Power System Analysis
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Lecture - 16
Power System Components and Per-Unit System (Contd.)

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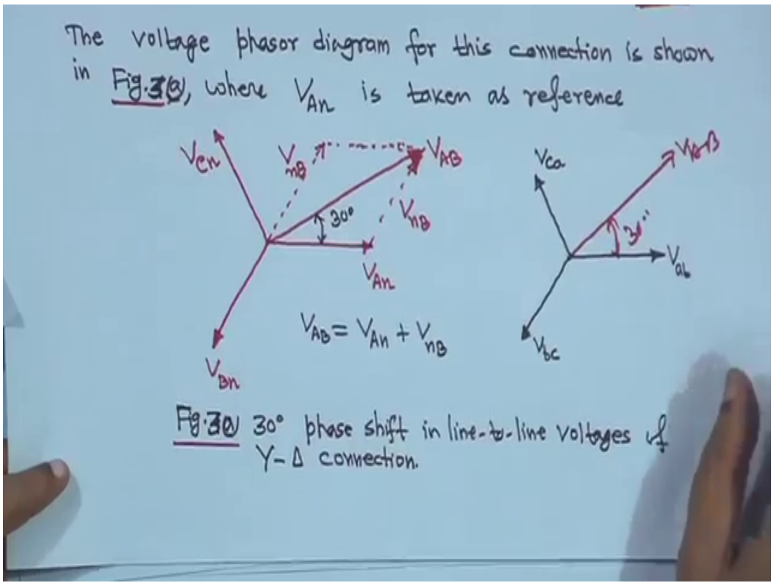

So, so we have seen that line to line voltage right, on the star side leading the line to line voltage delta side by angle 30 degree right. We will come to the per unit slowly and slowly.

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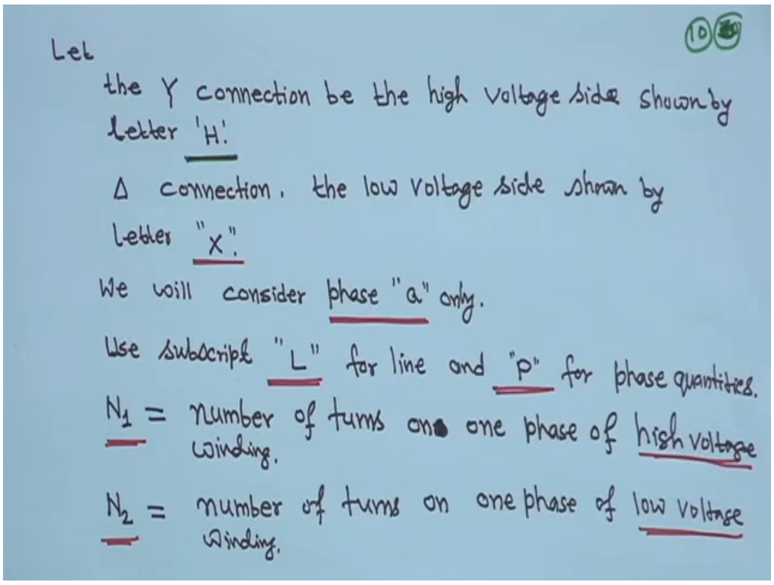

Now, this thing you have to understand little bit look. What we will do let the star connection, Y means the star connection be the high voltage side shown by letter H then for look, because why that A B C 3 phases are there. So, A B, B C, C A several times we have to utter those things instead of that. We are telling in the beginning also I said the star side should be always high voltage side. So, the star connection be the high voltage side shown by the letter capital H. Will high voltage with stand for H we will take H stand for high voltage right. The delta connection the low voltage side shown by letter X right. Will that that will consider X later we will come to that right.

So, star high voltage side we will take H, but low voltage side we will take x, but not L because L will stand for line right. So, delta connection the low voltage side will be shown by X right; that means, that; that means, your star side, that high voltage side we represent by H and delta side low voltage side we represent them by X right.

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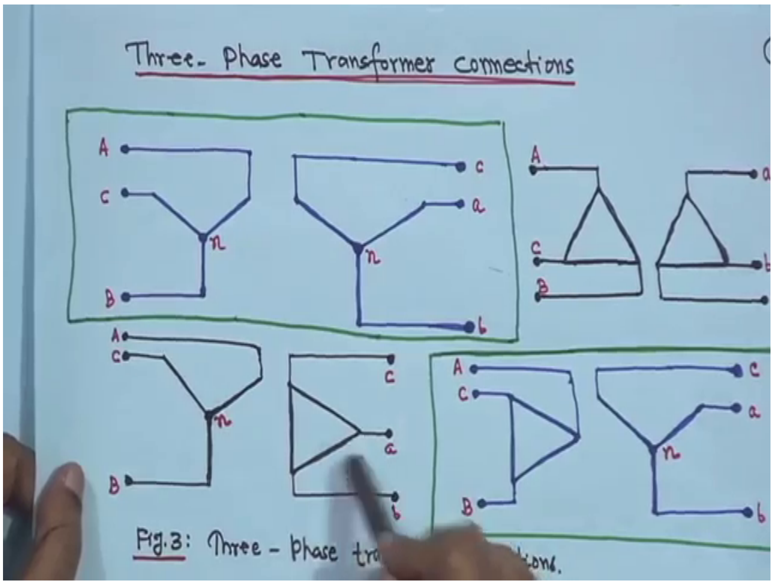

And the subscript L capital L will for line L stand capital L will stand for line for only for this purpose right. And P capital P for phase quantities, not for power just for phase quantities right. Not for power phase voltage phase current like this it is for this transformer only the P stands for phase quantity particularly voltage right. N 1 number of turns on one phase of high voltage winding right; that means, later will show; that means, this part this part, this part or this part number of turns is N 1. And delta side this side is number of turns in each phase is N 2 right. Therefore, N 1 number of turns on one phase of high voltage winding, we call high voltage instead of A B we say A N B N C N we will cause high voltage high voltage side. And N 2 is number of turns on one phase of low voltage winding. So, it is, it is number of turns means it is that turns in your either in A to N windings or B to N or C to N and for delta side it is either A B, B C or C A. So, N 1 is the number of turns on one phase of high voltage winding and N 2 is equal to number of turns on one phase of low voltage winding right. So, instead of N H or N X. We are making N 1 and N 2 right.

