Design Principles of RF and Microwave Filters and Amplifiers
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Module No # 1
Lecture No # 03
Symmetrical lossless network description for filter design

Welcome to this third lecture now, the same 2 port reciprocal network we enforce symmetry in the network. Generally filters, most of the times we design symmetrical filters the 2 port network is symmetrical. What do you mean by symmetrical? When I said this 2 port that I may say this is Z1 Z2 Z3. So in a T section as I already said that any 2 port network whatever may be the interconnection can be always represented like this now this Z1 Z2 different that is a symmetric network.

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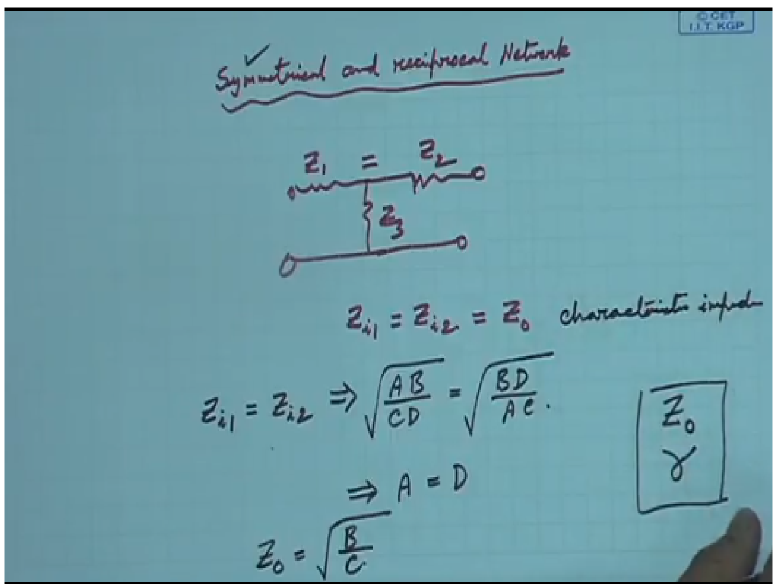

if I enforce symmetry then Z1 = Z2 so what happens to ZI1 my image impedance ZI1 and you see the expressions of ZI1 and Z12 already derived the moment Z1 and Z2 are same they becomes equal. So for a symmetrical network Z1 ZI2 are equal that we generally call Z0. What is Z0 a very very important concept called characteristic impedance.

So for a symmetrical network I have correct only one characteristic impedance for a non-symmetrical reciprocal network I have two image impedance but here it is one. So, what is the concept of characteristic impedance? that I terminate with characteristic impedance then the source with characteristic impedance Z0 will see that input impedance is also Z0, so it can deliver maximum power okay. But now so we see that we have characteristic impedance let us see what is his value in terms of ABCD parameters.

We already seen that when we enforce Z1 = Z2 that mean symmetry Z1 becomes ZI2 so that implies that we have already earlier derived ZI1 = AB by CD and this is BD by AC so that becomes A = D. You all know into port parameters transmission parameters if symmetrical network is equal to D it comes from here so what happens to value of Z0 it is simply B by C.

And so now you see and that for a symmetrical reciprocal network I have I can characterize the network by only 2 parameters 1 is that this Z0 and another we have already seen it is gamma. So Z0 and gamma are sufficient to characterize the network so by Z0 we will understand what will be our impedance level if there is an impedance mismatch etc and with gamma will enforce whether the frequencies passed or not whether attenuated or not.

So let now express the ABCD parameters we are so familiar with in terms of Z0 gamma that means from now on will specify only Z0 and gamma for filter etc specification so we should have a mapping between ABCD etc. So you can easily do that express ABCD in terms of Z0 and gamma because now our description will be in terms of that so you can start from that AD – BC = 1 and also A = D.

