Design Principles of RF and Microwave Filters and Amplifiers
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Module No # 2
Lecture No # 09
Microwave Filter implementation

Welcome to this ninth lecture of this course and also microwave filter design. So we are completing almost the design. So in micro filter design you see that up to this point we have seen the how to design RF filter with component values, lumped component values. But you know that those lump component values lose their values if you go to higher frequencies a capacitor can behave as an inductor an inductor, can be able and capacitor etc. because those lumped elements they are not of reliable values at high frequency.

Also it is difficult to make any value of lump components at high frequencies. Because here from the filter design you for a specific insertion loss characteristic you are designing a particular value. You are not sure whether that value that that lump component is available or can be fabricated because there can be fabrication difficulties.

But we know one thing that any lump component you can fabricate by a transmission line by a shorted open transmission line you can make any impedance value because we know that in a transmission line the input impedance that behaves either as a TAN function or cotangent function. So since TAN function extends from minus infinity to infinity plus infinity and cotangent function also have minus infinity to plus infinity variation.

So any value that means depending on the characteristic choosing the proper characteristic impedance and the proper link we can design any value of inductance or capacitance at high frequency that is done with the help of a transformation called Richard transformation. Will first see P Richard if I remember correctly P Richard's first introduce this transformation. So they make this microwave filters replacing the lamp component by stub transmission line stubs may be open stubs or short stubs as you all know you have all dealt with that you know my microwave classes.

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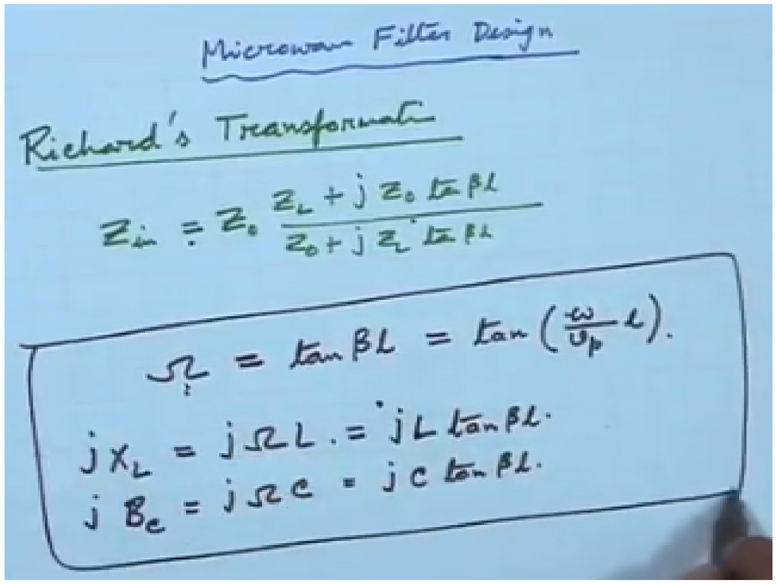

So with Richard's transformation we can attempt what Richard transformation says that you know that the input impedance of any transmission line with characteristic impedance Z0 and terminated by ZL. That is given by ZL + J Z0 tan beta L by Z0 + J ZL tan beta L all of you know this. And we have also seen this we have proved this also in the impedance transformer design we have extensively with these quarter wave transformer we have fabricated from these etc.

Now you see that what Richard has done, suppose that let us this TAN Beta is an important thing. So let us have a mapping that let us define a capital gamma capital this gamma and that let us call it TAN beta L. So TAN beta L we know in a transmission line what is this can be TAN Beta L where is the frequency, frequencies inside beta in transmission line a distributed transmission line the wave propagates by TM mode in TM mode this beta can be written as omega by BP.

Okay so you see that by this we are transforming the W plane to this gamma plane. This capital gamma so now we assume them that in the new plane this gamma is the angular frequency. So we can write what happens to inductors ZXL they will now will be calling previously we were calling J omega L now we will be calling gamma L and that is what J you see instead of this I can say L is there TAN beta L and in the susceptance PC that will be calling.

So how we are getting this let us see from this equation this is his transformation this is the result that with this transformation this is called the Richards transformation we get the new values are like this but let us see that what is this? So again I write is Z in is Z0 + J Z0 TAN beta L by Z0 + J ZL TAN beta L now when you know that we implement and inductor by a short circuited stub of electrical length L where electrical length Beta L and we implement a capacitor by an open circuited stub of electrical length Beta L.

