Millimeter Wave Technology
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Module 8
Lecture No 38
Millimeter Wave Systems (Contd)

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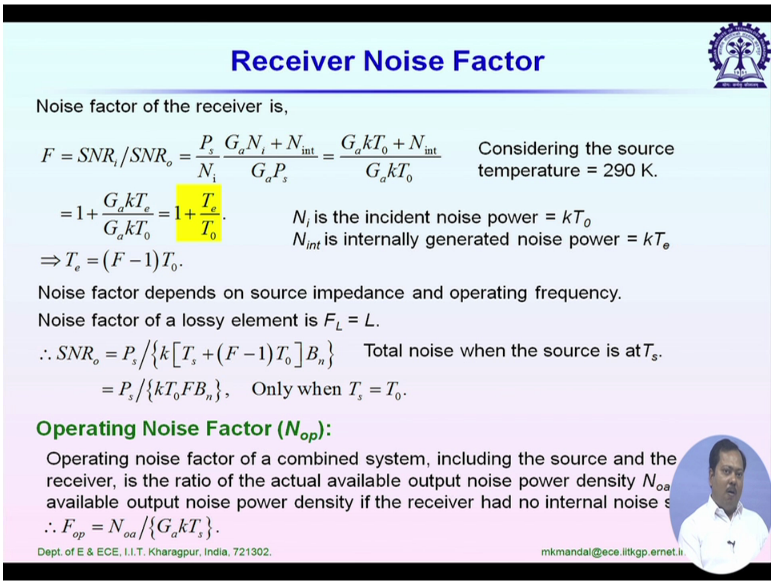

Receiver Noise Factor

Noise factor of the receiver is,

$$F = SNR_i / SNR_o = \frac{P_s}{N_i}\frac{G_a N_i + N_{\text{int}}}{G_a P_s} = \frac{G_a k T_0 + N_{\text{int}}}{G_a k T_0}$$

Considering the source temperature = 290 K.

$$= 1 + \frac{G_a k T_e}{G_a k T_0} = 1 + \frac{T_e}{T_0}.$$

N_i is the incident noise power = kT_0
N_{int} is internally generated noise power = kT_e

$$\Rightarrow T_e = (F - 1)T_0.$$

Noise factor depends on source impedance and operating frequency.

Noise factor of a lossy element is $F_L = L$.

$$\therefore SNR_o = P_s / \{k[T_s + (F-1)T_0]B_n\}$$ Total noise when the source is at T_s.

$$= P_s / \{kT_0 F B_n\},\quad \text{Only when } T_s = T_0.$$

Operating Noise Factor (N_{op}):

Operating noise factor of a combined system, including the source and the receiver, is the ratio of the actual available output noise power density N_{oa} available output noise power density if the receiver had no internal noise s

$$\therefore F_{op} = N_{oa} / \{G_a k T_s\}.$$

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Okay, now let us consider temperature T s instead of that standard value 290 Kelvin. And in that case let us see what happens, SNR how to calculate. So SNR at the output this is equal to P s divided by k into T s + F - 1 T 0 into B n. So the first-term this is due to this source temperature T s and the second term this is due to the effectiveness temperature T e, so if I now assume that T s = T 0, in that case P s by k into T 0 F B n. So sometimes we simply write down that SNR at the output this is equal to P s by T 0 F B n, but that happens only if T s = T 0. And if the source temperature is different than that 290 Kelvin or T 0, so we have to use this correction factor and this overall expression. We have to consider the effect of T s as well T e separately.

So considering a scenario like that, we can define the parameter, which we call the operating noise factor N op. So how we do it, operating noise factor of a combined system including the source and the receiver is the ratio of the actual available output noise power density, which is given by N oa to the available output noise power density if the receiver had no internal noise source. So that means in this case what we will be doing, we are considering actual temperature T 0, so T s and then calculating what is the available noise power density and output. And in the second case again we are recalculating it, but we are assuming the receiver itself is noiseless then what the effect of that left-hand side noise is and how it is being transferred to right-hand side, then we take the ratio.

So it is given by F op, this is equal to N oa, which is the available output noise power density so it includes both the contribution from left-hand side and due to the receiver itself and divided by G a k T s, so G a k T s you see here we did not consider any internal contribution of noise, so this is only due to the left-hand side.

