

Design Principles of RF and Microwave Filters and Amplifiers
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Module No #
Lecture No # 06
Introduction to Insertion loss based Microwave Filter Design

Welcome to this sixth lecture of this lecture series, that today we will see insertion loss based microwave filter design. Now already we have seen the motivation for going for this design in the previous lecture. Now today let me first define what is insertion loss? Insertion loss is its symbol is PLR and this is ratio of power available from source to power delivered to load by any subsystem.

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Insertion Loss based Microwave filter design

Insertion Loss $PLR = \frac{\text{Power available from source } P_{IN}}{\text{Power delivered to load } P_L} = \frac{P_{IN}}{P_L}$

Filter lossless.

Under matched condition (source & load) S_{21} .

$P_{LR} = |S_{21}|^2$

$= \frac{1}{|1 - |\Gamma(w)|^2|^2 |S_{21}|^2}$

Suppose I have any subsystem in this particular case it will be filtered but insertion loss is a generalize concept that is why let me call it a electronic network. It is a black box it is 2 port black box. Now this side it will be connected maybe to some other block but for this network this is the source side that, so it will be connected with this source. Similarly it may be connected to some other block but to this 2 port network that is the load. So it will be connected to another box which is called load.

Now let the power source is delivering P_{in} and the load is taking P_L . So insertion loss power available from source what is the power available from the source to these electronic network that is P_{in} and so I can write this as P_{in} and power delivered to load is P_L . Obviously you know that I do not know the impedance level of this so and I am assuming that in general this is ,there will be some loss here also but in this particular case, particularly our filter case, so this filter case there is no inside there is no power dissipation.

Because we will use only the reactive components and so the power is not dissipated inside so it is a lossless part here internally. But you know that due to that impedance mismatch there can be reflections here so the wave is going and then the wave can come back so there are reflections so all the power may not go here obviously the power that is reflected that will go back here, so here it will be something less power.

Similarly here also due to the impedance mismatch the wave is going like this but there may be reflections. So can I see that all this P_N P_L , so overall reflection coefficient so can I say that this is equal to $1 - \gamma$ which is a function of Ω . This γ is the reflection Coefficient, over all reflection coefficient which comprises of reflection here reflection coefficient here.

So by that I can define an overall coefficient between this and that is γ it is the function of γ and I know that the this ratio will be nothing but $1 - \gamma$, that means $1 - \gamma$ reflected power this γ please remember we have earlier discussed that this γ is a voltage reflection coefficient that is why we are taking this square thing, because we are taking of power.

Now if there is no mismatch here that means already impedance matching has taken place between source and this our filter network and also between load and our filter work then it will be like these. so there wont be this reflection co-efficient is 0 there but remember there is one more thing that is this network what about the power I gave. Yesterday also I said depending on the propagation constant in the pass band it will flow but in general a portion of that will go here.

In high frequency or microwave what is that, suppose if I give some voltage how much voltage comes here that I can express by a ratio called S parameter S_{21} . So I can see that under matched

condition so this is one and also I should then say that this is S21 square this S21 is this is port 2 of the electronic network this is port 1. So actually insertion loss is 1 by this into S21 Square.

So, under matched condition both source and load matched then PLR is nothing but S21 square here no there is a pit fall in measurements. Many times I see that in Indian engineer they say that what is insertion loss it is a S21 square. Please remember that if you have enforced the matching then only it is true otherwise you should also consider the reflection taking place both here and here. Many times when the measurement takes place this impedance mismatch is not taken care and people forget to incorporate this part, the reflection coefficient part ok.

So now I can say that I have define insertion loss that means physically what is this, it says that if this network was not there some power was flowing to the load. If I include this in the circuit the power will change, so this difference is basically the insertion loss. So due to the insertion of the this particular electronic network how much loss am getting in the circuit.

