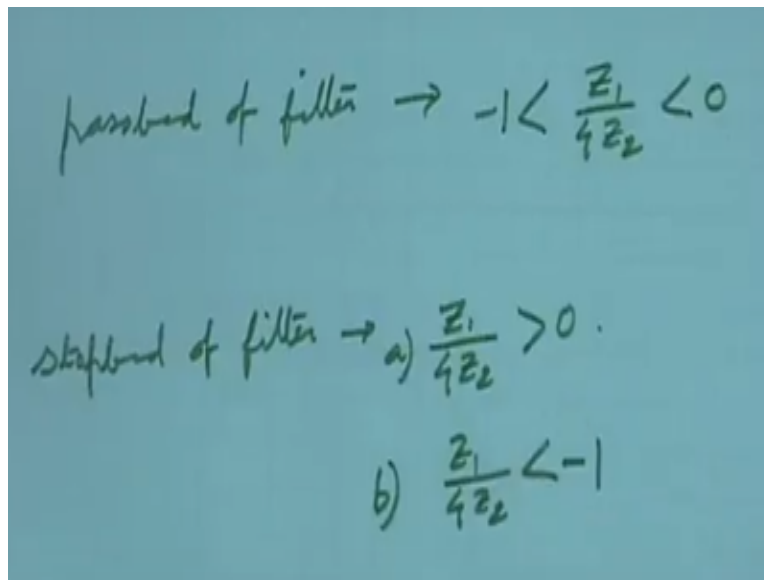


Design Principles of RF and Microwave Filters and Amplifiers
Prof. Amitabha Bhattacharya
Department of Electronics and EC Engineering
Indian Institute of Technology – Kharagpur

Module No # 1
Lecture No # 04
Constant K prototype filter design

Welcome to the fourth lecture, we have already seen that the Z_1 by port Z_2 an important ratio. Z_1 is the impedance of the series elements or a Z_1 by 2 the series elements and Z_2 is shunt elements that impedance that ratio in the pass band it should be in this range and in stop band it should be in this range. You know impedance is always a reactive element I is a function of frequency so in pass band we need to enforce this stop band we need to enforce this.

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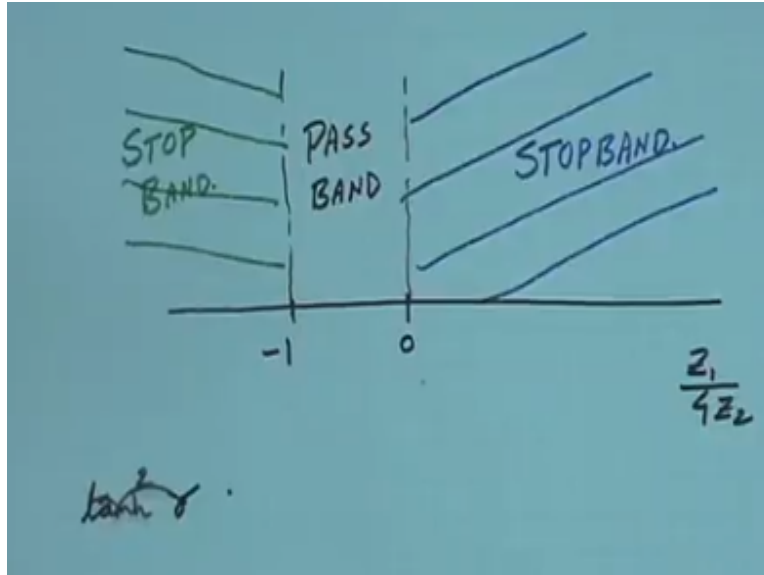


passband of filter $\rightarrow -1 < \frac{Z_1}{4Z_2} < 0$

stopband of filter \rightarrow a) $\frac{Z_1}{4Z_2} > 0$.

b) $\frac{Z_1}{4Z_2} < -1$

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So this boils down to this if I plot Z_1 by $4Z_2$ I have to point 0 and 1, so any Z_1 by $4Z_2$ greater than 0 is a stop band. So this entire thing up to infinity stop band, so let me write this is stop also Z_1 by $4Z_2$ less than -1 is stop band so this is also stop band, So I have only this zone this is my pass band. Okay, so now we got a filter we know that what I how I will have to choose the Z_1 and Z_2 so I get pass band.

