Power System Analysis
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Lecture – 13
Capacitance of Transmission Lines (Contd.)

So, today we will start that capacitance of 3 phase the double circuit lines right. So, come back to that earlier this thing we have seen for inductance as well as the capacitance that assuming that lines are transposed right.

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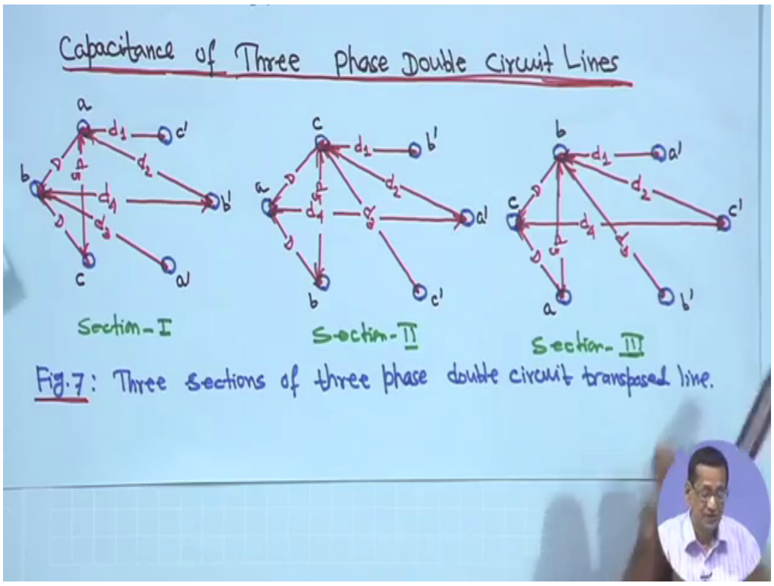

So, this is the configuration for the first transposition cycle this is the configuration for the second transposition cycle and this is the diagram for the third transposition cycle. So, section 1 section 2 and section 3 this arrangement we have seen everything before right all the distances are given that is a b c and this is a dash b dash c dash for first transposition cycle second 1 is your a b c and your a dash b dash c dash and for the third transposition cycle a b c and this is your a dash b dash and c dash right.

So, all the distances are marked here for letter also I have given that a to a c dash is is d 1 a to b dash is is d 2 and your b to b to a dash is d 3 and b to b dash d 4 and a b and as well as b to c this distance is equal from the symmetry this is d and d same is everywhere right. So, 3 sections sub 3 your 3 phase double circuit transpose lines right.

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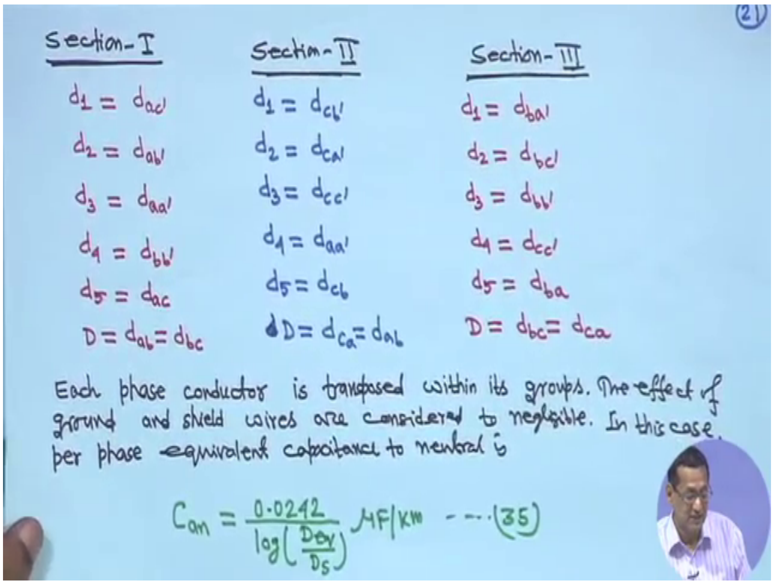

Now, the distances for your for your understanding and everything all are marked actually d 1 d 2 for the section 1 for the section 1, section 2 and section 3 for all these cases just for easy understanding I have marked everything that d 1 is equal to d a c dash d 2 D a b dash d 3 d a a dash d 4 b b dash and d 5 is equal to d a c and capital D is equal to D a b is equal to d b c similarly for section 2 and section 3 all the distances are marked right.

Now, each phase conductor is transpose within its group right. So, the within this group only the conductors are transposed right here and here also right. So, double circuit line it is right therefore, the effect of ground and shield wires have been neglected right because its effect is negligible. So, in this case per phase equivalent capacitance to neutral is C a n is equal to 0.0242 divided upon log D e q upon D s microfarad per kilometer in general you know log d upon r right, but there is a as it is a your double circuit line 3 phase double circuit line. So, it is basically log D e q upon D s microfarad per kilometer this is equation 35 and this we have seen before right.

Next your next you have to calculate D e q and D s you have to calculate D e q your just hold on you have to calculate D e q and D s right these 2 you have to calculate.

