


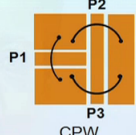
Millimeter Wave Technology
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Department of Electronics and Electrical Communication Engineering
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Module 5
Lecture No 24
Passive Components (Contd)

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T- Junction Power Divider

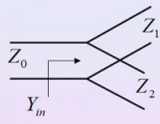


Microstrip line



CPW

T-junction power dividers.



Equivalent circuit.

- To match the input impedance, the condition

$$Y_m = \frac{1}{Z_0} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

- For equal power coupling, the output impedances for a 50 Ω input port are 100 Ω.
- The output can be matched to 50 Ω using a quarter wavelength transformer.
- No isolation between the output ports.

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Okay, so next topic is power dividers and couplers. So whatever we use at microwave frequency, so similar concept we can extend to millimetre wave frequencies, so only thing is that we have to do frequency components and also we need to keep in mind about the losses, so let us start with this simple T-junction. Frequently we use this junction to feed antennas particularly antenna arrays, it is shown in microstrip line pump CPW loop pump, so we have port P1, from this power is divided into Port P2 and Port P3 so in right-hand side figure same thing from P1 it is fed to Port P2 and P3. In right-hand side you can see these black lines, they represent wire bonding to keep the ground plane at same potential.

If you remember that even-odd mode problem, we do not expect even mode excitation because even mode it radiates it is we only want odd mode odd symmetry, so for this we are using this wire bonding to keep the ground plane at same potential. Now, in transmission line form, then we can represent this Port P2 and Port P3 by using 2 different transmission lines at the input impedance as seen by Port P1, then it becomes the parallel combination of the impedances of Port P2 and Port P3. Or we can write down Y_m , this is equal to $\frac{1}{Z_0}$ for port 1 equal to $\frac{1}{Z_1} + \frac{1}{Z_2}$. Now if Z_0 is 50 ohms, in that case if I want equal power split between this Port P2 and Port P3, we should have Z_1 equal to Z_2 .

And not only that $1/Z_1 + 1/Z_2$ that should be equal to $1/50$, so Z_1 and Z_2 each should be 100 ohms.

Now let us say, left-hand side we have a 50 ohms system, right hand side also we have 50 ohms system, so in that case how to modify the circuit? We can use $\lambda/4$ impedance transformer quarter wave impedance transformer, so we have to connect 2 impedance transformers one between port 1 and port 2 and other one between port 1 and port 3 and the characteristic impedance of this impedance transformer, it should be then square root of Z_0 into Z_1 for Port 2 and square root of Z_0 into Z_2 for Port 3. So for 50 ohms system all this 3, then the characteristic impedance of the impedance transformer, it comes square root of 50 into 100, approximately 70.71 ohms. And the length at a given operating frequency let us say 60 gigahertz, the length should be $\lambda/4$ per λ is the guided wavelength.

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Millimeter-Wave Interconnects

Simulation of a T-junction (Si substrate) Simulation of a bend (Si substrate).

Reactive T-junction loss – 0.24 dB @ 94 GHz
 Right angle bend – 0.11 dB @ 94 GHz

- Look at the reactive compensation used at the junctions.

K. J. Herrick, L. Katehi and R. T. Kihm "Interconnects for a multi-layer three dimensional architecture", Microwave Journal May, 2001.

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But the problem with this T-junction power divider is that Port P2 and P3 they are not isolated, so if there is any reflection let us say from Port P2 that power will come back to not only Port P1 into Port P3 also, so this is the problem with this T junction, and how we can modify it? We can modify it by using a resistor and the modified form is Wilkinson power divider. So you can see here, although it is showing on microwave design at lower frequencies, same principle we use at higher frequency millimetre wave, so left-hand side we have 50 ohms system, we have 2 impedance transformer they form one rectangle and in between we have a resistor of 100 ohms and right-hand side we have Port P2 and Port P3, they are again terminated in 50 ohms.