But another thing remains that we have found out that in pass band α is 0 and in stop band α is not 0 but what is this value that we have not found. So we will have to find the value of α in the stop band also we will find what is the value of β in the stop band because β is phase constant. So as the wave moves per unit length how much phase changes that also is a curiosity. And in pass band we have seen that α is equal to 0 but what happens to β here how β changes in pass band.

So that we have already seen because everything is embodied here $\tan \gamma$ hyperbolic square or let me start in a new $\tan \gamma$ hyperbolic square is Z_1^2 plus $4Z_1 Z_2$ by Z_1 plus $2Z_2^2$ square. We have already seen this number I think this so this already we have derived this that $\tan \gamma$ is here. Now we will try to see that under those stop band pass band conditions, that means under that Z_1 by $4Z_2$ that range what happens to α what happens to β both in stop band and pass band.

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$$Z_0 = \sqrt{Z_{oc} Z_{sc}} = \sqrt{\frac{Z_1^2 + 4Z_1 Z_2}{Z_2}}$$

$$\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}} = \sqrt{\frac{Z_1^2 + 4Z_1 Z_2}{(Z_1 + 2Z_2)^2}}$$

Condition -

$$\frac{Z_1^2 + 4Z_1 Z_2}{(Z_1 + 2Z_2)^2} < 0$$

Case I

$$Z_{oc} = +jX_a$$

$$Z_{sc} = -jX_b$$

$Z_1 + 2Z_2$

So this is the number now let us do a bit manipulation and from this we can find out that COS gamma these are all manipulation so that the thing become a bit simplified. COS gamma from 10 square I can always find COS gamma that will be 1 plus Z1 by 2 Z2. But we want to express in terms of Z1 1 by 4 Z2 because our condition is this condition is given by 4 Z2 say bit more manipulations. And we get that SIN hyperbolic gamma by 2 is this number Z1 by 4 Z2.

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$$\tanh^2 \gamma = \frac{Z_1^2 + 4Z_1 Z_2}{(Z_1 + 2Z_2)^2}$$

$$\cosh \gamma = 1 + \frac{Z_1}{2Z_2}$$

$$\sinh \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

So from here I came here from here again here so now I neither wholly gamma. So here you put this SIN hyperbolic gamma means alpha plus J beta by 2 is equal to root over Z1 by 4 Z2. Let me explain now SIN hyperbolic alpha by 2 COS beta by 2 plus J COS hyperbolic alpha by 2 SIN

beta by 2 is good. I now designate the two bands and because I have 2 stop bands every time instead of saying that let me call this is my stop band 1 and this is my stop band 2.

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$$\sinh\left(\frac{\alpha + j\beta}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\frac{\alpha}{2} \cos\frac{\beta}{2} + j \cosh\frac{\alpha}{2} \sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

Stopband I

$$\frac{Z_1}{4Z_2} > 0 \text{ and real.}$$

$$\Rightarrow \sinh\frac{\alpha}{2} \cos\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \dots (i)$$

and

$$\cosh\frac{\alpha}{2} \sin\frac{\beta}{2} = 0 \quad \dots (ii)$$

$$(ii) \Rightarrow \beta = 2n\pi, n \in \mathbb{Z}$$

I have only one pass band so no problem so I have got this. Let me see one by one stop band one what is the definition of stop band one what is stop band 1 Z_1 by $4Z_2$ is greater than 0. So Z_1 by $4Z_2$ is greater than 0 and real so this is greater than 0 and real. So can I say that if this is real, this is equal to this and this is 0. So my conditions are that this implies that \sinh hyperbolic α by 2 \cos β by 2 is equal to root over Z_1 by $4Z_2$ this is 1.

