantitative Characterization of Nonlinearity for Large Signal Amplifier (Contd.)

Welcome, to this second lecture on power amplifier design two, we have already seen the nonlinearity, its effect that it creates third order in the modulation product. Now we will quantitatively characterize that third order modulation product in this lecture. Now in that we have started with the two tone test, they assume that the assume tone is absent. So that means you put $A_2=0$ (()) (00:57) V_i was this there and here $V_0(t)$ that was this.

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Two Tone Test

Assume second tone absent: $A_2=0$

$$\begin{aligned}v_i &= A_1 \cos w_1 t + A_2 \cos w_2 t \\v_0(t) &= a_1 (A_1 \cos w_1 t + A_2 \cos w_2 t) \\&+ a_2 (A_1 \cos w_1 t + A_2 \cos w_2 t)^2 \\&+ a_3 (A_1 \cos w_1 t + A_2 \cos w_2 t)^3 \\&+ \dots\end{aligned}$$

We are considering upto third order non linearity and there if we make this $A_2=0$.

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Harmonic Terms


Amplitude

$$\frac{a_2}{2} (A_1^2 + A_2^2)$$

ω_1 $\left[a_1 A_1 + \frac{3}{4} a_3 (A_1^3 + 2 A_1 A_2^2) \right] \cos \omega_1 t$

ω_2 $\left[a_1 A_2 + \frac{3}{4} a_3 (A_2^3 + 2 A_2 A_1^2) \right] \cos \omega_2 t$

$$\frac{1}{2} a_2 \left[A_1^2 \cos 2\omega_1 t + A_2^2 \cos 2\omega_2 t \right]$$

$$\frac{1}{4} a_3 \left[A_1^3 \cos 3\omega_1 t + A_2^3 \cos 3\omega_2 t \right]$$


Again we can, this was our harmonic terms am writing. Here you put all $A_2=0$ value.

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Intermodulation terms

type	Amplitude
2 nd order IM	$a_2 A_1 A_2 \left[\cos(\omega_1 + \omega_2) t + \cos(\omega_1 - \omega_2) t \right]$
3 rd order IM	$\frac{3}{4} a_3 A_1^2 A_2 \cos(2\omega_1 - \omega_2) t$
3 rd order IM	$\frac{3}{4} a_3 A_2^2 A_1 \cos(2\omega_2 - \omega_1) t$
3 rd order IM	$\frac{3}{4} a_3 A_1^2 A_2 \cos(2\omega_1 + \omega_2) t$
3 rd order IM	$\frac{3}{4} a_3 A_2^2 A_1 \cos(\omega_1 + 2\omega_2) t$

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COMPRESSED AMPLITUDE

- ❖ Assume second tone absent,
 $\rightarrow A_2 = 0$
- ❖ Output at ω_1 is $(a_1 A_1 + 3/4 a_3 A_1^3) \cos \omega_1 t$
- ❖ At 1 dB gain compression point,

$$\frac{\text{actual gain}}{\text{linear gain}} = -1 \text{ dB}$$

$$\therefore 20 \log \frac{a_1 A_1 + 3/4 a_3 A_1^3}{a_1 A_1} = -1$$

$$A_{1(-1\text{dB})} = \left(\frac{4}{3} \frac{a_1}{a_3} \right)^{1/2} \sqrt{0.11}$$

Now if we do that $A_2=0$ then output at the desired frequency, I have only one by putting $A_2=0$, what I have made, that I have one frequency that frequency is ω_1 . So output at ω_1 if you look what is the output at ω_1 . So you see the output at ω_1 will be this, you see this part is absent but this is there. This is coming from that third order nonlinearity. So what will be the output of this $a_1 A_1 + 3/4 a_3 A_1^3$ so that's what I am saying, output of ω_1 is this.

At 1 dB gain compression point, we have this definition actual gain by linear gain = -1dB. So I put the actual gain that is this but at this been linear gain, then this was not there and this was simply one. And that according to the definition is -1 so by that I have related the A_1 its value, the A_1 is the input signal gain. That is given by this thing an important result.