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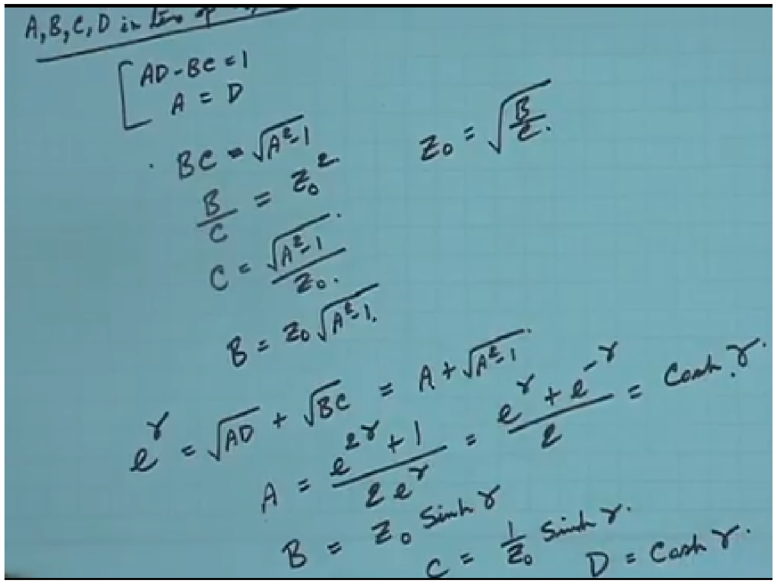

So with this you can now find out that let us decide at what will happen to them if we put this A = D we get BC = root over square – 1. And we know that B by C is Z0 square you see this one we have found as B by C so from this you can find out the value of C is root over square – 1 by Z0 and B is Z0 root over square – 1 but and what is A to the power gamma earlier we have seen it is AD + BC. Now in terms of this I can write it as A + root over square – 1 so this equation I can solve so A will be expressed in terms of A to the power gamma.

So by doing that I can see that a becomes A to the power 2 gamma +1 by 2 A to the power gamma that is nothing but A to the power gamma + A to the power – gamma by 2 which is nothing but COS hyperbolic gamma. So A is nothing but COS hyperbolic gamma now put this B is already expressed in terms of Z0 and this Z0 value.

You know let me write root over B by C so you can call for and see that B will be equal to Z0 SIN gamma please all of you do this exercises I am just writing you cannot memorize all these things but I am telling you the way so please do this manipulation it requires a bit familiarity with hyperbolic functions and C = 1 by Z0 SIN hyperbolic gamma and D = COS hyperbolic gamma.

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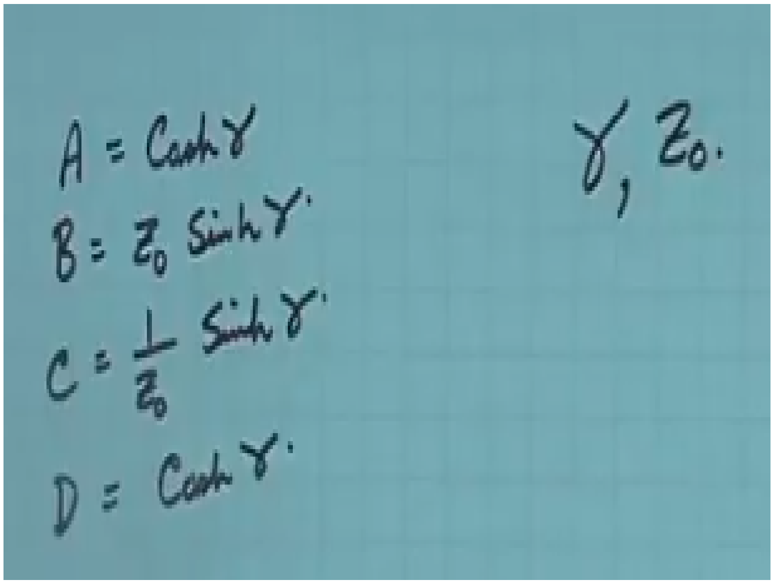

So you see that this expression that means now I can clearly write that A = COS hyperbolic gamma B = Z0 SIN hyperbolic gamma C = 1 by ZO SIN hyperbolic gamma and D = COS hyperbolic gamma so you see that actually here from also you can see that ABCD parameter was not needed because A = D for a symmetrical network.

And this is the description that any ABCD parameters I can express in terms of 2 quantities is gamma and Z0. So that is the beauty you know that we are in the right path what we will do now will put the filter condition. So for a filter as I said what is the filter that it will allow some bands to pass it will stop all other bands. So we demand that now 2 port network is in terms of gamma and Z0 what is our demand that if it is to be a filter so gamma in pass band should be lossless.