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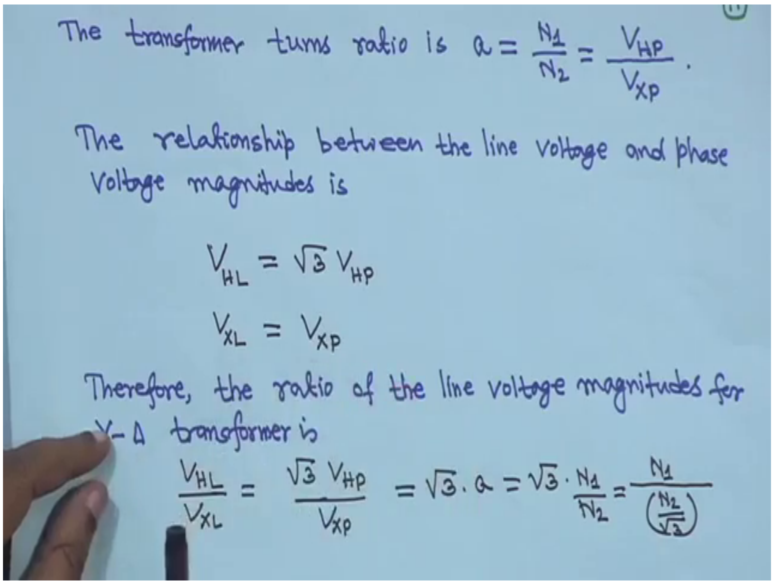

Now, little bit you have to understand here right. Now the trans transformer turns ratio is equal to a is equal to you know N 1 upon N 2 right. So, it is high voltage by low voltage side N 1 upon N 2 right. Is equal to this is equal to this is high voltage phase voltage by low voltage side phase voltage. This H stands for look, now H I told you H stands for high voltage and P stands for phase; that means, high V HP means, V capital H P means high voltage side phase voltage and X is the low voltage side and P phase V XP means low voltage side phase voltage So that means, this A is equal to your high voltage side phase voltage means if you take this one for any phase it will be V AN right. And low voltage your low voltage side delta side it is line and phase voltage same, it will be either V AB or V BC or V CA. So, instead of writing this we are writing high voltage side phase voltage; that means, it will be A to N or B to N or C to N and this is low voltage side phase voltage it will be V AB V a C or V CA, that ratio is N 1 upon N 2 that is V HP upon V XP. I hope you have understood this right. The relationship between the line voltage and phase voltage magnitude is V high voltage side line voltage, is equal to root 3 times V high voltage side phase voltage. I mean if this is your high voltage star side if this is the high voltage for example, one example, your V AB is equal to say right. Root 3 times you can take V AN right. Because they are balanced right. Magnitude wise I am taking or root 3 B N or root 3 C N. So, in general instead of telling like this we are telling V high voltage side line voltage right; that means, V AB V BC or V CA on the star side right. So, H stands for high L stands for line. So, V high voltage side line voltage is equal to root 3 times V high voltage side phase voltage; that means, this case if you take star side I mean V AB is equal to root 3 say V AN or root 3 V BN or root 3 V CN because this is balanced magnitude only and balanced right.

Similarly, for the low voltage side that is X stands for low voltage side line voltage is equal to low voltage side phase voltage. Because it is a delta this is your this side is your low voltage side. So, it is delta connection. So, line voltage and phase voltage both are same that is why we are making it as V XL V low voltage side line voltage is equal to V, X stands for low voltage low voltage side phase voltage. I think it is under it is now clear to you right. Therefore, the ratio of the line voltage magnitudes for star delta transformer is V HL upon V XL, V HL upon V XL is equal to root 3 V HP upon V XP is equal to root 3 into A right, and a is equal to N 1 upon N 2. That is root 3 into N 1 upon N 2 is equal to I am writing N 1 divided by N 2 upon root 3 later we will see what is the meaning of this right. So, V HL upon V XL is equal to basically root 3 into N 1 upon N 2 right. This is the high voltage side line voltage, by the ratio of the high voltage side line voltage to the ratio of low voltage side line voltage, that is actually root 3 into N 1 upon N 2 right.

Next, here I am telling only later we will see basically in dealing with your star delta or delta Y banks right.

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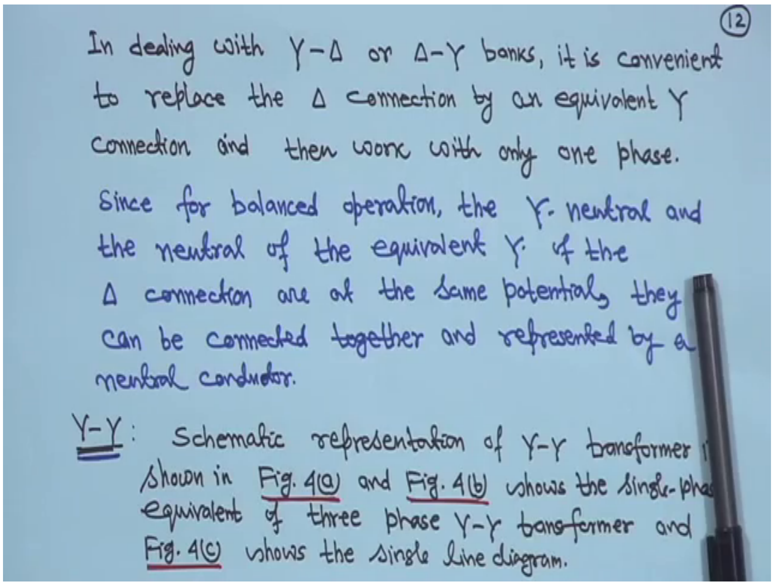

I mean star delta or delta Y banks it is convenient to replace the delta connection by an equivalent star connection. Later we will see that. That when is star delta transformer or delta star transformer, that delta can be represented by equivalent your what you call star connection right. And then we will work only with the one phase. Since for balance operation the star neutral and the neutral of the equivalent star of the delta connection are at the same potential. So, they can be connected together represented by a neutral conductor. So, when we are, when we are converting because it is a balanced system. So, when we are converting delta to an equivalent star right. So, that your neutral point of that delta of the star connection which is converted from delta, and the neutral point of the star they are at equipotential. So, they can be your what you call, they can be your represented they can be connected together such that represented by neutral conductor, such that they can be represented by single line diagram. Later we will see that right

Now, So far we have shown that schematic diagram without the windings. Now this first you consider the star, star. Star, star connection. So, schematic representation of star, star we have already seen now we will come little bit in detail. The star, star connection is shown in figure 4 a and 4 b I am coming to that shows that the single phase equivalent of 3 phase star, star transformer and figure 4 C shows the single line diagram.