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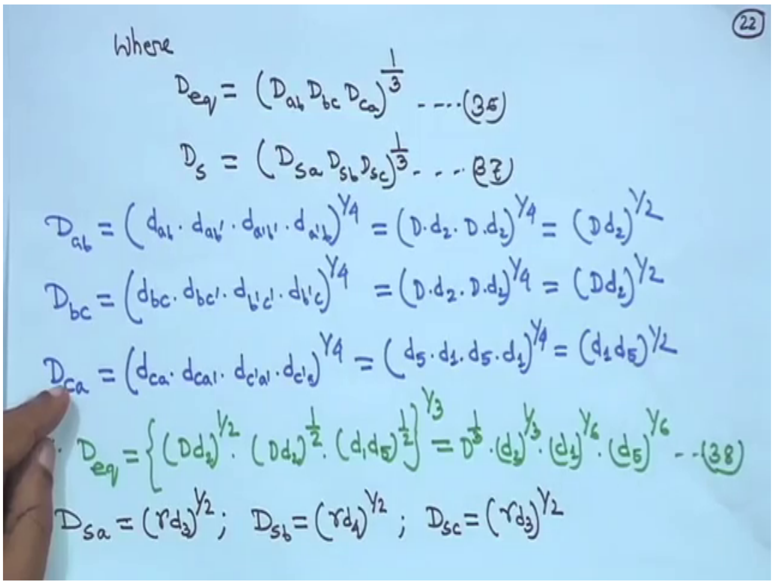

So, in this case first D e q you know earlier we have seen for inductance also capacitance also same thing D a b into d b c into d c a to the power 1 third right and D s is equal to same 1 same D s a into D s b into D s c to the power 1 third this equation is marked as thirty 6 and this 37 right. Now D a b d a b is actually D a b d a b dash d a dash b dash d a dash b just one I am showing that your D a b d a b is equal to a to b; that means, that is your that is your D a b then d your a b dash. So, D a b dash then your d a dash b dash. So, d a dash b dash and d a dash b and d a dash b right this to the power 1 by 4 right because 4 distances are there. So, basically from the symmetries is equal to d into d d 2 into d into d d 2 is equal to d d 2 to the half right.

Similarly, you similarly you calculate D b c right d b c also will become D d 2 to the power half right a b b c a b b c from the symmetry you can write it will be half, but a c will be D a b b c c a c a will be c a same thing c a will be same thing D c a d c a dash just take all the all the 4 combinations here right. So, d c a dash d c dash a dash right and d c dash a from the cycle one only from this figure only right to the power 1 by 4 that is actually will come d 5 into d 1 into d 5 into d 1 that is actually d 5 square d 1 square is equal to d 1 d 5 to the power half. So, D a b d b c d c a you have got it.

Now, this D a b d b c and d c a you substitute in this expression that is equation 35 if you do. So, if you do. So, D e q will become d d 2 to the power half into d d 2 to the one half into d 1 d 5 to the power half whole to the power they are all this thing to the power 1 third that is one third. So, is equal to it is d to the power 1 third then d 2 to the power 1 third then d 1 to the power 1 by 6 and d 5 to the power 1 by 6 right. So, this is actually equation thirty 8 right similarly D s a you take the self one from the symmetry only right r into d 3 because your what you call d a c will be r into d 3 to the power half D s b will be your this thing r d to the power to the power half and D s c will be r d 3 to the power half right just 1 minute. So, from your what you call from the symmetry you can make all this your what you call all the calculations right. So, it is because if you take 4 combinations r d 3 into r d 3 will come to the power 1 by 4 ultimately it will be r d 3 to the power half D s b will be r d 2 to the power half and D s c will be r d 3 to the power half right.

So,. So, next is once you get D s a D s b and D s c all these things you have got right then your D s will be D s a D s b D s c to the power the to the power 1 third. So, substitute all these 3 expression right.

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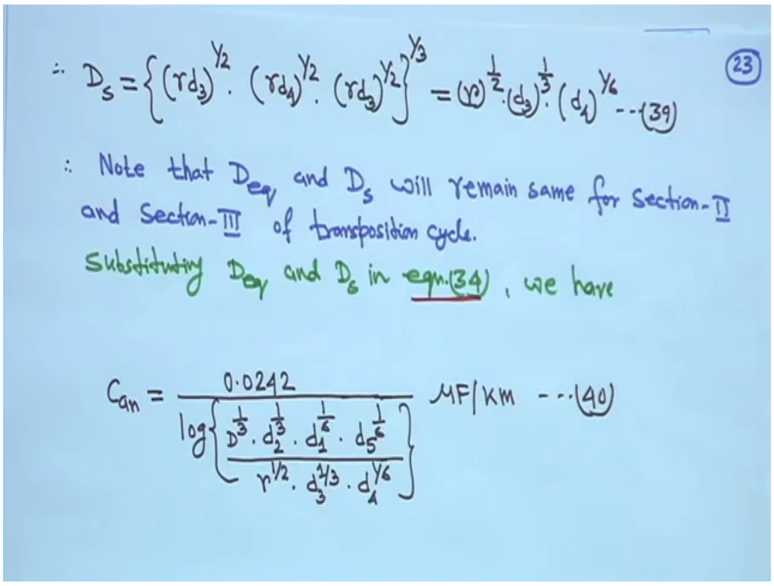

If you substitute then your; this one will be D s is equal to r d 3 to the power half into r d 4 to the power half into r d 3 to the power to the power 1 third. So, it is coming actually r to the power half then d to the power one third d 4 to the power 1 by 6 this is equation thirty nine right. So, if you see that transposition cycle that D e q and D s will remain same for section 2 and section 3 only conductor is changing position, but configuration remains same right therefore, for the second and third case also D e q and D s will remain same right. So, therefore, substituting the D e q and D s in equation 34; that means, this equation; that means, if you substitute this equation in 34 right. So, you can get C a n is equal to 0.0242 log all this expression after simplification it will come this much microfarad per kilometer. So, this is that equation 40.

So, this is basically one phase to neutral right it is a balanced system. So, b n C a n C b n C c n all the values numerical all the expression you will have the same values right. So, this is that line to neutral capacitance for the 3 phase double circuit line this is the expression right you need not remember anything because it is almost impossible to remember, but because d d c all these d 1 d 2 the way it had been taken you can take different way different terminology will come here different your that nomenclature whatever you take it will come. So, better is that you have to derive that and then you find out what is the value of this capacitance right now another thing is that next one is that effect of earth on the capacitance right.