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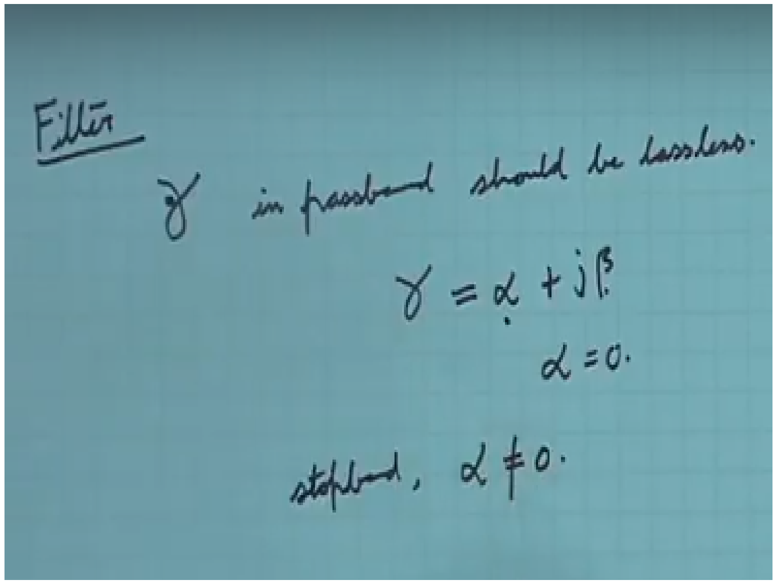

We want that gamma the propagation constant should be lossless then only the circuit won't attenuate power also not store power it will be able to deliver it to the port 2. So but you know that gamma is a complex quantity as I said that how we define that input volte ampere and output volte ampere their ratio that we call it A to the power 2 gamma.

So gamma is that now you are also familiar that generally this complex quantity we write as 2 part alpha + J beta, alpha is attenuation constant beta is the phase constant. For any electromagnetic wave this is true so when we say alpha in the pass band should be lossless. What we basically say that alpha should be 0 over the whole pass band that there should not be any attenuation. And what happens in stop band we want that it should be lossy ,so atleast alpha should not be = to 0.

Ok and we have already decided that to have this pass band alpha = 0 we will use only reactive elements. Now there is a theorem which will now see the theorem says that for a symmetric 2 port reciprocal network of pure reactances. If the characteristic impedance is a pure resistance that innovation constant is zero. If the characteristic impedance is pure reactance their attenuation constant is non-zero. So for this symmetric 2 port reciprocal this is a theorem of Everett symmetric 2 port reciprocal network made of pure reactances.

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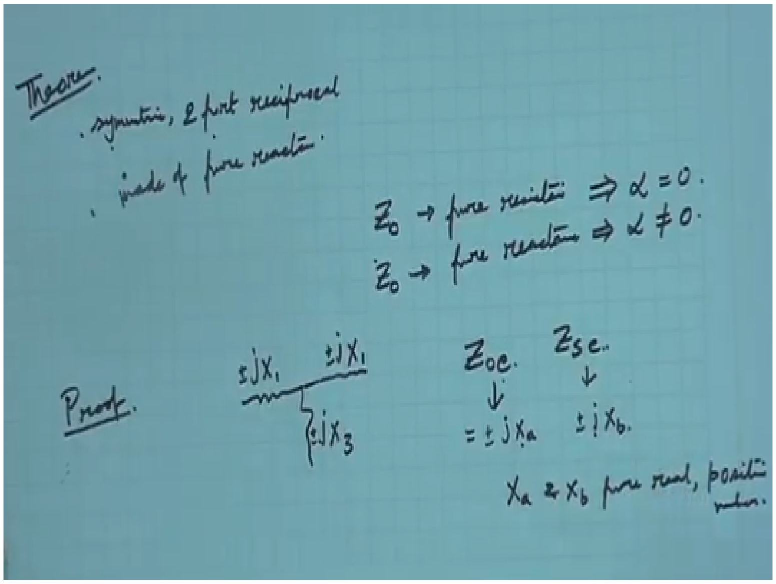

The theorem says that if Z0 is pure resistance then alpha = 0 if Z0 is pure reactants alpha is non-zero let us prove this theorem. So will now prove this theorem since the network is made of pure reactances can I say that network these are all instead of Z they are JX1 it is a symmetric network JX1 and let us say X3 instead of Z1, Z2, Z3 now this may be plus minus whatever if this is the case then can I say that ZOC and ZSC. Which we already saw they are also pure reactances. We have seen that Z0 = root over ZOC into ZSC open circuit and short circuit cases.

