Design Principles of RF and Microwave Filters and Amplifiers
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Module No # 2
Lecture No # 08
Filter transformation

Welcome to this eighth lecture on microwave filter design now we have seen how to design various low pass prototype filters and that time we have assumed in the prototype that source and load resistances they are unity except in when in Chebyshev filter when n is even. Now we also that time made a low pass prototype with omega C the cutoff frequency is 1 hertz. Now we will see how to scale this up so that topic is called as filter transformation.

Now one transformation is impedance we need to change to the actual source and load resistances and also in frequency we need to transform from omega C is equal to one to the actual given cutoff frequency and also we want to transform the nature of filter from low pass to either high pass or band pass or band stop. So one by one will see this three types of transformation.

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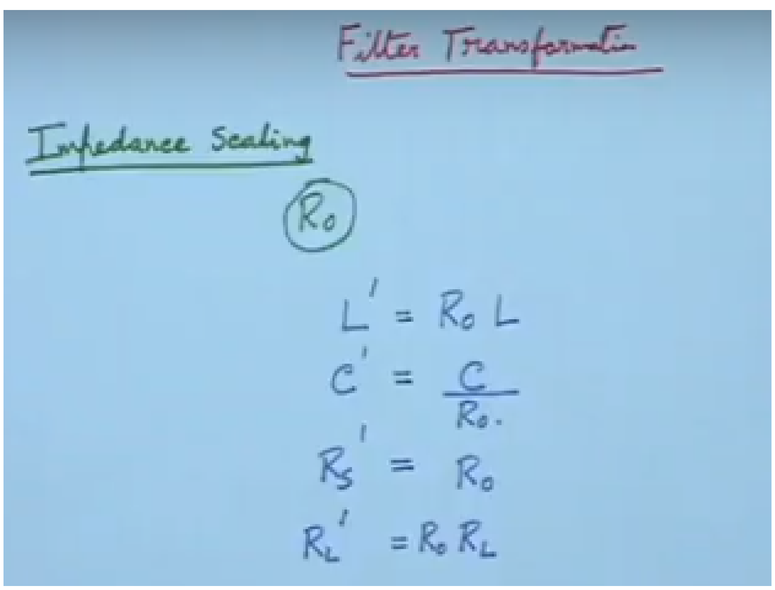

The first one is called impedance scaling when we want to change the source and load resistances to the actual given values specified values. So a source resistance RO, suppose if the source resistance is R0 then I can be multiplied by multiplying the impedance of the prototype by this value R0. So by this all the maybe reactive, may be resistive they needs to be scaled up.

So, when the multiplier in the impedance that means source resistance from 1 ohm to odd R0 ohm we are doing that time let the in after impedance scaling the impedance is becoming prime quantities. So a L dashed, suppose in low pass prototype it was L. So after impedance scaling it will become L dashed. So the multiplier will be R0 L similarly if originally it was C then here it will be C by R0. Similarly if it as R0 in the actual prototype then RS dashed will be like this.

If it was RL in the original or in the prototype then after scaling it will become R0 RL. So this is impedance scaling, so whenever we have impedance scaled from 1 to R0, 1 ohm to R0. We have this R11 to RS we need to this, so this is impedance scaling then we will the second variety that frequency scaling for low pass filters. So in omega C in prototype is 1 hertz but we need to change to some other value omega C from 1 to omega we need to scale.

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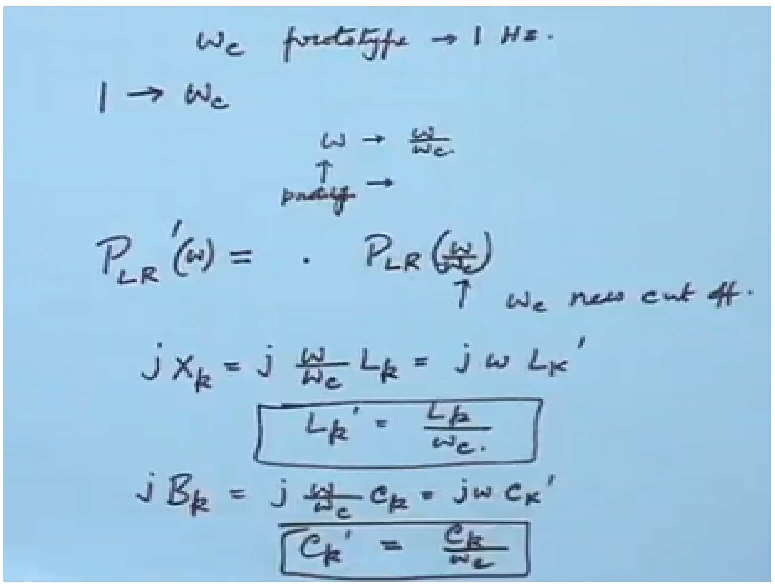