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FRACTIONAL IM PRODUCTS

Fractional 2nd order IM product

$$\text{IM2} = \frac{\text{output amplitude of the 2nd order intermodulation output}}{\text{amplitude of fundamental output}}$$

$$= \frac{|a_2| A_1^2}{|a_1| A_1} = \frac{|a_2|}{|a_1|} A_1$$

So if I know my system, my non-linear system, so I know $A_1 A_3$, So you do this, if I know $A_1 A_3$, I can calculate what is my gain compression point. That means what is the level of the input signal at which I will start getting gain compression point. Now also let us calculate the fractional second order IM product, intermodulation product. What is second order intermodulation product, output amplitude of the second order intermodulation output divided by amplitude of fundamental output.

So second order intermodulation input 1 case 1 we made $A_2 = 0$, we do not have. But this is the harmonic term. This is the actual term linear amplitude of fundamental output. So I get this, so Intermodulation product of second order IM2 it is I am calling fractional because I am normalizing with fundamental output that you see there is linearly with this A_1 , A_1 is the level. So IM2 where is the linearly with the level.

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FRACTIONAL 3RD ORDER IM PRODUCTS

$$IM_3 = \frac{3/4 |a_3| A_1^3}{|a_1| A_1} = \frac{3 |a_3|}{4 |a_1|} A_1^2$$

- ◊ IM2 rises linearly with input signal
- ◊ IM3 rises linearly with square of input signal

In dB scale,

- ◊ Slope of IM3 is twice that of IM2



So fractional third order IM products, IM product is this and the fundamental one is this so it is A_1 square, so I can say IM2 just before what we have seen second order intermodulation product fractional part rises linearly with input signal. Because previous one when we have seen that it is simply proportional to A_1 . These are constants, system constants but IM3 rises linearly with square of input signal, so that is the thing. Now if I convert the same thing to dB scale, slope of IM3 is twice that of IM2. Because of the A_1 square I can say that slope of IM3 is twice that of IM2.

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POWER INPUT – OUTPUT PLOT

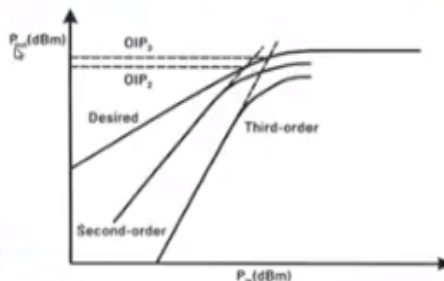


Illustration of extrapolated nonlinear amplifier intercept points.

- ◊ **Both side power is in dB**
- ◊ **Slope of desired signal → 1**
- ◊ **Slope of IM₂ product → 2**
- ◊ **Slope of IM₃ product → 3**

So if I put it in that transfer characteristic, input output in the dB scale. Then you see the linear ones is going like this, second order is going like this, the third order its slope is twice. So it will

go like this. So you see that at a particular value I say that it will be significant. Or it will be significant to the input signal, you see that second order is always lagging.

But this third order intercept since its slope is much higher than second order intermodulation. So at some high level, it will catch up and it will be of same level as the desired signal.

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3RD ORDER INTERCEPT POINT

- ❖ **At low input, IM3 product is small**
- ❖ **IM3 slope is thrice, so at high input, it catches up.**

At low input, IM3 product is small. So it is negligent but IM3 slope is thrice, so at high input it catches up.

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NTH ORDER INTERCEPT POINT

The input signal level where the extrapolated values of the desired signal and the n^{th} order inter modulation product are equal is called the n^{th} order input intercept point S_{IIPn}

- ❖ **Similarly there can be output intercept points OIP**
- ❖ **OIP s are more useful to deal with**

Nth order intercept point let us generalize the concept. The input signal level where the extrapolated values of the desired signal and the nth order intermodulation product are equal is called the nth order input intercept point S_{11P_n} . It is the signal level, that's why we are designating with S. But what is the concept, add this value of the input, my intermodulation product is catching up.