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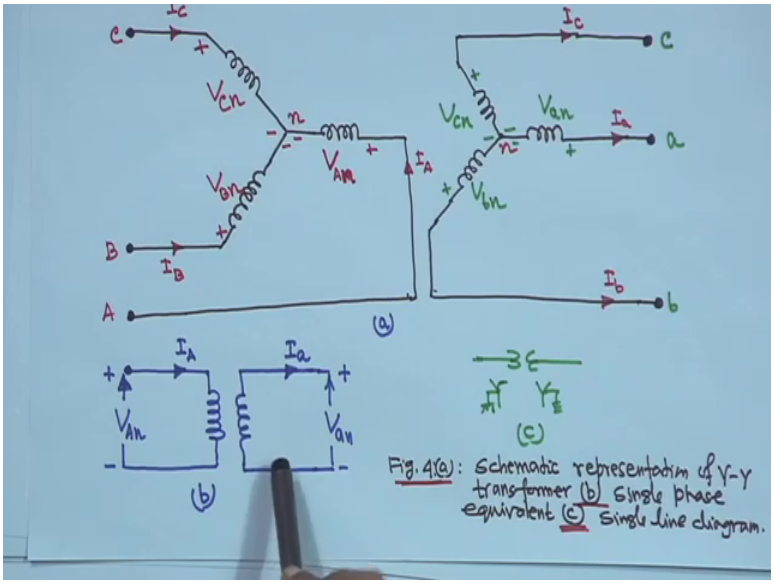

Now as it is a star, star as it is a star, star. So, this is your A B C this is neutral N this is N earlier in the diagram somewhere I showed that the very first in any one, but here it is correct. So, here it neutral here it is neutral right. So, everywhere polarity is marked plus minus plus minus plus minus. So, this volt this is V AN this is V BN this is V CN and current it is I a this is your I B and this is I c right. And this is your this is star, star connection only star, star right. And this side is there your other side also this current is this secondary side. So, current is going out. So, I c this is I a this is I B phase c phase a phase b right. And as it is a balanced system. So, this can be represented by an equivalent single phase only, single only phase a is drawn. So, this is plus minus this is the polarity look even if I do not mention polarity, if arrow is here; that means, it is positive right. And if nothing is no arrow will be here at the bottom. So, it is negative whenever arrow tip is showing means it is a positive. Even if I missed anywhere, but if arrow is there means it is a positive. So, this is the current I a and that is the primary that is the your primary side and this is that I a is small a subscript is small a right. And this is this V AN, this is V AN. This is A this is V AN right. This is A V AN. So, A capital N and this is V AN small N this is your V AN, V AN this small a and equivalent one as star side here it is not shown, but here it is not shown, but star side is always grounded this is the star, star and this is the star, star transformer equivalent circuit representation right. This is a single line diagram transformer star, star, but later when you study the load flow studies we will see much. But here magnetizing component will be neglected for this representation right.

So, the first one is that schematic representation of star, star transformer. This is the single phase equivalent and this is the single line diagram of the transformer right. Now will come to star delta. Now this is star delta right. Here this side is star side right. So, we are not talking about much about high voltage or low voltage.

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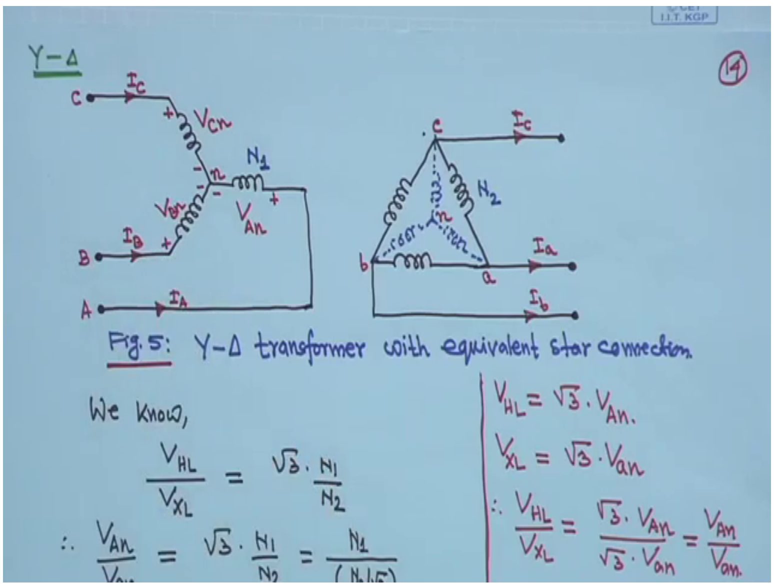

Now, but this side is always high voltage, this side is always low voltage right. So, for each phase that number of turns N 1 here also N 1, N 1, but one phase is N 1. And this delta side N 2, N 2, N 2 here it is saying N 2 right. And it is star side same as before plus minus plus minus plus minus. So, this is line to neutral voltage V AN for phase V BN this is V CN. And similarly for delta side it is a, b, c right. And this is equivalent star I will come to that later on right. And your turns per phase is N 1 here also line and phase same voltage. So, it is N 2 right. Now we have we have just discussed no that V high voltage side line, voltage by V low voltage side line voltage is equal to root 3 into N 1 upon N 2 just now we have seen right. No equation number is given here; that means, V high voltage side line voltage right. V high voltage side line voltage see, here I make it is equal to root 3 times say V AN right. Any line voltage V AN is the phase voltage A N, V AN is the phase voltage. So, high voltage side is star side this is star side. So, this is V AN is equal to root 3 V AN and V low voltage side line voltage is equal to root 3 into V AN right; that means, this V AN actually equivalent star right. We are assuming this delta can be made an equivalent star that is why that is why in reality this reality this star is not there for mathematical representation this is an equivalent star and neutral is available. Right. N neutral is available; that means, for delta case if this equivalent star is available then V your X is stands for low voltage side. So, low voltage side line voltage right; that means, that means V AB V BC or V CA any of these right. Anyway is equal to root 3 times into V AN. That is the equivalent star that is the your line to neutral that is the phase voltage right.

So, this one you can write therefore, V HL upon V XL, that is V HL upon V XL is equal to root 3 V AN divided by root 3 V AN means V capital A N divided by root 3 V small a n this; that means, V AN is the phase voltage of the star side and this V AN this is that your equivalent delta has been converted to equivalent star that is the phase voltage of this equivalent star V small a n, V small a n root 3 root 3 will be cancel is equal to V capital A N upon V AN

