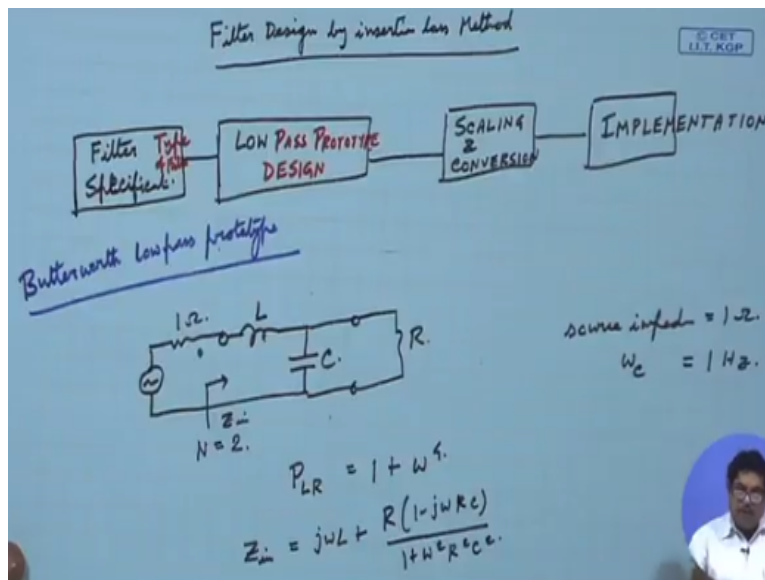


**Design Principles of RF and Microwave Filters and Amplifiers**  
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**Module No # 02**  
**Lecture No # 07**  
**Prototype Low Pass filter design**

Welcome to this lecture on filter design micro filter design by insertion loss method. Now as I said that we can specify there are some restrictions on specifying the insertion loss now once that is done and we chosen some way to represent that then we need to know how to design and implement the filter. So I am now giving you the whole process flow that first from the filter specification you need to understand that what will be the PLR or insertion loss requirement what type of filter I will do etc.

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That we have already seen once we have that we do it this is an important step so this will understand that whatever may be the filter, filter may be band pass filter, may high pass filter, may be band stop filter, may be low pass. We always first do a low pass prototype design. So the first step is important one, low pass prototype this is unlike the image parameter method. Here whatever may be the thing we have decided about what will be the insertion loss which with polynomial will do that means what type of filter I will use is and the pass band stop it is characterize many times it is not low pass.

But we always start with low pass prototype design that is why we are calling it prototype. Once we have done that then we will do some operations called scaling and conversion to make it to the specification that the specifier or user is given. So this is another important step in this microwave filter design and finally we will see how to implement it implementation of the that means on a actual microwave circuitry how we implement it. So filter design by insertion loss method mainly have these four important blocks.

In any design you have this at from specification you start to decide something then here you do first a loss pass prototype then you have some scaling and conversion operations and then finally you think how to implement it if required will see that for implementation and we need to some minimization is required about the length etc that will see.

So we the low pass prototype design here in this part already we have decided about type of filter that means whether it is butterworth or a Chebyshev or elliptic or the linear phase that we have decided so we are going here now. First we start with butterworth low pass prototype now please see that here, we have a source then we have a source assistance that we assume to be always one.

And then we have and LC circuit and then we terminate it with a resistance load. So the source resistance is one this is one ohm, so we can say that source we have assumed in the prototype always source impedance as one later we will see that we can go to any actual source impedance but in prototype we have this is L this is a C in the series (()) (05:31) in the shunt term we put C and this.

Now this is a single section if required you have multiple sections etc. So source impedance is one Ohm and also in prototype design the cut off frequency in low pass is taken as one hertz so  $\omega C$  is one hertz. So our and you see we have design L and C so that we get a Low pass circuit with this cut off frequency. And you see the number of elements N is equal to 2 so that will determine the PLR.