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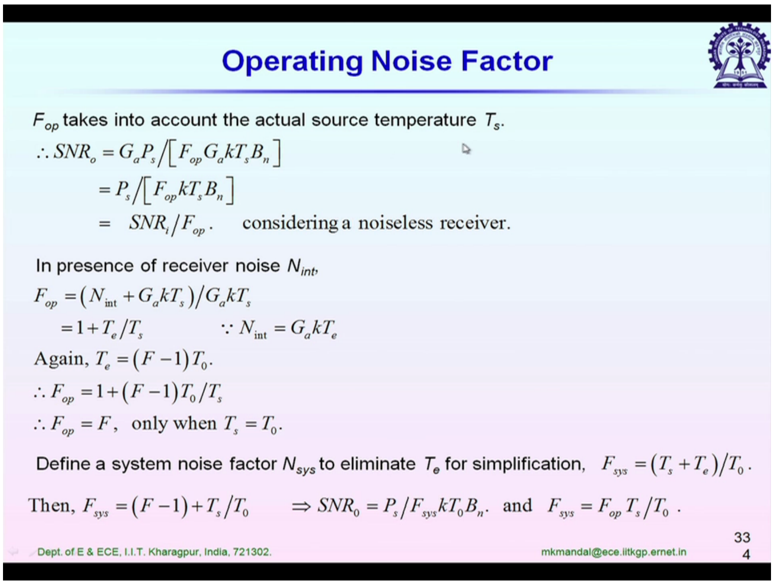

Operating Noise Factor

F_{op} takes into account the actual source temperature T_s.

$$\therefore SNR_o = G_a P_s / [F_{op} G_a k T_s B_n]$$
$$= P_s / [F_{op} k T_s B_n]$$
$$= SNR_i / F_{op}. \quad \text{considering a noiseless receiver.}$$

In presence of receiver noise N_{int},

$$F_{op} = (N_{int} + G_a k T_s) / G_a k T_s$$
$$= 1 + T_e / T_s \qquad \because N_{int} = G_a k T_e$$

Again, $T_e = (F - 1) T_0$.

$$\therefore F_{op} = 1 + (F - 1) T_0 / T_s$$
$$\therefore F_{op} = F, \ \text{only when } T_s = T_0.$$

Define a system noise factor N_{sys} to eliminate T_e for simplification, $F_{sys} = (T_s + T_e)/T_0$.

Then, $F_{sys} = (F - 1) + T_s / T_0 \quad \Rightarrow SNR_0 = P_s / F_{sys} k T_0 B_n.$ and $F_{sys} = F_{op} T_s / T_0$.

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So F op it takes into account the actual source temperature T s. So why we are doing it because let us say you have designed the receiver, you have the noise figure or let us say noise factor of whatever specified by manufacturer that is for standard value 290 K. And now for your system, which may be it may operate let us say for space application, it can operate at -50 - -60 Kelvin, -50 - -60 degree centigrade or it can be at very high temperatures scenario, so for that case you calculate what is the SNR for your receiver chain.

So then SNR at the output this is equal to G a P s this is the output power divided by output noise power, we are expressing it in terms of F op so then this is equal to simply F op G a k T s into B n over a noise bandwidth B n, or G a cancel out so P s by F op k T s B n, now P s divided by k T s B n, so this k T s B n this is actually the noise present at my left-hand side of the receiver over a bandwidth B n. So simply then P s divided by k T s B n, this factor we can call SNR at the input side, so we can simply write down F operating that is equal to SNR at the input divided by SNR at the output, not at standard temperature but at any given temperature, so that is how we can define another parameter F operating noise factor.

So now in presence of receiver noise N internal, in the previous one when we calculated k we put simply K T s B n, here we did not consider effect of internal noise generated inside the receiver, now we are introducing that. In that case F operating this is equal to N internal plus so this you see noise factor this is SNRi by SNRo, so in terms of noise simply then we can write down noise at the output divided by noise at the input. So noise at the output this is G a k T s + internal noise divided by noise at the input. G a k T s so this is 1 + T e by T s, when we are putting N internal = G a k T e. Now already we have seen that T e in previous expression, T e = F - 1 into T 0, so we can also express F operating that is equal to 1 + we are simply just putting the value of T e in the previous expression, so F - 1 into T 0 by T s. In this expression, it simplifies to F or F operating it becomes F only when T s = T 0.

