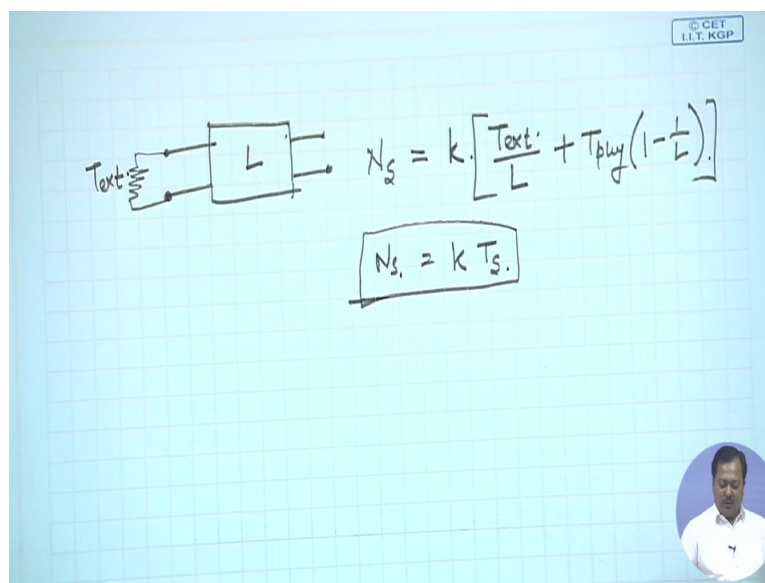


Millimeter Wave Technology
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Module 8
Lecture No 37
Millimeter Wave Systems (Contd)

So, now we are considering some external noise which is incident from left-hand side and altogether we want to express the system by some equivalent noise temperature, then how to calculate it?

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So we have a 2 port network which is providing some internal noise and it is also connected to some external source of noise, which can be given by $T_{external}$. So in that case then the total noise power at the output N_s , this is equal to $k T_{external}$ by L , this is for the contribution from left-hand side + $T_{physical}$ into $1 - L$, whatever we have seen we can represent it by some effective noise temperature T_s , $k T_s$, so N_s this is equal to $k T_s$.

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Antenna and Source Noise

Circuit generated internal noise,
 $N_{int} = kT_{phy}(1-1/L)$, L is the loss (ratio).

Let this noise power is generated by a resistor connected at the source. Then the corresponding equivalent input noise temperature,

$T_e = LN_{int}/k = T_{phy}(L-1)$

Example:
 Let a 3dB ($L = 1.995$) matched attenuator at 290 K is connected to a RF source. Then, T_e that must be added to the source equivalent noise temperature is 288.5 K.

If T_{ext} is equivalent noise temperature of the source, the total noise power at the output is,
 $N_s = k[T_{ext}/L + T_{phy}(1-1/L)] = kT_s$.

For a transmitter, $L=1/\rho_a$, T_{phy} is the antenna temperature and T_{ext} is the noise temperature of the transmitter just before antenna.

Available power for a noisy system (T_{phy} is the physical temp. and not equivalent noise temp.).

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Now consider one antenna, so here you can see here so N_s it is equal to k into T_s . Now consider 1 antenna which is connected to a transmitter, now antenna it will have some efficiency. Let us say antenna efficiency is given as 90%, then and if any power we feed to antenna it will be attenuated by antenna and this value it can be given by that 90%, if we convert it to fraction 0.9. Or we can say L loss due to antenna; this is 1 by ρ_a or 1 by 0.9. Then T_{phy} is the antenna temperature let us say, so this is the physical temperature of the antenna and T_{ext} is the noise temperature of the transmitter just before antenna. So in that case we can calculate, then what is noise power available from this antenna and source, which can be given by $k T_s$.

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Receiver Noise Temperature

A noisy network with source and load (G_a is the available power gain).

The rms voltage generated by a noisy resistor R_s (Rayleigh-Jeans approximation),
 $e_s = \sqrt{4kTR}$.

So, maximum noise power delivered to a matched load is $P_i = \left(\frac{e_s}{2R_s}\right)^2 R_s = \frac{e_s^2}{4R_s} = kT$

Gain of the network, $G_a = \frac{e_o^2/4R_{out}}{e_s^2/4R_s}$, depends on source resistor but not on R_L .