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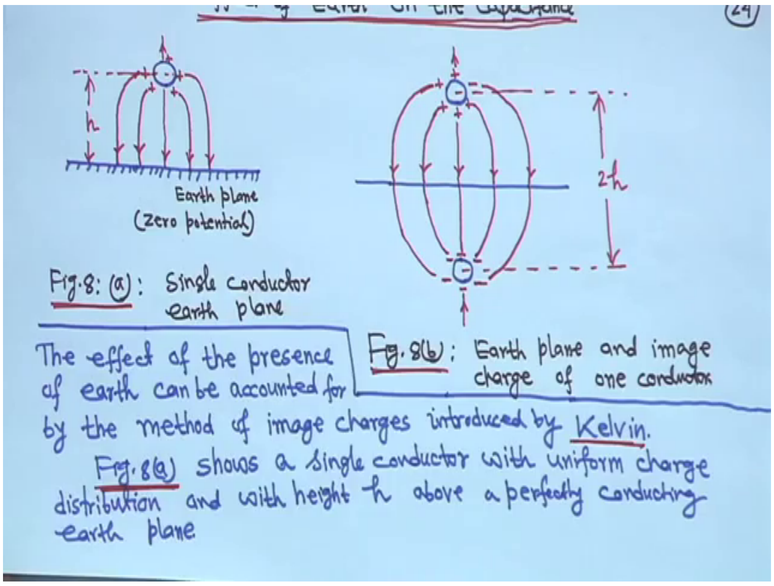

So, this is effect of earth on the capacitance say this is your this is your that earth plane assuming that this is 0 potential and you have a conductor here right a uniform charge is uniformly distributed right assume that these are the positive charge right.

So, this is a single conductor earth plane 0 potential here and you will use that introduce scale means what you call that image charges. So, the effect of presence of earth can be your accounted for the method of image charges introduced by Kelvin right. So, figure this one figure 8 a figure 8 a shows a single conductor with uniform charge distribution and with height h that above a perfectly conducting earth plane that is this conductor is above the ground that height is h actually and you have to include that you have to call the effect of the earth right now what; what one can do is that we will make it a image conductor just suppose this is this is a charge q say we will take another image conductor whose if it is a plus charge q then it will be minus q and if it is from here to here it is h you will imagine that it is also below the ground it is also here at height h. So, total height is from here to here is 2 h and this is actually Kelvin has introduced this concept right.

So, if we assume that this conductor right this one it has your it has a positive charge q coulomb if this has a q coulomb then an equal amount of negative charge minus q coulomb is induced in your induced earth; that means, it is here we will assume if it is q then here it will be minus q another thing is the electric field lines will originate from the positive charge right that is you know right on the conductor and terminate on the negative charges that is why it is made like this it is made like this originating from the positive charge assume this is a positive charge and terminating here at the negative charge right. So, that is and figure 8 b; that means, this one; that means, this one figure 8 b; that means, this one earth plane and image charge of one conductor shows that the earth is replaced by image conductor lies directly below the original conductor right. So, the electric field above the plane is the same that is I mean whatever is the electric field above here; here also it is same; that means, if you try to take potential between any 2 points above this above this right.

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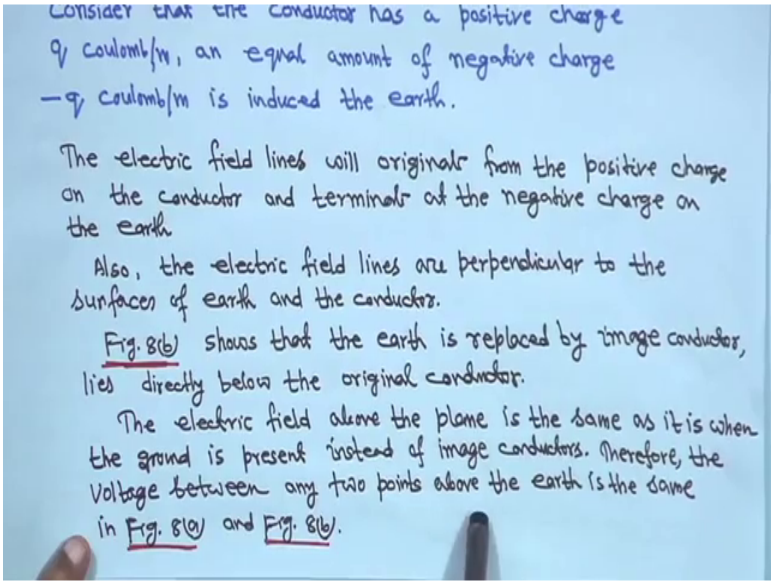

Here we assume that they are same actually that is why I have made it that electric field above the plane is same as it is when the ground is present instead of image conductors right therefore, the voltage between any 2 points above the earth is the same as in figure a and figure b right.

So, for any 2 points you take whatever the voltage will be here; here also this thing above this above this plane that anywhere any 2 point the voltage will same point of course, the voltage will be same right there this is the this is how this is that considering this image conductor you have to find out what is the effect of the earth on the Capa; this thing what you call on the capacitance right. So, to derive such thing for example, that capacitance of a single phase line considering the effect of earth right.

