Design Principles of RF and Microwave Filters and Amplifiers

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Module No # 01
Lecture No # 02
Concept of Image Impedance and propagation constant

Welcome to this second lecture on concept of image impedance. Now, I hope you agree with me that any 2 port network can always be represented by either a T section or a PIE section without losing a generality I take that my 2 port network that means transmission matrix characterization is ABCD that is a T section the same analysis hold for PIE section. So now I have that T section, this is Z1, this is Z2, this is Z3 all this are impedances, complex impedances.

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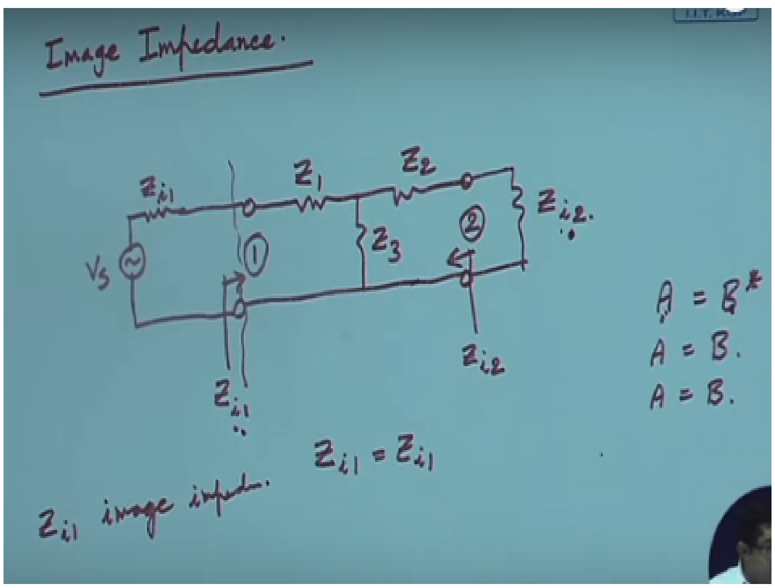

So this is many port 2, this is my port 1, this is the internal description of the network. Now, I want when will I excide the port 1 with that voltage BS and I have some internal impedance let us call that internal impedance ZI1last time I called it ZS this time I am calling it ZI1 is simply change of number this is my port 1 this is my port 2.

Now you see all of you are familiar with maximum power transfer theorem says that if the load impedance complex is a complex conjugate of the source impedance then maximum power gets transferred. I think that you have noticed that in low frequency particularly this VLSI people etc., when they work upto gigahertz range they do not give any consideration to these maximum power transfer theorem.

Because in baseband unless you go upto radio frequency and transmit it you have plenty of power so you are more concerned with your voltage maximization that is why you design a good C amplifier with very high voltage game but voltage gain is not necessarily mean a maximum power gain but we when we go to radio frequency we know power microwave power is very precious to produce microwave power lots of complete complicated circuits are required.

So also when power is received by a receiver in radio frequency it is very small amount so its life and death for RF engineer or RF circuits to maximize power. So maximum power transfer theorem always is the design of RF circuits always we try to pay regard to the maximum power transfer theorem. So can I have this whole thing suppose this I will terminate by some load impedance let the load impedance is called ZI2 this one I called ZI1 source impedance is ZI2.

Now the idea of image impedance at this point obviously if I look at I will get some impedance. Now if this impedance is equal to ZI1 then I can from the source I can have maximum power transfer ok. Now according to maximum power transfer I suppose these I am looking at some Zin or something Zin now Zi is complex conjugated ZI1 star then I know that maximum power transfer will takes place.

But as I said that our consideration now is filter which is lossless network so this Z1, Z2, Z3 there is no R involved ideally. So they are complex, but they are generally they will be either real or not real either all of them will be pure imaginary term there will be pure reactance. So in that case I say that now I say that I will look into here ZI1 then ZI1 is equal to ZI1 suppose any complex conjugate for pure imaginary things it is equal.

As you know that suppose I have to complex two number A is equal to B star suppose a is a complex number B is a complex number if A is pure real and B is pure real then I can say A is equal to B. Similarly, if A is pure imaginary and B is pure imaginary then on low I can say A is equal to B. Since we know that this will be my all these are pure reactances this may be a pure resistance so that is why I can call that my demand is ZI1 here should be equal to ZI1 here.