That means becoming same as the original signal or the linear signal, where the extrapolated values of the desired signal and the nth order intermodulation product becoming equal. That means the linear signal that is becoming equal to the intermodulation signal.

So there can be input nth order intercept point. Similarly there can be output intercept points. Output points are OIP. Input intercept points are called IIP. Generally OIP is more useful to deal with because we have amplifier output, we are concerned with. But if intercept point is specified in terms of input, you can also do that.

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VARIOUS IIPs

S_{11P_n} are values where fractional $IM_n = 1$

$$A_{11P_2} \times \frac{|a_2|}{|a_1|} = 1 \Rightarrow A_{11P_2} = \frac{|a_1|}{|a_2|}$$

$$A_{11P_3}^2 \times \frac{3}{4} \frac{|a_3|}{|a_1|} = 1$$

$$\Rightarrow A_{11P_3} = 2 \sqrt{\frac{|a_1|}{3|a_3|}}$$

Now let us calculate these values, these values were it will catch up. So what is this? Definitely from the definition I can say that S_{11P_1} are values where fractional Intermodulation product should be one. Because fractional, if you remember, if you see your notes, fractional intermodulation product what we have defined that where it is the intermodulation product of nth order divided by the fundamental signal level.

So if, where they are touching at the intersect point are becoming equal. So at that point fractional IM n is 1. So I can do that because already we have done that in the modulation thing. So If you do that you see, that at these value the second order intermodulation is becoming , so this the value where second order intercept is coming, here the third order intercept point is coming at this value. So this is given by a1 by a2 and this is given by a1 by a3.

So A1 if you remember that is the linear part, A2 is the coefficient, my system coefficient for second order non linearity, this is third order non linearity. Now we have already said that we are concerned about IIP3 and 1 dB, this D should be a smaller D. But in the Power point I think in the title page it comes like this. Now comparing input 1 dB gain compression point and because we have already seen what is input 1 dB gain compression point.

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IIP₃ AND 1 DB GAIN COMPRESSION

Comparing Input 1 dB gain compression point and A_{IIP3}

$$A_{1(-1dB)} = \left(\frac{4}{3} \frac{a_1}{a_3} \right)^{1/2} \sqrt{0.11}$$

$$\rightarrow A_{(-1dB)} = A_{IIP_3} \times \sqrt{0.11}$$

$$A_{(-1dB)}(dB) = A_{IIP_3}(dB) - 9.586 \text{ dB}$$

$$A_{IIP_3}(dB) = A_{-1dB}(dB) + 9.6$$


If you remember this was our thing and now we have seen AIIP3. So if you compare the comparison is this. That A1 IP3 is 1dB gain compression point + 9.6 that means let me show those graph, so I can say or so you see this was our point of this point was our. This was our 1 db gain compression point let us say in the input side what that is saying that if I go further 9.6 db my third order intercept will come and at that point the linear signal and third order intercept point becoming equal.

You see the message is this is simple mathematics but the message is that the third order intercept point that comes when input signal level corresponding to the 1 db gain compression point is added by 9.6 db just remember this is a db relation.

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OIP₂ AND OIP₃

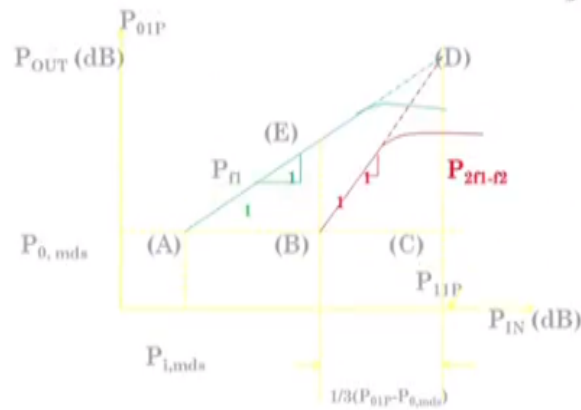
$$A_{OIP_2} = \frac{|a_1|}{|a_2|} \times |a_1| = \frac{|a_1|^2}{a_2}$$

$$A_{OIP_3} = \sqrt{\frac{4}{3} \frac{|a_1|}{|a_3|}} \times |a_1| = \sqrt{\frac{4}{3} \frac{|a_1|^3}{|a_3|}}$$