And this is the measured response of this Wilkinson power divider, you can see S₁₁ input impedance matching from Port 1, it is quite good. S₂₁ equal to S₃₁ and they are approximately -3 dB, it shows equal powers split and isolation is 3-2 it is also down, at operating frequency dipped below 40 dB, we have good isolation. So ideally what should be the S parameters of 1 Wilkinson power divider? From all the ports we should have 0 reflection that means S₁₁, S₂₂, S₃₃, they should be 0, from Port 1 to Port 2 and from Port 1 to Port 3 and vice versa they should be equal, but we have a change of phase 90 degree, but this phase difference is same for both these ports that means angle of S₂₁ is equal to -90 as well as angle of S₃₁ that is again -90 so that we do not have any phase difference between these 2 output ports. So this is one basic property of power divider, we do not have any phase difference between the output ports.

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Millimeter-Wave Interconnects

Simulation of a T-junction (Si substrate) Simulation of a bend (Si substrate).

Reactive T-junction loss – 0.24 dB @ 94 GHz
 Right angle bend – 0.11 dB @ 94 GHz

- Look at the reactive compensation used at the junctions.

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Now, let me show you some millimetre wave circuit. So this is showing one T-junction realised in CPW technology at 94 gigahertz on silicon wave front, so left-hand side this is Port P1 and right-hand side Port P2 and P3 you can see the impedance transformer $\lambda/4$, we have different signal line width, in this middle part is having 70.7 ohms characteristic impedance for that signal line width is smaller end the slot width is higher. And you can see here, a photograph is given here of the fabricated structure, so for this one, these black lines they actually are showing the wire bonding to keep all the grounds at same potential. Now, when we design something at so high frequencies like 94 gigahertz we have to be very careful about the bends and junctions, they are always associated with some inductance and capacitance.

So for this example right hand side, we are using a corner bend and this bend it is associated with some inductance and capacitor, usually it is represented by a low pass T network to series inductors and 1 shunt capacitor, so we have to nullify the effect and for that we need some sort of reactive compensation. You can use any full wave simulator to determine the exact dimensions of this reactive compensation; you can see for this bend intentionally the characteristic impedance of this bend section is quite high to nullify that low pass filtering effect. And not only that, this bend can generate higher (())(8:45) modes, for CPW it can generate even mode. So again we are using wire or wires, you can see in right hand side photograph, these black lines they are showing the wire bonding to keep ground plane at same potential.

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Wilkinson Power Divider

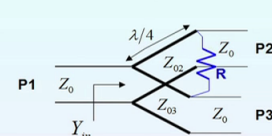
Unequal power division:

If the power ratio between the ports $K^2 = P_3/P_2$,
Then, the design equations are

$$Z_{03} = Z_0 \sqrt{\frac{1+K^2}{K^2}},$$


$$Z_{02} = Z_0 \sqrt{K(1+K^2)},$$

$$R = Z_0 \left(K + \frac{1}{K} \right).$$




Transmission line circuit.

Multi-port power divider:



A 8-way Wilkinson power divider.

Wideband power divider:



A wideband Wilkinson power divider.

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Similarly we can also design Wilkinson power divider on silicon. Now, on Wilkinson power divider unequal power division that is also possible, in that case we have to use this design equation. For example, if the power ratio between the ports P3 and P2 they are given K square equal to P3 by P2, then the design equation for Z 03 and Z 02 they are given by these 2 equations and the required resistance in this case it is not 100 ohms, but given by R equal to Z 0 into K + 1 K. Here is one example of 1 is to 8 Wilkinson power divider, if you look inside it comes with PCB form with packaging. This is the input port, it is 1st divided into 2 ports using one single Wilkinson power divider, then again it is divided into 2 and finally we have 8 output ports.

We can also design wide band Wilkinson power divider, this is a very standard technique for bandwidth enhancement of any couplers and power divider that you use multiple of the

components, but obviously in that case design equations will be different. So here is an example, it shows a 3 section wideband Wilkinson power divider, so you can see it is in micro strip line technology and the thickness of these strips of the different section they are different.