That means source impedance and these input impedance looking at this port should be equal but you see this ZI1 is a function of this load resistance ZI2. But now so why it is called this ZI1 is called image impedance if I can find an impedance load impedance Zi2 so that if I terminate this network this is already given network and I look at here and see that the input impedance is ZI1 then I know that I can transfer maximum power from the source to this network this to this port one of this network.

now why ZI1 is called an image impedance because if at this plane I look so I looking at this side I am getting to the right side and getting and ZI1 looking at left side I am seeing the impedance ZI1. So this side is image of these that is why this is an image impedance it so ZI1 is an image impedance. Same thing here and here I want that if I look at here I should look at some output impedance that should be equal to Zi2.

So at port 2 if I look to this side I am getting ZI2 if I look this side I should get ZI2. So if I can find PR to ZI1 ZI2 so if I terminate by ZI2 then I get here ZI1 impedance similarly here if I terminate by ZI1 and excite here I should see here is ZI2 these two pairs are called image impedance. Since this is an a symmetrical network because ZI1 is not equal to ZI2 I will have two impedance ZI1 and ZI2. Let us see that whether this image impedance can be represented in terms of this impedance of this network.

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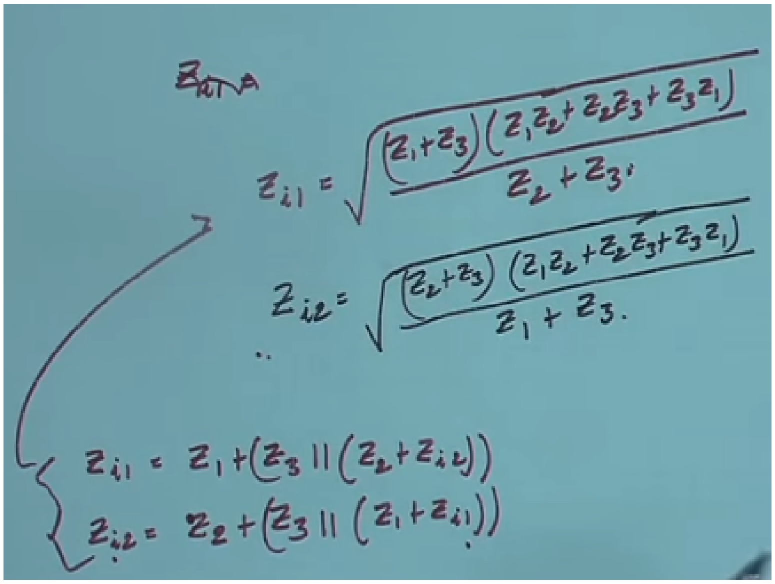

So the same diagram I can write that ZI1 or do like this so this ZI1 I can write as what is ZI1 obviously it is Z1 plus you see Z3 parallel to Z2 plus ZI2. Likewise what is ZI2 it is Z2 plus Z3 parallel to Z1 plus ZI1. Now these two equations you see ZI1 here I have ZI2 I have ZI2 here. I have ZI1 I have two equations so I can solve for ZI1 and ZI2 in terms of Z1, Z2, Z3.

If I do that upon solving this I get ZI1 is equal to root over Z1 + Z3 into Z1 Z2 + Z2 Z3 + Z3 Z1by Z2 + Z3 and ZI2 is Z2 + Z3 into Z1 Z2 plus Z2 Z3 + Z3 Z1. So you see that this image impedance can be represented in terms of the component impedances of the T section. Now always we won't be knowing Z1 Z2 Z3 as I said that let us consider two port network as a black box. But we can do measurements and always find these image impedances.

How, you know that any measurement requires either an open circuit or short circuit of the one of the port. So for any impedance measurement you look to do this also you have seen that if you want to find any 2 port parameter you need some port condition either short or open etc etc., So if we measure measurement of image impedance, let us say that port one we measure impedance when port 2 open.