What was the PLR expression for the maximally flat you see for maximally flat it was  $1 + K$  square omega by omega C to the power 2N. So here this PLR will take as since we are taking N is equal to 2 and omega is already one so one + omega to the power 4. So PLR we have specified and it is realizable because it is an even function of Omega as not only that function of omega square which is also required that even powers of omega.

So it is omega 4 so it satisfies I can be realized now let us look at what is the input impedance I am looking from the source. So I can write  $Z_{in}$  will be  $j\omega L$  in parallel combination of C and R once I have  $Z_{in}$  what is the characteristic impedance immediately. What is the reflection Coefficient I can immediately write that reflection coefficient will be  $Z_{in} - 1$  by  $Z_{in} + 1$ . And once I have this I can find out PLR it is  $1 - |r|^2$  that is  $1 - \frac{Z_{in} - 1}{Z_{in} + 1} \frac{Z_{in}^* - 1}{Z_{in}^* + 1}$  if you simplify you get  $\frac{Z_{in} + 1}{2} \frac{Z_{in}^* + 1}{Z_{in} + 1}$ .

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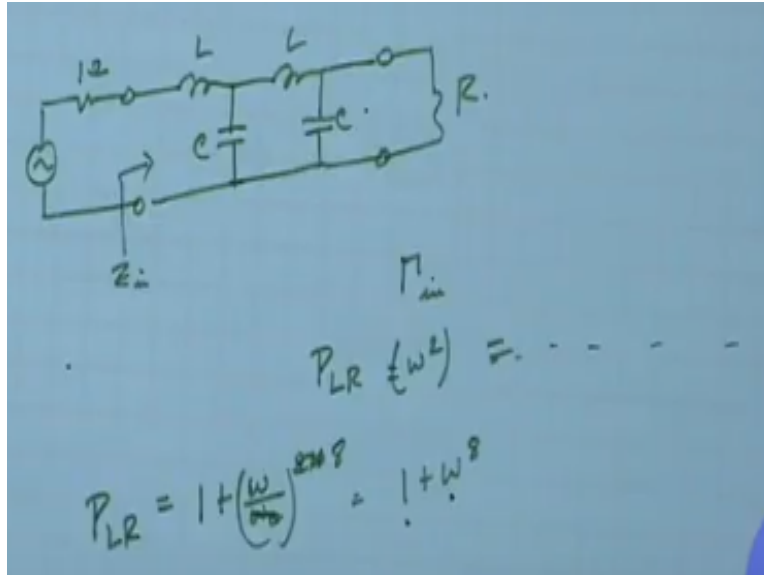
$$P_{LR} = \frac{1 + \omega^2 R^2 C^2}{4R} \left[ \left( \frac{R}{1 + \omega^2 R^2 C^2} + 1 \right)^2 + \left( \omega L - \frac{\omega R C^2}{1 + \omega^2 R^2 C^2} \right)^2 \right]$$

$$= 1 + \frac{1}{4R} \left[ (1-R)^2 + (R^2 C^2 + L^2 - 2LCR^2) \omega^2 + L^2 C^2 R^2 \omega^4 \right]$$

$P_{LR} = 1 + \omega^4$   
 $R = 1$   
 $R^2 C^2 + L^2 - 2LCR^2 = 0$   
 $L^2 C^2 R^2 = 1$   
 $C = L = \sqrt{2}$

Now  $Z_{in}$  is a complex quantity  $Z_{in} + Z_{in}^*$  will be twice real so we know that we already have the expression for  $Z_{in}$ . So if we put  $Z_{in} + Z_{in}^*$  you see only the real term is this one because this is imaginary, this is also imaginary. So we can immediately write this will be  $2R$  by  $1 + \omega^2 R^2 C^2$  and another term is this  $Z_{in} + Z_{in}^*$  magnitude square that immediately we can write that.

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So now once I have this I have this PLR expression I can immediately find out what is PLR? PLR turns out to be  $1 + \omega^2 R^2 C^2$  by  $4R$ ,  $R$  by  $1 + \omega^2 R^2 C^2$  to the power 4. So this is the PLR expression from the circuit from my this circuit I have found the PLR.