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Millimeter-Wave Interconnects

Simulation of a Wilkinson power divider (S_0 substrate)

Power divider loss - 0.4 dB@94 GHz

K. J. Herrick, L. Katehi and R. T. Kihm "Interconnects for a multi-layer three dimensional architecture", Microwave Journal May, 2001.

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This is one example of Wilkinson power dividers at 94 gigahertz designed on silicon, so we have many reactive compensations along with the wire bonding and this black dot, it represents the resistor. Usually we use thin film nickel chromium resistor at high frequency millimetre wave frequency, so this is a typically shows a loss of 0.4 dB, which is considered a very good value at 94 gigahertz. So loss of 0.4 that means instead of S_{21} equal to - 3, we are having S_{21} equal to - 3.4 dB, S_{31} equal to - 3.4 dB.

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Passive Components: Image Guide

Symmetric parallel-coupled guides:

Image guide Even mode excitation Odd mode excitation

• For a lossless coupled lines matched at four ports, the voltages on the lines are

$$V_A(z) = (V_0/2) [\exp(-j\beta_e z) + \exp(-j\beta_o z)]$$

$$V_B(z) = (V_0/2) [\exp(-j\beta_e z) - \exp(-j\beta_o z)]$$

Even Odd
 $V_0/2$ $-V_0/2$
 $V_0/2$ $V_0/2$

Z = 0 Z = l

P4 B P3
 β_e, β_o
 P1 A P2

Coupled IG

Millimeter wave and optical dielectric integrated guides and circuits, S.K. Koul, Wiley. 221
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So next we go to couplers, we can design couplers in micro strip line technology as well as using any other wave guiding system like rectangular waveguide, image guide, in all the guiding systems we can design couplers. Let me give the 1st example in image guide where we will consider 2 coupled image guide, they are parallel to each other as shown in this figure. So if you recall, we have TM to y and TE to y mode and just like microstrip line, we can divide for our analysis we can consider the even mode excitation and the odd mode excitation. In even mode excitation this is the plot of E_y along x, so E_y it is maximum on inside the image guide and it is having minimum value in between two. So if you remember for the microstrip line case, we can use one PMC between these two coupled lines, here also almost similar thing.

And for the E_y creation in odd mode, they have positive and negative value on left hand and right-hand side, so coupling can be controlled by controlling tuning the gap between 2 guides as well as length of this coupled line. So this is showing a typical top view of a directional coupler using image guide.

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Image Guide Coupler

IG coupler with straight bends.

- At Z_2 coupling reduces to a negligibly small value.

IG coupler with curved bends.

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Now for practical realisation as I said, whenever we design any bend we have to be very careful, each and every bend it is associated with some reactance and at lower microwave frequencies usually we do not consider its effect is negligibly small, but if we go for high-frequency design typically above 60 gigahertz we have to consider their effect. So you can use any full wave simulator to characterise and to minimise this reactive loading effect, for this example we are showing different types of termination, so from this coupled section how we are dividing power among this port P1 to P4. For the 1st one we are using the straight Junction, for the 2nd one we are using a rounded Junction, the rounded Junction it shows somewhat improvement in terms of bandwidth.

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Image Guide Coupler

Overall S-parameters in terms of effective length l_e ,

$$|S_{12}| = |\cos(\pi \ell_f / 2L)|$$

$$|S_{13}| = |\sin(\pi \ell_f / 2L)|$$

where $L = \frac{\pi}{\beta_e - \beta_o}$

$$\ell_f = \ell_o + [2/(\beta_e - \beta_o)] \Delta\phi = \ell_o + (2L/\pi) \Delta\phi$$

where $\Delta\phi = \int_{z_1}^{z_2} [\beta_e(z) - \beta_o(z)] dz$

l_o is the length of straight coupled section, $(2L/\pi)\Delta\phi$ is the correction factor due to connecting arms.