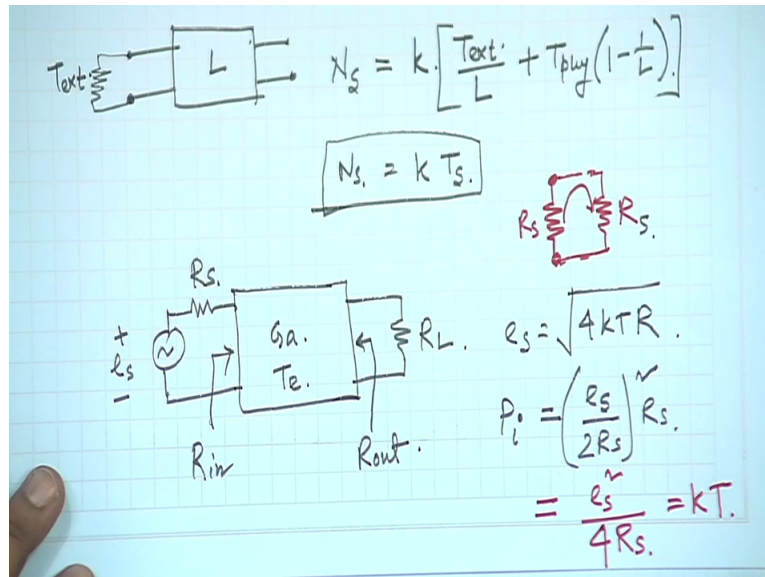
So, the output noise power from the device, $N_o = G_a k T_s + N_{int} = G_a k (T_s + T_e) = G_a k T_o$.

T_o is the operating noise temperature.
 Note that T_o and T_e (G_a) both depend on R_s .

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Let us consider a receiver now and we are going to calculate the overall receiver noise temperature. So what we will be doing, let us first consider one amplifier, a 2 port network again.

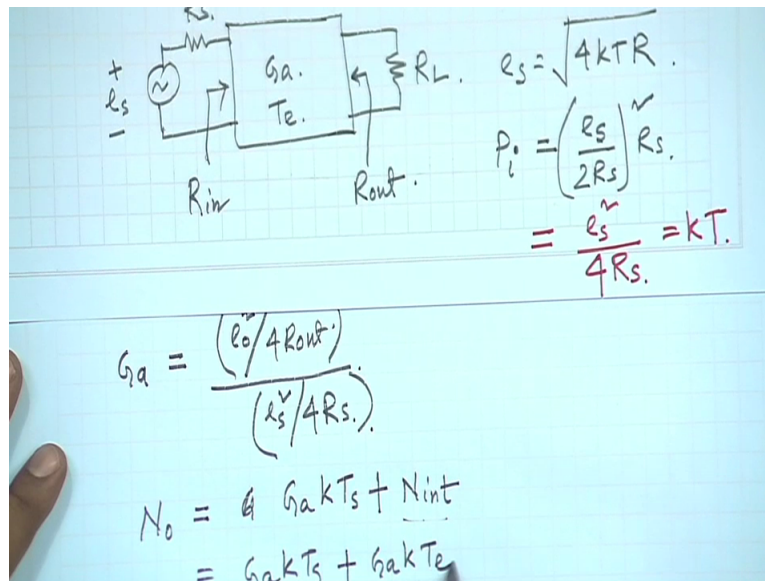
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So the amplifier amplification factor is given by G_a , it is sometimes called the available gain and T_e is its effective noise temperature. Now it is connected to some external resistance, which is given by R_L at right-hand side and left-hand side it is connected to a source of internal resistance R_s . And if the source power is 0, in that case the power which is incident from left-hand side, it is only the noise power. Also, the input and output resistances they are also given for the amplifier. Now the noise voltage available because of the noise power generated by R_s that can be given by $e_s = \text{square root of } 4kTR$, so maximum noise power delivered to a mesh load, we are considering one mesh scenario that is P_i this is equal to e_s by twice R_s whole square into R_s .