Now this we can little bit simplify and see that this I can write as because ultimately my actual PLR the butterworth PLR that is  $1 + \omega^4$ . So let me write it in terms of  $\omega^4$  and  $\omega$  independent term etc. If we do that you will get like this so you compare this with our specified PLR which is  $1 + \omega^4$  this is butterworth this is the thing I got so now by comparing I can say that what is immediately is observable is what is the value of  $R$  because this is  $\omega^2$  this is  $\omega^4$ .

So 1 is equal to this that means we should be this term  $1 + 4R^2 C^2 \omega^2$  that should go to zero if we compare this two so  $R$  becomes 1 ok. Then if I  $\omega^2$  terms I see that this also will go to zero because in our butterworth thing we do not have this  $\omega^2$  here we have that this should go to zero so I should get  $R^2 C^2 + L^2 - 2LCR = 0$ . And by comparing  $\omega^4$  I can see that  $L^2 C^2 R^2$  should be 1.

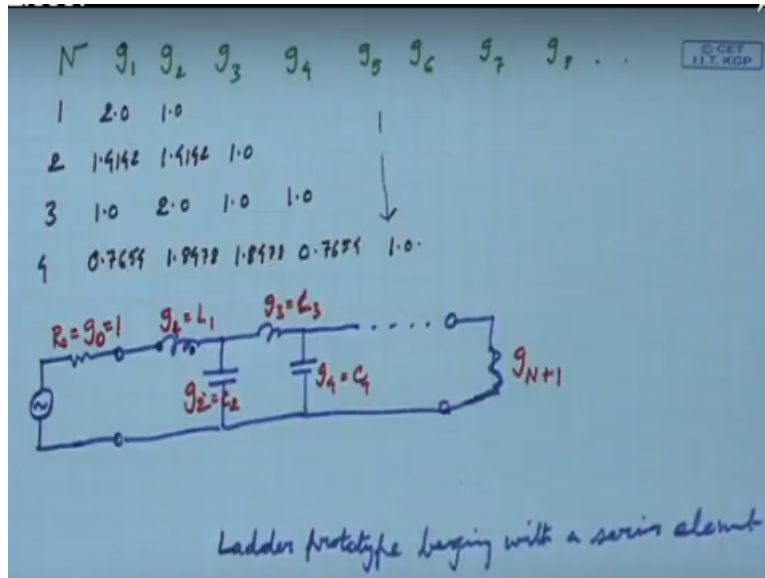
So these are the three equations are or this is R already solved so I can put this values. So I have two equations I need to solve LC that I can do and if you do that you get C is L is root 2. So I have solved because see in my assumed circuit there are unknowns LCNR and I have found that R is equal to now 1 C is equal to root 2 and L is equal to root 2. So this way you can do you can see that always this part is true I have taken this this is N is equal to 2.

Suppose I have N is equal to 4 If I specify that order N is equal to 4 so that time will have like this is 1 ohm then I have another LC N is equal to 4 then this side again I will have this R this is L this is C I will find out what is  $Z_{in}$  from  $Z_{in}$ , I will find out what is gamma in, from gamma in what is PLR. So I will have PLR as a function of omega square here, and I know that if I have N is equal to 4 what will be the PLR specification from the butter worth.

So that I know it will be  $1 + \omega$  by omega C whole to the power 2 N. So depending on N here is equal to 4 means  $8 \omega$  C is 1. So it is  $1 + \omega$  8, so you will have this expressions here you compare so you will see that the non-omega term the DC term that should be 1 and omega square omega 4 omega to the power 6 will be 0 and the components and the coefficient of omega 8 term that should be 1 so by that you have solve.

But obviously you see this is becomes a bit cumbersome the process is valid but it becomes very cumbersome if I go for higher and higher order. So for the large N the method is nor very practical but what people have done people have tabulated these values that people have solve it because polynomial is known that is known so prototype you see that we know all the source impedance 1 ohm omega C 1 hertz so I can solve for this.