So if you have output intercept point this is the second order output intercept point it is given by this. This is the third order intercept point this is given by this. Since we know our system that means we know A1 A2 we can always calculate this. Now what is the utility of third order intercept point.

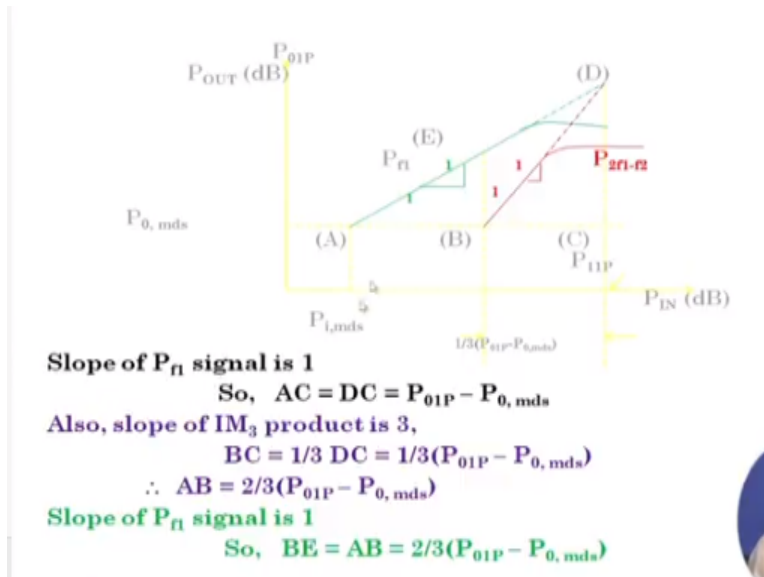
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UTILITY OF 3RD ORDER INTERCEPT POINT



You see that this is my actual linear graph obviously this is extra plotted here because it is already going to the intercept point now I know that this slope here I am showing the red one is showing the third order intermodulation signal. So at this point you see it is catching up so this is the third order intercept point.

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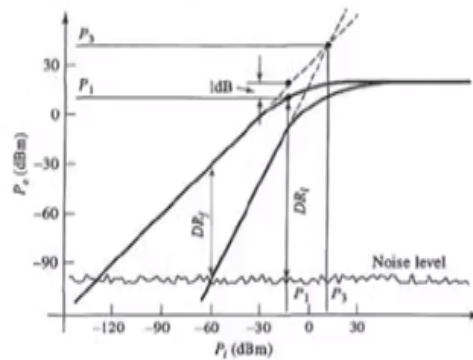
And you see this slope is one, this slope here it is one, now what will do we will show by mathematics that slope of this signal is one so I can say you see this $AC = DC$. Because if slope is one then DC and AC they are equal but I know what is DC ? DC is nothing but P_{01P} minus $P_{0,mds}$. Because this is the point $P_{0,mds}$ so this is here you write P_{01P} minus $P_{0,mds}$ also we

know slope of this three so BC this BC is one third DC that is one third this. So from this I have found AC, I have found BC I have found AC, I have found BC so I can find what is AB?

AB is two third POip minus Pomds also slope of PF1 signal is one, so from that I can also find out that AB is nothing but equal to B what is B this so this is two third this. So from this actually I have an important relation that or go later that B is the range that previous you see B. So what is B? B to E so this is the range where the third order intercept point is coming that is what B is the range of IMC product where just started coming.

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SPURIOUS FREE DYNAMIC RANGE



Illustrating linear dynamic range and spurious free dynamic range.