And \cos hyperbolic α by 2 \sin β by 2 is equal to 0 this is 2. So these two conditions now from 2 if this is there we know that \cos hyperbolic α cannot be 0 so \sin β by 2 should be 0 and what is \sin β by 2 0 I know that β then becomes β by 2 I know that β then becomes $2n\pi$ where n is an integer and from. So β value I know so what is \cos β by 2 now \cos β by 2 is ± 1 .

So let me write what is \cos β by 2 that will be ± 1 whole to the power n so putting this into putting this value to 1 that will say that α is 2 into ± 1 whole to the power n \sinh hyperbolic Z_1 by $4Z_2$ so I got the non zero α value in the stop band 1. Now let us see the stop band 2 what is stop band 2 this is my stop band 2 that means Z_1 by $4Z_2$ is less than -1 so stop band to Z_1 by $4Z_2$ is less than -1 . So this is imaginary this root over Z_1 by $4Z_2$ is pure imaginary.

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$\rightarrow (i) \Rightarrow \alpha = 2 (-1)^n \sinh^{-1} \sqrt{\frac{Z_1}{4Z_2}}$
Stopband II
 $\frac{Z_1}{4Z_2} < -1, \quad \sqrt{\frac{Z_1}{4Z_2}} \rightarrow \text{pure imaginary}$
 $\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} = 0 \dots (iii)$
 $j \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \dots (iv)$
 $\beta = (2n-1)\pi, n \in I.$
 \downarrow
 $j \cosh \frac{\alpha}{2} = \pm \sqrt{\frac{Z_1}{4Z_2}}$

So let me see this quantity so this is pure imaginary so this should go to 0 and this should be equal to this so I can write SIN hyperbolic alpha by 2 COS beta by 2 is equal to 0 let me call that earlier I want to set so three and J COS hyperbolic alpha by 2 SIN beta by 2 is equal to root over Z1 by 4Z2 this let me call 4 okay. So these three implies what that we know that if SIN hyperbolic alpha by 2 that cannot be 0 here. So COS beta by 2 is 0 that means beta is 2n - 1 this is 1 PIE N belongs to integer.

So if that is case what is the value of SIN beta by 2 you know that it will be one so J of this is this so can i say that J so now putting it here J putting this value here J COS hyperbolic alpha by 2 is plus minus root over Z1 by 4Z2 so that becomes alpha is equal to 2 COS hyperbolic root over Z1 by 4Z2. So this is the value of alpha in pass band 2 ok. Now we see what happens to pass band, we know alpha is 0 that we already seen but what happens to beta.

So you see this again alpha is 0 so COS hyperbolic alpha that this term become 1 so we have this term only so J SIN beta by 2 is equal to root over Z1 by Z2 so SIN beta by 2 is Z1 by 4Z2 magnitude. So I know what is beta in pass band beta is 2 SIN inverse Z1 by 4Z2. So in pass band alpha is this beta is this in various stop bands I have so once I have this now let us design a prototype filter. So let us take again I am taking T network you can also take a PIE network. So in pass band we know that the Z1 and Z2 should be opposite sign let.

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passband

$$\alpha = 0.$$

$$j \sin \frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

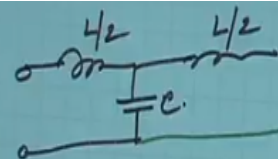
$$\sin \frac{\beta}{2} = \sqrt{\left| \frac{Z_1}{4Z_2} \right|}$$

$$\beta = 2 \sin^{-1} \sqrt{\left| \frac{Z_1}{4Z_2} \right|}$$

Let us take Z_1 is inductive that means the picture will that I have inductance here, capacitance here, again inductance here let us call this L by 2 this is L by 2 this is C . So Z_1 is what Z omega L Z_2 is 1 by J omega C and what is the that important number Z_1 by $4 Z_2$ this will be $-\omega^2 LC$ by 4 . In pass band we know this number is so pass band is where this number is between -1 and 0 .