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So that table it is tabulated now that tabulation need a bit understanding that is why let us see that people have tabulated it in micro books you will see that in (( )) (17:42) book it is given for butterworth filters I have and they have given the value suppose if I have only one element if I have a 2 element, if that tables are available now you see that when I view it was LCR but here are  $G_1, G_2, G_3$  etc so how to interpret that so that concept is that this is called ladder prototype.

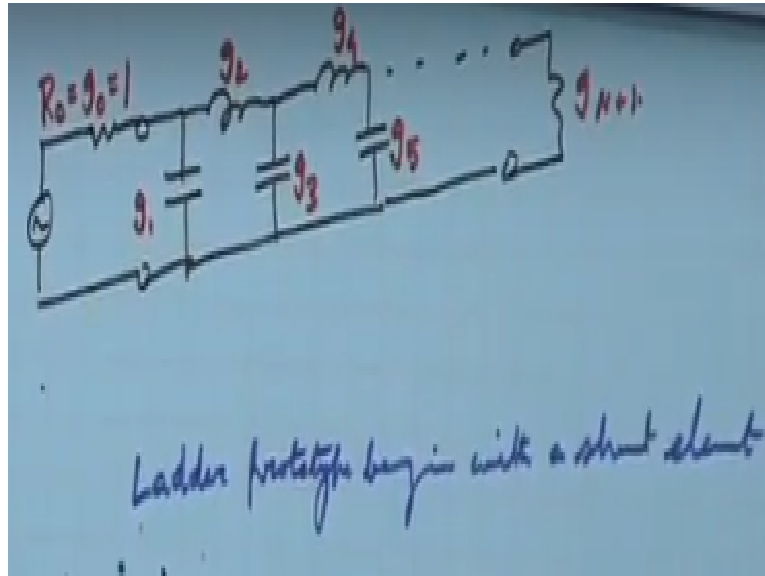
So you have seen in any synthesis problem you have this type of prototype because this is your reference with reference to this you design. So you know first element is like this then so you see that what I designed that is called as ladder prototype now there are 2 varieties. You see this in the first one I can have a series inductance or I can have a shunt capacitance now if I have series induction this is called as ladder prototype beginning with a series element.

And in this case you see the nomenclature is this I know that this is actually the resistance but they call it to have this table general this is your first element  $G_1$  sorry this is not first one because in prototype always this value is known  $R_0 = G_0 = 1$ , so this they call it  $G_0$  then this one they call it  $G_2$ . So basically, I can say  $G_2$  will be your if that  $G_0$ .

So this  $G_1$  this is  $L_1$  then this one they should call  $G_2$  this is  $G_3$  this  $G_4$  and the last one will be how much because we are starting here there  $N$  capital  $N$  order so this is an extra one so  $G_{N+1}$

so this is ladder prototype beginning with the series element so you see that this is your L1 now this will be worth this you can call as C2, this is your L3, this is your C4 like that.

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There can be another variety that ladder prototype beginning with a shunt element. So in that case the circuit will be like this. So here we see that  $R_0 = G_0 = 1$  then this will be your  $G_1$ , this will be  $G_2$ , this will be  $G_3$ , this will be  $G_4$ , this will be  $G_5$  this will be  $G_{N+1}$ . So we can now summarize that in both these cases what we get is  $G_n$  or  $G_0$  (()) (23:22) for what it is the generator resistance for first type that let me call that this is first type and this one let us call that means second type.

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$$g_0 = \begin{cases} \text{generator resistance (1st type)} \\ \text{generator conductance (2nd type)} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance if } g_N \text{ is a shunt capacitance} \\ \text{load conductance if } g_N \text{ is a series inductance} \end{cases}$$

$$g_k \quad k=1 \text{ to } N = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

So please note that when we have series element type we call it second type when we have shunt element we call it first type. So with reference to that you can write that so in the first type when I have shunt here what is this circuit this, so I need to change this circuit because here generally they in the second type they do like this that instead of a voltage source they represent a current source here so that means here we will have  $G_0 = G_0 = 1$ .