So they will be because we have seen their various combination of these they will be also pure reactances. So let us consider that ZOC, let us call it either + or – the pure reactance this is let us call + or – JXB. That means XA and XB pure real positive numbers you agree that it should be. Because since I have incorporated JXA and XB are positive real numbers now let us see there can be several cases.

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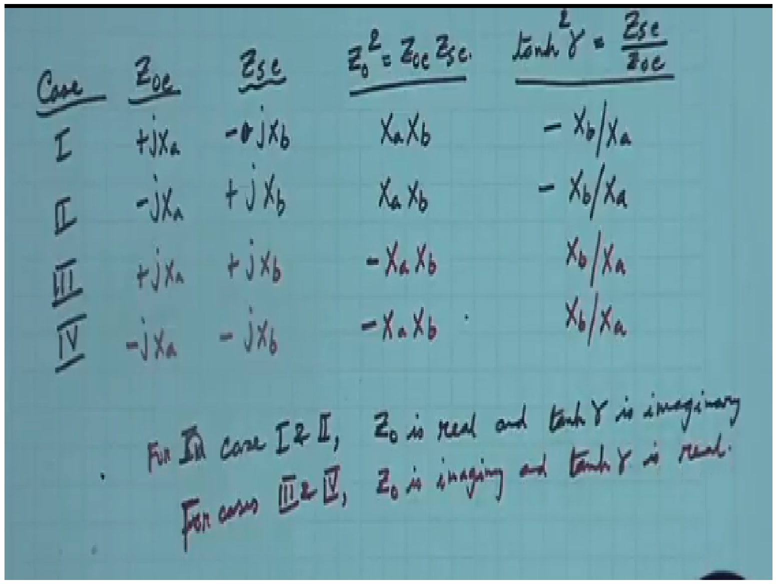

Case 1, 2 am proving the theorem, is important for filter design now let me say what is ZOC what is ZSC then let me we know Z0 square = ZOC, ZSC and we now 10 the square is ZSC by ZOC so we will make table.

So case 1 is let us say that this ZOC and ZSC they are of same sign so it is + JXA it is -JXB then what will be Z0 square the it will be simply XAXB and what will be these ZSC by ZOC it will be – XB by XA case 2 this is –JXA this +JXB this will be XAXB this will be – XB-XA this is + JXA + JXB this is – XA XB this is XB by XA this is – JXA – JXB – XAXB XB by XA.

So you see that time I said that I have some possibilities so I have taken all permutations of these. From here you can see that case 1 and 2, Z0 is real that means Z0 it is a characteristic impedance it is real means this is a pure resistance and TAN hyperbolic is imaginary. Because TAN this square is – so TAN hyperbolic will be an imaginary number. And in case 3 and 4, I have marked that by rate you see Z0 is imaginary and TAN hyperbolic is real. So let me write this thing that in case 1 and 2 Z0 is real and TAN hyperbolic is imaginary.

In our power cases 3 and 4, Z0 is imaginary and TAN hyperbolic gamma is real. Still I have not proved the theorem because theorem said that if Z1 is real, alpha will be 0, if Z1 is imaginary there will be non-zero alpha. I have not reached there. So I need to do that to prove the theorem so part 1 is this then let me the write what is TAN hyperbolic gamma? Tan hyperbolic gamma is nothing but TAN hyperbolic alpha + J beta.

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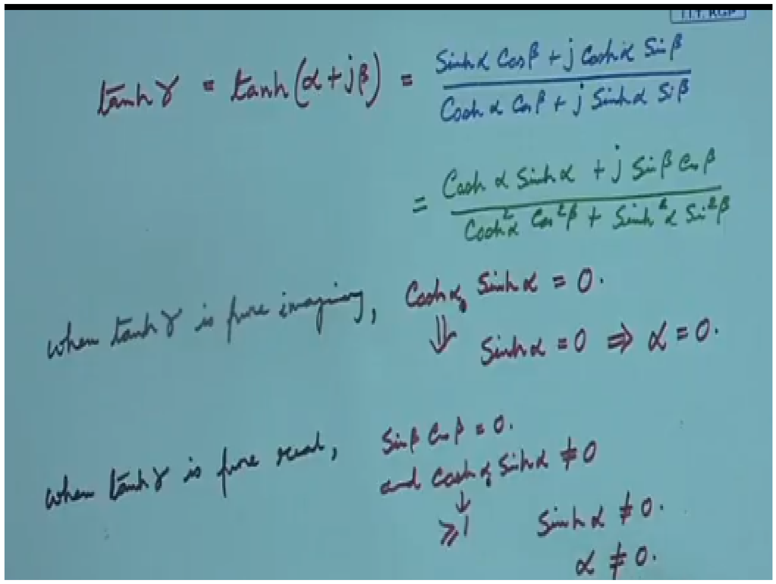