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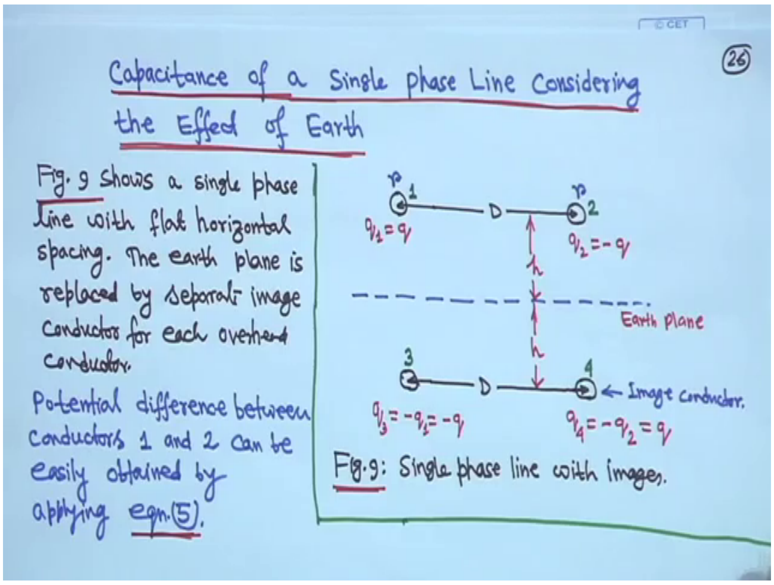

So, we will show here only the single phase 1 3 phase one we will be leaving up to you to do this right. So, we are assuming that single phase line having their radius r r right and there are 2 conductors if charge is here q 1 is equal to q then naturally here q 2 will be minus q right and this image charges if it is q 1 equal to q here will be minus q. So, we are taking q 3 is equal to minus q 1 is equal to minus q image charges this is the image charge of this one similarly for this one image charge of this one is this one this is the earth plane here it is q 2 is equal to minus q; that means, image charge will be plus q. So, q 4 is equal to minus q 2 is equal to q right.

So, and distance between these 2 conductors is d therefore, distance between the image conductor is also d from the ground height is h. So, this side is also h. So, totally 2 h right; so, first you have to find out the potential difference between conductors 1 and 2 you have to apply the equation 5 earlier we have given no equation 5 I am writing once again right for your thing. So, this is for your 4 conductors effectively because this is of course, this is that this is this is 2 conductors single phase, but 2 image conductors are there. So, you have to consider the 4 conductors.

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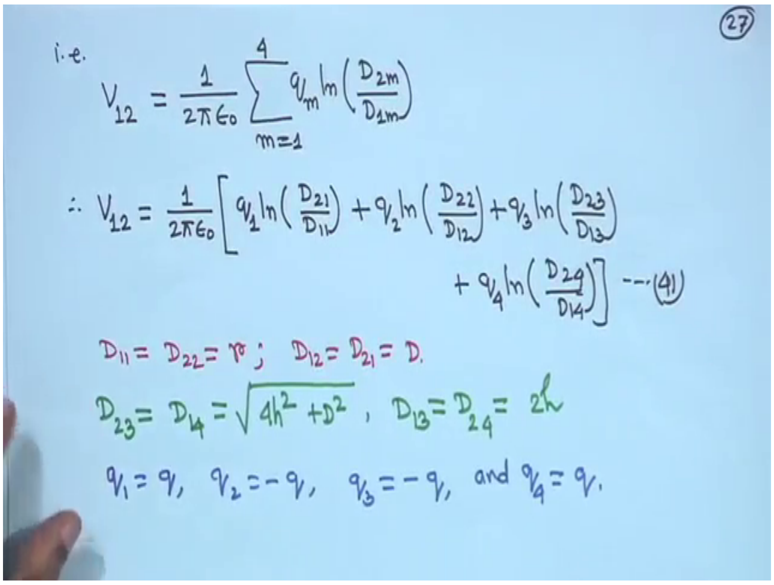

So, if you apply equation 5 then generally what will happen the voltage between this one and 2 these 2 conductors V 1 2 you can write you apply equation 5 right if you go back to equation 5 just let me see if I can quickly find out equation 5 just 1 minute just hold on this equation 5 this is actually equation 5 it is general formula V k I is equal to 1 upon 2 pi epsilon 0 that m is equal to 1 to n q m l n d I m upon d k m volts right. So, here 4 conductors are there including 2 image conductors. So, n is 4, right.

And you have to find out k is equal to 1 I is equal to 2 that is V 12. So, here your here same 1 upon 2 pi epsilon 0 m is equal to 1 to 4 and here it is equal to k is equal to your 1 and I is equal to 2; that means, k is equal to 1 it is d 1 m if I is equal to 2 d 2 m right.

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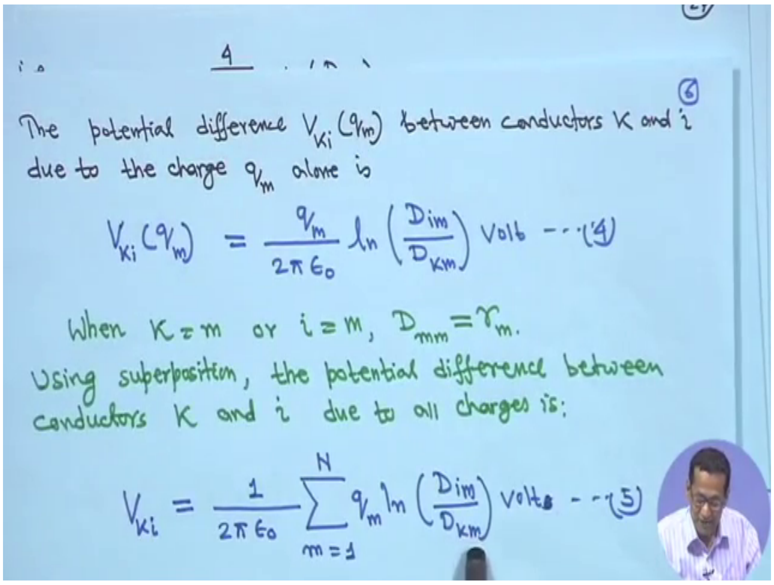

Therefore, this equation becomes equation 5 becomes 1 upon 2 pi epsilon 0 m is equal to 1 to 4 q m l n d 2 m upon d 1 m now you expand this you expand this right if you expand this it will come V 1 2 upon 2 pi epsilon 0 q 1 l n d 2 1 upon d 1 1 plus q 2 l n d 2 2 upon d 1 2 plus q 3 l n d 2 3 upon d 1 3 plus q 4 l n d 2 4 upon d 1 4 this is equation 41.