So let us see those first that when we have shorted Stub shorted means load side we short that means Z1 = 0. So what happens to input impedance this is like this I have an Z0 I was this equation is for terminating with ZL now ZL become 0 so what is my input impedance, this is my length L here the propagation constant is beta. So Zin will take the value Z0 then you see that J Z0 TAN beta L by Z0 so that will be J Z0 TAN beta L.

Now you see Richard transformation shows that this Z for inductor this is to be J L Tan beta L. So I need to choose this Z0 as L that means to have a to fabricated JXL that means an inductor with reactants XL. I need to choose what this stub it will be a shortage stub with L as the characteristic impedance and Length is L and its Beta is given by that omega by BP. 4

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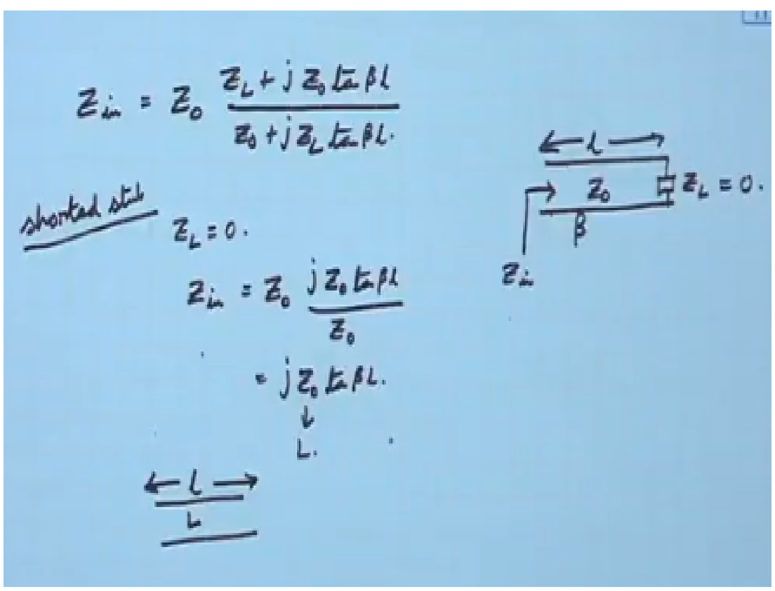

So similarly so this part we have proved that how which had got this similarly for let us do this thing again write Zin = Z0 ZL + JZ0 Tan beta L by Z0 as JZL Tan beta L this time let us put an open circuit ZL = infinity and this is Z0 this is as before L and also the propagation constant is beta. So what happens what is Zin value under this condition Zin is Z0 this is infinity so divide so you get 1 by J TAN beta L.

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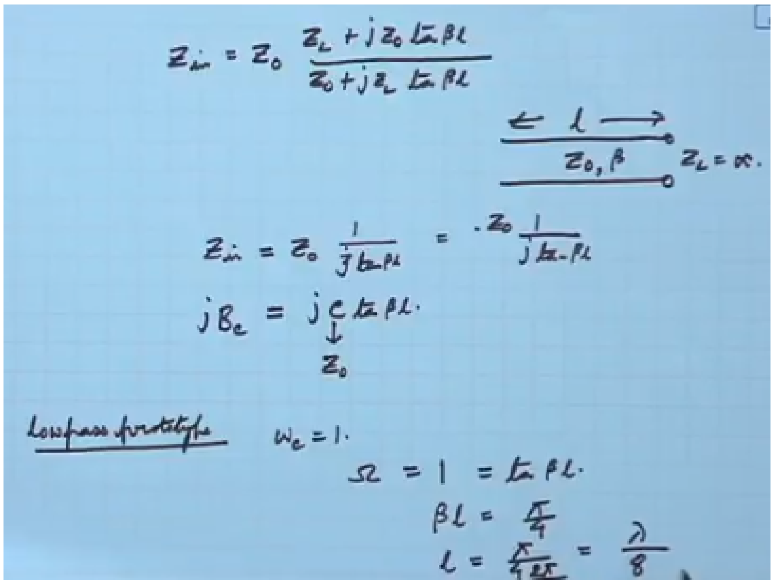

So that is 1 by J TAN beta L Z0 then I can write Z1 is capacitor inside Zbc that I can write as JC TAN Beta L, so again it shows that I need to choose this has Z0 value at .C So that is what he has done that you see he has chosen a characteristic impedance of the line will be now chosen as C and length is L also for low pass prototype you see, the cut of frequency was omega C = 1 so in Richard thing the mapping is to this omega plane so omega is capital omega is now 1.