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$Z_1 = j\omega L$
 $Z_2 = \frac{1}{j\omega C}$



$$\frac{Z_1}{4Z_2} = \frac{-\omega^2 LC}{4}$$

passband $\rightarrow -1 < \frac{-\omega^2 LC}{4} < 0$

\downarrow

$$1 > \frac{4\pi^2 f^2 LC}{4} > 0$$

$\frac{1}{\pi^2 LC} > f^2 > 0$

$f_c = \frac{1}{\pi\sqrt{LC}}$

LOW PASS FILTER

So you write that one is greater than these greater than 0 so you can write 1 from here you can follow that 1 is greater than $4 \pi^2 f^2 LC$ by 4 greater than 0 omega is $2 \pi f$. So we have written that or 1 by $\pi^2 LC$ is greater f^2 greater than 0 . Now this whole

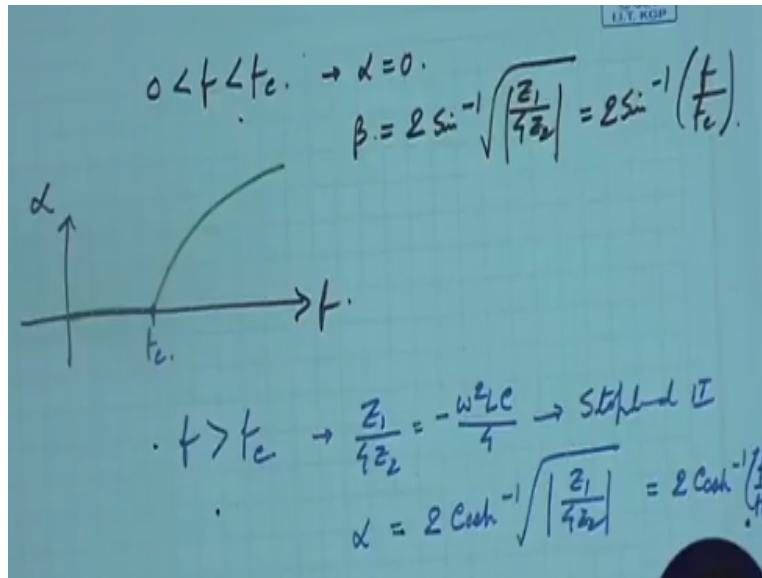
thing I can name as what FC so FC is what FC is 1 by PIE root over FC this is LC square so this FC square so FC is this all of you know this from prototype.

So what type of filter is you see that in my pass band extend from FC 0 to FC. So the frequency 0 to FC are unattenuated and over FC it is suppressed or starts attenuating so this is a low pass filter. If instead of this I had interchanged this some capacitance with some inductance I could have a high pass filter all those you know that then we can have band pass low pass.

But what I did from image impedance concept I have brought you here let me finish now that what happens when we have seen the alpha is zero in pass band. So between 0 to F to FC the alpha is 0 and what is beta we already derived you see your expressions. I can write it as $2 \sin^{-1} \frac{F}{F_c} \sqrt{\frac{Z_1}{4Z_2}}$ I know now $Z_1 = 4Z_2$ values so this becomes $2 \sin^{-1} \frac{F}{F_c}$ very good.