Also we have seen it previously, you see it simplifies to this SNR at the output, in this expression it simplifies to k T 0 F B n only when T s= T 0. But this case also, F operating = F that specified only when TS = T 0, otherwise not. So now let us say T s is not equal to T 0, so then we have to consider all the effects of T s, T e and T 0 is already given or specified for the components whatever we are using so, so many noise temperature we have to deal with. We can simply eliminate T e by introducing one model parameter, so define a system noise factor N system to eliminate T e for simplification, so F system how we are defining, F system that is equal to T s + T e by T 0. So putting the value of T e here, F system this is F - 1 + T s by T 0.

Or we can write down then SNR at the output that is equal to P s divided by F system k T 0 B n. You compare this expression with the previous one for a noiseless receiver. So for noiseless receiver SNR at the output is it is equal to P s by this factor, but here this is T s by F system not F operating if we have a noisy receiver, which is the practical scenario, so F system where it is related to F op by F op into T s by T 0. So if we know the F value given for T 0 then we can easily calculate F system from this expression F op into T s by T 0.

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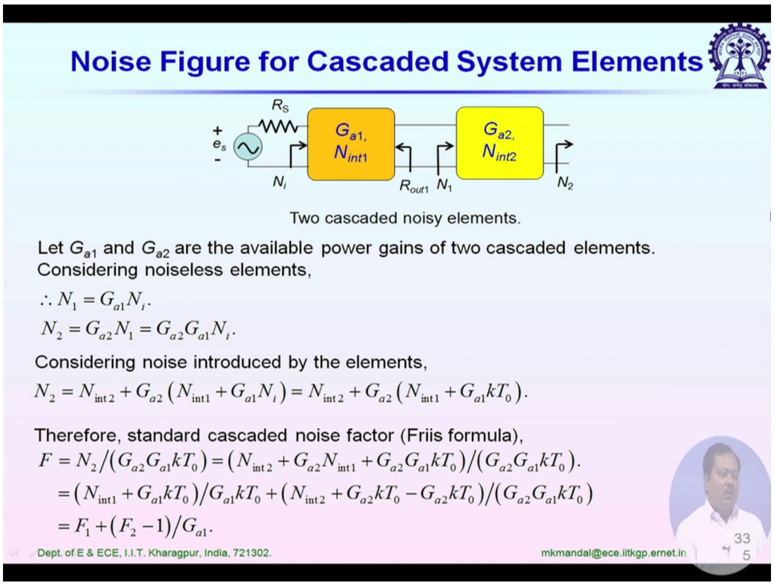

Noise Figure for Cascaded System Elements

Two cascaded noisy elements.

Let G_{a1} and G_{a2} are the available power gains of two cascaded elements. Considering noiseless elements,

$$\therefore N_1 = G_{a1} N_i.$$

$$N_2 = G_{a2} N_1 = G_{a2} G_{a1} N_i.$$

Considering noise introduced by the elements,

$$N_2 = N_{\text{int}2} + G_{a2}\left(N_{\text{int}1} + G_{a1} N_i\right) = N_{\text{int}2} + G_{a2}\left(N_{\text{int}1} + G_{a1} k T_0\right).$$

Therefore, standard cascaded noise factor (Friis formula),

$$\begin{aligned} F &= N_2/(G_{a2}G_{a1}kT_0) = (N_{\text{int}2} + G_{a2}N_{\text{int}1} + G_{a2}G_{a1}kT_0)/(G_{a2}G_{a1}kT_0). \\ &= (N_{\text{int}1} + G_{a1}kT_0)/G_{a1}kT_0 + (N_{\text{int}2} + G_{a2}kT_0 - G_{a2}kT_0)/(G_{a2}G_{a1}kT_0) \\ &= F_1 + (F_2 - 1)/G_{a1}. \end{aligned}$$

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Now noise figure for a cascaded system, we already calculated equivalent noise temperature for a cascaded system, now we are going to calculate noise figure or noise factor for cascaded system. So basically overall SNR calculation we can use any of these 2 approaches, noise figure approach or noise temperature approach. Noise temperature approach already we know, now we are going to see the noise figure one. So we are considering a simple scenario, we have let us say just 2 amplifiers in Cascade, it can be any other components also, simply we have to calculate its gain. The first one let us say gain is G a1 and it is providing some internal temperature N internal 1, so if I express in terms of noise temperature you remember we expressed it by T e 1, then N internal 1 that is equal to k T e 1.