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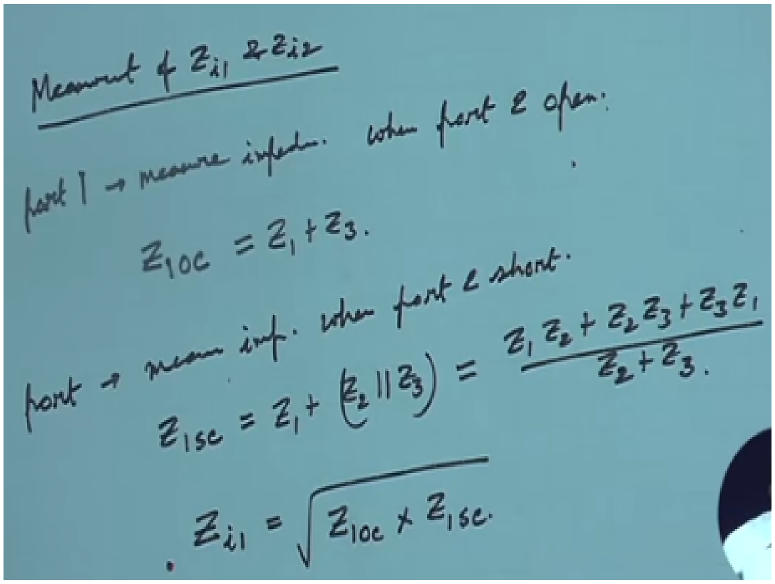

We call that measurement as Z1 since we are doing at port 1 open circuit Z1OC means I am measuring the impedance input impedance at port 1 with port 2 open. So if we look at the circuit if I open circuit this what will be Z1OC it will be simply Z1 + Z3. Similarly if we measure impedance with port 2 with sorry again port 1 measure impedance when port 2 is short it now let me short this port. So I call Z1SC second port is shortened you look at the circuit If I short it, it will be Z1 + Z2 Z3 parallel.

So Z1 + Z2 parallel Z3 Ok now what is this, this is Z1 Z2 + Z2 Z3 + Z3 Z1 by Z2 + Z3. Now you observe the image impedance terms already I have solved ZI1 can I just compare can I say that Zi1 is equal to Z1OC into Z1SC. So by measurement I can always find Z1 OC I can find Z1 SC. I know what image impedance is immediately I can calculate from these.

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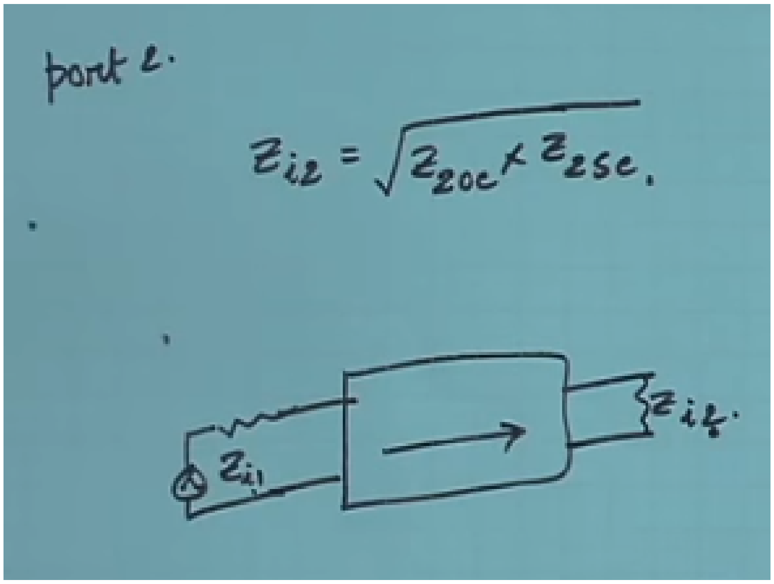

Similarly instead of port 1 if i measure in port 2 by once open circuiting port 1 find the input impedance at port 2 and then again you short the port 1 and measure the input impedance and port 2. So port 2 things if we do you will see the same thing that ZI2 can be expressed as Z2 OC into Z2 SC. So this shows that in image impedances can be always obtained from short or open circuit measurements on any network.

So we can easily do this suppose I am given a network I can always find this Z1 OC Z1 SC Z2 OC Z2 SC and find out tis Zi1 Zi 2 and then I choose a source with that internal impedance that I want and terminate or choose the load as Zi 2 I know I can achieve maximum power transfer. Already I said so that means I can have maximum power transfer is guaranteed if I use image impedance as the terminating impedance at both the ports.

Now we know we have said that we will be using the two port network as only lossless components, that means you do not use any hard so their own many internal loss there. So, by terminating with image impedances I assume maximum power transfer no loss in the circuit lossless. So image impedance is an important thing performance measure of the power transport power transmission that is taking place to a network so you see that we can specify something on it later when we will design a filter.