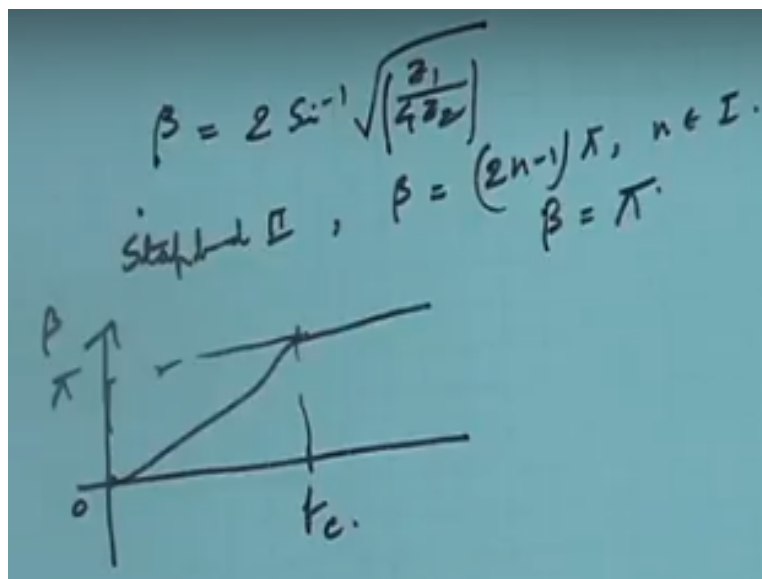
So I can draw that versus F I can plot alpha. So before plotting let me see the stop band that what happens when F is greater than FC, F and F greater than FC $Z_1 = 4Z_2$ is $-\omega^2 LC$ by 4 so you put in our case this. So this is F is greater than FC we at which stop band 2 do you see that you will here because of this minus omega square LC so actually we are $Z_1 - 4Z_2$ minus omega square LC so we are in this part and in this part we know that what is alpha, alpha will be $2 \cos^{-1} \sqrt{\frac{Z_1 - 4Z_2}{Z_1}}$ and this is $2 \cos^{-1} \sqrt{1 - \frac{F^2}{F_c^2}}$.

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So now you can plot alpha that there will be FC so upto this FC alpha is 0 this is your pass band this is 0 but then after that it will have this type of variation COS hyperbolic F by FC so that is like this. So alpha is rising here with a COS hyperbolic inverse variation. Now what happens to beta we have seen in pass band where is beta. We have already seen derived that in pass band beta is 2 SIN inverse root over Z1 by 4Z2 so and in stop band 2 beta will be 2n - 1 PIE.

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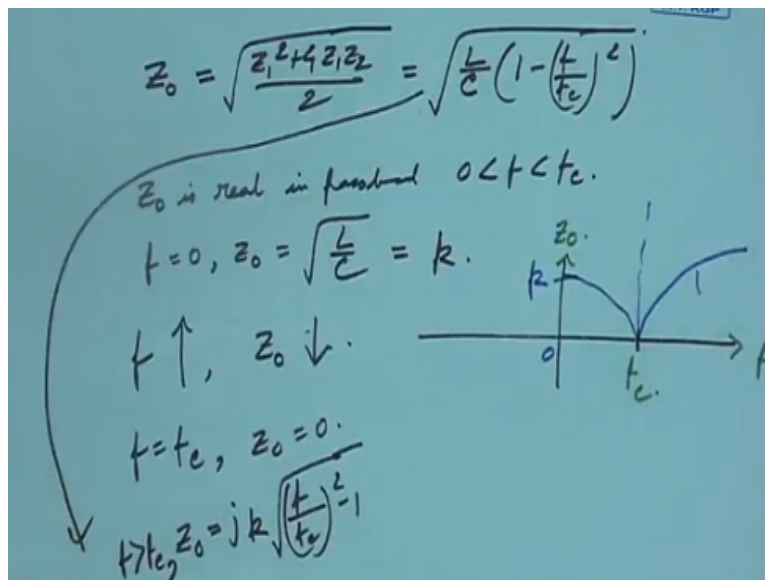


So you can take that beta is simply PIE so if I plot FC, so these 2 SIN inverse variation, so this will be something like this and then it will stay at PIE this is beta this is your PIE. So this will be the beta variation, what does that mean beta variation means you see that if I have if my base

band signal or if my RF signal that as a frequency spread that means instead of that means instead of a single tone.