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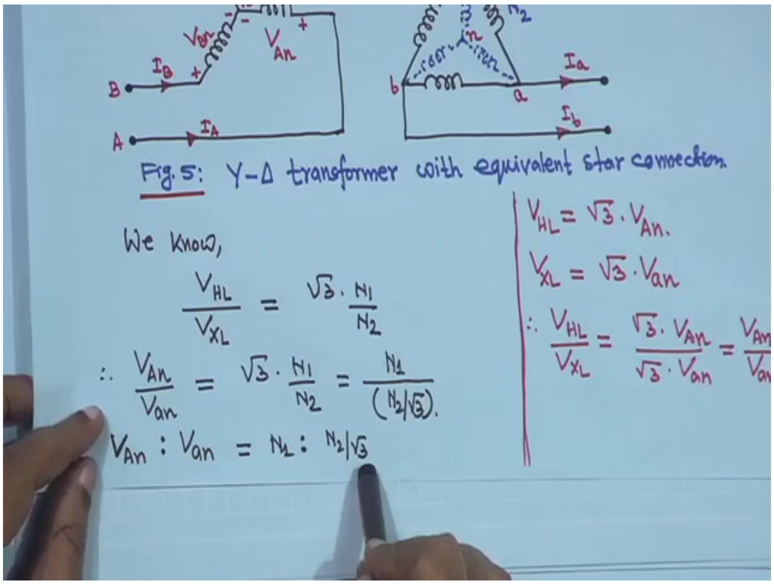

This much you have understood? Therefore, this V HL upon V XL V HL upon V XL is equal to V AN upon V AN; that means, this V AN V capital A N upon V AN is equal to root 3 into N 1 upon N two; that means, it is N 1 divided by again we are writing N 2 upon root 3 that is V AN is to V AN is equal to N 1 is to N 2 upon root 3. As if this turn was N 2 I mean meaning is something like this.

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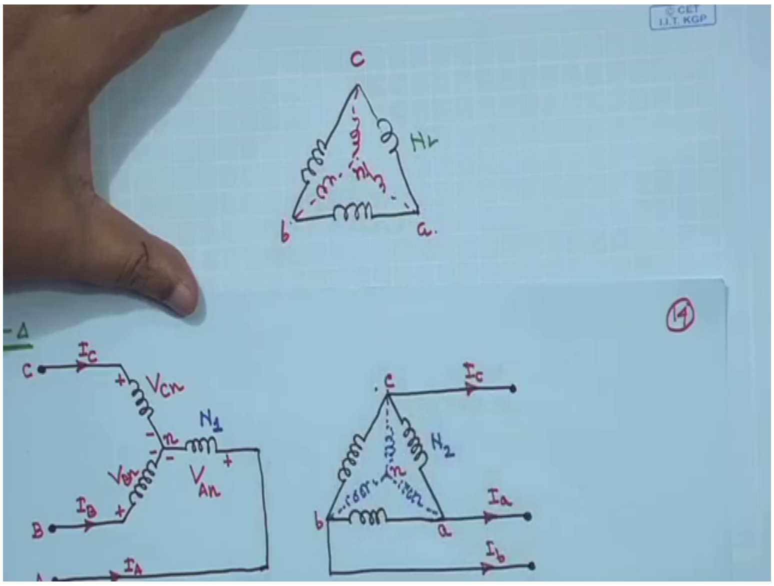

I am redrawing for you here right. Just see this one, this one right. This is your this point is a this is a this is your b and this is c An equivalent star is like this right. Equivalent like this. And this is neutral this is neutral right. And; that means, turns is turns is N 2 this is N 2 in each phase, but as we have taken V AN upon V N 2 by root 3, as if turns of equivalent star actually I am making it here. These turns of equivalent star actually it has become N 2 by root 3 right. So, this have been as if, as if after converting into star connection instead of N 2 it has become N 2 by root 3. That is why, that is why this V AN upon instead of writing this N 1 is to N 2 upon root 3 the ratio of line to neutral voltage on star side to the line to neutral voltage on the delta side is equal to N 1 is to N 2 upon root 3.

Hope this you have understood, no? If you have understood this things are fine. I have tried all these things to tell you right. I hope you have understood, the meaning of H and X all have been made it right. And this one will help you actually to understand it right. Similarly for delta star also meaning is same not doing it for you, but meaning is same. As if this equivalent star that is each phase the number of turns has become N 2 upon root 3 right. Reality it is not there, but from mathematical point of view it is like right, that the meaning is like this right.

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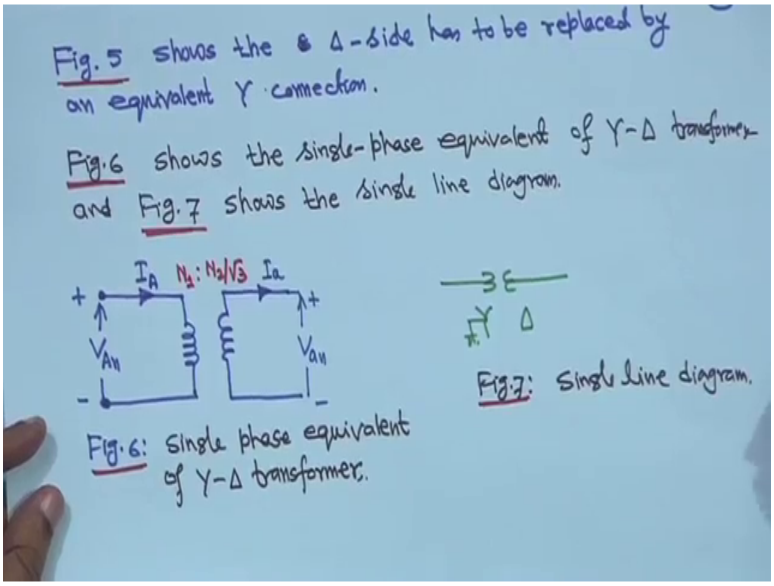

So that means, figure 5, now this is that this is that delta side if you replace by star it is like this, that is why here I have written that figure 5 shows the delta side has to be replaced by an equivalent star connection right.

Now, figure 6 shows the single phase equivalent of star delta transformer, and figure 7 shows the single line diagram. Now as this side is star this is also we are taking equivalent star for single line diagram, but not delta right. That is why that is why this side is A N capital A N plus minus this is V AN I a this side turns is N 1, but this side is N 2 by root 3 and I a because this is this is it is equivalent star is taken, equivalent star is taken right. And this side is V small a n right. And this is your single phase equivalent star delta transformer diagram. And this is that your single line diagram. Star is always grounded not shown there, but you have to show a star is always grounded. And this is delta this is single line diagram of the star delta transformer. I hope you have understood this right.

So, as and once again, as if it is an equivalent star from the delta N turn is N 2 by root 3, but not N 2 right. It will be N 2 by root 3 right. Next is now we will come to the per unit system right.

So, after making all these things, now per unit system will be easier right. So, power system quantities because you have 4 quantities mainly. Current voltage impedance and power. These 4 quantities are there right. So, power system such as current voltage impedance or power, are often expressed in per unit values right. For example, suppose if base voltage is 220 k V right.