So that is equal to TAN Beta L so this gives us what is the length L can I say that this implies Beta L is PIE because TAN PIE by 4 is 1 so from that I can solve for L = PIE by 4 by what is beta beta is 2 PIE by lamda. So I get lambda by 8 so it says that all this lengths when we choose this that this is the length L these all lengths at omega C is lambda by 8. So at the cut off frequency the line so I can say that when you are doing Richards transformation this lambda is the wave length at omega = omega C.

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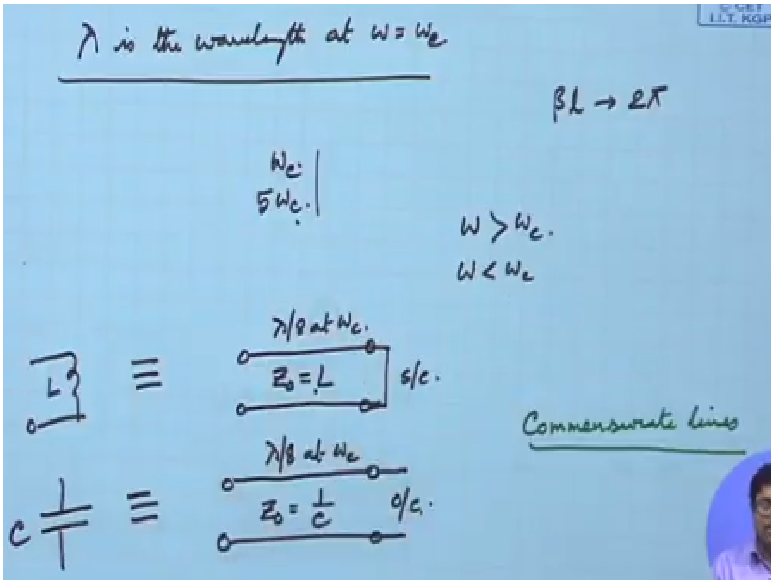

Now obviously this means that at other frequencies these lines their impedances is will change so lamb they wont represent the prototype lumped inductance and capacitance properly. But there is periodicity that after every I have omega C then at five omega C again the element value will match. This is because this beta L by electrical length by beta L that is a periodic function of 2 PIE since am having lambda by 8.

So I am having this variation that it is designed for omega C at other frequencies it wont match but at 5 omega C again it will match okay. So filter is function differ at omega greater than omega C or omega less than omega C those value it will be different. But that you can tackle by other technics by broad banding etc that are advanced techniques.

So now we can say that by Richard's transformation in the low pass filter if there was a inductance L that you can represent by in a distributed circuit by a short head thing you take the characteristic impedance of the transmission line as L same value and this is lambda by 8 length at omega C.

Similarly a capacitor of value see that with Richard transformation again the length is same you see that is the beauty that all LC they will be of same length but what is the value of Z0 this will be 1 by C and you make this open circuit this is a short circuit now since all this length are of same length all the stubs are of same length but they are of different variety either short circuit or open circuit depending on it these lines are called commensurate lines.

So you need not bother about these line lengths and if required suppose 1 length is not very small or very large you can use this periodicity and go to higher values so that higher or lower value so that you can get that same thing. So that means with this in the prototype design whenever this is a lumped L or lumped C you can represent like this this.

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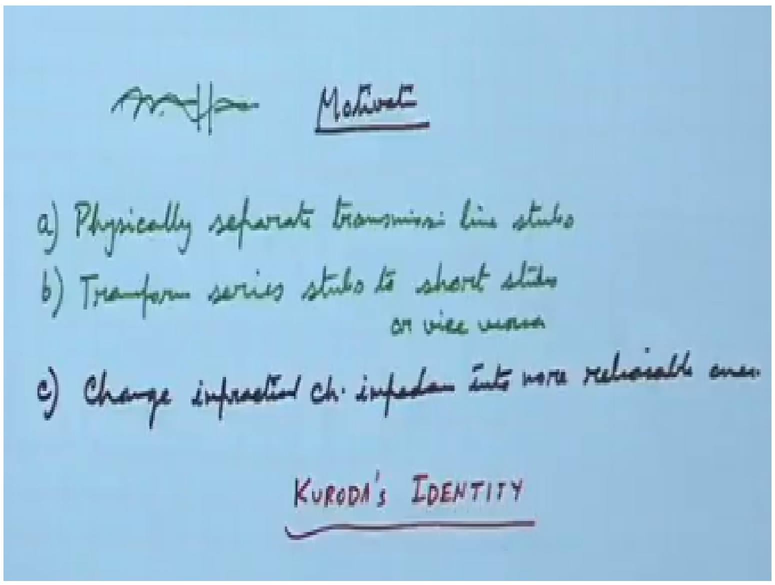