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$IL = 10 \log_{10} PLR \text{ (dB)}$
 $\Gamma(\omega) \rightarrow \text{even fn. of } \omega$
 $\Gamma(\omega) = \Gamma(-\omega)$
 $Z(\omega) = R(\omega) + jX(\omega)$
 $R(\omega) \text{ is even fn. of } \omega$
 $X(\omega) \text{ is odd fn. of } \omega$
 $\Gamma(\omega) = \frac{Z(\omega) - Z_0}{Z(\omega) + Z_0}$
 $= \frac{R(\omega) - Z_0 + jX(\omega)}{R(\omega) + Z_0 + jX(\omega)}$
 $\Gamma(-\omega) = \frac{R(\omega) - Z_0 - jX(\omega)}{R(\omega) + Z_0 - jX(\omega)} = \Gamma^*(\omega)$

So in general, this insertion loss in IL this is expressed in db, so is remember that $10 \log_{10} PLR$ is the insertion loss in db now here I will take a property of any microwave network you may not be familiar with this in graduate level we do not teach that but in post graduate the microwave technology courses there we prove one point that gamma omega for any two port network that should be always an even function of omega.

So what does that mean, that means always this is a property believe me, or if you want to see this is proved in any microwave engineering course. So in any book particularly the our recommended book this book microwave Engineering by David Pozar it is already recommended in your course. So in this book this is derived that always gamma is event function of Omega that means $\gamma(\omega) = \gamma(-\omega)$. We will utilize this property now let me write what is gamma omega.

Gamma Omega is the impedance of the network either this side or this side looking at input impedance or output impedance here - Z_0 by $Z_0 +$ if the characteristic impedance of the this filter section is Z_0 then source, then this $Z(\omega)$ is the source impedance if I am talking of this reflection then this $Z(\omega)$ is the load impedance.

And here also there is another property that any impedance, source, load etc., or the input impedance there the $Z(\omega)$ it can be retained as $r(\omega) + jx(\omega)$ that is the part of this impedance and reactive part of impedance. And $r(\omega)$ is always again is an event function of ω and $x(\omega)$ is odd function of ω . This property are also true as I said that gamma omega is event function similarly this is also true so we will write that ok this will break into that $r(\omega) - Z_0 + jx(\omega)$ and $r(\omega) + Z_0 + jx(\omega)$ now what is gamma of $-\omega$.

Because I want to test what is the nature of this PLR. So we are going there that is insertion loss has some specific functional characteristics with respective ω . So, that we are trying to see, so let me see what is this this is utilizing this properties that $r(\omega) - Z_0 - jx(\omega)$ by $r(\omega) + Z_0 - jx(\omega)$ but we know that this two should be equal. So if I say that then basically what is this right side can also retain as the complex conjugate of gamma omega. You see that this is real so at this minus this by this minus this.

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$$|\Gamma(\omega)|^2 = \Gamma(\omega)\Gamma^*(\omega) = \Gamma(\omega)\Gamma(-\omega) = |\Gamma(-\omega)|^2$$

$|\Gamma(\omega)|^2$ is an even fn. of ω .

$$|\Gamma(\omega)|^2 = a + b\omega^2 + c\omega^4 + d\omega^6 + \dots$$

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

M, N real polynomials in ω^2 .