The second component, it is G a2 gain and internal noise generated N internal 2. So N 1 whatever falling from left inside to the second component, so this is the output noise power from the first component. So if I now first consider a noiseless component that means N internal 1 or N internal 2 is 0, in that case N 1 this is equal to N i multiplied by gain of this amplifier, so that is the noise available at the output, we consider a noiseless system. So that output is falling on the second amplifier, so N 1 = G a1 N I similarly, N 2 = G a2 N 1 that is equal to G a2 G a1 N 1. Now let us consider the effect of N internal 1 and N internal 2, so considering noise introduced by the elements N 2 that is equal to N internal 2 + G a2 multiplied by whatever noise coming from left-hand side, N internal 1 + G a1 N i or we can write down this is equal to N internal 2 + G a2 into N internal 1 + G a1 k T 0, so N i we are representing by k T 0.

Then the standard cascaded noise factor this is also the Friis formula, second form for noise factor, F = N 2, so output noise power divided by G a2 G a1, k T 0. So we can now put the values here N 2 already we calculated in the previous one, N internal 2 + G a2 N internal 1, G a2 G a1 k T 0 divided by G a2 G a1 k T 0. So if I consider the first 2 terms, N internal 1 and G a1 k T 0, so G a2 this is cancel out and this is for the we are considering this second and third term. Now if I consider the first term N internal 2, with that we are simply adding and subtracting G a2 k T 0, so original term is N internal 2, then + - G a2 k T 0 divided by this, so it simplifies to N internal 1 + G a1 k T 0 by G a1k T 0, this is F 1 + F 2 - 1 by G a1.

So this is for a 2 component system, if we have n number of components we can simply add other terms so then the third term it will be F 3 - 1 divided by G a1 G a2, similarly you can define it for nth number of terms. So again what we see that the contribution of first component it becomes very important, so for all other components it is being divided by the gain of other preceding components. So that is why whenever we use an amplifier, it should be a low noise amplifier just immediately after the antenna to decrease the overall noise figure of the receiver. So now we learn how to calculate the overall noise figure of a receiver, noise factor of a and if we simply convert it to decibel scale then we will call overall noise figure of the receiver. So it will determine the noise floor below which the receiver it cannot detect any noise power any signal power.

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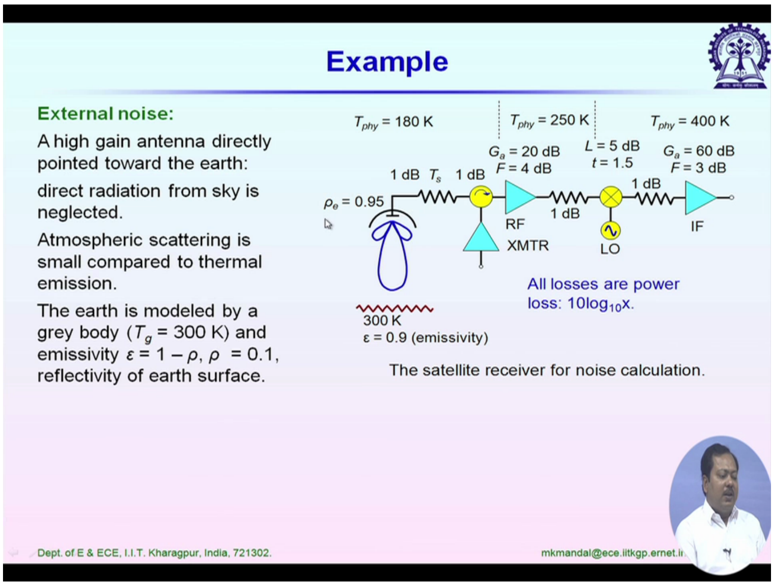

Example

External noise:

A high gain antenna directly pointed toward the earth:

direct radiation from sky is neglected.