And that is equal to SIN hyperbolic alpha, COS beta + J COS hyperbolic alpha, SIN beta by COS hyperbolic alpha, COS beta + J SIN hyperbolic alpha, SIN beta. Now numerator denominator both are complex number A + JB so I generalized the determinator. So that becomes after some simple manipulation COS hyperbolic alpha COS hyperbolic alpha SIN hyperbolic + J SIN beta, COS beta by COS hyperbolic alpha square COS square beta + SIN hyperbolic square alpha and SIN square beta. So denominator is pure real now can talk of that.

When TAN hyperbolic gamma is we have seen in this case TAN hyperbolic is imaginary. So TAN hyperbolic is pure imaginary COS hyperbolic alpha, SIN hyperbolic alpha. So TAN hyperbolic alpha to be in a imaginary I require this part should go to 0 now here you see COS hyperbolic alpha always is greater than 0 so this is e to the power alpha + e to the power minus alpha by e to the power by 2 so it cannot be 0 COS hyperbolic alpha cannot be 0.

So the only possibilities this implies SIN hyperbolic alpha is equal to 0. What is SIN hyperbolic alpha that means e to the power alpha is equal to e to the power minus alpha that alpha is equal to 0. So where is the theorem, said that when Z0 is pure resistance alpha is equal to 0. You see Z0 is pure resistance from this I got alpha is equal to 0 so, okay let us see the other option. When

TAN hyperbolic gamma is pure real what I get you see this thing is pure real that means this part should go to 0.

I demand SIN beta, COS beta should be zero and also COS hyperbolic alpha, SIN hyperbolic alpha non-zero because this this also may go to zero then whole thing will be zero. But I require that this is nonzero but COS as I said COS is always greater than one so no problem it demands as COS this is always greater that equal to one. So what we demand is SIN hyperbolic alpha is nonzero so that means alpha nonzero means the attenuation constant is nonzero that mean there is a attenuation.

So that proves the theorem that when so what it shows let me go to my cases that it this case when Z0 is real there will be no attenuation and when Z0 is these there will be definitely attenuation. So can I not say that if I enforce this condition this is my pass band condition and this is my stop band condition of a filter. So I will do that, so but you also observe here that so this is my filter pass band I will go from here but here I have one demand that ZOC and ZSC they should be of opposites sign.

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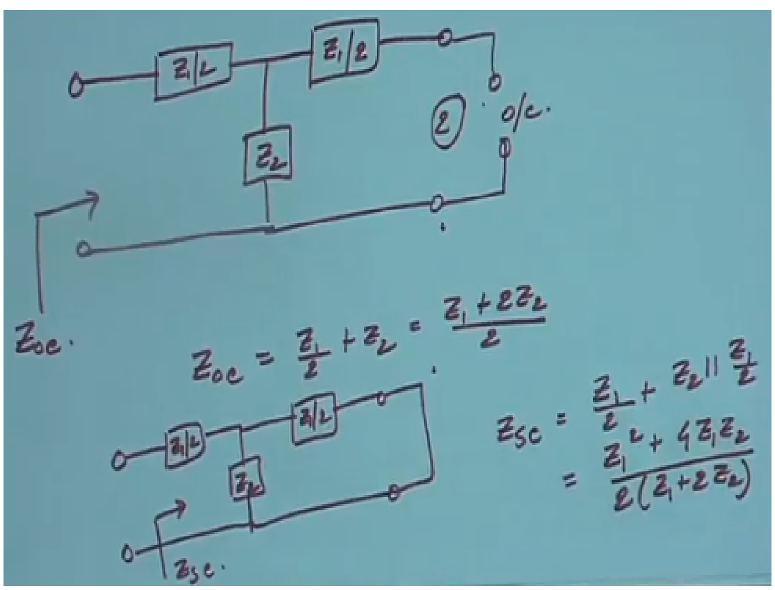

Now ZOC ZSC they are that means that or I will see that, so let me now see my T section I can also see it with PIE section but the so it's a symmetrical and instead of earlier I was calling Z1 now people call it Z1 by 2 because there are 2 in series so okay. Now remember obviously Z1 and Z2 they are pure reactances we want to derive the propagation constant of this gamma of this network already we have done TAN gamma so from there we will try to see enforce the pass band condition.