Now, d 1 one is equal to d 2 two is equal to r the radius of the conductor right and d 1 2 is equal to and d 1 2 is equal to d 2 1 is equal to d because compared to r d is very large. So, that is why d 1 2 is equal to d 2 1 is equal to d now d 2 3 and d 1 4 they are equal that is d 2 3 d 2 3 I mean this diagonal 1 and d 1 4 this diagonal 1 and this diagonal 1 d 2 3 and d 1 4 is equal to this is basically under root d square and here it is 2 h. So, under root d square plus 2 h whole square; that means, root over d square plus 4 h square right that is why d 2 3 is equal to d 1 4 is equal to root over 4 h square plus d square right and d 1 3 is equal to d 3 4 d 1 3 is equal to 2 h is equal to d 2 4 is equal to 2 h that is d 1 3 is equal to d 2 4 is equal to 2 h right now here everything is defined here everything q 1 q 2 q 3 q 4 all are defined therefore, q 1 is equal to q q 2 is equal to minus q q 3 is equal to minus q and q 4 is equal to q. So, all these things you substitute in equation 41 here you substitute and simplify right therefore, your V 1 2 is equal to 2.

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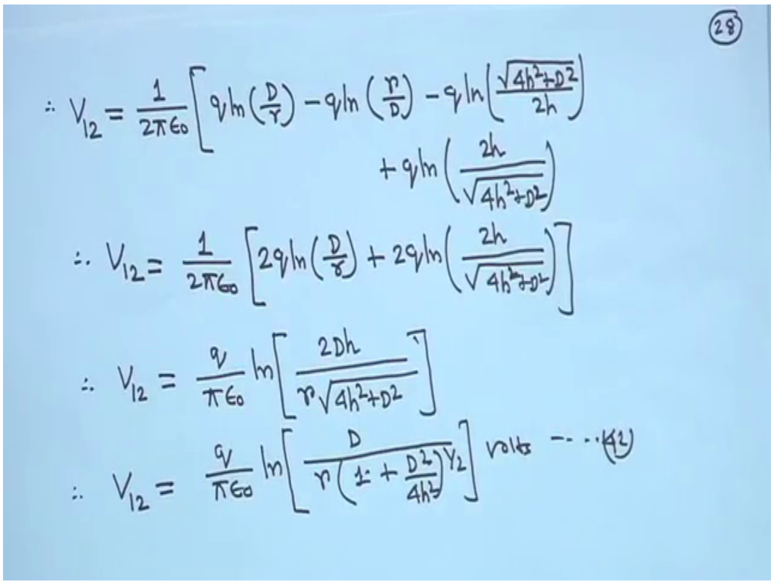

After substitution it will come like this V 1 2 is equal to 1 upon 2 pi epsilon 0 q l n d upon r minus q l n r upon d minus q l n root over 4 h square plus d square divided by 2 h plus q l n 2 h root over 4 h square plus d square right.

So, therefore, V 1 2 is equal 2 pi epsilon 0 it will come after simplification 2 q l n d upon r plus 2 q l n 2 h root over 4 h square plus d square therefore, V 1 2 is equal to your q by pi epsilon 0 l n 2 d h divided by r root over 4 h square plus 2 d square right. So, these 2 these 2 and these 2 this 2 will take common this 2 will cancel this 2 this two. So, and if you l n it will be basically 2 d h divided by r root over 4 h square plus d square right therefore, V 1 2 is equal to q by pi epsilon 0 l n d divided by r in bracket 1 plus d square upon 4 h square to the power half volt this is equation 42 right. So, this is the voltage thing if it is if earth effect is not there ground effect is not there then earlier it was just d upon r, but now it is r into 1 plus d square upon 4 h square to the power half; that means, this factor had been multiplied with the r right.

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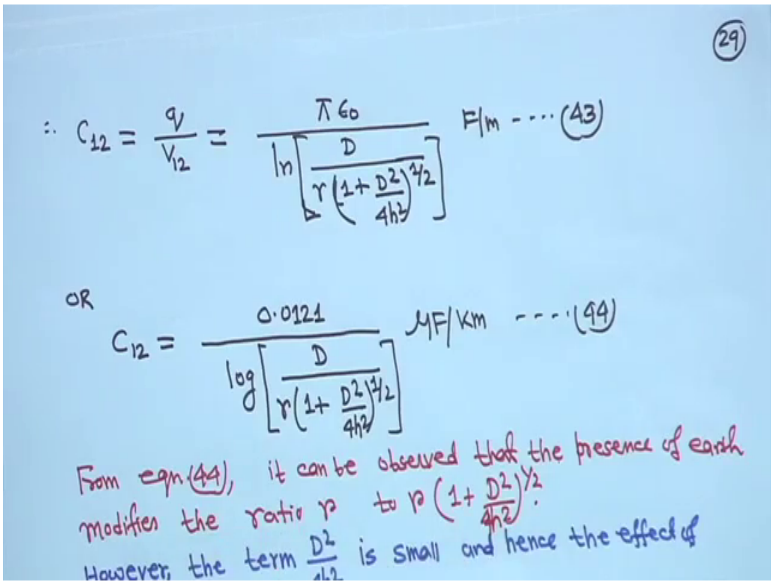

Therefore C 12 is equal to q upon V 12 right is equal to pi epsilon 0 divided by l n d upon r into 1 plus d square upon 4 h square to the power half farad per meter this is equation 43, now you convert it to log then instead of natural log then C 12 is equal to 0.0121 divided by log d upon r into 1 plus d square upon 4 h square to the power half microfarad per kilometer.