So that means you see if we need to frequency dependence of the filter from omega we need to change by omega by omega C. So we need to replace the omega in the prototype this is the prototype in the prototype thing that to be scaled by omega by omega C then only it will be the cutoff frequency will be change because cut off is one. So you see then omega is equal to omega C this thing again becomes one.

So that is the scaling that means if original insertion loss as a function of omega was this we need to make that omega by omega C this will be then actually given by the change quantity PLR dashed which is PLR with new omega. So this is the scaling so here this omega C is the new cut off frequency. So when we do this that means when we make from omega prototype we change by omega by omega C all the impedances they also change.

So let us say that now the reactive impedance ZXK that will now be called Z omega by omega C original LK. So this in the new nomenclature will say J omega LK dashed after transformation. So the new value of LK dashed is nothing but LK just to compare here LK by omega C. So that means when we change the cutoff frequency all prototypes LK they should be divided by omega C o get the new LK dashed values.

Similarly we know that if we have the susceptance JBK then that is J omeaga by omega C, CK so that is J omega CK dashed from here if we compare what will be CK dashed CK dashed will be CK by omega C. So you see that all capacitances of the prototype they need to be divided by omega C to get this.

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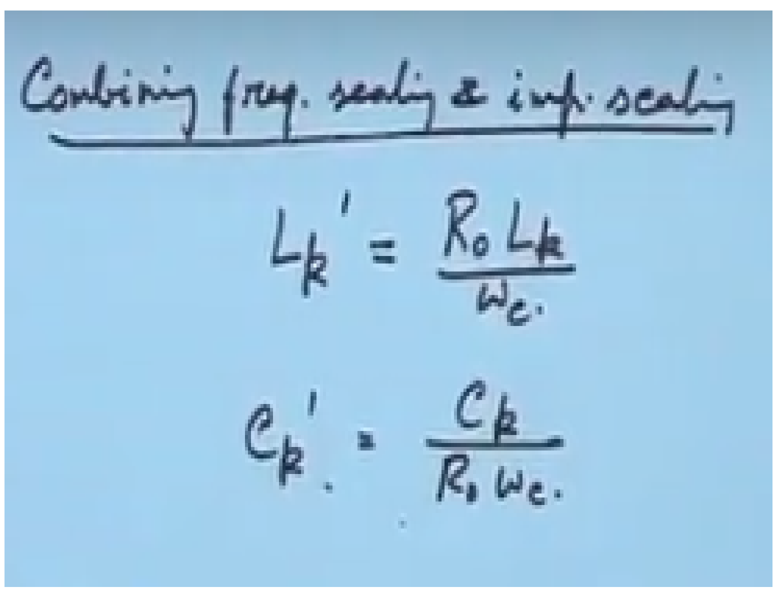

Now when we combine the impedance scaling as well as the frequency scaling so combining frequency scaling and impedance scaling will get LK dashed is R0 LK b omega C and CK dashed is equal to CK by R0 omega C. The reason is you can see that in case of impedance scaling the L dashed is to be multiplied by R0 and C dashed is to be divided by R0 here for frequency scaling both LK and Ck to be divided.

So here you see both are divided by omega C By 1 is multiplied by R0 another is divided by R0. So this is frequency scaling and impedance scaling. Now we see that how the past behavior changes low pass to high pass transformation out prototype is always low pass. But let us see what was our prototype insertion loss PLR versus omega that was 1 here -1 this is 0 this is some value given by the PLR minimum that we can in pass band that we can.