BE is the range where IM3 product has just started coming

So I now know that this is the range that I should avoid because now the third order intercept point is coming so I should stop. That means I do not need to go upto the 1 db gain compression point I need to see that when this third order intercept point has started coming up then I need to stop and that is my that is called spurious free dynamic range. So my actual dynamic range is not this previously we have given the definition of output dynamic range with this.

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EXPRESSION OF DR_F

$$\begin{aligned}
 DR_f &= \frac{2}{3} (P_{01P} - P_{0,mds}) \\
 &= \frac{2}{3} [P_{01P} + 174 \text{ dBm} - 10 \log B \\
 &\quad - F(\text{dB}) - X(\text{dB}) - G_A(\text{dB})]
 \end{aligned}$$

That our actual thing that 1 db gain compression point minus PO mds but DRf we are saying it is here and that we can P POip expression poip then PO mds for that we have already derived the expression -174 plus all those so that if I put from a If any system Poip is specified I can find the spurious free dynamic range. Usually this spurious free dynamic range is smaller than the dynamic range for a single tone system.

So that means if I have single tone I can use dynamic range but if I have a band of frequencies and if have the non-linearity system then I know that not all the full dynamic range is usable. If I restrict myself within spurious free dynamic range DRf then I know that I will never go to non-linearity.

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EXAMPLE ON DYNAMIC RANGE

An amplifier has available power gain of 40 dB, 500 MHz bandwidth, noise figure of 7 dB and 1 dB gain compression point of 25 dBm. Calculate dynamic range and spurious free dynamic range assuming detector SNR requirement to be 3 dB.



So let us see an example of dynamic range, and amplifier has a available power gain of 40 db, five hundred megahertz of bandwidth noise figure of 7 db and 1 db gain compression point is given at 25 dbm.

Calculate dynamic range and spurious free dynamic range assuming detector SNR requirement to be 3 db. That means what i say that above noise level if it is three db more I say that is ok there is a signal.

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SOLUTION TO PROBLEM

$$\begin{aligned} P_{0, mds} &= -174 \text{ dBm} + 10 \log(500 \times 10^6) \text{ dB} \\ &\quad + 7 \text{ dB} + 3 \text{ dB} + 40 = -37 \text{ dBm} \\ DR &= P_{1 \text{ dB}} - P_{0, mds} \\ &= 25 \text{ dBm} + 37 \text{ dBm} = 62 \text{ dB} \end{aligned}$$

Third order Intercept point is

$$\begin{aligned} P_{OIP} &= 25 \text{ dBm} + 9.6 \text{ dB} = 34.6 \text{ dBm} \\ DR_f &= 2/3(34.6 + 37) = 48 \text{ dB} \end{aligned}$$

So solution is given here we can first find out what is PO_{mids} minus 174 this is the bandwidth then this are given the noise figure etc etc., you see that PO_{mids} is -37 dbm so if any signal is -37 dbm then my system deducts it. So dynamic range we can easily get because $PO_{1\text{ db gain}}$ compression point 25 dbm that it specified I have calculated this so minus of minus this so dynamic range is 62 db.

But also calculate what is third order intercept point because you will have to calculate spurious free dynamic range. So first calculate third order intercept point the moment I know 1 db gain compression point at 9.6 db so my third order intercept point is this. Now I know what is my spurious free dynamic range it is two third of this PO_{ip} plus this minus this PO_{mids} .

Just previously we have seen got that formula PO_{ip} minus PO_{mids} two third of that. So that you can calculate the two third of this so that turn out to be 48 db. So it says that even though the dynamic is range is 62 db I should from the noise floor or maximum usable deductible range PO_{mids} I should go only upto 48 db I can change the level but above that if I do then I will have the distraction in the system etc

Because third order intercept point will come that means third order intermodulation product will also present with me. It will also have the significant value I won't be able to distribute between them so that will be distracting in my amplifier output. So that completes the this analytical characterization of the non-linearity then we will see that this are all calculation but if in any amplifier we want to measure it how to measure it that we will see in next lecture. Thank you.