$$P_{LR} = \frac{1}{1 - \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

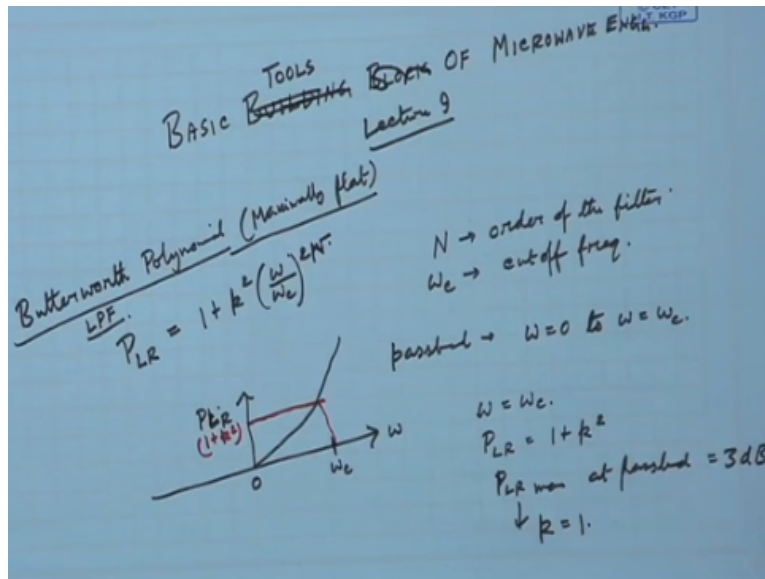
So we saw that minus gamma minus omega is nothing but gamma conjugate omega if we have this so we can say that what happens to this term, gamma omega square because this is there in the insertion loss expression. So gamma omega square is gamma omega into gamma this is equal to gamma mega is equal to gamma minus omega this is by definition and then we have just seen that this is nothing but, gamma minus omega.

So I can write it as gamma - omega whole square because I can replace this with or I can write like this gamma of - omega and gamma of - omega, so that is gamma - omega whole square. So I can say, you see the property that gamma omega whole square can I say is and an event function of Omega. So what does it mean that, gamma omega square when I will synthesizes as I said from the start that actually in this insertion loss base method we specify the insertion loss which is nothing but attenuation part that how it will go that in pass band, stop band etc.,

For that we require to have this, but these says that this can be of that I event function of omega means allot function components odd components are 0 that means it can have only this a+b omega square + c omega 4 + d omega 6 etc., no omega 1, omega 3 like that terms. So we can say that this gamma omega square can be represented by two polynomials. Where M and N are real polynomials in omega square M and N are real polynomial.

You see this is reflection coefficient so all obviously this will be $m\omega^2 + n$ because this is this plus something. So what is then PLR is $1 - M\omega^2$ by $M\omega^2 + N$ and that this $1 + N\omega^2$ by $N\omega^2$. So, it is says that you can specify anything but I will be able to realize a filter only this insertion loss is specified in this given form that means if you specify something with ω etc then it won't be realizable.

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Now based on this specification there are various choices. So already I the student to refer to another NPTEL course basic building blocks of Microwave Engineering in lecture 9 basic building blocks basic tools of Microwave Engineering my course, there in lecture 9 actually that was in respective impedance matching and there we have seen in details the properties of various polynomials which was used for synthesis that time it was impedance transformers but those are also valid for filters.

So please brush up your knowledge of basic polynomial functions like Butterworth polynomial, Chebyshev polynomial, elliptic polynomial maximally flat which is nothing but Butterworth etc., So here we start with Butterworth polynomial so if insertion loss is specified in the form of Butterworth polynomial you know Butterworth is also called as maximally flat because given the order compare to any other polynomial function it has.

Suppose if it is ordered in up to N derivatives are all zero so that is why called as derivative flat response. So Butterworth polynomial so this is the flattest possible pass band so that means if we specify the insertion loss in the pass band by Butterworth filter. Then we can say that we will get a very flat response which is desirable in the pass band.

Obviously always zero cannot be achieved that attenuation constant zero but will achieve a very flat pass band and by specifying a level we can say that ok my pass band is I am not attenuating this all the frequency components in this band not more than this amount. So for a low pass filter the specification if we follow Butterworth, you know Butterworth polynomial is given like this.

So already ω^2 it is a function, so you see that it is a function only ω^2 so it is realizable from this property of M and N that ok I can realize it so let us try to realize this and N is the order of filter that you know sorry I can make it capital N . So N is the order of the filter ω_c is the cut of frequency low pass filter cut of frequency will be loss pass filter angler given by ω_c .