If I have a multi tone things a band of frequencies present in my voice and suppose it is getting transmitted if you put this filter there will be dispersion. Because this is not a linear function of frequency so various frequency components will be reaching the transmission thing but since the length of filters are small it is not a problem but if you go to higher in frequency suppose you cross V gauge range then even this is significant and you will start having dispersion etc.,

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So you need to design it from a micro filter view point this is the problem these low frequency filter design and what is Z_0 also another important point these we have seen alpha beta but now we will have to see what happens to Z_0 but characteristic impedance because our whole impedance matching is depending on that we said that the filter is terminated by Z_0 filter is both in the source and load side but what is the Z_0 value.

So we have already seen that Z_0 is $Z_1^2 + 4Z_1Z_2$ by 2 what is this if you put our those values that Z_1 is equal to $j\omega L$ Z_2 is equal to $1/j\omega C$. you will see that it is L by C into $1 - f^2$ by f_c^2 Z_0 is real in pass band that we have already seen from that theorem.

So 0 to F to FC now at F is equal to 0 what is Z0 you see from that expression it is root over L by C and that generally is this filter design people RF filter design they call it K a constant you see LYC is constant and Z0 here is independent of frequency but that is only at DC but at any other frequency even in pass band Z0 is changing as we are increasing F, as F is increased Z0 you see getting decreased.

if you are increasing this part is increasing so that Z0 is decreasing and at F is equal to FC what happens to Z0 F is equal to Z0 is 0. So I know that variation I can plot it and also let me complete that in pass band in stop band what will be Z that means when F is greater than FC Z0 will be $J K$ is constant into F by FC whole square -1 just put that in this expression for F when F is greater than FC you will have bring this J out and from here you get F greater than FC Z0 is this.

So if we see the plot this plot will be suppose I am plotting Z0 in the X axis F here there will obviously a FC here. So we will see the graph as so this you see that this is your stop band so it is starting from the value of K gradually decreasing to 0 and then increasing so it is a dangerous thing. You see at FC it is very good resonator it is a good filter it has good output etc but in other places it as a characteristic impedance which is not a constant value K.

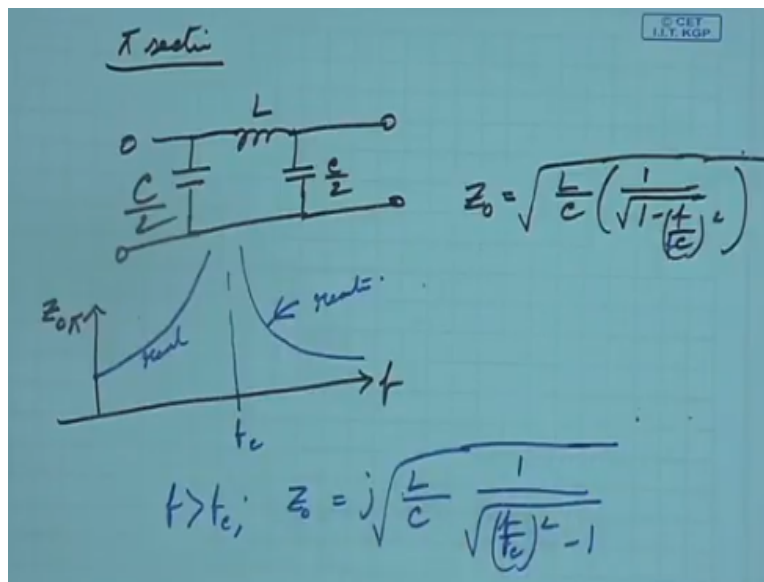
So you will have the load impedance and source impedance chosen according this characteristic impedance of the filter. So at that only frequency it is matching all other there is mismatch so there will be power loss due to reflection in the circuit this is a problem of this constant K filters low pass filter. So what they do they put try to improve this impedance matching that means instead this graph so tricky they try to flatten it that is why M derived section they can flatten it to very good amount but you need some extra hardware for that.