So this is called as identity but one more thing that is tat in high frequency another comes that this lumped elements suppose I have 1 L in a filter than a C now their distance is that also a matters because there is phase difference between them when you are putting with this then that distances etc may not be feasible and I need to sometimes I need to put some more gap or some less gap so some redundancy, redundant line needs to be incorporated.

So that these gaps etc they are become feasible and there are sizable gaps between them some math sections need to be put so more practical micro filter implementation requires these because sometimes I need to physically separate transmission line stubs. Then sometimes in fabrication if I have this short circuit open circuit sometimes I need make for if there is a large circuit that all the thing should be short circuit or all the things should be open circuit because that makes fabrication easier.

But then I need to have transformation you know that can be easily done because any suppose I have a implemented a inductor by a short circuit now I can also implement that by an open circuit because the equations are from I can have only the length etc they will have different values so sometimes we need to do that that is why this is another need for practical design.

Transform series stubs to short stubs or vice-versa that means sometimes opposite some short stubs may be needed to change to series stubs then sometimes change impractical characteristic impedances into more realizable one. Now these 3 are the practical things so motivation for using another technique which is called Kuroda's identity.

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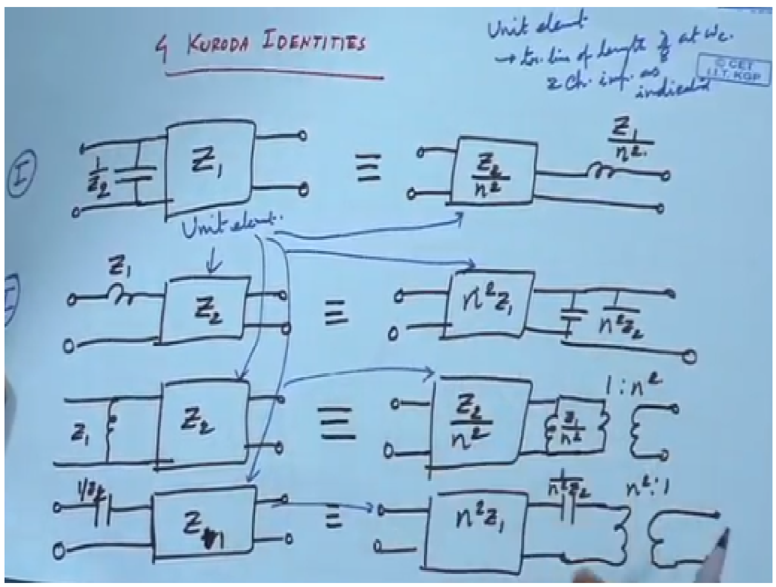

Kuroda's identity now Kuroda he gave these things are 4 Kuroda's identities but he did that suppose I have a lump filter I want to convert it to an inductor that means basically these shunt open circuit I want to make as a series short circuit stub.

This is so what he did he said with this you add let us say this was of Impedance 1 by Z2 he said add a Z1 this in Kuroda's nomenclature it is called unit element these are redundant element is putting and showing that this is equivalent to putting Z2 by N square some line with characteristic impedance Z2 by N square and then in series with a inductor of characteristic impedance in N square.

Similarly this is called is Kuroda's identity first type, Kuroda's identity second type is like this that you have a series inductor this is equivalent to pieces that you have this value is Z1 so here is saying this is equivalent to 1 by N square Z2 this one. So this is second type, then third type is you have instead of in series inductor you have a shunt inductor of value Z1 he said put with is a Z2 and this equivalent to Z2 by N square then a you want to keep all inductor but y u need to have a transformer.

So this one is saying Z1 by N square its characteristic impedance and this is 1 is to N square terms assured transformer and is forth variety is you have it was a shunt you have a series capacitance then you put Z2 sorry this is 1 by Z2 this is Z1. So this he says this will be N square Z1 then you can written this 1 by n Square Z2 but then with a transformer whose ratio is M square is to M.