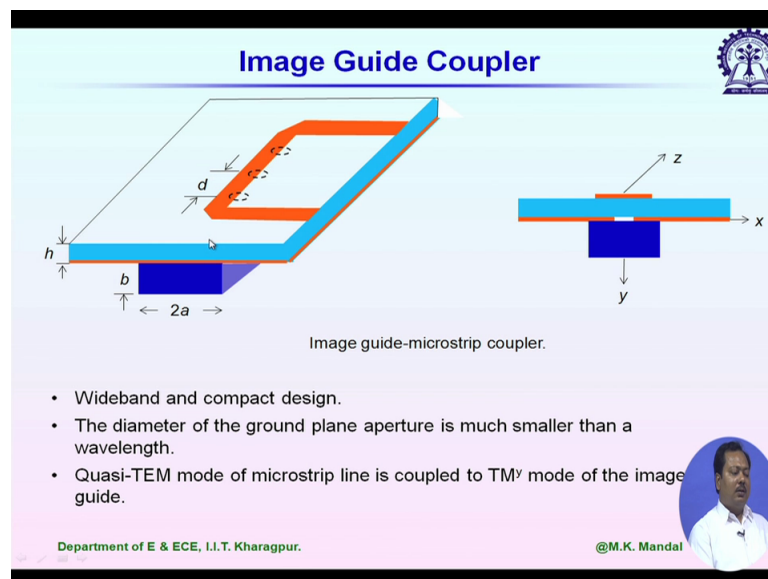
- The bandwidth of the coupler, usually, is very small $\sim 3\%$.
- Wideband coupler can be designed if $(\beta_e - \beta_o)$ is independent of frequency (using an inset).
- Effective coupling length decreases for a broadside coupled structure.

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Then the overall S_{21} and S_{31} can be represented in co-sinusoidal and sinusoidal form, it is $\cos(\beta_e l)$ by twice L , where capital L it is equal to $\frac{\pi}{\beta_e - \beta_0}$, so β_e and β_0 , they represent the phase constant for even mode and odd mode excitation and l , this is the effective coupled length and this is $l_0 +$ some we have correction factor. So this correction factor is because of this bend actually, so l_0 became the basis you can see this is the straight path straight coupled path and in addition to that due to this bending we have to add some correction factor, so this is twice L by π into $\Delta\phi$, where $\Delta\phi$ this is indication $\int_1^2 \frac{\beta_e - \beta_0}{dz}$.

But the problem of this coupler that the bandwidth is very small typically 3 percent or even then that, but wideband coupler multi-section and using some sort of inset, wideband design is possible and the effective coupling length decreases for a broadside coupled structure. Here, we are using side-by-side coupled structure, so if one is above another one in that case we have higher coupling level.

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I am showing you another example, here Port 1 and Port 2 they are in microstrip line technology whereas, Port 3 and Port 4 they are in image guide technology. So this particular coupler involves 2 different types of transmission line system wave guiding system and now the coupling is being controlled by some slots in the Common ground plane. So on top we have microstrip line and below the ground plane we have the image guide and now by controlling the diameter of the slot and the position of the slot we can control the coupling level, so it is usually very wideband and compact. Quasi-TEM mode of microstrip line is coupled to TM_y mode of the image guide. So for TM_y mode if you remember so for a

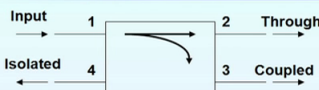
TM to y we have the highest UI component which is perpendicular to this ground on near the ground and also we have a company along Z inside the image guide which has highest value on dielectric air interface.

So from microstrip line, mainly from this perpendicular electric field to this perpendicular UI component, this power is coupled through this slot in between this image guide and the dielectric of the microstrip line.

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Directional Couplers





Power flow in a directional coupler.

To characterize a directional coupler, define

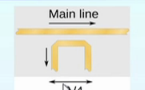
Coupling $C = 10 \log \frac{P_1}{P_3} \text{ dB}$,

Directivity $D = 10 \log \frac{P_3}{P_4} \text{ dB}$,

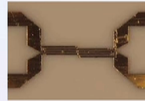
Isolation $I = 10 \log \frac{P_1}{P_4} \text{ dB}$.