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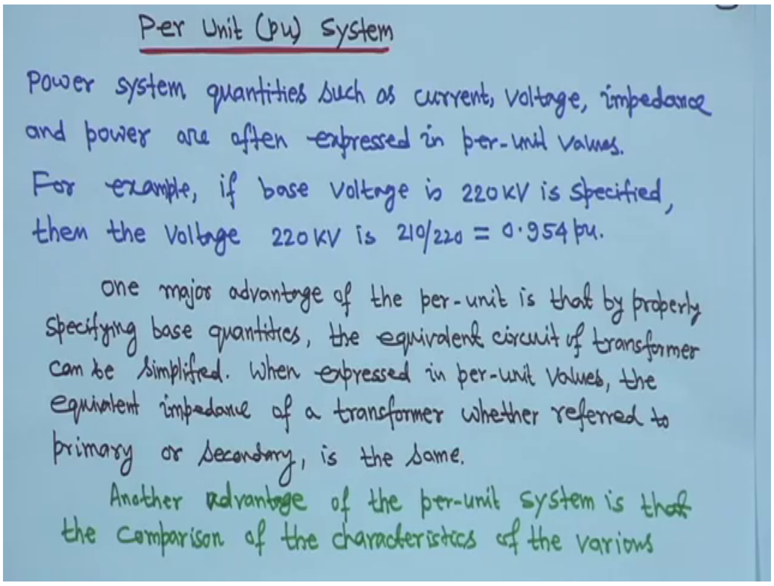

Suppose any line any suppose, you have a 220 k V transmission line that is base voltage. Suppose you are measuring the voltage of the transmission line it is a line to line voltage suppose it is 210 k V therefore, in per unit it will be 200 by 220 that is equal to 0.954 per unit it will become a dimensionless quantity right. So, one major advantage of per unit is that it is properly specifying is that by properly specifying base quantities, the equivalent circuit of transformer can be simplified right. If you choose the proper base quantities which express in per unit values, the equivalent impedance of transformer whether referred to primary or secondary is the same. This we will see later we will prove it. That equivalent when you convert everything in per unit that equivalent impedance with refer to primary or secondary it will remain same. This is another this is major advantage right.

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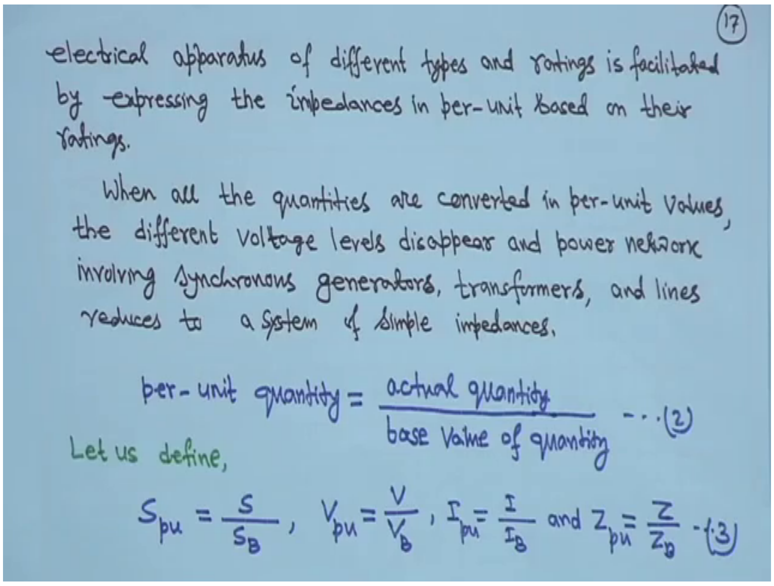

Another advantage of the per unit system is that. The comparison of the characteristic of the various electrical quantities of different type and ratings right. And the rating is facilitated by expressing the your impedances in per unit base on their ratings. I mean transformer synchronous generator lines you are choosing base quantities you can you can transform all these quantities into per unit values. This is a major advantage right. And when all the all the quantities are converted in per unit values that different voltage level will disappear in your circuit diagram. Because that suppose at generating station say may be your voltage is stepped up from generator terminal voltage say 10.6 k V. 10.6 k V to 220 k V then from 220 k V line will go then 220 k V by say, 132 k V then 132 by

66 k V. Then 66 by 33 k V and so on. Then we will come to downstream the distribution system. So, all instead of mentioning all these voltage level if you choose an appropriate base and all the voltage accordingly will be transformed then the voltage in the in your circuit in your circuit diagram that voltage levels I mean different voltage level will disappear. This is a major advantage right. So, it will be just all these things can be represented by simple impedances right.

So, per unit quantity, per unit quantity is equal to actual quantity divided by the base value of quantity, this is equation 2 right. So that means, we are making the things dimensionless and accordingly judiciously you have to do it, such that all electric laws will be valid after transforming to the per unit system. So, suppose power S pu S upon S B remember this thing right. I will come to that S is a complex quantity, but S B is a real quantity, we will come to that. Similarly V pu is equal to V upon V B right. S B is the base power V B is the base voltage. So, V upon V B right. It will be per unit similarly per unit current I pu is equal to I upon I B I B is the base current and Z pu is Z upon Z B Z B is also base impedance S B V B I B and Z B these all are real quantities right. This is equation 3.

I will I have given the nomenclature right.

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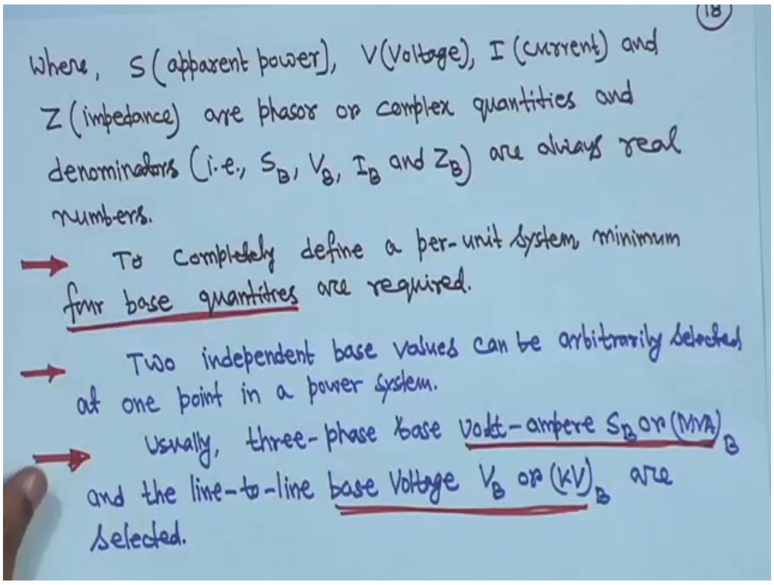

So, actually where S in general, that is your S is apparent power that is volt ampere or kilovolt ampere or mega volt ampere. That is V a k V a or MVA V is the voltage it may be in voltage in voltage or in k V I is the current ampere or kilo ampere and Z impedance in ohmic values right. Are phasor or complex quantities, because apparent power voltage current and impedance are phasor or complex quantities right. So, particularly voltage and current they are actually phasor quantities right. And that is why written either phasor or complex and your denominators example S B V B I B and Z B all are always real numbers they are real quantities right.