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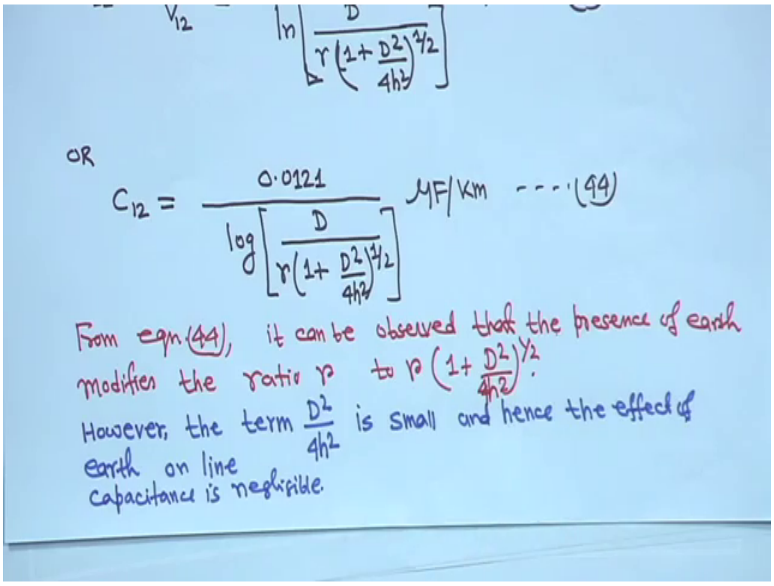

This is equation 44; now in equation 44 it can be observed that the presence of the earth modifies the ratio r to r into 1 plus d square upon 4 h square to the power half right. So, earlier if you earlier the effect of earth was not there it was r only, but now with this earth a factor is multiplied that is 1 plus d square upon 4 h square to the power half right. However, this d square by 4 h square term actually very small right and hence the effect of earth on line capacitance is negligible actually this term this d square by 4 h square actually because conductor lies much above the your you know much above the ground right compared to this d because d is the spacing between the 2 conductors and h is that height of the conductor from the ground.

So, d square upon 4 h square will be much smaller right that is why its effect is negligible. So, this is for your; what you call this is for your single phase single phase line. So, for 3 phase 1 I will suggest that you should try yourself right.

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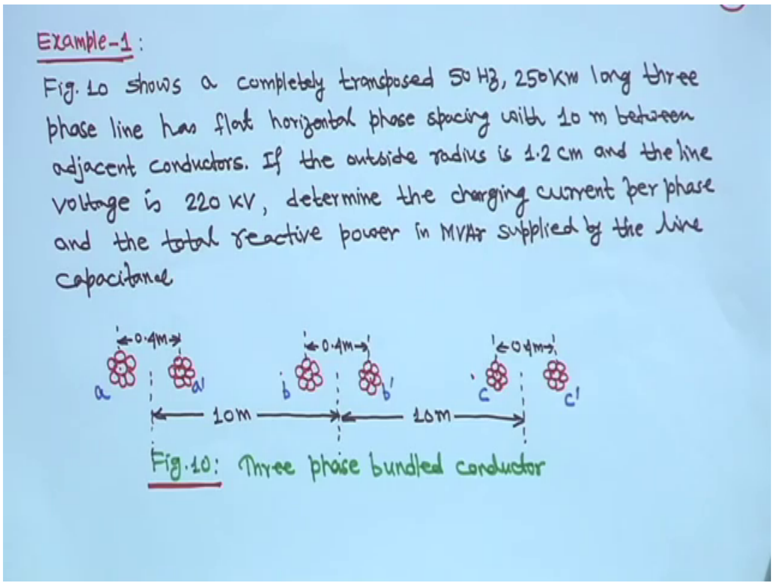

Now, come to that example right. So, this is this is a; this is that this is figure 10 the 3 phase bundled conductors right 3 phase bundled conductors. So, figure 10 shows a completely transposed 50 hertz 250 kilometer long 3 phase line has flat horizontal phase spacing with 10 meter between adjacent conductors right if the outside radius is 1.2 centimeters; that means, this is that this is called this is the central strand and 6 more are there around that, but outside radius is given here no question of r dash or anything right. So, if the outside radius is 1.2 centimeter that is given and the line voltage is 2 twenty k V you have to determine the charging current per phase and the total reactive power in Megavar supplied by the line capacitance because whenever you consider that whenever you consider that you know line capacitance the charging capacitance basically it injects actually that your reactive power right. So, particularly for long transmission high voltage long transmission line you have to consider the charging capacitance; that means, it injects what you call that reactive power.

So; that means, you have to find out the total reactive power in Megavar supplied by the line capacitance. So, in this case this is the this is the configuration that it is bundled conductors, but a a dash your in each phase 2 such say 2 such conductors are there right a a dash b b dash c c dash. So, between these 2 distance is point 4 meter here also here also it is same and the distance 10 meter and 10 meter right.

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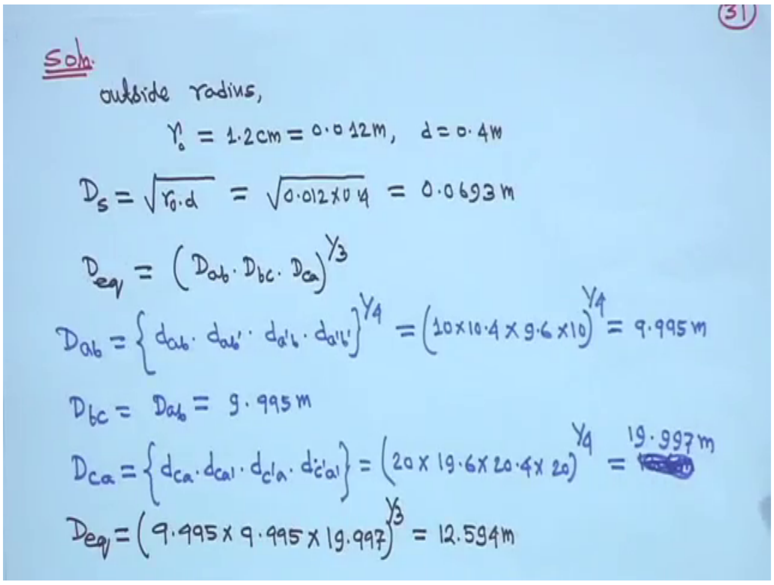