So in the second type this is a conductance so we say that generator conductance in the second type then what is  $G_{N+1}$  this is in the first type it is load resistance if  $G_N$  is a shunt capacitance here load conductance if  $G_N$  is a series capacitance please understand that here it is ladder network so this  $G_{N+1}$  what it is it depends on depending on the number if I have finished with a previous one  $G_N$  if it was a shunt capacitance then this will become a load resistance.

If it is a series capacitance shunt capacitance sorry this is not series capacitance series inductance. If this is a series inductance last one then this will be conductance ok. And in general these are the two  $N$  values what is in between let us call that  $G_k$  where  $k$  is varying from 1 to  $N$ , so this is inductance for series inductors and capacitance for shunt capacitors.

So the moment that has been done you see this generalization now looks meaning they are specified various values as will take. Now it is up to us which design we are following



accordingly we will choose whether it is an inductance if it is inductance this is capacitance this is induct if this is capacitance this is inductance capacitance like that it alternate.

So will see suppose you can have a design that a maximally flat low pass filter is to be designed with a cut off frequency of 8 gigahertz, and a minimum attenuation of 20 db at eleven gigahertz. You know that means specifying that what is your pass band how much and in stop band how much is the thing how many elements are required.

So you can go on do that we will have a tutorial there will see this problem. Similarly you can have Chebyshev filter design prototype filter so let me just briefly say this at Chebyshev low pass prototype. So again omega C is 1 only thing is the PLR is now  $1 + K^2 T_N^2(\omega)$  square TN square omega .Now TN this Chebyshev polynomial they have a property that  $T_N(0)$  is equal to 0, if N odd and plus minus N even.

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Chebyshev low pass prototype

$\omega_c = 1$

$P_{LR} = 1 + k^2 T_N^2(\omega)$

$T_N(0) = \begin{cases} 0, & N \text{ odd.} \\ \pm 1, & N \text{ even.} \end{cases}$

$P_{LR} = 1$  at  $\omega=0$ , for  $N$  odd  
 $= 1 + k^2$  at  $\omega=0$ , for  $N$  even.

$N=2$

$T_2(\omega) = 2\omega^2 - 1$

$P_{LR} = 1 + k^2 (2\omega^2 - 1)^2 = 1 + k^2 (4\omega^4 - 4\omega^2 + 1)$

⑧  $= 1 + \frac{1}{4R} [(1-R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4]$

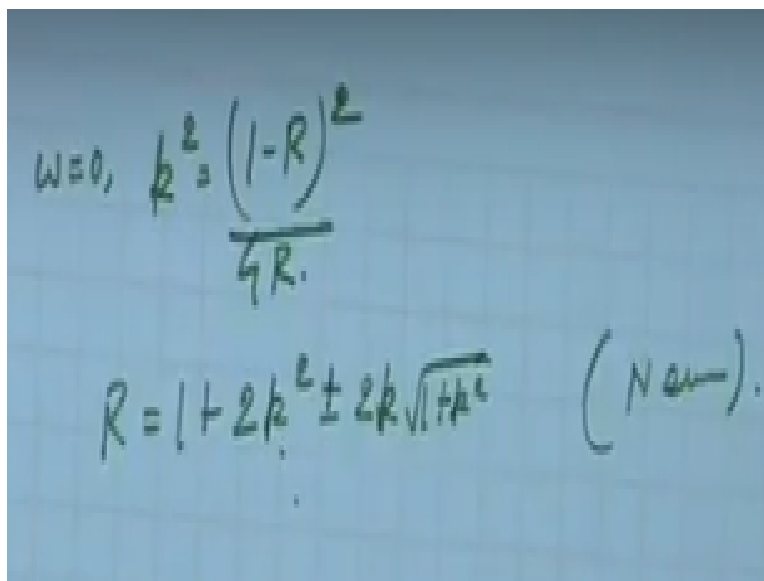
So accordingly your PLR will be 1 at omega is equal to 0 for N odd you see because N odd TN is 0 so PLR becomes 1 and this will N even it will be  $1 + k^2$  at omega = 0 for N even. So it is up to you that if you choose the N value because N values will come from all those specifications that in stop band I want this much attenuation in pass band I want these from that N will come the moment N comes you can choose what is the PLR values from there.