So you see here what we are considering, so R_s this is the noise source, I want to extract noise power from the source or we can see in other way. We are connecting a resistor here, so some noise power will be delivered to the resistor, then we are considering what is the maximum noise power that is being delivered to this resistor and that happens according to maximum power transfer theorem when this R this is equal to R_s or we can write down this is equal to R_s . So in that case if I want to calculate P_i , this is simply I^2 into R , then the current component went through it, this is e_s by twice R_s whole square multiplied by R_s or we can write down this is equal to e_s^2 divided by, $1 R_s$ is cancelled out so $4 R_s$ and this is equal to kT , which already we have seen.

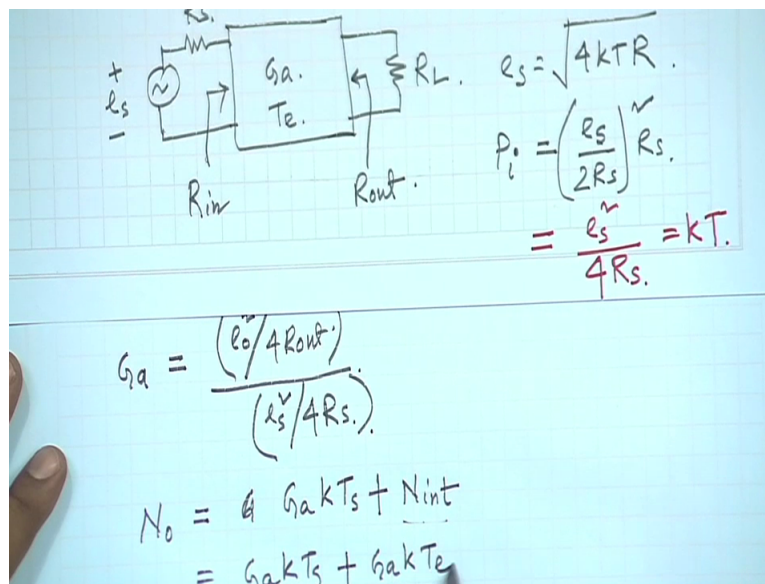
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Now, gain of the amplifier G_a we can express in terms of the input noise power and output noise power. So output noise power is e_o^2 divided by 4 into R_{out} and the input noise power this is e_s^2 divided by 4 into R_s . So one interesting thing then Gain you see for C amplifier also if we want what to calculate the overall gain of any C amplifier it depends on R_L , but simply if I say what is the gain of a C amplifier, we do not consider R_L , we take just the open circuit at output voltage and then calculate what is the gain of the amplifier. Now if you change R_L , obviously the available power to R_L it will change, but when we call the G_a or gain of the amplifier then we do not consider R_L , but this gain it depends on the source resistor R_s .


So here also then this G_a it depends on R_s , but not on R_L . So output noise power if I want to calculate at output terminal let us say, we are denoting it by N_o so that is equal to $G_a k T_s$, so $k T_s$ this is the incident power from left-hand side, which is being multiplied by gain of the amplifier G_a . In addition to this, the amplifier itself it will produce some noise, which is given by N_{int} . N_{int} this factor we can also express in terms of $T_{effective}$ noise temperature, which is given T_e , so we can also write down that this is equal to $G_a k T_s + N_{int}$ this is equal to $G_a k T_s + G_a k T_e$ or this is equal to $G_a k T_s + T_e$.

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Let us represent it by effective temperature $G a k T_0$, so T_0 this is called the operating temperature operating noise temperature. So you see here, so this is the amplifier and left-hand side we have a noise source connected to this amplifier and right-hand side we have a load resistor connected here, then we calculate it we expressed already a $G a$ in terms of the input and output noise power and from that we calculate that we calculated the output noise power from the device, so output noise power what we see that it contains the contribution due to the external noise source, which is $G a k T_s$. In addition to that, this amplifier itself produce some noise, which is given $G a k T_e$, so all together this effect we can represent by some operating noise temperature T_0 , so N_0 then that is equal to $G a k T_0$, So T_0 , T_e , $G a$ all functions of R_s , but they do not depend on R_L .

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Receiver Noise

For a three element cascaded section, total excess output power,
 $N_{int} = G_{a3}G_{a2}G_{a1}kT_{e1} + G_{a3}G_{a2}kT_{e2} + G_{a3}kT_{e3}$.