So instead of ABCD we can specify image impedances and that will solve one many of our problems but think one point that I have image impedance here I have image impedance here also you see with this I require to know that ok by this Zi 1 and Zi 2 terminations Zi 1 here and Zi 2 here I have to ensure that I am giving maximum power am delivering to this load.

But I am assuming that here there is no loss but the power is flowing in this direction it may so happen since I am using reactive elements power may be locally confined that is not flowing there. So I need to also see how propagation is taking place inside this 2 port network. So we need to have the transmission of power to the network also that we will next see that this is called propagation of power.

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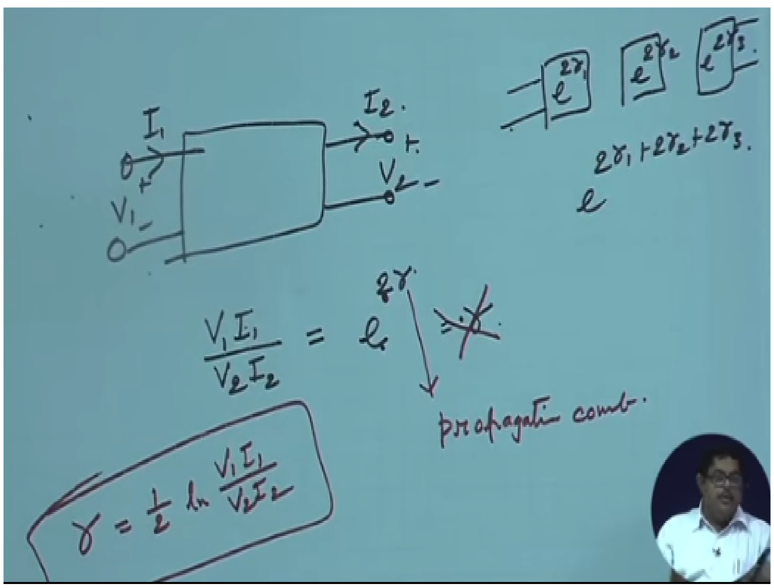

So what we define that again to two network I have this V1, I have V2, I have I1, I have I 2 as before now let me define V1 I1 by V2 I2 is volt ampere the concept you have learnt in you electrical circuit class. So input volte ampere these are complex quantities input volte ampere and output volt ampere. What is this ratio that will be something now I want to ensure that is fully making the power transmission possible.

So we call this I can name it any number this will be some number you see input by output volt ampere but we have certain advantage if we instead of defining any number here we write it as some exponential factor e to the power two gamma why because you see when I will cascade many such networks this one will have some this ratio e to the power 2 gamma 1 this will have E to the power 2 gamma 2 another will have E to the power 2 gamma 3 etc.,

Now from this input to this input if I want to find what is this transmission ratio of all the volte ampire if I express it exponential factor the final thing will E to the power 2 gamma 1 + 2 gamma 2 + 2 gamma 3 but if I do not use this exponential factor. If I just write it as gamma suppose then I will have to work out and I will have to work out and I will have to find out what is the magnitude and phase all these things here.

But exponential factor makes simply be an addition in if it is an absolute value it would have been some multiplication. We always prefer addition to multiplication that is why it initially people did like that they put this as propagation constant. But now with after some learning people understood that if we represent this ratio is it exponential factors and also you see I have taken a factor two here why because many times will be interested to see what is the voltage ratio what is the current ratio.

But this is actually a volt ampere ratio which is for that it is actually product of voltage and current. So I have taken two gamma, this gamma is called propagation constant. So what is the definition of propagation constant you see gamma is equal to half LN V1 I1 by V2 I2 a very important definition propagation constant you see it shows that how input power is propagating through the network inside the 2 port network.

So my job is now to find out what is this e to the power 2 gamma ratio is equal to V1 I1 by V2 I2 is equal to in terms of ABCD parameters AV2 plus BI2 into CV2 plus DI2 by V2 I2 also I know V2 is equal to ZI 2I image impedance it is terminated with image impedance so if you do that finally you can solve that this ratio will turn out to be this simple manipulation put thus and you know the value of Zi1 ZI2. So you will get this will be simply this or e to the power gamma is equal to root AD plus root BC.