So port 2 let us make that this is open circuit condition so that time we know we call it ZOC and what will be that ZOC = Z1 by 2 that is equal to Z1 + 2 Z2 by 2. Now I put that port 2 is shorted this is Z1 by 2, Z1 by 2Z2 and this is ZSC. So what is ZSC it is you can easily see Z1 by 2 + parallel of Z2 and Z1 by 2 so that if you do it will become Z1 by 2 + and Z2 parallel Z1 by 2 and that is equal to Z1 by square + 4 Z1, Z2 by 2 Z1 + 2Z2 so I got Z0c and ZSC.

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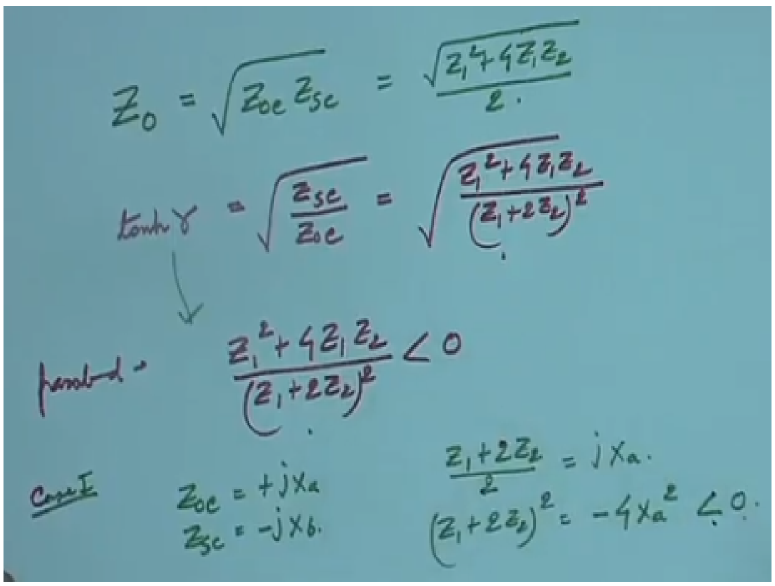

So I can easily now find what is the characteristic impedance Z0 I know always this is true I have expression for them say. So if you put it becomes Z1 square + 4Z1 Z2 by 2 already we have in that table that this gamma becomes pure imaginary. So this is the stop band there ZOC and ZSC there should be opposite side so, because that only makes, you see ZOC and ZSC their opposite side makes the characteristics impedance real otherwise they are of same type this is imaginary.

And what happens to the other, so characteristic impedance we have find what is the other parameter of the 2 port symmetrical reciprocal network gamma. So what is TAN gamma we know that is ZOC by ZSC and if you do this that will become root over Z1 square + 4Z1 Z2 by

Z1 + 2Z2 square. Now this we want this TAN gamma should be imaginary in the pass band. So our demand that in the pass band Z1 square + 4Z1 Z2 by Z1 Z2 + 2Z2 square should be less than 0.

Consider this case 1 in the table will now confine ourselves to this case 1 and 2. In case 1 ZOC is + JXA this is - JXB so let us put it in case 1 ZOC + JXA ZSC is - JXB so what happens to 2Z1 + 2Z2 by 2 this 1Z1 + 2Z2 by 2 becomes JXA and Z1+ so this 2Z2 whole square that becomes - 4XA squared we know XA is positive real number so Z1 + 2Z square that is less than 0. So Tan beta that means in pass band we know this TAN beta value is this is less than 0.