$\therefore T_e = N_{int} / kG_{a3}G_{a2}G_{a1}$

$= T_{e1} + T_{e2}/G_{a1} + T_{e3}/G_{a1}G_{a2}$. (Friis law for cascaded system)

Now, let a receiver is modeled by a constant gain G_o over a band B_n . The receiver input power is P_s , receives a noise P_n from the antenna. Then considering matched scenario,

$$SNR_o = \frac{G_o P_s}{G_o P_n + B_n N_{int}} = \frac{SNR_i}{1 + B_n N_{int} / G_o P_n}$$
 Radar equation for the receiver.

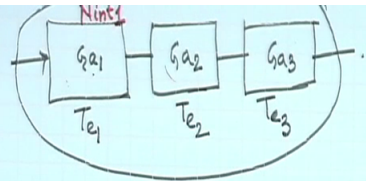
Considering available power gain G_a and the above definition,

$$SNR_o = \frac{G_o P_s}{G_o k T_e B_n} = \frac{P_s}{k T_e B_n}$$
 (in terms of operating temperature)

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And in this case also we assume that the output R 1 is mesh to this and so it is the worst-case scenario, this N 0 this is the highest available noise power from the amplifier on the noisy source. Now let us consider several 2 port components they are connected in Cascade, so that means what we are considering let us say we have 3 amplifiers; the first one gain is given by $G a 1$ and the effective noise temperature is given by $T e 1$.

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$$N_{int} = G_{a3}G_{a2}G_{a1}kT_{e1} + G_{a3}G_{a2}kT_{e2} + G_{a3}kT_{e3}$$

$$= kT_e G_{a3}G_{a2}G_{a1}$$

$$\therefore T_e = \frac{N_{int}}{kG_{a3}G_{a2}G_{a1}}$$

$$= T_{e1} + \frac{T_{e2}}{G_{a1}} + \frac{T_{e3}}{G_{a1}G_{a2}} \dots$$

It is followed by another amplifier for which gain is $G a 2$, effective temperature is $T e 2$. Again we have one more $G e 3$, effective noise temperature $T e 3$ and we want to calculate what is the overall noise contribution of this system, so if we know for 3 we can do it for N components in Cascade. Then N internal we can write down this is equal to, so you see this

first amplifier whatever noise it will produce let us say that is given by N_{int1} , so this N_{int1} it will be amplified by G_{a2} , further it will be amplified by G_{a3} , but whatever noise produced by the third component this component is not being amplified by left-hand that is G_{a1} or G_{a2} . Then the total noise contribution N_{int} ; for the first one this is $G_{a3}G_{a2}N_{int1}$, $G_{a2}N_{int2}$, N_{int3} .

So N_{int1} this is the noise produced by the first one and it is being amplified by all these 3 amplifiers. Then for the second one $G_{a3}G_{a2}N_{int2}$ and for the third one this is $G_{a3}N_{int3}$. So if we represent it this overall contribution, if we represent it by some effective temperature let us say T_e , in that case we can write down this is equal to kT_e . T_e is the overall equivalent noise temperature, then kT_e into $G_{a3}G_{a2}G_{a1}$, so therefore T_e this is equal to $N_{int} / (kG_{a3}G_{a2}G_{a1})$. So what we are assuming, as if one resistor of equivalent noise temperature T_e is connected to left of this system and system itself is noise less so that is why we are multiplying by 3 gain factors G_{a3} , G_{a2} and G_{a1} .

So in terms of the individual noise temperature we can also express it, so you just put that value of N_{int} whatever we obtained in this first equation. So if we put this value here, so it becomes $T_e1 + T_e2$ by so remaining term is $G_{a1} + T_e3$ remaining terms $G_{a1}G_{a2}$. For N components if we have N number of components simply we can extend it, so it will become for N th term $T_e n$ and below we have $G_{a1}G_{a2}$ to till $G_{a(N-1)}$, so this is called the Friis law for cascaded system.