Atmospheric scattering is small compared to thermal emission.

The earth is modeled by a grey body (T_g = 300 K) and emissivity $\varepsilon = 1 - \rho$, $\rho = 0.1$, reflectivity of earth surface.

All losses are power loss: $10\log_{10}x$.

The satellite receiver for noise calculation.

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So minimum SNR that should be higher than this also we have, we learned what is link-margin, this value is not sufficient for any wireless system so above that we have to maintain link margin of 3 dB to 10 dB. Now let us take one example with numerical values, let us say one antenna it is directly pointing towards Earth, it can be a radiometer itself, this whole receiver chain together with antenna. So in radiometer application it is actually one type of passive imaging, what is done, simply the power whatever radiated by the other surface it is connected by antenna and then it is converted to some effective noise temperature. And now if we point towards different directions, so depending on the dielectric constituents of the Earth surface and the roughness of other surfaces, the effective noise temperature will change since the input power will change, so that is how we can map any given Earth surface and it is very popular in remote sensing applications.

We can monitor sea level, we can monitor ice level and many other things, so this is one such application. So antenna is likely pointing towards Earth and the Earth's temperature physical temperature is given as 300 Kelvin, so it is not also it is not a perfect black body because it is characterised by some emissivity Epsilon, so emissivity for black body equal to 1. Now emissivity it can be given by 1 - Rho where Rho is the reflectivity, so if Epsilon is 0.9 that means Rho = 1 - 0.9, which is 0.1, so a good emitter is a bad reflector. So this antenna is connected to a circulator by using a section of transmission line may be a cable or waveguide section. Starting from antenna to circulator they are at a physical temperature given by 180 Kelvin, it is quite low but it may happen in space.

The loss of this connecting section is given as 1 dB, we also have some loss from this circulator which is given by 1 dB and look at the antenna, antenna efficiency is 95% or Rho e it is given as 0.95. Immediately after the circulator, we have a low noise amplifier so gain of this low noise amplifier is given as 20 dB and the corresponding noise figure not factored in dB. Noise figure is 4 dB, again we have some loss due to connecting sections that is 1 dB and next we have one mixer, it is down converting the millimetre wave signal coming from left-hand side. So for the Earth this LNA and the connecting cables they are at physical temperature to 50 Kelvin, but right-hand side maybe they are sealed inside one package for them they are at thermal equilibrium of physical temperature 400 Kelvin, it is quite high.

So just after mixer we have a connecting section then the IF amplifier, which is obviously low frequency amplifier, so for this low frequency amplifier noise figure is given as 3 dB, gain 60 dB. Now when we convert this noise figure to noise factor obviously, log base is 10 we have to take 10 to the power, but we are dealing with power not voltages. So that means we have to divide all these values by 10 not 20 for example, if I want to convert this 4 dB noise filter to corresponding noise factor what we have to do, 4 by 10.4, so it will be 10 to the power 0.4. Now what we have to do, we have to calculate what is the overall SNR that also we can calculate if we can calculate whatever the overall effective noise temperature T e of this system.

So we will start from the source basically, before antenna we have source the Earth surface, so the earth surface temperature is given 300 Kelvin. Now what happen you can see, if I look at the power whatever received by antenna, some part is due to emission due to the Earth itself and another part, which is coming from the dwelling radiation from atmosphere that is being reflected by Earth surface coming to antenna. But since it is Rho is 0.1 it is a poor reflector and looking at the roughness of Earth surface, we are simply neglecting that reflected power, in practice also it is negligibly small. We will be considering then only the effect due to this emission from Earth surface itself, which we are considering as a grey body.