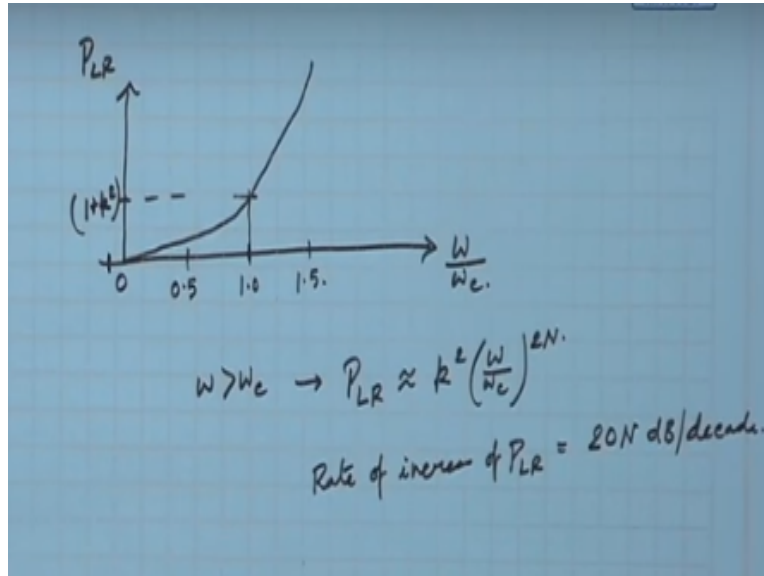
Now pass band if you see this pass band extends from $\omega = 0$ $2\omega = C$ and at the bandage that means suppose I am plotting ω and PLR then 0 this is ω_c . Now from here it will be having some stop band so PLR will change but what is the PLR at this point we can find from here so at $\omega = \omega_c$ $PLR = 1 + K^2$.

So, I can specify now you know butter worth response since we have taken like, Butterworth response is like this, it is maximally flat very flat. But in pass band I will have some attenuation. But this the maximally flat one, but I should know that what is the maximum attenuation I am having so this value is what $1 + K^2$. So suppose I want that ok no more than 3 db attenuation or no more than .1 db attenuation.

So I will put that at I know that at ω is equal to ω_c at cut off in the pass band the maximum attenuation takes place here in the Butterworth polynomial and that value is K^2 from that I can always find K . So for a example if we choose that this our insertion loss maximum value is the PLR maximum at pass band I wont tolerate more than 3 db. So if this is 3

db what happens to K that $1 + k^2 \log_{10}$ of this is 3, so from that you can find out K is equal to 1 under this condition.

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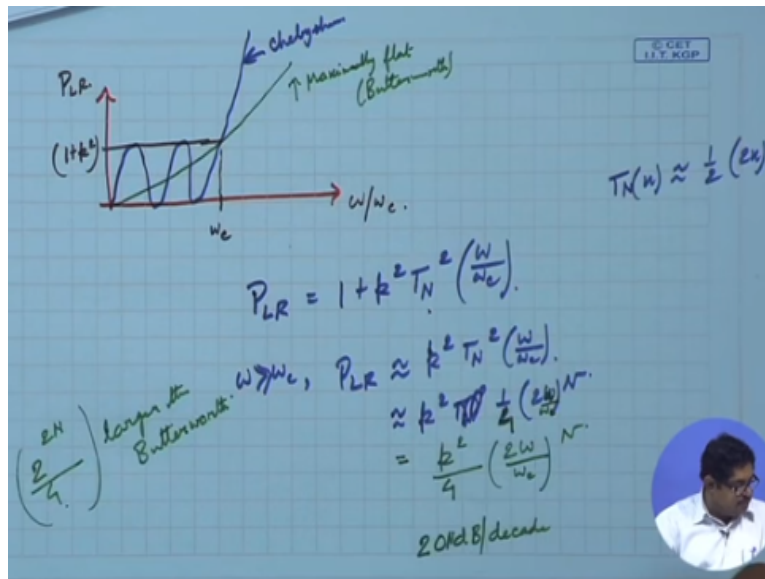
So we can plot this PLR versus omega by omega c normalize with respective omega C. So I know the values will be like this by .5, 1, 1.5 etc so this is point where cut off will takes place and so what is the value at 0 let us also see the at 0 what is PLR ? this is 1 so PLR is 1 here and PLR this value is $1 + K^2$ also what happens to this polynomial.