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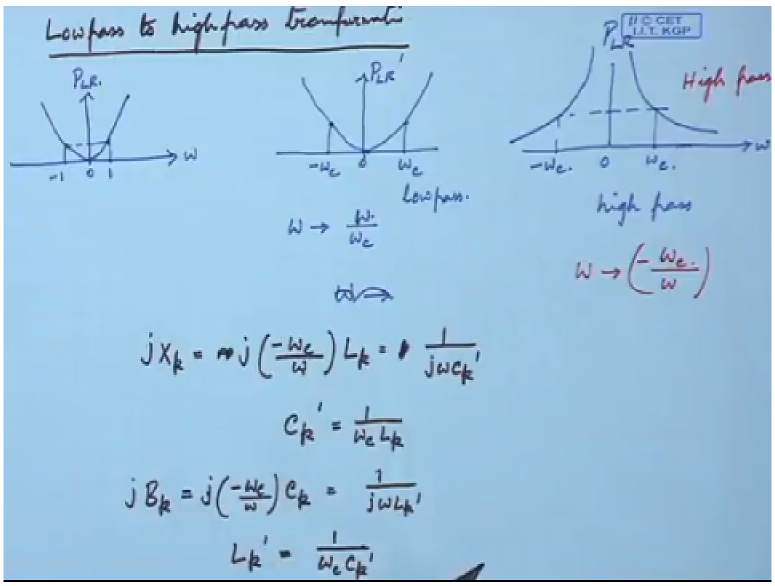

Now if we want to make this two characteristic like this that instead of 1 I want to make it omega C and minus omega C this is 0 this is the new you can say PLR dashed. Then what I need to do you see that if I change this that we have already seen that if I make omega change to 1 by omega by omega C then when omega is equal to omega C I have 1 so this point is being mapped to omega C when omega is equal to minus omega C this point is mapped to here when omega is equal to 0 this point maps to this point.

So this is simply frequency scaling as we have already seen now we want to change the past behavior that means what we want high pass means the PLR should be like this that this our omega this is a new PLR you can say double dashed for high pass this was still low passed with a changed omega but this is high passed. Here I want that suppose this is my level tolerable getting in pass band.

So then I want this is my omega C and this is my minus omega C. So I want that at zero value the PLR should go to infinite at omega C PLR will come to certain value and thereafter fall. Similarly at minus omega and thereafter it should fall that means I need a frequency variation inverse to this type of thing. So we map instead of this the new map sorry here you write the low pass to high pass transformation is obtained that here I instead omega that here I instead of this omega comes down and this omega comes C up.

But also when I do this actually you know that high pass to low pass means the inductance, inductor and capacitor they will be interchanged now to have them in but all the inductance and capacitances values L and C they need to be positive. So to do that I need to add a negative here so minus omega is changed to minus omega by omega C this becomes a high pass filter.

This is high pass at lower value it is cutting but it is giving so now we can find out what are the change in reactants and values inductor and capacitors components values. So again JXK which I will n write of J of minus omega C this one minus omega C by omega LK and that I will give a name. So it is obvious that omega has come here so this has become a capacitance. So I will call it 1 by J omega CK dashed the new quantity CK dashed so from here you cow an see what is CK dashed?

CK dashed is nothing but if you compare this CK dashed is 1 by omega C LK. So the previous prototype LK that has changed to CK dashed. Similarly, I can do if susceptance of the capacitor JBK that will be J minus omega C by omega into CK and that is one by here you see J omega LK dashed. So by comparing I now find the LK dashed is 1 by omega C by CK dashed. You can easily check that if I have not taken this minus then always this LK dashed and CK dashed wont be negative.

So that will create an would not be positive that will create problem. Because CK dashed LK dashed components values they always need to be a positive that corresponding reactances may become negative etc. But this component values make it realizable we need to add this minus sign. Now if we include here already we have included frequency scaling as well as pass transformation. We can include already we have seen the impedance scaling also.

So with impedance scaling so that means I can say that impedance scaling class frequency scaling that means from 1 to omega C plus low pass or LP 2 HP all this put together then my new CK dash value is nothing but 1 by R0 omega C LK and LK dashed value is R0 by omega C Ck.

So a inductor changes to CK dashed when I make this is well known but with this scaling you need to need to know that if I want to make from a low pass prototype to a high pass filter the inductance in the low pass filter prototype will be changed to a capacitor with this value and the capacitor in a low pass filter will be changed to inductor with these value.

So in tutorials we will see problems with this and now this is low to high pass transformation. We now see that what happen to low pass to band pass transformation. So originally again let me draw the PLR of the prototype this was the original PLR this is omega, this is 0, this is + 1, this is -1, this was the pass band.