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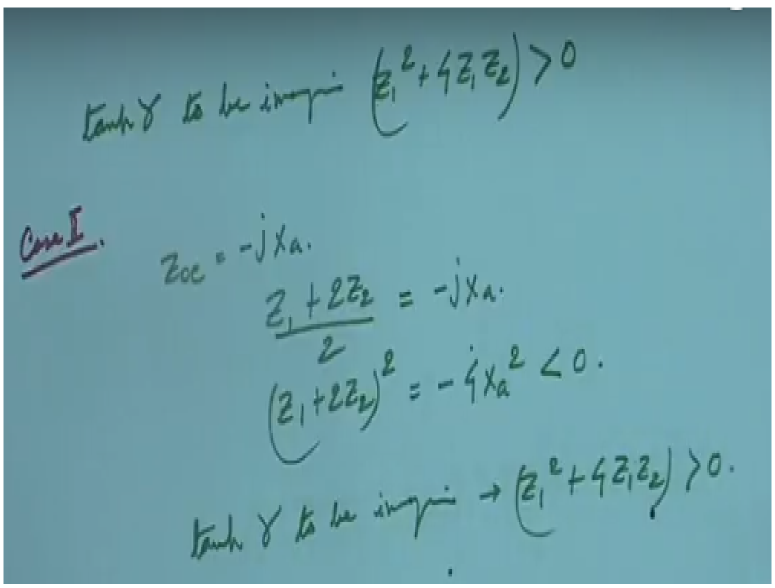

So this whole thing less than 0 these need to be positive to satisfy this so we can say that TAN gamma to be imaginary Z1 square + 4Z1 Z2 that needs to be greater than 0. So this is from case one that means first case from pass band we know this. Let us see case 2 there we have seen ZOC is - JXA this is case 2 ZOC is - JXA ZSC + XB.

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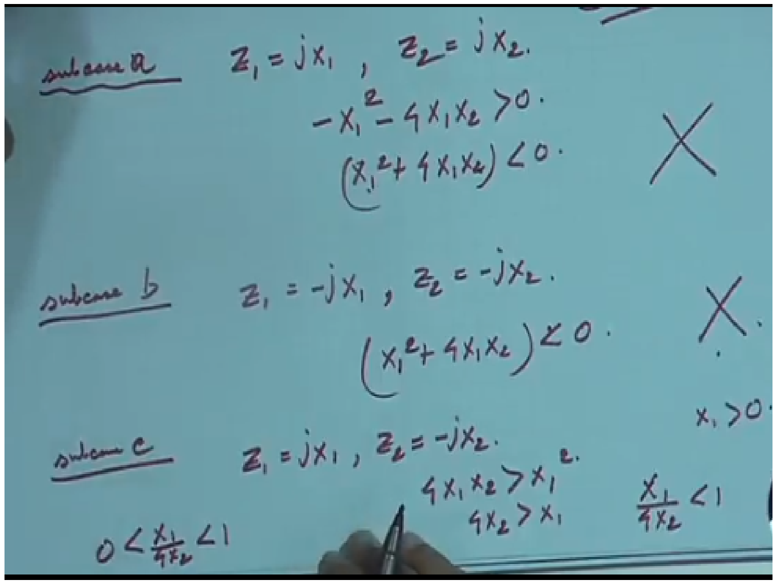

So with this what happens to Z1 + 2Z 2 by 2 that becomes – JXA, so what happens to the square of this 2Z2 square that is again - 4XA square that is again less than 0 so same condition that here also TAN gamma to be imaginary we require Z1 square + 4Z1 Z2 greater than 0. So the condition in the pass band if we want to have pass band without any attenuation then condition is Z1 square + 4Z1 Z2 greater than 0. Now Z1 and Z2 they are reactive elements.

So again there will be various sub cases so I can say sub case 1 a sub case A because there will be various cases of Z2 Z2. We should remember that Z1 = JX1 let us chose this Z2 = JX2. Then we can write what happens to Z1 square + 4Z1 Z1 that is - X square - 4X1 X2 greater than 0 or X1 square + 4X1 X2 is less than 0. So since X1 and X2 are positive real numbers this is not possible so I say that sub case is not possible similarly subcase B there will be Z1 = - JX 1 is equal to -JX2 again we will get the same condition X1 square + 4X1 X2 is less than 0.

So again not possible sub case C here let me take here as ZX1 Z2 as opposite SIN JX2 if you do this what is this value Z1 square always we are evaluating this Z1 square + 4Z1 Z2 that should be greater than 0. This is the condition to achieve the pass band. So we are doing this and getting this 4X1 X2 is greater than JX1 square now as X1 is greater than 0. So 4X2 will be greater than X1 or X1 by 4X2 greater than 1, also X1 is greater than 0.