So this is the problem so what we will do we will see that if I try replicate the design to microwave filters I will have problem due to these. I can have M derived etc that may solve the problem but then it is bulky. What will see that you can design the whole thing completely from a different stand point that is called insertion loss view point insertion loss filter design and all modern micro wave filter design are based on these.

But I introduced this concept because in B.Tech sometime this though this is part of the text syllabus in our present day this is not taught with so emphasis but these are the impedance image impedance concept characteristic impedance concept all comes from here which later we use for transmission line and other things. So unless and until this impedance part is clear, propagation constant is clear later understanding the microwave filter design microwave amplifier that will become problem that is why I took so much classes extra to revise.

But before completing I say that I have always taken T section you can always take an PIE section. So if we take a PIE section just what happens let me see this constant K filter PIE so my filter will be like this. Again I decide this is L so this is L and these two I take C by 2 this is a symmetrical PIE filter.

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So it is Z_0 expression if you now derived it will be who taught it f arrow top it f Z_0 pie actually that time I have said Z_0 you can call I Z_0 T section also Z_0 PIE that will be FC so this part here also note that upto this part this is possible this is real and this is the reactants part because it is pure imaginary but in the same graph since we are showing we should that this portion is reactive this portion is real to resistive.

And here this portion is reactive this portion is real this is in pass band so you see here in the T section it was coming like this in PIE section increasing like this and you can find that when f is

greater than FC this Z_0 that is even by from here just when F is greater than FC so L by C into 1 by root over F by FC whole square - 1 . You just take J out that become this.

So other α β 's are same but these characteristic impedance of a PIE section is different from characteristic impedance of a T section ok. So you know that if we change this L and C when we get high pass filter then we put cascade of low pass filter with high filter you get band pass filter or in one arm instead of single L .

We can take a series of L and C here also we can take a series of L and c parallel combination of L and C and that gives different filters notch filters etc of stop band filters as it is sometime called notch filter that thing that design you can easily do but all this at low frequency is ok RF frequency is ok upto 1 gigahertz I say 1,2 gigahertz is ok.

But above that if you go see these will pose a series problem you need to have impedance matching M section for that etc which created lot of problem we wont go into that we will see that instead of you can simply do it if you consider insertion loss. Because whatever we say ideal filters is pass band it should not have loss but always it will have loss if we calculate that loss and enforce that in pass band I need insertion loss to be within these and in stop band I needed to be enormously high compared to pass band that is filter.

So how in power terms we are saying that here we are saying that in terms of that propagation constant should be lossless that means attenuation constant should be zero. So instead of that I can that in terms of power that insertion. So we will have to define what insertion loss that means in power transfer what is the loss happening if I introduce this filter and that if we can keep insertion loss to minimum over a band that will be pass band and very high at other band so we can do microwave filter design by that even all low frequency filter can design by that.

So that is the more modern technique and then you can have various functions to specify your pass band and stop band. So any arbitrary specification you can make and you can synthesize the filter of that. That is the huge story of research still it is evolving that various specifications because when you go to space when you go for various modern electronic equipment you have a

(()) (34:09) of specifications of filters I want these to be cut to this much I want nearby it should have this you given this.

So various shapes are available and you know if you know how to do by insertion loss you can incorporate all these things. We will see some typical cases some Chebyshev, Butterworth etc shape which or if various elliptic shapes that how people so design filters from that insertion point from here the next class. So this more or less classes completes the low frequency part or RF part of the filter design microwave which starts from 1,2 gigahertz and mandatory for above 3 gigahertz.

You cannot have this design because of this problem also the your TM mode apart from that other mode starts propagating there TE, TM modes etc and also you cannot consider that your distributed line is considering as I said that TM OM other waves will start coming. So you need to consider real wave picture voltage and current does not have any meaning there in equivalent since you have voltage current but microwave things are based on electric field and magnetic field etc.

So filter design there will be very complicated if we transform this network theory concept voltage current concept but as I said in terms of insertion loss it will be very easy also we will see that makes the whole thing very computer program friendly and by that now a days all modern design are insertion loss. Thank you.