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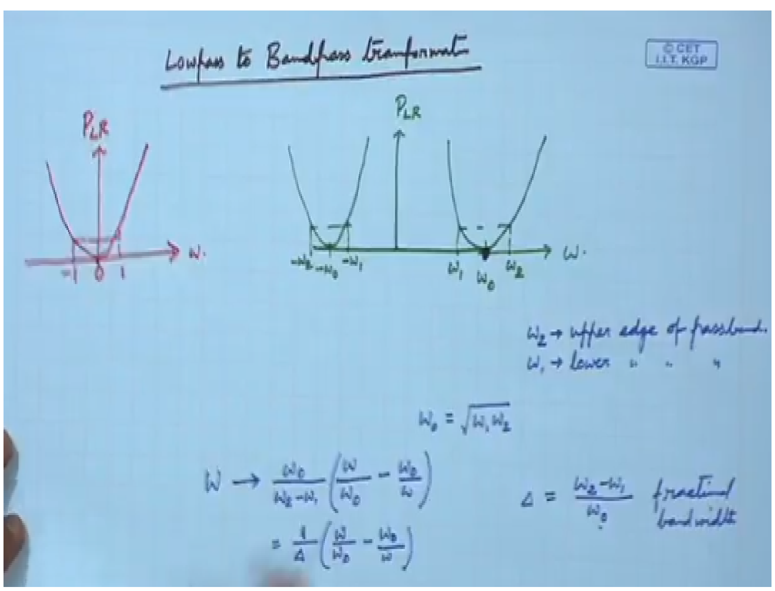

Now what I want as a band pass I want that it should be this is my new PLR omega then you see I want that it should go to omega 0 and also there will be a high frequency upto which it will be pass let us call this omega 1. Similarly here it will minus omega 0 this is the minus omega one, this is minus omega 2. So this is thing now for this we want to find out what is omega 1 omega 1 is the edges of the pass band. So I can write omega 2 is the upper edge of pass band, omege 1 is the lower edge of pass band.

And omega 0 we can take either as arithmetic mean of omega 1 and omega 2 or we can take it as geometric mean of omega 2 now both possibilities are there. But equation transformation will become easier if we take it at geometric mean that is why we generally take it as omega 0 is equal to omega 1, omega 2. If we take that and the transformation we call that omega of prototype should be changed like this omega 0 by omega 2 – omega 1 into 1 omega – omega 0 by omega 0 by omega.

Now we can ask how i got this before that let me simplify this bit let us also called that this term let us call delta this is no omega term here this is a constant. So let me denote it by a constant delta is equal to omega 2 – omega 1 by omega 0. You know what is this? This is nothing but delta is nothing but the fractional band width because band with is F2 – F1 everything is multiplied by 1 PIE here or this is angler frequency.

But when we take the ratio it is same as fractional bandwidth now F2 – F1 by F0. So this delta is this so in terms of that side I can write 1 by delta into omega by omega 0 – omega 0 by omega. Now you see that I want this point the minimum attenuation point in the minimum insertion loss point in the pass band that should be mapped to omega 0. So you see when omega is equal to 0 if I put then this is 0. I want that omega is equal to 0 or it is actually scaled version of this so omega 0.

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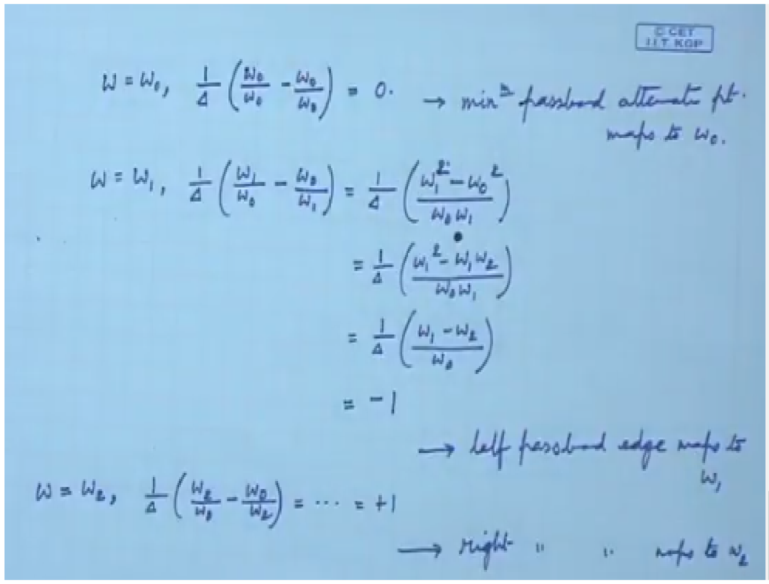