For a loss less coupler, $I = D + C \text{ dB}$.


Ideal coupler : $S_{11} = S_{22} = S_{33} = S_{44} = 0$.



Microstrip line Directional coupler



Lange coupler



Directional coupler in waveguide technology.

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Now, direction coupler in microstrip line technology so in this example, in the 1st example we are using simply a coupled microstrip line. So in this figure right-hand you can see this is the mainline, with this we have the coupled microstrip line here. In general, direction coupler it is a 4 port network, Port 1 shows the input port, Port 2 we call it through port, Port 3 is the coupled port and Port 4 is the isolated port. So if we feed at Port 1, most of the power of it will appear at the through and a fraction of the power it will appear at the coupled port. Usually, the phase difference between the through and coupled port is 90 degree. And not only that, we have some other types of couplers for directional couplers the power coupled to coupled port is quite small, sometimes it is 10dB or 20dB down in comparison to through port.

And they are typically used to monitor the power level, let us say you are transmitting through one antenna and you want to monitor the power level, in that case we can use 1 directional coupler and they also have some other uses. Some important parameters like coupling C, we define coupling C as 10 Log P 1 by P 3 in dB, where P 1 this is the incident

power, P_3 this is the coupled power available at Port P3. Similarly, we can define directivity so isolated how isolate it is with respect to the power coupled to couple port, so it is defined by $10 \log \frac{P_3}{P_4}$ in dB and then the isolation $10 \log \frac{P_1}{P_4}$ in dB, for a lossless system it can be shown that I is equal to $D + C$ in dB that means the isolation in dB scale that is equal to directivity + coupling. And now for ideal coupler then, all these ports they should be properly matched or the input reflection, it should be 0, so S_{11} , S_{22} , S_{33} , S_{44} , all should be 0.

Not only that, for ideal isolation again S_{41} it also should be 0. So not only in microstrip line technology already we have seen that it can designed in image guide, it also can be designed in NRD, it also can be designed in rectangular waveguide technology. So for this particular example it is showing the 3 ports and the isolated ports actually terminated inside and from outside you can only see 3 ports of this 4-port directional coupler in waveguide technology. Now, in case if we need higher coupling level, so as I said that usually for microstrip line or other directional coupler, power available at coupled port it is quite small typically 10 or 20dB down compared to through port. Now in case let us say we need equal power rating between the coupled port and the through port.

So what determines this coupling level? For microstrip line, that gap between 2 lines it determines the coupling level, if we decrease the gap then couple power will increase. Now for equal power spitting gap requirement is so small that simply cannot be fabricated. So in that case we have an alternative that is called Lange coupler so instead of one section multiple sections are used as shown here and then they are wire bonded so that the effective coupling between 2 lines increases, so this figure shows a typical Lange coupler to increase the coupling level.

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Hybrid Couplers

Conventional branch-line coupler: A square structure with four ports labeled P1, P2, P3, and P4. The top and bottom horizontal arms have characteristic impedance $Z_0/\sqrt{2}$. The left and right vertical arms have characteristic impedance Z_0 . The length of each arm is $\lambda/4$.

Conventional rat-race coupler: A circular structure with four ports labeled P1, P2, P3, and P4. The outer circumference has characteristic impedance $\sqrt{2}Z_0$. The inner circumference has characteristic impedance Z_0 . The length of each arc between adjacent ports is $\lambda/4$.

Analysis procedure:

- Use even-odd mode analysis procedure for four port networks.
- Obtain the overall ABCD matrix in even- and odd-mode excitations.
- Obtain the even- and odd-mode reflection and transmission coefficients: $\Gamma_e, \Gamma_o, T_e, T_o$.
- Using formula, convert them to S-parameters.
- Apply the conditions (input matching, coupling, isolation and phase relationship) to obtain the impedance and length values.