So, it is; so, what we what we have to do is first the outside radius is given. So, it is already given outside is 1.2 centimeter. So, it is a 0.012 meter right and this is the d d is given d is equal to 0.4 meter between these 2 a a dash or b b dash or c c dash d is equal to 0.4 meter right and outside radius means you need not do anything else right. So, directly it is given that 0.012 meter and now, you have to find out D s right. So, outside radius we are taking r 0 right, so, D s is equal to r 0 into your r 0 into d root over right actually if you what you call if you try to find out that what will be D s right general.

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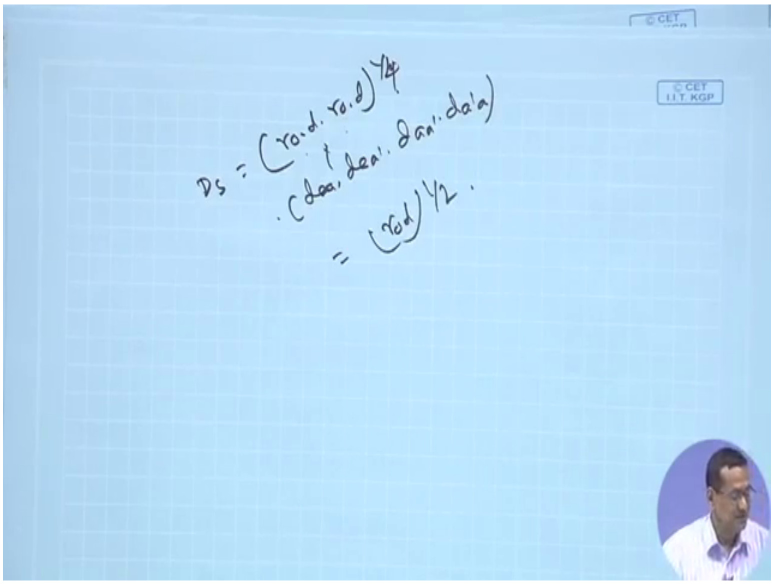

So, basically it is your D s is equal to your r 0 into d into r 0 into d to the power 1 by 4 right 1 by 4 basically it is d a a into d a dash then d a dash into your d a dash a right. So, basically it is r 0 d into the power 1 by 4. So, that is your r 0 d to the power half right. So, that is why D s is equal to r 0 [FL] this 1 into 0.4. So, it is 0.0693 meter right.

Now, D e q earlier we have seen D a b into d b c into d c a to the power 1 third. So, from the symmetry of this configuration D a b will be equal to d b c right, but d c a will be different right. So, D a b is equal to take all the possibilities that D a b that is distance D a b into D a b dash then d a dash b into d a dash b dash right. So, that way it will your D a b is equal to D a b is equal to here center to center is given middle point rather middle point midpoint of these 2 conductor and these 2 are given; that means, D a b actually equal to 10 meter right I mean a to b is 10 meter right therefore, D a b is equal to your 10 meter and D a b dash d a to b dash right a to b dash a to b is 10 meter and b to b dash is point four. So, therefore, D a b dash is equal to 10.4 right similarly d a dash b if you take d a dash b that is your here to here that is a dash b right.

So, from here to here it is your what you call this is your center to center is 10 meter. So, this you have to find out a dash to b dash. So, from it is midpoint. So, this is 0.2 and this is 0.2. So, it will be 10 minus 0.2 minus 0.2 that is your 9.6 that is why it is d a dash b is equal to 9.6 and again d a dash b dash a dash b dash again same thing a dash b dash again 10 meter. So, it is into 10 to the power 1 by 4 because 4 distances are there. So, is equal to 9.995 meter and d b c is equal to D a b is equal to 9.995 meter right. So, similarly your d b c is equal to D a b both are same from the symmetry a b to b c same now you calculate.

Similarly, you calculate D c a. So, if you calculate d c a it will be d c a into d c a dash d c dash a into d c dash a similarly all the distances d c will be find 20 from this from this figure you will find that d c a is equal to 20 meter then d c a dash 19.6 and d c dash a 20.4 and d c dash a dash 20 to the power 1 by 4 is equal to 19.997 meter next you calculate D e q equal to 9.995 right into 9.995 into 19.997 to the power 1 third that is equal to 12.594 meter that is D e q right, but if you if you see that approximate values for D e q can be calculated quickly which is very close to exact value if we assume that you neglect all this thing.

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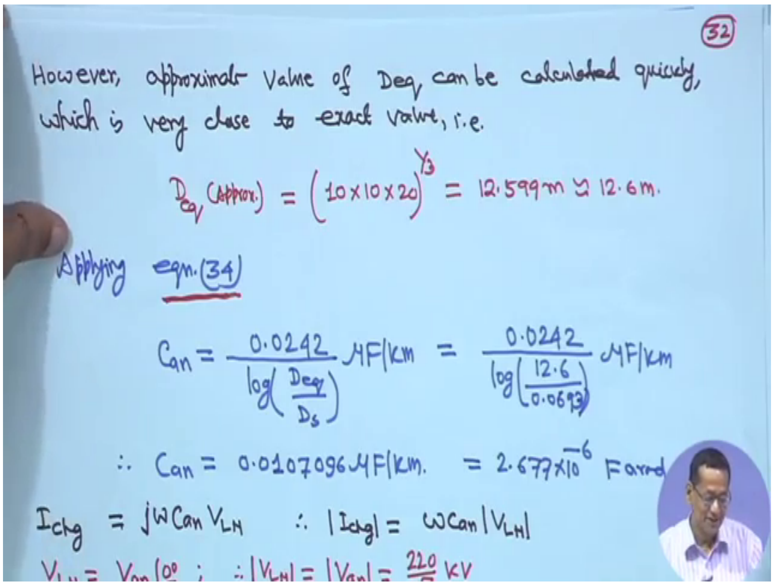