So in all the cases M square is equal to 1 + Z2 by Z1 so this each box represent and unit element they are called unit element basically it is same as a transmission line of unit element all these are unit elements these are all what is unit element this is a transmission line of length lambda 8 at omega C same as what we have seen the feature transformation case and characteristic impedance in data transformation was ALRC here characteristic impedance as indicated in the identities.

So Z1 is the characteristic impedance of these box Z2 is the characteristic impedance Z2 by N square is characteristic impedance etc. and lumped inductor and capacitor represent stubs of inductor represent the stubs of short circuit and capacitor represent the stub of open circuit respectively. Now all this can be proved. We will just see how to prove this first identity let us say so first identity.

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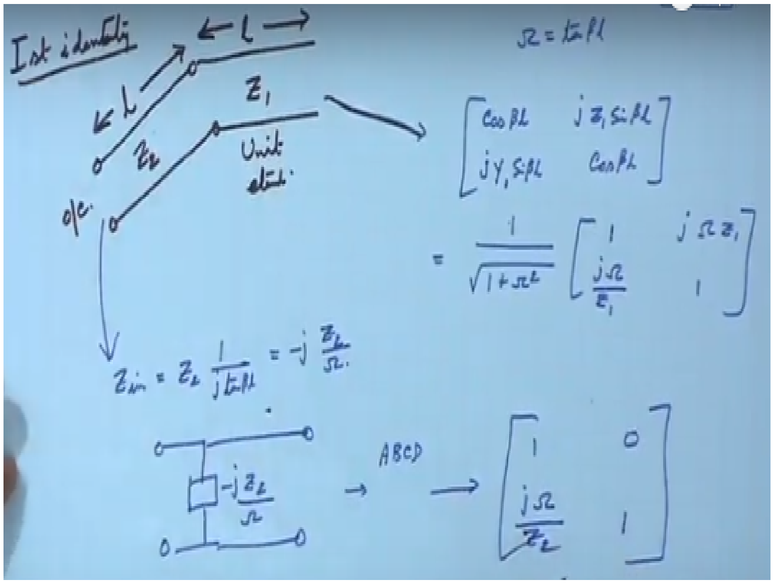

You see that I have basically you see here I have shunt capacitance so that means I have a shunt stub open circuit stub and this value is 1 by Z2 characteristic impedance. So these stub 1 by Z2, Z2 and this length is lambda by 8 length this is a OC stub with this I have a Z1 again the length, all length you see are L which is lambda by 8 and this is my unit element unit element.

Now this if I have a transmission line of length L and characteristic impedance Z1 I know it is ABCD matrix for this one particular one ABCD matrix will be COS Beta L this we have done earlier that how to find ABCD matrix so in earlier in NPTEL lectures you refer now in terms of Richard transformation I can because there we have seen can be TAN beta L is equal to capital Omega so 1 by root over 1 + Omega square into capital omega 1 J omega Z1 then J omega by Z1 and 1.

Just you put then you get then this is remember capital omega is TAN beta L with this I can write in this and also this open circuited stub. What is the input impedance of this open circuited stub that I can write as Z2 1 by J TAN Beta L and this is in terms of this -JZ2 by omega. So now what is the this thing is nothing but like this -J Z2 by Omega. So what is it ABCD parameter ABCD will be simply 1 0 is to 1 you see sorry ABCD C will be Z2 and this is 1.

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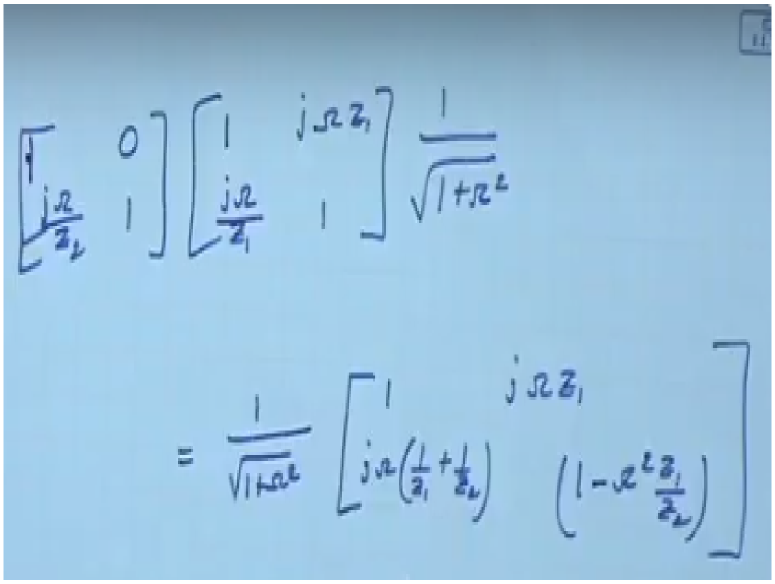