When it is omega is equal to omega 0 this point should map here you see omega is equal to omega 0. So let me put it here that this is my transformation keep it what omega is omega 0 what happens to this new term 1 by delta then omega 0 by omega 0, - omega 0 by omega 0 so it is 0. So I can say that this minimum pass band point maps to this omega 0 point here. Now when omega is because I can say that minimum pass band attenuation point maps to omega 0

When omega is equal to omega1 what happens 1 by delta then omega 1 by omega 0 – omega 0 by omega 1 so this is 1 by delta omega 1 square – omega 0 square by omega 0 omega 1 then you can find out that what is omega square you see already we have put this so 1 by delta omega 1 square – omega 1 omega 2 by omega 0 omega 1. Now that you know that you know 1 by delta so I can omega - 1 omega 2 by omega 0 and what is delta again you see delta is omega 2 – omega 1 by omega 0.

So can I say that this will be -1 that means it says that the left pass band edge I have put here omega is equal to omega 1. So the left pass band edge that will mapped to the omega 1 that means this point mapped to omega 1. So let me write the effect of this the left pass band edge maps to omega 1similarly that if I put omega is equal to 2 then the whole thing 1 by delta omega 2 by omega 0 – omega 0 by omega 2 that becomes +1.

So I can say that right pass band edge maps to omega 2 and you get a graph like this. So this is exactly here similarly you can also find out that if I make now this completes this part. Now you see if we map again omega is equal to omega 0 you will see that minimum pass band attenuation again maps to minus omega 0 minimum pass band attenuation point maps to minus omega 0.

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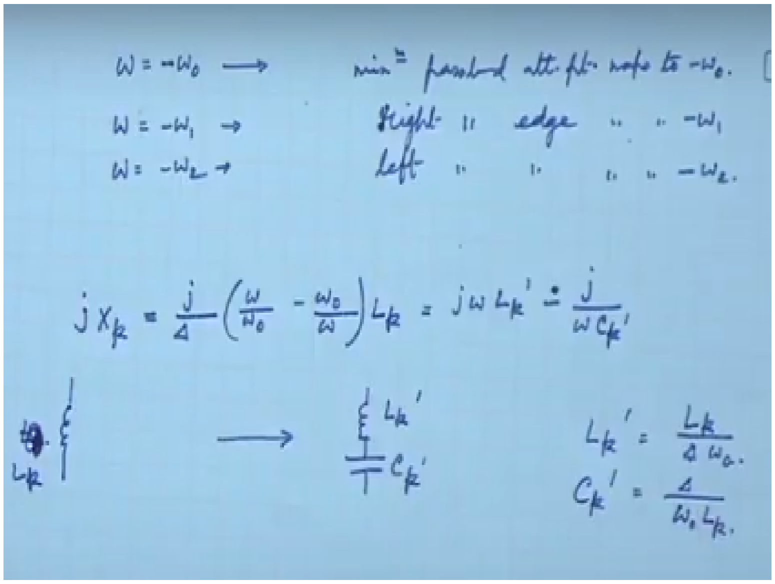

If you take omega is equal to omega 1 then you see that the, if you maps to – omega 1 then right pass band edge instead of left pass band edge for positive frequency. Right pass band edge maps to minus omega 1, omega is equal to minus omega 2 you will see left pass band edge maps to minus omega 2. So you get a graph or plot like this so transformation is correct now we will have to find out what is the corresponding component values.

You see so with that transformation now an inductor will be like this J by delta. Omega by omega 0 minus omega 0 by omega into LK so that will give you j omega LK dashed – J by omega CK dashed. So basically you see a series inductor becomes a series LC circuit with a new value of inductor, and new value of capacitor. So you can see that in the originally I have a LK now I am having an L now that is after transformation that is becoming a LK dashed and CK dashed in series.