If we assume that instead of subtracting this point 4 and this that if you take 10; 10 into 20 I mean if you look at the diagram that if you take that this is 10; this is 10 and between these 220; if you take then you will find that 10 into 20; one third is equal to 12.599 meter and this one we got actually this 12.599 and this one you got 12.594. So, 599 and 594 there is almost no difference it is approximately 12.6 meter this is also approximately 12.6 meter hardly any difference.

But from the class room purpose or the numerical purpose you have to solve this, but from the research purpose you can take this one right. So, that is your that is no difference of course, right, but you have to see that your approximately they are same, but anyway if you apply now equation 34 C a n will be 0.0242 divided by log D e q upon D s microfarad per kilometer you substitute D e q and D s here.

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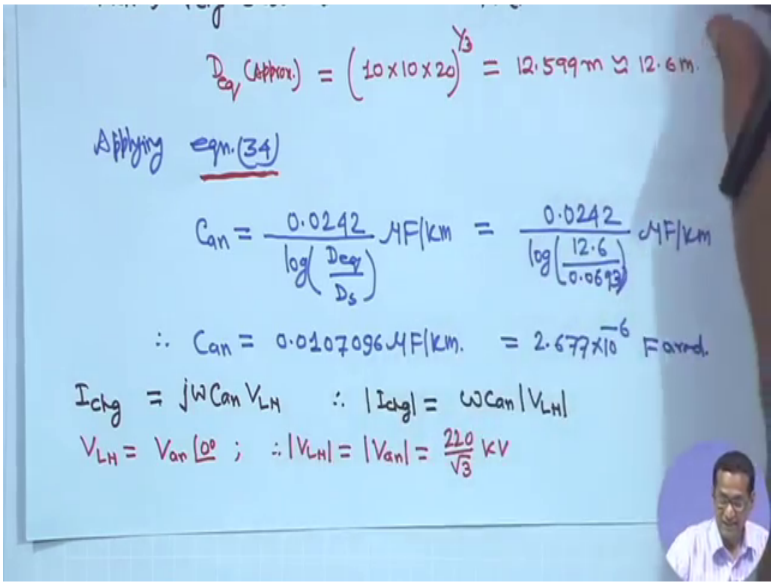

All this you will get C a n is equal to 0.0107096 microfarad per kilometer is equal to if you write in farad 2.677 into 10 to minus 6 farad right because your line length is 250 kilometers I think when we took that that line length is 250 kilometer right. So, it is your microfarad per kilometer that this value you have to multiply by 250 right. So, that will be your 2.677 10 to power minus 6 farad now charging current I is equal to j omega c n into V line to neutral voltage.

Because you are taking C a n that line to neutral capacitance right into V l stands for line to neutral voltage and j the char capacitance in your generally leading current. So, j omega C a n V l n. So, magnitude of I charging is equal to j will not be there here because it is magnitude omega C a n V line to neutral. So, V line to neutral if you V a n is the reference Phasor then it will be V a n angle 0 therefore, V mod V l n is equal to magnitude V a n is equal to 220 upon root 3 kilo volt right because line to line voltage you are given 220 KV, so, 220 upon root 3 because you need line to neutral voltage V l n right then magnitude of charging current right.

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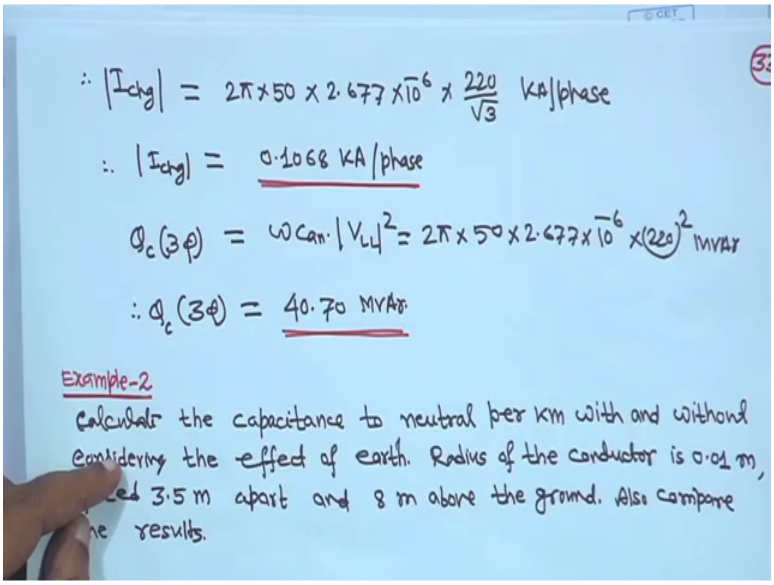

Magnitude of charging current is equal to 2 omega 2 pi f into 2.677 10 to the minus 6 into 220 by root 3 kilo ampere per phase that is I charging current will be 0.1068 kilo ampere per phase right now q c 3 phase is equal to omega c n into V line to line voltage square right. So, basically it is V square in general V square upon x c x c is equal to 1 upon omega c. So, here also it is equal to V line to line square by your x C a n right that is actually one upon omega C a n it will go to your what you call numerator. So, omega C a n into V l l square substitute omega 2 pi a your C a n also 2.67 10 to power minus 6 and V l l is 2 twenty whole square mega var right this is actually 40.70 mega var. So, this much var is injected.

Thank you.