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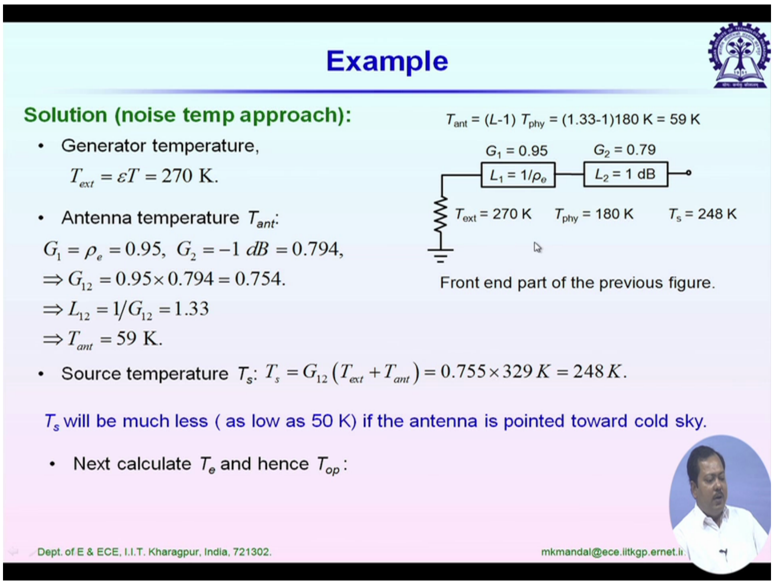

Example

Solution (noise temp approach):

- Generator temperature,
  $T_{ext} = \varepsilon T = 270\text{ K}.$
- Antenna temperature T_{ant}:
  $G_1 = \rho_e = 0.95,\ G_2 = -1\ dB = 0.794,$
  $\Rightarrow G_{12} = 0.95 \times 0.794 = 0.754.$
  $\Rightarrow L_{12} = 1/G_{12} = 1.33$
  $\Rightarrow T_{ant} = 59\text{ K}.$
- Source temperature T_s: $T_s = G_{12}\left(T_{ext} + T_{ant}\right) = 0.755 \times 329\,K = 248\,K.$

T_{ant} = (L-1) T_{phy} = (1.33-1)180 K = 59 K

Front end part of the previous figure.

T_s will be much less (as low as 50 K) if the antenna is pointed toward cold sky.

- Next calculate T_e and hence T_{op}:

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So this effective radiation we will be representing by resistor, so this is the figure, so for this part what we are doing we are considering the first part starting from Earth surface to circulator, which are at a physical temperature 180 Kelvin. Now how to calculate T external? T external is simply Epsilon into T physical, so temperature of Earth surface is given as 300 Kelvin into Epsilon 0.9 T external it becomes to 70 Kelvin, we are using temperature approach. Now antenna temperature T antenna, so antenna loss is given by 1 by Rho e because gain a 0.95, then loss it is 1 by 0.95 then followed by antenna we have a section of waveguide for which again loss is 1 dB, so we calculate then what is the corresponding gain, so it is 1 by 10 that is 0.1, so 10 to the power 0.1.

So overall gain G 1 into G 2 for this 2 cascaded system, so G 1 into G 2, so G 1 you see 0.95, G 2 0.794, G 12 then it become 0.754. Now loss L 12 it is simply 1 by overall gain, which is 1.33, so we calculate then T antenna it is equal to we already have seen the formula; T e = T physical into L - 1, so put it here, T antenna it becomes 59 Kelvin. Then the source temperature T s for these 3 components altogether, this is equal to G 12 multiplied by T external because you see noise generated by this external source or the resister it is being propagated through antenna and that waveguide infection, so T s = G 12 into T external + T antenna or put the values, it becomes 248 Kelvin, so T s is 248 Kelvin.

Here we did not consider any direct contribution of sunlight, actually sky is very cold if you point your antenna towards earth you will receive more power compared to if you just point your antenna to the cold sky. So cold sky temperature is typically very low equivalent noise temperature is 50 Kelvin, so this source part calculation is over, next part we have to calculate T e and hence the operating. So we have to consider the contribution due to LNA, then connecting section, Mixer, connecting section and then the IF amplifier oh before that we have circulator also. So among these, mixer specification it is given its temperature ratio you see here L = 5 dB that means conversion loss is 5 dB and temperature ratio is 1.5, so actual its operating temperature is 290 multiplied by 1.5. Okay, let us take 5 minutes break, then we will continue the calculation.