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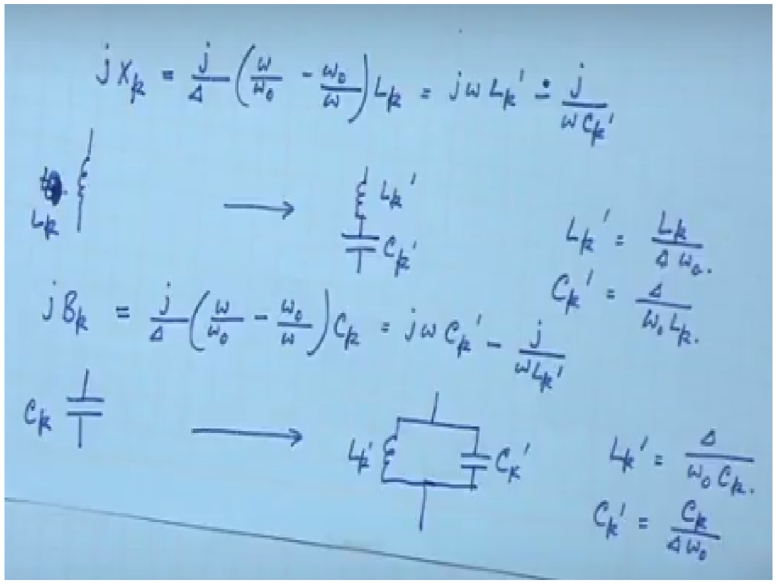

And what is the value of CK dashed if you just compare LK dashed is nothing but LK by delta L0 let me call it LK itself. So LK dashed is LK by delta omega 0 delta into omega 0 and CK dashed that is equal to delta by omega 0 LK. Similarly, the capacitor in the prototype so JBK is susceptance that will become now J by delta omega by omega 0 minus omega 0 by omega CK. So that is J omega CK dashed minus J by omega LK dashed so a you see that here a shunt capacitor transform to a shunt LC circuit with elements LK dashed and CK dashed.

So if I draw again I had a CK dashed so if I draw again a low pass prototype but now am having a shunt LC with LK dashed and CK dashed. What are their value LK dashed just compare here you will get delta dash is delta by omega 0 CK. CK dashed is equal to CK by delta omega 0. So also you can check that both these series resonant circuit and parallel resonant circuit they their resonance frequencies as omega 0. So at omega 0 you see that omega 0 you see that whole circuit resonance that is why PLR you get as 0.

So this is the band pass transformation now we want to just I am giving the band stop transformation in band stop we have omega just inverse of these delta was 1 by delta here omega delta is this omega by omega 0 – omega 0 by omega to the power -1.

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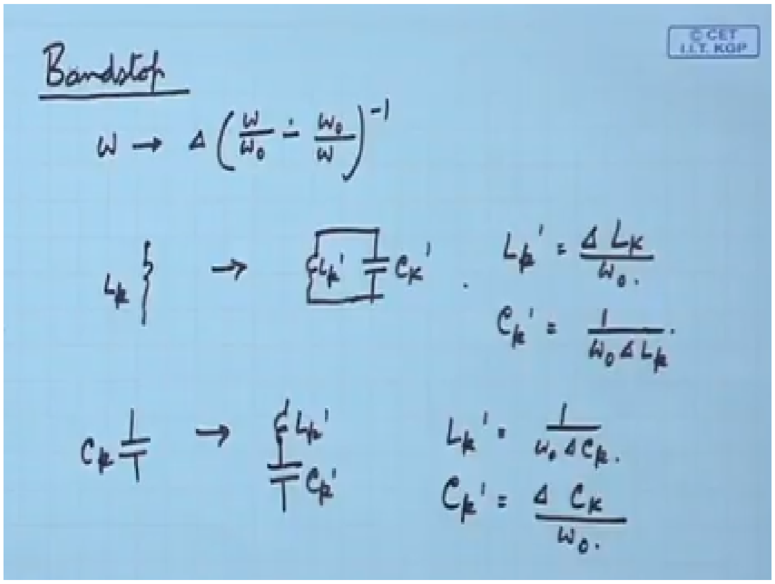

So you can just check that you know band pass characteristic that will be something like this that instead of this you will have that at this point this will be high so just inverse of this just you can check it and there if you have an LK that will be changed to a parallel resonance circuit LK with LK dashed with CK dashed. Where LK dashed will be delta LK by omega 0 CK dashed is equal to 1 by omega 0 delta LK.

And a capacitance CK that will be given by a series resonance circuit LK dashed CK dashed where LK dashed is equal to omega 0. Delta CK and CK dashed is equal to delta CK by omega 0 that is all. So you now have high pass band, band stop anything you want to band pass you can do. So with this now we were in a position to find out what is a actual filter specification. If it is a band stop or if it is a high pass then from the prototype we can design it.

But then there will be some issues with high frequency implementation of microwave implementation of that will take up in next class. Thank you