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Receiver Noise

For a three element cascaded section, total excess output power,

$$N_{int} = G_{a3}G_{a2}G_{a1}kT_{e1} + G_{a3}G_{a2}kT_{e2} + G_{a3}kT_{e3}.$$

$$\therefore T_e = N_{int} / kG_{a3}G_{a2}G_{a1}$$

$$= T_{e1} + T_{e2}/G_{a1} + T_{e3}/G_{a1}G_{a2}; \quad (\text{Friis law for cascaded system})$$

Now, let a receiver is modeled by a constant gain G_o over a band B_n . The receiver input power is P_s , receives a noise P_n from the antenna. Then considering matched scenario,

$$SNR_o = \frac{G_o P_s}{G_o P_n + B_n N_{int}} = \frac{SNR_i}{1 + B_n N_{int} / G_o P_n}. \quad \text{Radar equation for the receiver.}$$

Considering available power gain G_a and the above definition,

$$SNR_o = \frac{G_a P_s}{G_o k T_e B_n} = \frac{P_s}{k T_e B_n}. \quad (\text{in terms of operating temperature})$$

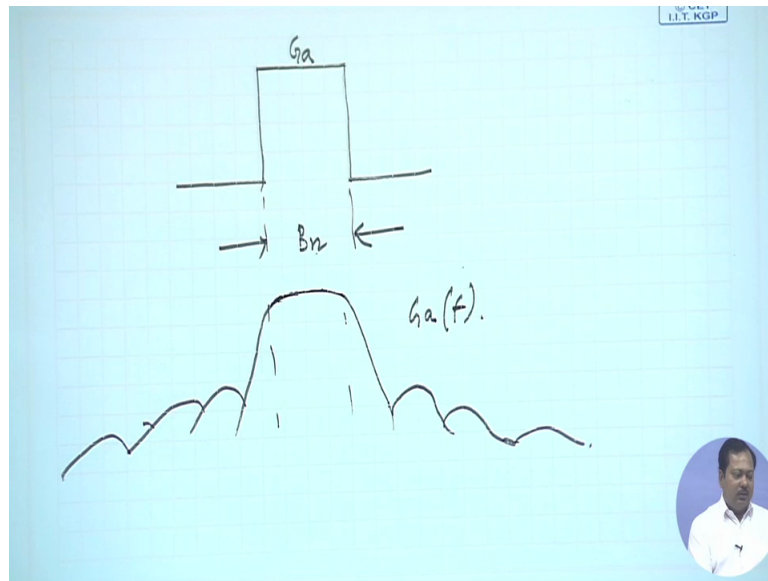
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So what we see that if we have many Cascade connections, the contribution of noise from the first component it is dominant over all others. That is why when we design any receiver, the first component is chosen which will have minimum contribution of noise, so that is why we use low noise amplifier typically, which will provide very low noise and after that whatever components we are having their contribution is smaller compared to the first component. So now let us consider a receiver, it is modelled by a constant gain G_0 over a band B_n , we are introducing now some bandwidth given by B_n . Then the receiver input power is given by P_s and it receives a noise power P_n from the antenna, then considering matched scenario so always by default we will be considering matched scenario, so whatever power is coming from left-hand side so it is being observed right-hand side half of that observed in the load.

So then SNR at the output of the receiver that is equal to you see this is G_0 SNR is signal-to-noise ratio, so signal at left-hand side is P_s it is being multiplied by G_0 , so at output available signal power is G_0 into P_s . And what is the output noise power, so left-hand side we have receive P_n noise, it is being amplified by G_0 times by the receiver chain + we have some internal noise contribution because of the receiver cover bandwidth B_n , B_n multiplied by n internal and it can be given by equal to SNR at the input divided by $1 + B_n N_{\text{internal}}$ divided by $G_0 P_n$. How we come here, simply you divide both numerator and denominator by $G_0 P_n$, so then the top one it becomes P_s by P_n , which is SNR at the input.

So this equation is called Radar equation for receiver, it is one important parameter. Now considering available power gain G_a and this above definition, we can also express SNR at the output, so in terms of operating temperature this is equal to G_a into P_s divided by $G_a k T_0 B_n$, so this is the operating temperature and simply then this is equal to P_s by $k T_0 B_n$, so you remember recall this T_0 is the operating temperature. Now noise bandwidth, so what if we are many times calling noise bandwidth B_n and we are using it to calculate overall noise power, then what value should we take for B_n for any given channel. Let us say we are using a channel bandwidth 1 gigahertz so should we take simply $B_n = 1$ gigahertz.