When omega is ($\omega > \omega_c$) (24:49) into to the stop band that means Omega by omega C is the large number then can I say that for omega greater than omega C I can say PLR is approximately $K^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$. So what is the rate of increase of PLR in the stop band from this I can say that rate of increase of PLR is how much it is $20n$ db part decade. This is well known if you have this expression you can always say this.

So we see that here at attenuation that alpha increase monotonically with frequency. But we know that it is at lower side of the pass band it is not much but at bandage that means near cut off it is increasing but I can always specify that where it will be level fixed lamp, and this rate of interest is $20n$ db per decade. So how much you want to achieve this rate so that basically by that you select what is the order of this filter?

Order means you will have to find how many sections you need to put ok. So, third order fourth order means you will have this. Now let us go to the another design that instead of this if I say that in the low pass filter unlike this Chebyshev ok I want this rate to increase further, that I want this should be sharper than this I do not mind, obviously you cannot choose everything. So I will say I want a very sharp price compare to Butterworth here I do not mind if instead of monotonically increasing the in pass band the insertion loss repels.

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That means I can have high low but do not cross this limit, so that is possible Chebyshev is sharpest one or much sharper than our butter worth. So I have this PLR here omega by omega C let we fix my $1+k$ square this is designer will choose and this is my omega C. So a Butterworth polynomial it can be so let me first draw the maximally flat one, this is my Butterworth maximally flat or butter worth now this is Chebyshev.

Chebyshev as various depending on the order that Chebyshev various but it repels whether there will be one repel 1 cycle or 2 cycle 3 cycle that depends on order. But you see always they are confined between this and this, but so what is the advantage you see that here it was at monotonically increasing here I have repel. But I can specify that ok B within this limit so in pass band it is never crossing the limit. Once it is going away from pass brand it is much sharper than Butterworth that is it is advantage.

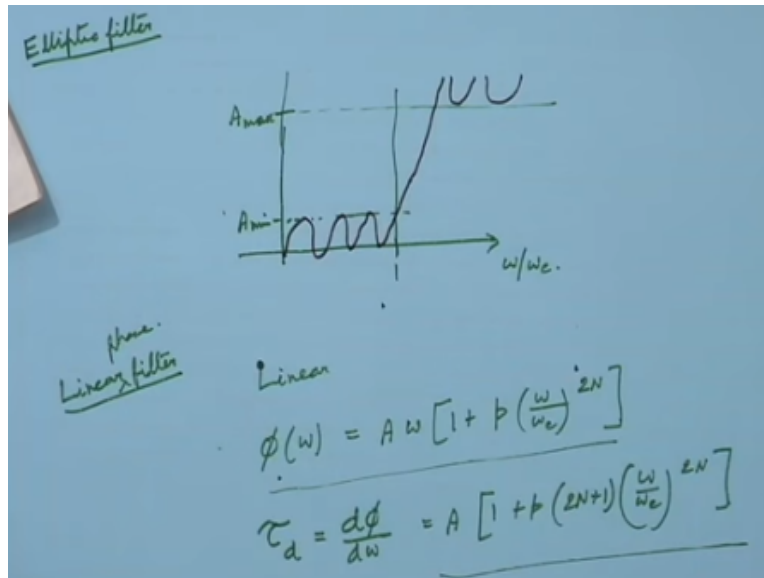
So if you want your prime concern is after pass band when I entering stop band I will have a very good cut off and very sharp cut off. You opt for this Chebyshev shape polynomial, so again you can determine the polynomial by this K^2 and what is this rate of increase if we see the so in this case first me let me write down the Chebyshev polynomial from $1 + k^2 T_n^2(\omega/\omega_c)$ (29:49).