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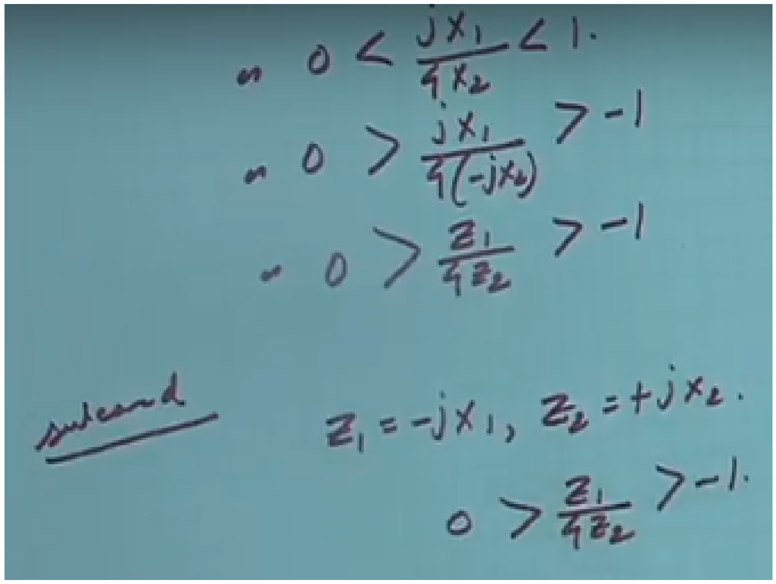

So combining these the condition is that 0 less than X1 by 4X2 less than one or I can say or 0 less than JX1 by 4X2 is less than 1or greater than JX1 by 4 – JX2 is greater than -1 or 0 greater than Z1 4Z2 is greater than -1. So this is possible that means here if we and enforce this condition it boils down to Z1 by 4Z2 to this ratio should be between 0 and -1 and let us see the sub case D.

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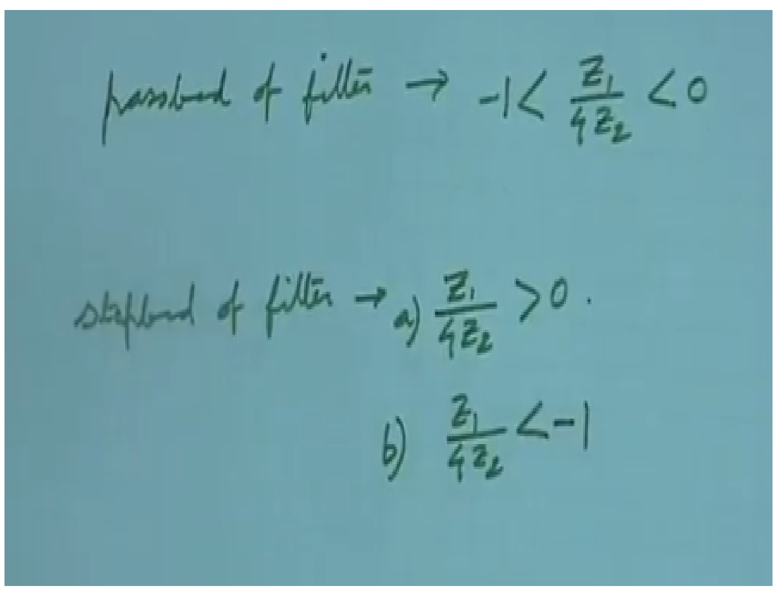

That will be Z1 = - JX1 Z2= + ZX2 again if you do you do you get the condition that 0 Z1 by 4Z2 n – 1. So this shows that, you can have the pass band of filter should be in the range this Z one by four Z2 this number should within this range. And where is the stop band of the filter, there are two possibilities for that we have already seen that Z1 by 4Z2 that is greater than 0 or Z1 by 4Z2 that is less than 1.

This is mathematics we will see in the next lecture what is the implication of that but what we have done here, we have found out that if you want to have pass band the two reactances their impedances is Z1and Z2 this Z1 by 4Z2 should be within this range then only you can have pass band and if you choose Z1 by 4Z2 either this or this you get stop band of filter Thank You