To completely define a per unit system, minimum your 4 base quantities are required that is voltage current your impedance and power these 4 quantities are required right. Therefore, 2 independent base values can be arbitrarily selected at one point in a power system. At least you have to choose 2 base values generally we have seen that we choose 2 things one is base your volt ampere another is base voltage, these 2 generally we choose right. So, usually 3 phase base voltage 3 phase base volt ampere S B or MVA B that is MVA Base right. And the line to line base voltage V B or k V base are selected. Generally MVA Base or line to line voltage k V base generally we select first right. For the base quantity these 3 actually for any system right.

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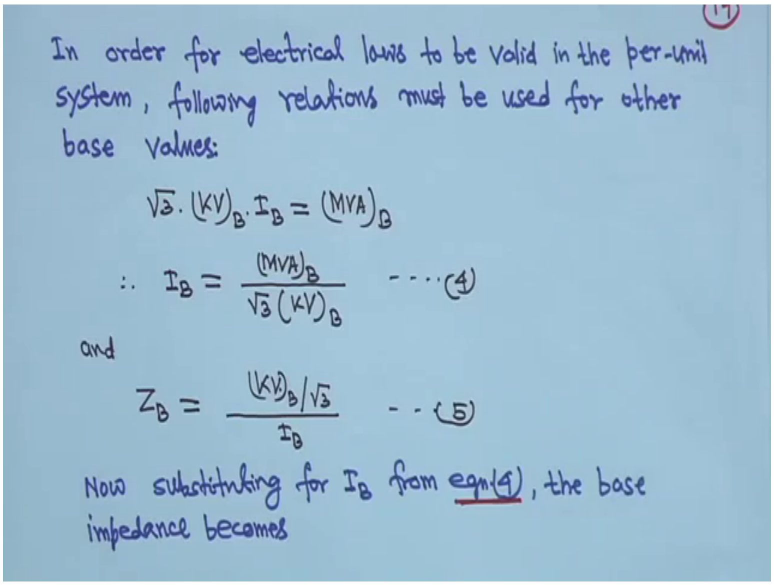

Therefore in order for electrical laws to be valid in the per unit system right. Following relations must be used for other base values. For example, the base MVA is a 3 phase 3 phase base volt ampere, I told you this is your 3 phase volt ampere and this base voltage is a line to line voltage, and base impinges is a 3 phase base voltage and MVA; that means, that means root 3 k V base into current I B base is equal m V base MVA Base right. That is I B is equal to base current from here you can calculate MVA Base divided by root 3 into k V base the suffix B is used for base this is equation 4 right. And Z B is equal to k V base upon root 3 your into I B. Because k V base is equal to line to line voltage right; that means, this k V base you divide it by root 3. So, it will be phase base voltage divided by I B, this is equation 5 this is the base impedance right.

Now, substituting for I B, that I B from equation 4, this I B from equation 4 you substitute here. This I B you substitute here, if I substitute here. You will get if you substitute if you substitute, you will get Z B is equal to k B square base upon MVA Base,

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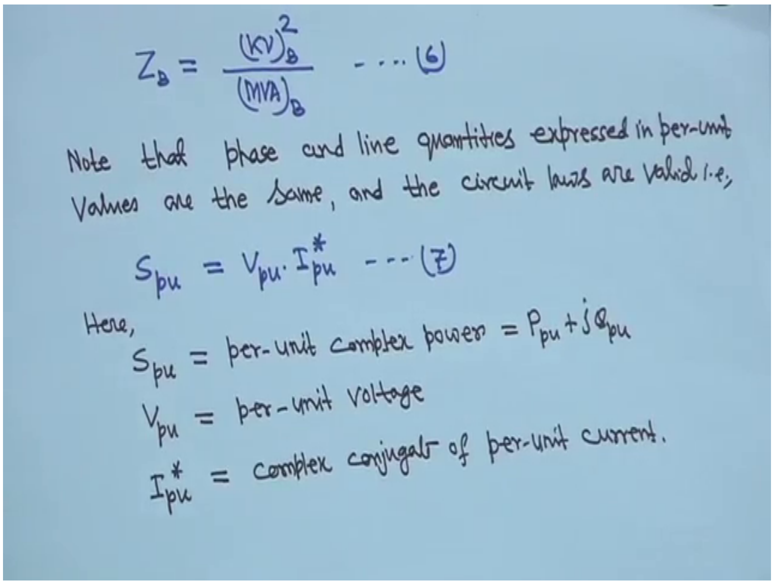

This is equation 6. This unit is ohm, it is unit is ohm right. Note that phase and line quantities expressed in per unit values are the same, and the circuit laws are valid. This is true now power is equal to S pu is equal to V pu into I pu conjugate right. So, why this conjugate comes I will come to that when we will study the your load flow studies this is conjugate comes to capture the your power factor angle, when voltage and currents are purely sinusoidal right. We will come to that for load flow studies. So, S pu is equal to V pu into I pu that S pu is per unit power that is P p u plus j Q P u right. All per unit, V pu is equal to per unit voltage and I pu star, that is complex conjugate of per unit current right. So, star means that complex conjugate.

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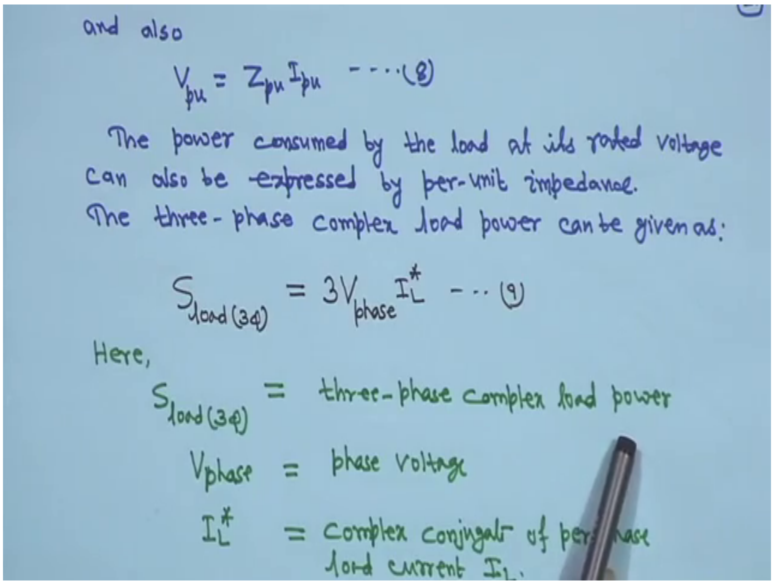

And also V pu is equal to Z pu into I pu this is equation 8. These are the standard electrical equations right. Now the power consumed by the load that is rated, voltage can also be expressed by per unit impedance right. The 3 phase complex power load power can be given as S load suffix is load bracket is 3 phase, 3 phase power is equal to 3 V phase voltage into I L the star the conjugate that load current conjugate 9 right. Where S load 3 phase is equal to 3 phase complex load power right.