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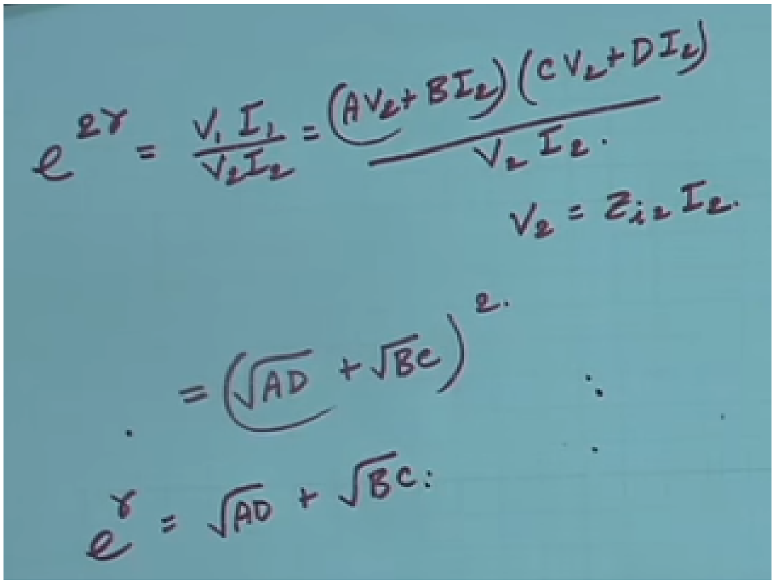

Now here you see this propagation constant I have expressed in terms of ABCD parameters. One more thing is remaining I have already said about characteristic impedances is characteristic impedances also expressible in terms of ABCD parameters. Let us see I have the same 2 port network I have I1 here, I have V1 here, and I want this should be ZI1 and here this should be terminated by ZI2 and this is V2 this is my I2.

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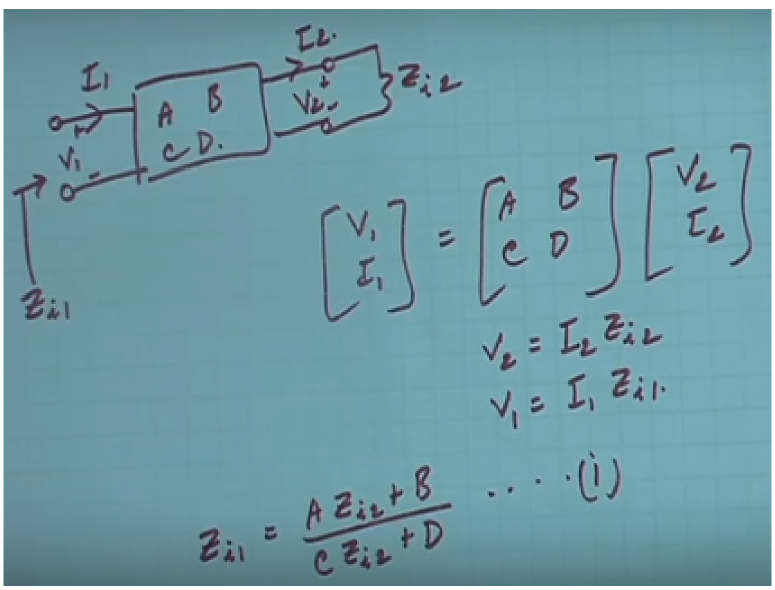

So I can write I know this is ABCD so V1I1 is equal to ABCD the definition of transmission parameters also I have V2 is equal to I2 ZI2 and V1 is equal to I1 ZI1. So put these equations and find out what the Zi1 you will see you will get AZI2 + B by CZi2 + D let me call this for timing equation 1. Now reverse the picture that same transmission line this time I am putting the excitation here.

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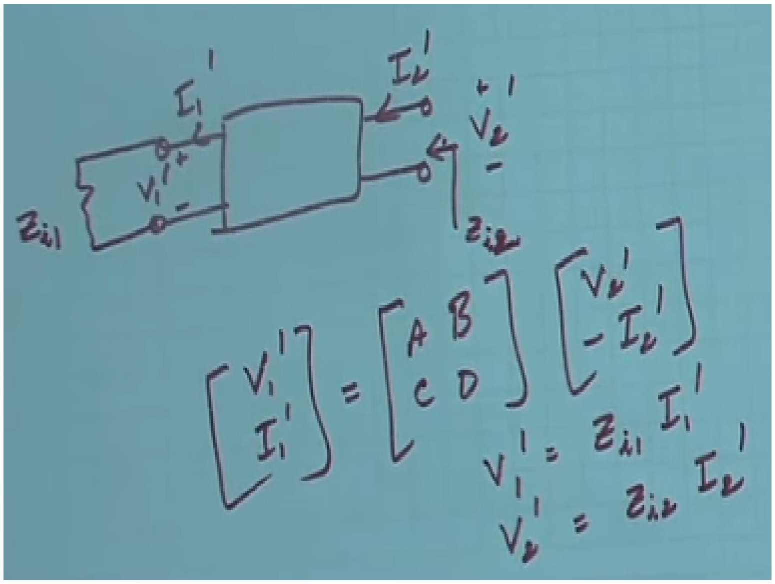