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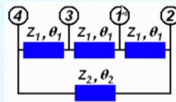
Next is hybrid coupler, so in the previous example typically they are used for very low coupled power. Now, for equal power spitting we prefer hybrid couplers and hybrid couplers they do not involve any usually we use properties of transmission lines to obtain that coupling criteria and in addition to that we can add some phase requirements. For example, the output power between the output through port and couple port let us say we need a phase difference sometimes 0 degree, sometimes let us say we need 90 degree, sometimes let us say we need 180 degree depending on application and we can design all of them by using hybrid couplers. So the 1st example is a branch line coupler as we call it, you can see again they are 4 port couplers; Port P1, this is the input port, Port P2 and P3 they are the output ports, P4 this is the isolated port.

And for conventional design, power incident at Port P1 it will be equally divided between port P2 and Port P3 and there will be phase difference of 90 degree between port P2 and Port P3. If we have equal power splitting, in that case if we solve using the required criteria of s parameters, in that case it can be shown the length of each arm at resonant frequency it should be $\lambda_g/4$, where λ_g is the guided wavelength and not only that, the characteristic impedance of this top and bottom line that should be $Z_0/\sqrt{2}$ this is for equal power split. So for Z_0 equal to 50 ohms then should be 35.35 ohms and left and right branch they should be of 50 ohms Z_0 . So if I feed a power at Port P1 then $\angle S_{21}$ it is actually - 90 and angle of S_{31} that is - 180 and overall we have a phase difference of 90, so we have 90 degree phase difference between Port 2 port P3.

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Rat Race Coupler (RRC)



Configuration of a reduced length RRC.


$$[S] = -\frac{j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Characteristics of a RRC:

- (i). A signal applied to port 1 will be evenly split into two in-phase components at ports 2 and 3, and port 4 will be isolated.
- (ii). If the input is applied to port 4, it equally splits into two components with a 180 deg phase difference at port 2 and 3, and port 1 will be isolated.
- (iii). When operated as a combiner, with input signals applied at ports 2 and 3, the sum of the inputs will be formed at port 1, while the difference will be formed at port 4.

M. K. Mandal and S. Sanyal, Reduced length rat-race coupler, *IEEE T-MTT*, Dec. 2007.

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Next example is Rat race couple of, rat race coupler is very useful in radar applications. This has interesting features this one, so if you feed at Port P1 you see port trajectory is little different than the branch line coupler because of its property. If I feed at Port P1 then power is divided equally between port P2 and Port P3 and Port P4 will be isolated, so the farthest port is isolated. And not only that, if you use Port P1 as the incident port then port P2 and P3 they will be in some phase, so angle of S₂₁ equal to angle of S₃₁. Now, if I use Port 4 as the incident port, in that case again power will be divided between Port 3 and Port 2, but this time there will be a phase difference that is - 180. So Port 1 and Port 4 anyone of them can be used as the input port, but we have 2 different phase relationships.

Now, if we use the output ports for example, Port 2 and Port 3 as the input ports together, so we have led us say a power source and we are feeding to Port 2 and Port 3 then their addition will appear at or submission will appear at Port P1 so that is why sometimes we call this one as the sum port and their difference, it will appear at Port P4 so we call it sometimes the difference port or Delta Port. So that means if we have 2 signals, we can simply do their addition and subtraction by using a Rat race coupler. So this is the S matrix for one ideal lossless rat race coupler, ideally all 4 ports should be matched, so S₁₁, S₂₂, S₃₃, S₄₄, all the diagonal elements are 0 and you can say S₁₂ and S₁₃ are 1 multiplied by - J by root 2, so power is equally divided with the same phase.

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Hybrid Couplers

Conventional branch-line coupler.

Conventional rat-race coupler.

Analysis procedure:

- Use even-odd mode analysis procedure for four port networks.
- Obtain the overall ABCD matrix in even- and odd-mode excitations.
- Obtain the even- and odd-mode reflection and transmission coefficients: $\Gamma_e, \Gamma_o, T_e, T_o$.
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- Apply the conditions (input matching, coupling, isolation and phase relationship) to obtain the impedance and length values.