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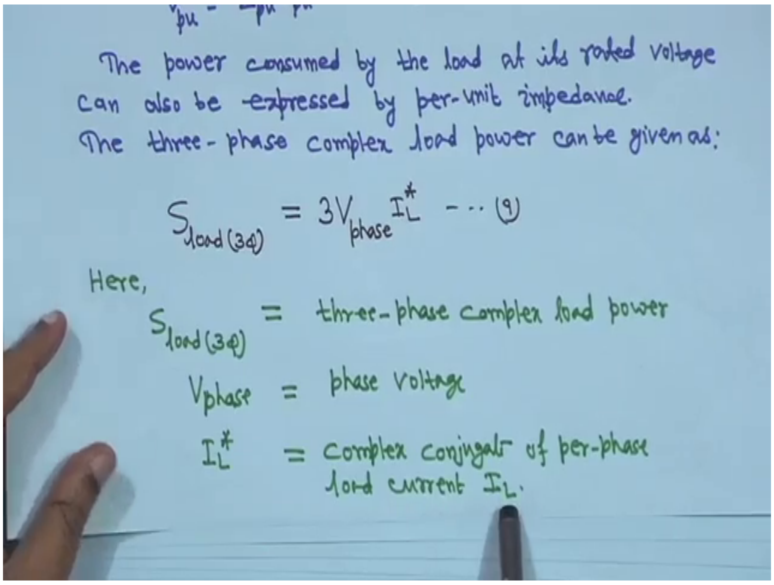

V phase is equal to phase voltage, and I L conjugate the star is a complex conjugate of power phase load current I L right. So, load power also can be made like this right. Is equation 9 V phase into I L this is a phase load phase current right. Per phase current. So, this is for 3 phase that is why 3 is multiplied understandable right.

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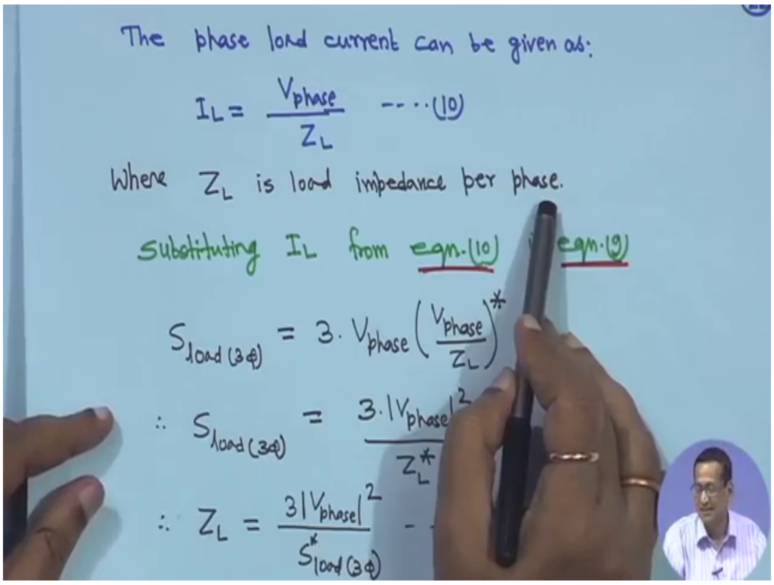

Now, the phase load current can be given as, I L is equal to V phase upon Z L Z L is the load impedance this is equation 10. Where Z L is the load impedance per phase right.

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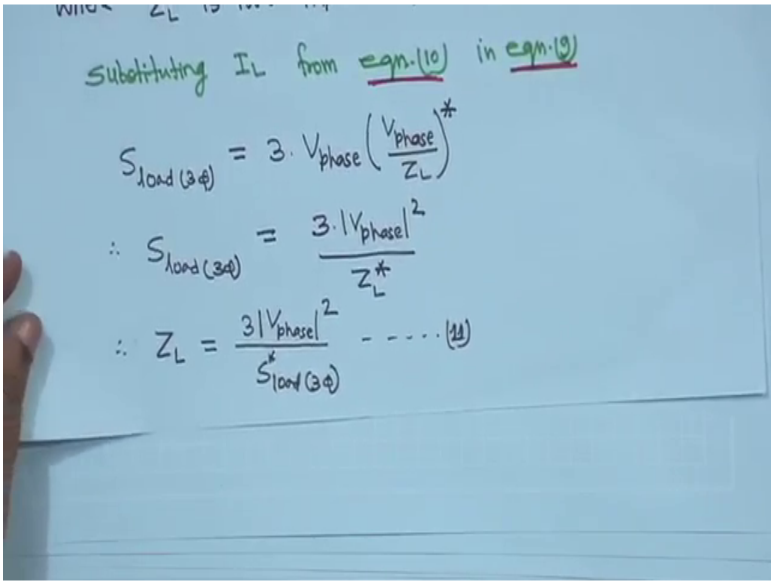

Therefore, substituting I L from equation 10 in equation 9; that means, here in equation 9 you substitute I l; that means, S load 3 phase will be 3 into V phase, then V phase upon Z L star that is conjugate right. Is equal to V phase look V phase upon Z L conjugate means V phase conjugate divided by Z L conjugate. So, V phase into V phase conjugate will be 3 into V phase magnitude square right. Divided by Z L conjugate right. I hope you know this no V phase V, V into V conjugate is V square right. So, V phase into V conjugate magnitude V phase square divided by Z L conjugate.