So for the N is equal to 2 prototypes you can now find out that suppose if we choose,  $N = 2$  let us say. So for  $N = 2$  let us say that T2X the Chebyshev polynomial second order that is  $2x^2 - 1$ . So PLR will be immediately 1 plus you see it is N is equal to 2 so N even so TN is plus minus 1 and it is  $1 + K^2$ . You see N even  $1 + K^2 \cos^2 2\omega$  square  $- 1$  whole square. So that is  $1 + k^2 \cos^2 4\omega - 4\omega^2 \cos^2 + 1$ .

So already we have seen that own change these our basic this if we have assume this design this is the same design. So from there we have chosen that Zin etc, so that expression finally we have derived what is the PLR this expression PLR this is for that design.

So this is one side and then for the Chebyshev we have this so you can equate that means I am writing that one plus this one this equal to I can write that one plus already that time we have derived this  $1 - R \cos^2 \omega + R \cos^2 \omega + L^2 \omega^2 - 2LCR \cos^2 \omega + L^2 \omega^2 C^2 R^2 \omega^4$  this already we have done.

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$$k^2 = \frac{(1-R)^2}{4R}$$

$$R = 1 + 2k^2 \pm 2k\sqrt{1+k^2} \quad (\text{New}).$$

So now we can compare and from that you can find out various values. So at you know at omega is equal to 0  $k^2 = 1 - R$  whole square by  $4R$ . So from here you can solve for R as a function of K square for K. So K is what this is an N even K is what, K is a ripple level so that will be specified that I want ripple not to cross this once I know that I have K value so I can find out R similarly can do.

So these things can be done it hand so may be in exams etc will tell you to do for small values but, if larger once we do  $N$  is equal to 3,  $N$  is equal to 4,  $N$  is equal to  $N$  will give you the tabulated values the element  $G_1$ ,  $G_2$ ,  $G_3$  values will get ok. But you understand so that you can find out from the table which  $G_1$  to apply what is your ladder prototype etc.

So now you see that if you specify some  $K$  value here then  $R$  is not 1 as was the case in Butterworth in the Chebyshev  $R$  is not 1. So  $R$  as some value that means you see from the basic design there is a problem that yes so I have taken these as 1 ohm source impedance is 1 ohm. But your this  $R$  that is not coming as 1 ohm it as some value, so there will be some reflections here also depending on that  $R$  value  $Z_{in}$  may not be 1, so there may be reflections here.

So filters generally not matched you have faced the music in image parameter base method here also there will be mismatch. But I again refer you to the ninth lecture of basic tools of microwave engineering where we have learnt how to tackle impedance matching problem. So you can always design with this filter quarter wave transformer or you can also add an extra additional filter element.

Suppose you are having any even you see this problem comes in  $N$  even but it  $N$  is odd then this is not there  $R$  is equal to 1 that we have already seen. So you can also do that because for odd  $N$ ,  $R$  is 1 for Chebyshev so you try to make  $N$  odd there. So you can design for various ripple values these tabulations are available you should know you should not memorize or think this.

But if you understand this simple concept by which you can design this way for other filter types also the tables are available or you can make your own table once by programming etc. you can first make and then you can go on designing whenever the need arises. So this makes the whole thing step forward that whatever may be the filter you first design the prototype.

Now we have not covered that actual filter would not be like that it will be either high pass or it will be band pass or stop band it also its cut off frequency is not always 1 hertz it will have some

other values. So source impedance also need not be 1 ohm as we have assumed in the prototype it can be any 50 ohm 100 ohm those are typical values.

So now from this design once we have got this prototype low pass filter. We will learn in next lecture how to scale up that and how to get the actual filter thing that is called filter transformation that will take up in the next class.