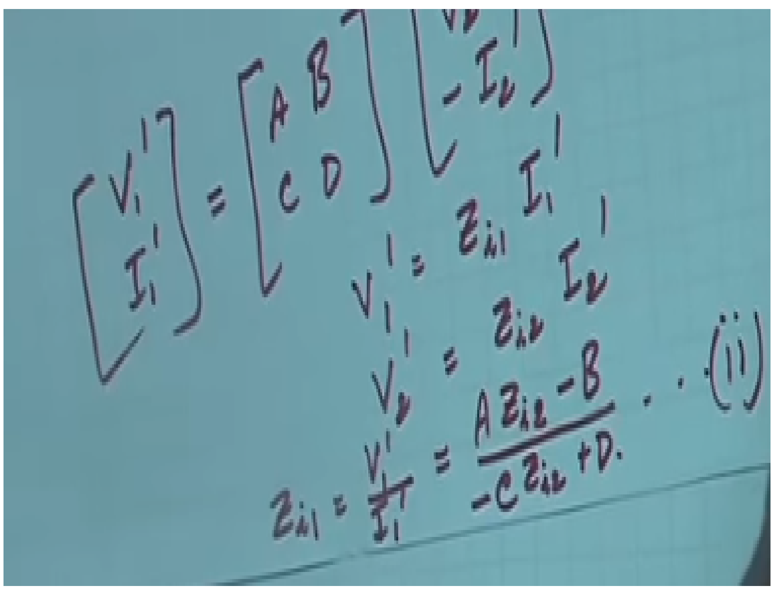

So I am looking at it here I will get ZI2 and this is my V2 dashed as before this is my I2 dashed as before and from here I am taking terminating you to Zi 1 this is my V1 dashed this is my I1 dashed so here again I can write that V1 dashed I1 dashed is equal to ABCD V2 dashed minus I2 dashed and what about the ports V1 dashed is equal to ZI1 I1 dashed V2 dashed is equal to ZI2 I2 dashed. Then find out that what is your ZI2.

So or you find what is your ZI1 which is nothing but V1 dash by I1 dash that will turn out to be AZI2 - B by - C ZI2 + D. So this let me call equation 2, you have equation 1, you have equation 2 to solve for ZI1. If you solve even so from 1 & 2 you can solve for ZI1 and that will be equal to or ZI1 will be equal to AB by CD and ZI2 will be equal to root over BD by AC.

Now I am happy because I know that ZI1, one of the image impedance can be expressed in terms of four ABCD parameters ZI2 also I can express in terms of ABCD parameters and I have already seen that propagation constant, gamma you see this propagation constant that also I can express in terms of ABCD parameters. ABCD parameters completely characterization 2 port network I say equivalently I can say two image impedance ZI1 ZI2 and e to the and gamma these three also characterizes a network.

But what is the beauty if I have ZI ZI 2 I know what is impedance level of the excitations of the network that means what is the source impedance what is the load impedance they are according to the power matching. So that no maximum power sorry they are according to the maximum power will flow and by putting conditions and gamma I will be able to say whether these frequency will pass or not. So instead of ABCD parameters this is a better description of a 2 port network if I want to design a filter.

And already I have seen that I can do the measurement of image impedances that time I said in terms of the by opening and shorting the port. I will also have to prove that I can do this for propagation constant also because this is a new thing that time I didn't say these. So that I will do now that measurement of image impedance and propagation constant. So what we will do the same network this is port 2 this is open circuit and I am looking here at let me call this ZO1.