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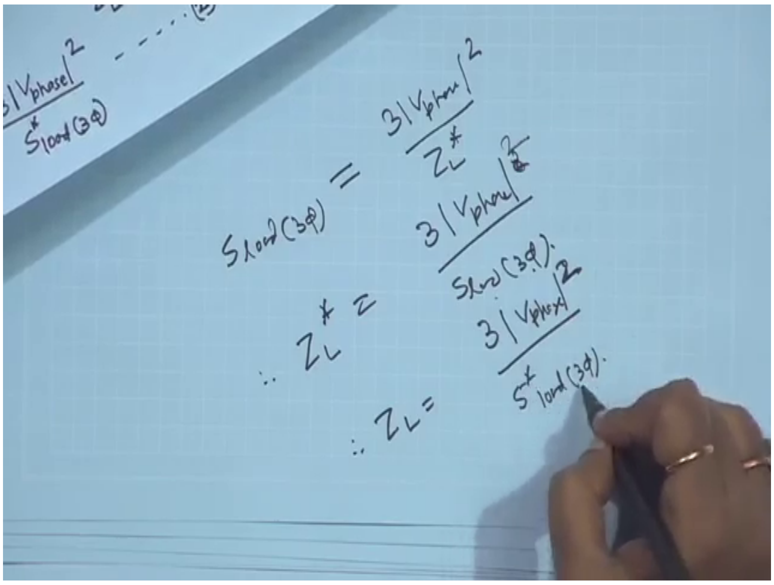

That means; that means, z; that means, Z L here I am writing for you that S load 3 phase is equal to 3 V phase square divided by Z L conjugate; that means, Z L conjugate is equal to 3 V phase square divided by S load 3 phase; that means, Z L is equal to 3 this is voltage magnitude. So, nothing to deal with that square as it is if we take conjugate on both sides you take conjugate on both side it will be S your conjugate load 3 phase right. So that means, this equation we are writing this equation we are writing here then we are writing Z L conjugate equal to 3 phase voltage square divided by S load 3 phase; that means, take conjugate on both sides Z L this is a real quantity. So, nothing to do with that. So, 3 V phase square, but this is a complex quantity. So, S conjugate load 3 phase right.

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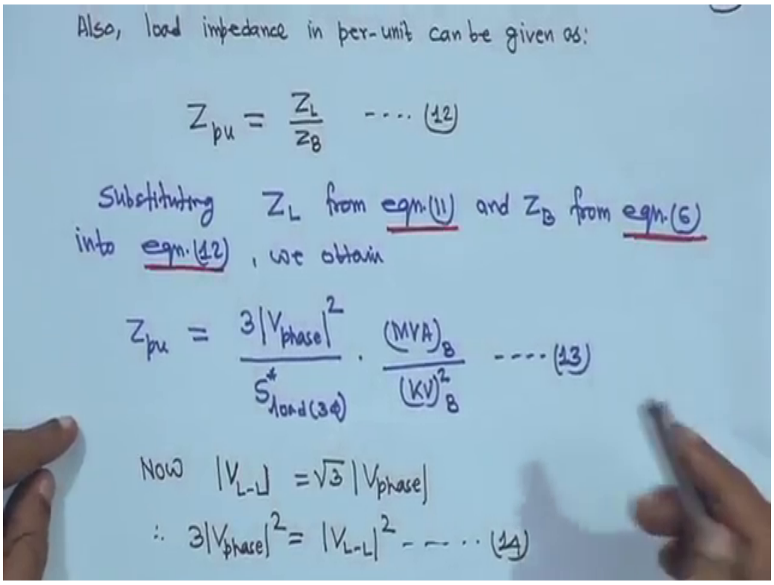

Therefore this we are writing Z L is equal to 3 your magnitude V phase square S conjugate load 3 phase that is your equation number 11 right. So, also load impedance in per unit can be given as Z pu upon Z L upon Z B as usual this is equation 12. Now substituting Z L from equation 11 right. And Z B from equation 6 into equation twelve. So, from equation your from equation your 11 this expression of Z L this is equation 11 this Z L you substitute here and from equation 6, from equation 6 this Z B k V square base upon k V square base upon MVA Base 6 you substitute in equation 12 right. If you do So, if you do So then, your this one it will become Z pu is equal to 3 into V phase square magnitude square divided by S conjugate load 3 phase into MVA Base upon k V square base this is equation 13 right.

So, now magnitude V line to line voltage L dash L I have made is equal to root 3 times phase voltage magnitude of phase voltage right; that means, square it on both sides and bring this on this side. So, 3 into magnitude of V phase square is equal to magnitude V line to line voltage square, this is equation 14 right. This is equation 14. And next using equation 13 and fourteen; that means, these 3 into V phase square is equal to magnitude V LL square you substitute here replace this 3 phase V phase square by this V LL square because this is equal to this one.

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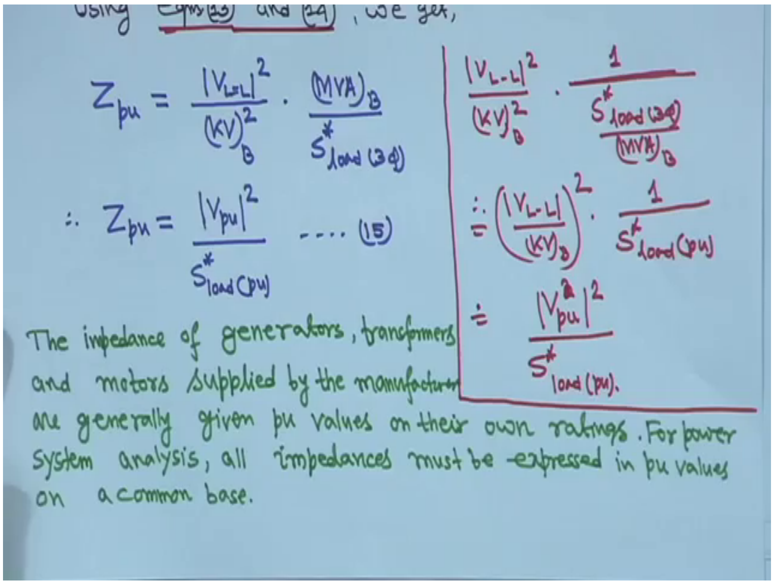

So, if you do so, if you do so, it will come Z pu is equal to V line to line voltage square divided by k V square voltage into MVA Base by S star load 3 phase right. So, simplify it will becoming like this right. Now here your what you call this is actually this one that V LL square upon k V square into this one can be written as one upon S star load 3 phase I upon MVA Base. This equation I am rewriting to a right hand side for your understanding, that magnitude of V LL square upon k V B square it is and this one we have this part this part we are writing one upon S star load 3 phase by MVA Base. So, in bracket we are writing line to line voltage magnitude on k V B whole square into and this is your S load this conjugate 3 phase by MVA Base; that means, we are writing this part this S star that is conjugate load now P u no question of 3 phase now because this part is divided by the base right. That is why it is pu, and this is also V line to line voltage by k V base. So, this is also V pu, this is you are writing actually magnitude V pu square upon S square this conjugate that load per unit power. That is why this Z pu is equal to we are writing this equation is equal to Z pu is equal to V pu square by S conjugate load P u this is equation 15 right. So, so this part simple calculation is made it here for your understandable understanding that Z per unit is equal to V pu square upon S conjugate this is conjugate load and it is per unit load right. Load per unit load, but it is conjugate right. This is equation 15, so.

Thank you.