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So whatever definition the significance of B_n till now we have used, it is like one simple window function which has some value over the given bandwidth and out of this band it has 0 value; but it is not the practical scenario. So let us consider any practical case so in this expression when we use some value of B_n , so that means let us say we are considering the ideal scenario, this is the 1 gigahertz channel bandwidth it is given B_n and then model it by a constant gain G_a , when we talk about the gain of the amplifier or gain of the receiver G_0 whatever and out of this band this G_a is exactly 0. But for any practical system consider any band pass filter or anything, so for that it is not constant but it becomes a function of frequency and we have contribution due to you see other frequency components, so it will vary.

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Noise Bandwidth



In the noise power $G_a k T_0 B_n$, B_n is the equivalent noise bandwidth considering a rectangular band of constant gain G_a and a constant temperature T_0 .


A real system is defined by its transfer function $H(j\omega)$ and $T_0(f)$.

Actual output noise power,

$$P_n = \int_0^{\infty} |H(j\omega)|^2 k T_0(f) df.$$

Let the T_0 is constant over the band and mid-band gain, $G_a = |H(j\omega_0)|^2$.

$$B_n = P_n / k T_0 G_a = \int_0^{\infty} |H(j\omega)|^2 df / |H(j\omega_0)|^2.$$




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So gain G_a it is a function of frequency and bandwidth B_n , then what value we should take for this type of practical scenario, so what we do here there is a way of calculation. For a real system obviously it will be defined by its transfer function H , which is a function of $j\omega$ and the corresponding temperature, which previously we consider constant over the frequency, but practically it should be a function of frequency. Then actual output noise power we should consider all the frequency bands starting from 0 to infinity and we integrate it over 0 to infinity $|H(j\omega)|^2 k T_0 df$. Now if T_0 is constant over the band and let us say mid-band gain is given by $G_a = |H(j\omega_0)|^2$ in that case so we are assuming T_0 is constant otherwise, it is becoming complicated, for simplification this assumption.

So B_n this is equal to then total noise power P_n divided by $k T_0 G_a$ or 0 to infinity this integration value divided by $|H(j\omega_0)|^2$. So from this expression we can calculate B_n value, but even then you see here we consider T_0 is constant over this given bandwidth. Next noise factor, so total noise contribution it can be calculated in terms of noise temperature, also sometimes another parameter is used to quantify the noise contribution that is called noise figure, we can calculate it for any given 2 port devices, so usually if it is a lossy device with some insertion loss let us say L in dB, and then noise figure in dB it simply becomes L . If you buy any active component like an amplifier or some active mixer, usually the noise figure value will be written or given by the manufacturer, it can be also measured in lab experiment by using spectrum analyser and standard noise source values or VNA Vector Network Analyser.

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Receiver Noise Factor

Noise factor of the receiver is,

$$F = \frac{SNR_i}{SNR_o} = \frac{P_s}{N_i} \frac{G_a N_i + N_{int}}{G_a P_s} = \frac{G_a k T_0 + N_{int}}{G_a k T_0}$$

Considering the source temperature = 290 K.

$$= 1 + \frac{G_a k T_e}{G_a k T_0} = 1 + \frac{T_e}{T_0}$$

N_i is the incident noise power = $k T_0$
 N_{int} is internally generated noise power = $k T_e$

$$\Rightarrow T_e = (F - 1) T_0.$$

Noise factor depends on source impedance and operating frequency.
 Noise factor of a lossy element is $F_L = L$.

$\therefore SNR_o = P_s / \{k [T_s + (F - 1) T_0] B_n\}$ Total noise when the source is at T_s .
 $= P_s / \{k T_0 F B_n\}$, Only when $T_s = T_0$.