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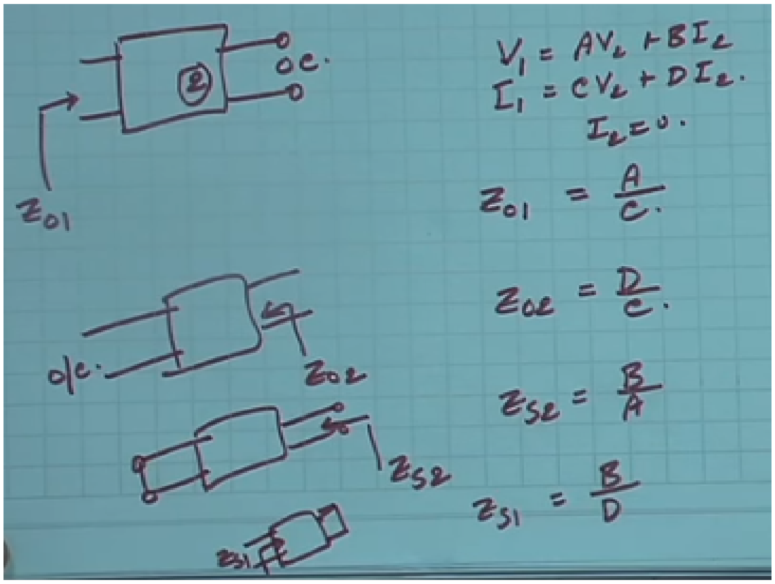

So I know V1 equal to AV2 plus BI2 I1 is equal to CV2 plus Di2 etc and open circuit means I2 is equal to 0. So if you enforce that is Zo1 that will be A by C then you short circuit so or open circuit this port purpose you open circuit and measure here Z02 so Z02 that will be turn out to be D by C then you do that short circuit port one and measure the from this port you measure ZS2, you will see Zs2 will turn out to be B by A.

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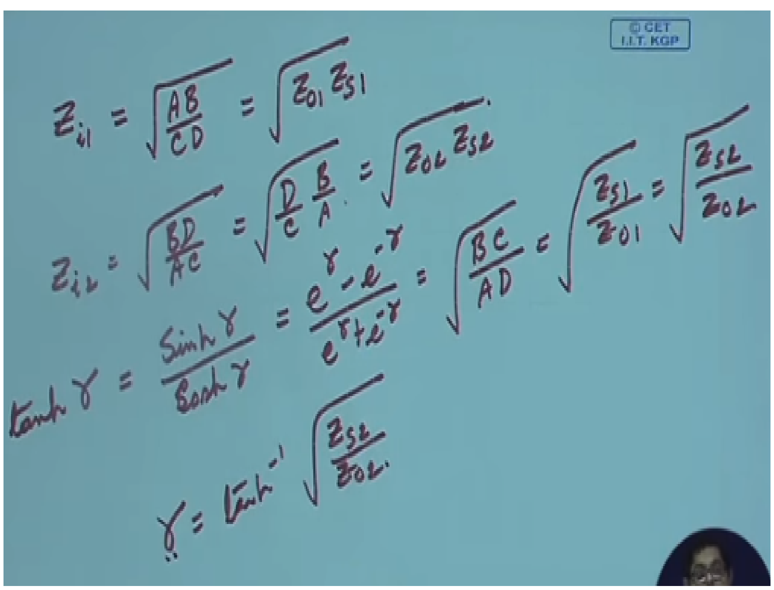

And which one I missed 2, ZS1 this, so you short circuit this port and measure here ZS1 ZS1 will be B by D. Once you have that you can immediately write because already we have seen ZI1 is equal to the Zo1 into Z02 etc., So you will get that is equal to AB by CD and that is nothing but ZO1similary Zi2 is equal to root over BD by AC and that is D by C into B by A that is nothing but ZO2 ZS2. And you see what is TAN gamma, TAN hyperbolic gamma all of you are familiar with hyperbolic functions.

So this is E to the power gamma minus E to the power minus gamma by E to the power gamma plus and that is nothing but BC by AD that is ZS1 by ZO1 and or Zs2 by ZO2. So you see that gamma can be expressed completely in terms of short circuit and open circuit measurement. So I can measure image impedance by open circuit, short circuit measurements. I can also measure gamma by open circuit short circuit measurements.

Previously I showed that ZI1 ZI2 gamma the completely characterizes the network reciprocal 2 port network. Now I have now I have shown that they also can be measured so you do not have a difficulty any 2 port network, lossless network you can represent like this. So an alternate description for characterization of 2 port network is in terms of ZI1 and ZI2 and gamma I think in the next class will introduce another criteria all symmetrical network and we will simplify this procedure. Thank you