$$\begin{bmatrix} 1 & 0 \\ \frac{j\Omega}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & j\Omega Z_1 \\ \frac{j\Omega}{Z_1} & 1 \end{bmatrix} \frac{1}{\sqrt{1+\Omega^2}}$$

$$= \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega Z_1 \\ j\Omega\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right) & \left(1-\Omega^2 \frac{Z_1}{Z_2}\right) \end{bmatrix}$$

This you can check that this one already we have seen earlier so now the composite this whole thing composite thing ABCD matrix will be multiplication of these two. So I can write first I will have this is first that means this into this ABCD that is the beauty these 2 are in cascade. So I can easily write that 1 0 J omega by Z2 1 into 1J omega Z1 J omega by Z1 1 into that scalar multiplier 1 + big omega square.

So this 1 by root over 1 + gamma square then 1 J Z1 J omega I by Z1 + 1 by Z2 and 1 – omega square Z1 by Z2 this is the left hand side now right hand side if you look at the first identity the first unit cell and then Z1 by N square. So this I keep this basically LHS I can say Left Hand Side ABCD matrix composite is this in right hand side I have this in its L length L characteristic impedance Z2 by N square then just look at the figure this is L and this is Z1 by N square.

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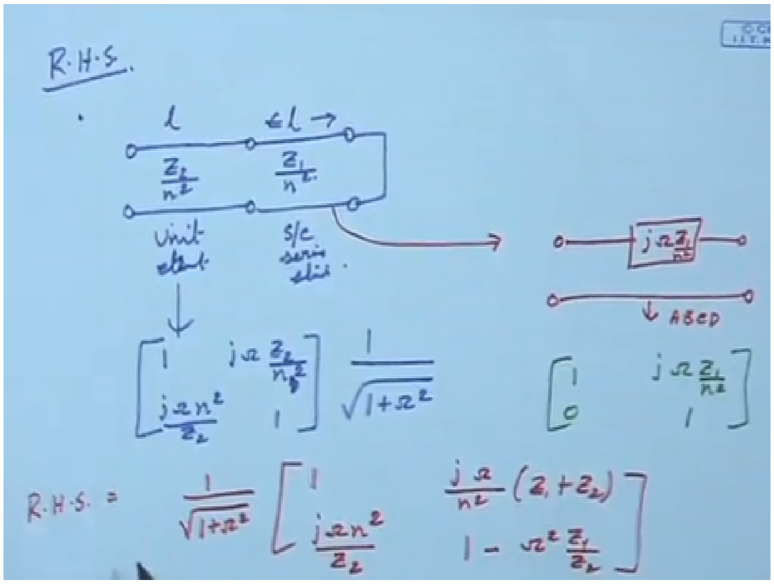

So this is unit element sorry this is short circuit series stub so I made it series stub so for unit element ABCD matrix there will be 1 am sorry its square J omega N square by Z2 1 by root over 1 + omega square. And this one short circuited stub again you see it is equivalent to this that J omega Z1 By N Square series Stub so you know that ABCD matrix its ABCD will turn out to be 1.

This thing we have done earlier just refer there so the composite so RHS will be composite of this this is the first one in this one if you do that you get 1 by 1 + Square then 1 J omega by N square Z1 + Z2 J omega N square by Z2 1 – gamma square Z1 by Z2 now you see this is RHS this is LHS. So see LHS is these RHS is these so on this 2 becomes identical only when I choose that in value that N square = 1 + Z2 by Z1. That is why Kuroda has done that and in all these cases this N square 1 + Z2 by Z N.

So with this you can now have all the permutations given and so if you want to design any implementable filter upto Richards transformation you come then you take the appropriate Kuroda's identity because you see which one you need. Because if you want to if you want need if you have a shunt capacitance but you want to convert with series 1.

You can have this just add this unit element and in series with that you can get it similarly if you have this you can use this, this also popular these 2 these 2 sometime you use but generally inductance and they are not generally in this fashion but some band pass etc they are there but low pass high pass generally we have this type of thing so you can use these 2 identities.

But if you can refer to here and always do this so we will see some implementation of these in the next lecture that how we go about these transformations. Thank You