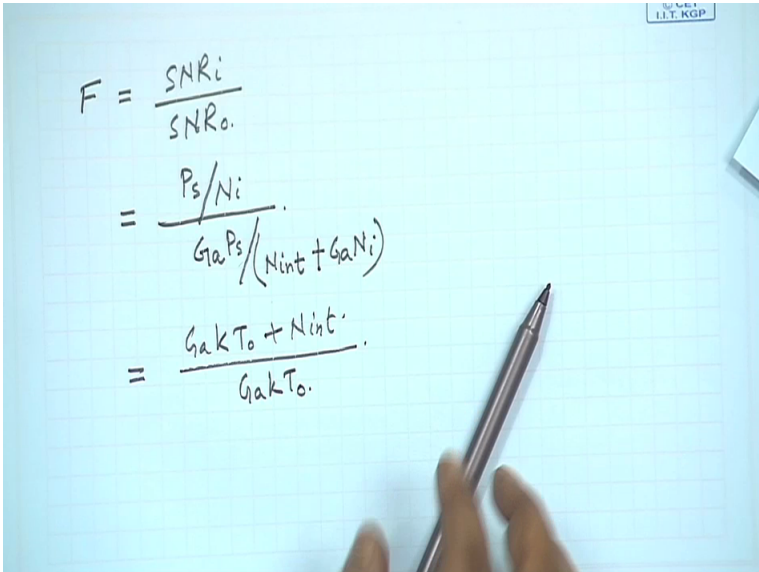
Operating Noise Factor (N_{op}):

Operating noise factor of a combined system, including the source and the receiver, is the ratio of the actual available output noise power density N_{oa} to the available output noise power density if the receiver had no internal noise source,
 $\therefore F_{op} = N_{oa} / \{G_a k T_s\}$.

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So in general then what is first let us define noise factor of a 2 port network. Noise factor F, this is the ratio of signal to noise ratio at the input terminal divided by signal to noise ratio at the output terminal, so SNR at the input divided by SNR at the output. Now signal to noise ratio obviously is better at left-hand side compared to right-hand side at the output, why? Because at output side we have additional noise generated by the component itself, so that is why F this is always greater than 1. If we express it in dB, we call it noise figure and that is always a positive quantity, so for a given system then already we did it, let me express mathematically.

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$$F = \frac{SNR_i}{SNR_o}$$

$$= \frac{P_s / N_i}{G_a P_s / (N_{int} + G_a N_i)}$$

$$= \frac{G_a k T_0 + N_{int}}{G_a k T_0}$$

So then the noise factor F this is equal to SNR at the input terminal divided by SNR at the output terminal, so at the input terminal let us say the power already whatever we assumed in previous example P_s is the incident power and let us say N_i is the incident noise from left-hand side, we are considering per unit hertz. So this is the SNR at the input side, now we can calculate SNR at the output side so then the available power is the application factor G_a multiplied by P_s divided by total noise, total noise is some noise internally generated inside the 2 port device + the contribution from left-hand side, which is $G_a N_i$. So this is equal to $G_a k T_0 + N_{\text{internal}}$ divided by $G_a k T_0$. Now it is a function of temperature, you see what value we should take for T_0 , so if the noise from my left-hand side it changes, obviously this value will change.

So then if we buy let us say one amplifier from market, some noise figure or noise factor is given with that then what it denotes, because it becomes a function of input noise. So usually a standard temperature is considered to calculate noise factor and hence figure that is 290 Kelvin. So if I do not consider simply actual scenario practical scenario, by noise factor we mean the noise contributed by the component it has been calculated at a given $T_0 = 290$ Kelvin. So from this then you see if I put N_{internal} that is given by $G_a k T_e$, T_e is the equivalent noise temperature so then it becomes simply $1 + T_e / T_0$. So T_e , this is the effective noise temperature of the 2 port device itself, T_0 this is for standard calculation we will consider 290 Kelvin.

Then we can express T_e also in terms of F , so $T_e = (F - 1) T_0$ this is another important expression. So if F is given, we can calculate what is the effective noise temperature T_e , if T_e is given we can calculate what is the noise factor F , so they are related by this important relationship. This noise factor depends on source impedance and operating frequency obviously, if we have a lossy network as I discussed it will provide you some insertion loss or power will be attenuated by that 2 port network. If the loss is given by L then simply its noise factor is equal to L . And now we are considering a different scenario, let us say this source temperature is not T_0 , it is given by T_s whatever we considered in the previous case. Okay, so let us take a break then we will consider a practical scenario with a given temperature T_s thank you.