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Now you consider for example a Port 2 and Port 3, we have - 1 and 1 and multiplied by - J by root 2 and magnitude is same, but phase they are out of phase. Now in this conventional design I have discussed about equal power split, unequal power split also possible so in that case we have to derive the corresponding equation and then the impedance requirement will be different. For a conventional rat race couple if we do analysis for equal powers split, then the arm length it is $\lambda/4$ this longest arm length this is $3\lambda/4$, so altogether it has a circumference length of $3\lambda/4 + 3\lambda/4 = 3\lambda/2$ for this bottom part and the characteristic impedance of this line is uniform throughout and it is given by $\sqrt{2} Z_0$, so it is 70.71 ohms, this is using transmission line theory.

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Analysis of the Rat-Race Coupler

RRC configuration

Even-mode circuit.

Odd-mode circuit.

A section of a transmission line

$$\begin{bmatrix} \cos \theta & jZ_0 \sin \theta \\ jY_0 \sin \theta & \cos \theta \end{bmatrix}$$

A stub as shunt admittance.

$$Y = -jY_0 \cot \theta \quad (\text{short circuited stub})$$

$$= jY_0 \tan \theta \quad (\text{open circuited stub})$$

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Now, how we do analysis of it we simply use even-odd mode analyses technique, so I will skip the analyses technique just I am showing the 1st step, what we have to find out? We have to find out the even mode and odd mode equivalent circuit of it, and then we obtain the ABCD matrix equivalent ABCD matrix in even mode condition and a odd mode condition. So if you look at the circuit, we can place the symmetrical we have to find out a plane of symmetry first, so the symmetrical plane bisects the coupler and Port 1 and 2 will be at one side, 3 and 4 will be at another side. So in even mode condition this plane will be replaced by PMC or open circuit condition, in odd mode condition this plane will be replaced by a PEC or short-circuit condition, then we have the corresponding equivalent circuit as shown here.

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Analysis of the Rat Race Coupler

$$\Gamma_e = \frac{A_e + B_e/Z_0 - C_e Z_0 - D_e}{A_e + B_e/Z_0 + C_e Z_0 + D_e} \quad T_e = \frac{2}{A_e + B_e/Z_0 + C_e Z_0 + D_e}$$

Assuming, $Z_1 = Z_2 = Z$ and the matching, isolation and phase conditions result


$$\tan \frac{\theta_1}{2} \tan \frac{\theta_2}{2} = -1,$$

$$Z = Z_0 \sqrt{\left\{ 3 - \frac{1}{2} \left(\cot^2 \frac{\theta_1}{2} - \tan^2 \frac{\theta_1}{2} \right) \right\}}$$

The S-parameters are:

$S_{21} = 1/X$	Where,
$S_{31} = 1/X$	
$S_{24} = -1/X$	
$S_{34} = 1/X$	

$$X = \left[2 \cos \theta_1 + j \frac{1}{2} \sin \theta_1 \left(\frac{Z}{Z_0} + \frac{2Z_0}{Z} \right) - j \frac{Z_0}{Z} \cos \theta_1 \cot \theta_1 \right]$$

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Now, it looks like a bird transmission line and 2 point circuit stub, right-hand side 2 short-circuit stub. We can obtain theoretically, what is the S₂₁, S₁₁ for this given two port network, so by using even mode and odd mode equivalent circuit we can actually simplify a 4 port network to 2 port system then we do the analysis, find out the ABCD parameters in even mode and odd mode, then find out what is the overall reflection coefficient, transmission coefficient both in even mode condition and odd mode condition, then we impose the rat race condition that means this matrix basically, S₁₁ should be 0, S₂₂ should be 0 and all this equal power splitting condition, phase difference condition, if we impose those conditions then finally we have the design equations.

So we have basically 2 equations and this is the general solution and that means here we assume that Theta 1 can be arbitrary anything, not just 90 degree and 90 degree is a special case from... This equation if you put Theta 1 equal to Theta 2 equal to 90 degree, in that

case you will find out that Z equal to 70.71 for a 50 ohms system. So we will take a break then we will discuss about termination and transition, thank you.