And we choose like this so for large N again I can say that when ω is greater than ω_c PLR is $K^2 T_n^2(\omega/\omega_c)$. Now at high value of ω/ω_c T_n Chebyshev polynomial it is approximated very good approximation is this so this becomes $K^2 T_n^2$ sorry K^2 into half 2×10^{20N} so this is K^2 by 4×10^{20N} here I will write 2×10^{20N} . So 2×10^{20N} e to the power N .

This is T_n so T_n^2 (31:23) so here you now tell what is the slope I have came here because I want it is sharper slope so slope is 20 db by decade 20 ndb, so same as Butterworth. Then you can say that how I am getting it and what is the advantage, at advantage is that always this value of T_n it is always 2 to the power $2N$ by 4 times insertion loss of any Chebyshev filter is 2 to the power $2N$ by 4 times larger than Butterworth.

Now obviously if you go for $N = 1$ that means the single section Chebyshev then you do not have any advantage but the moment you go for $N = 2$ or $N = 3$ etc., So $N = 2$ means you have 2 to the power 4, sixteen by 4 that means 4 times larger the Butterworth always you will have the value. So even if the slope are not much different but you are getting more value of insertion loss so your alpha is increased. Then we will see there is another filter called another very popular filter particularly in CD ROM etc this is used this called elliptic filter.

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Now if you want you have seen that butter worth and Chebyshev same rate of price but elliptic is the maximum rate of price it can give but what it will you will suffer that it has ripple both in pass band and stop band. So Chebyshev you see it monotonically increases once it is in the pass band it had ripple in once it is at attenuation of stop band. Here it is in pass band it is ripple but it does not have any ripple.

Elliptic have ripple in both so that means here it will be something like these how many it depends on the order so it goes and then here again it ripples but no problem if you specify that ok at least everywhere I want this attenuation in the stop band this is your stop band, this is your pass band. So here as before you can specify that ok instead of pass band it is beyond this and stop band attenuation should be like this.

Then you go for elliptic and there is another one sometimes in a filter if you have if your bass band signal or if you want to possess the signal any bass band signal is not a single tone it as a sprayed of frequency suppose when am talking, talking up to 20 Kilohertz or roughly from 4 kilohertz to 20 kilohertz there will be the voice sorry 20 hertz to 20 kilohertz. So you see these different frequency if they are attenuated differently then there is problem in the reconstruction.

So sometimes that means a linear system I want filter should act as a linear system, sometimes I can tolerate that is not a problem, but in some application, I cannot tolerate that, that time I want

that linear phase filter. Sorry this is linear phase filter, so linear phase means what that my phase should be specified like this $1 + P \omega$ by ωC whole to the power $2N$.

So if we specify that P is a constant but I should have this type of variation so this is a as you can guess but in phase am specifying a Butterworth type polynomial this is nothing but a Butterworth type polynomial. So this is the phase of the voltage transfer function of the filter that should satisfy these under this condition the group delay because that is the measure of whether I can tolerate or not.

As a group when the bass band signal is moving so bass band signal or any RF signal with a frequency spread that what is the group delay that means what is the maximum delay between the maximum phasing delay and minimum phasing delay so that is given by $D \pi$ by $D \omega$ and you see it trans out to be $1 + b 2n+1 \omega$ by ωC whole to the power $2N$.

So this is a again you it is a maximally flat response so that may be tolerable so if you use the linear phase filter you have this. Like that there can be other specification, other polynomial still this is search is going on, people are using various newer functions from mathematics taking new functions and implementing that to get a more desired attenuation characteristic etc.,

So we have seen that how to do it so now we will try to see how a microwave engineer will implement this. Because this mathematics or this specifications is one thing but finally as a engineer we should implement that for that we need to have some mechanism there is nothing fundamentally new. This is all about filters but unless and until we engineers plan how to design how to implement it.

We do not consider our job finished scientist up to this they stop but we engineer will always go on and try to make that ok if I want to make it and taste it I should know after this what will happen that will see in the next class the implementation of this filters. Thank you