Power System Analysis

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Lecture - 22
Characteristic and Performance of Transmission Lines (Contd.)

Next you come to the voltage regulation right. So, voltage regulation of the transmission line may be defined as the percentage change in voltage at the receiving end of the line right.

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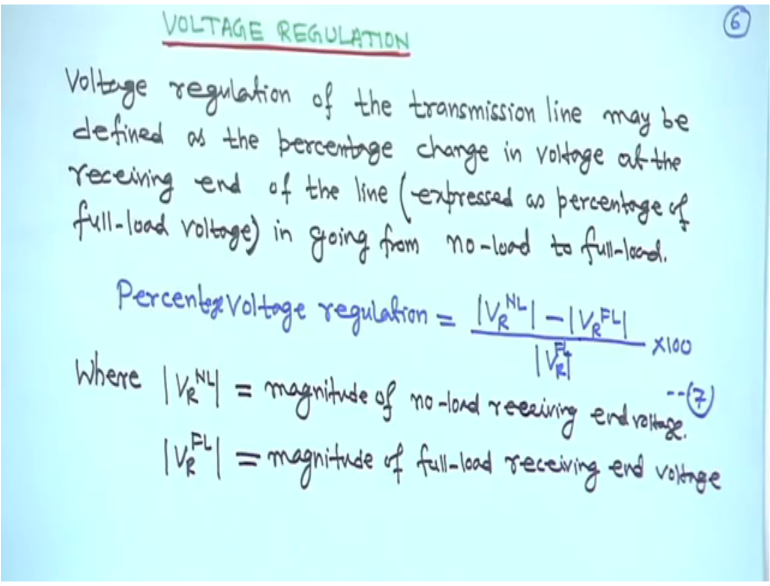

Expressed as percentage of full load voltage, in going from no load to full load; that means, that is your percentage voltage regulation it can be given as this magnitude load everywhere, magnitude is there wherever we take it will take magnitude only, but again and again I will not say magnitude magnitude, I will just simply I will say receiving end sending end like this right. So, it is actually V R N L that is the receiving end no load voltage magnitude minus this is receiving end F L there is a full load voltage magnitude, divided by with respect to your receiving an full load voltage.

So, V R F L magnitude into hundred; that means, where V R N L everywhere mod is there. So, magnitude of no load receiving end voltage and V R F L magnitude of full load receiving end voltage right. Now this is that percentage voltage regulation right. So, it is defined as the percentage changes voltage at the receiving end of the line in going from no load to full load that means that at no load right.

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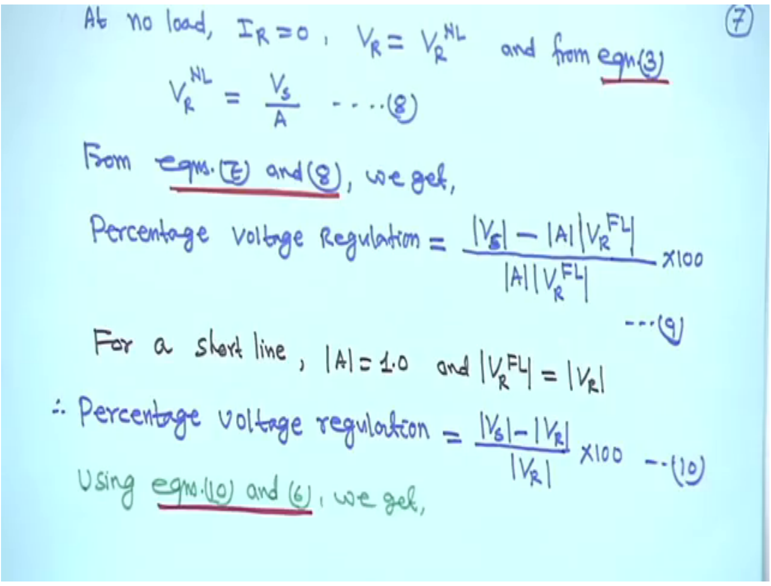

I R is equal to 0, at no load I R is equal to 0; that means, you are just one minute just hold on. So, at no load when I R is equal to 0; that means, this equation you put I R is equal to 0.

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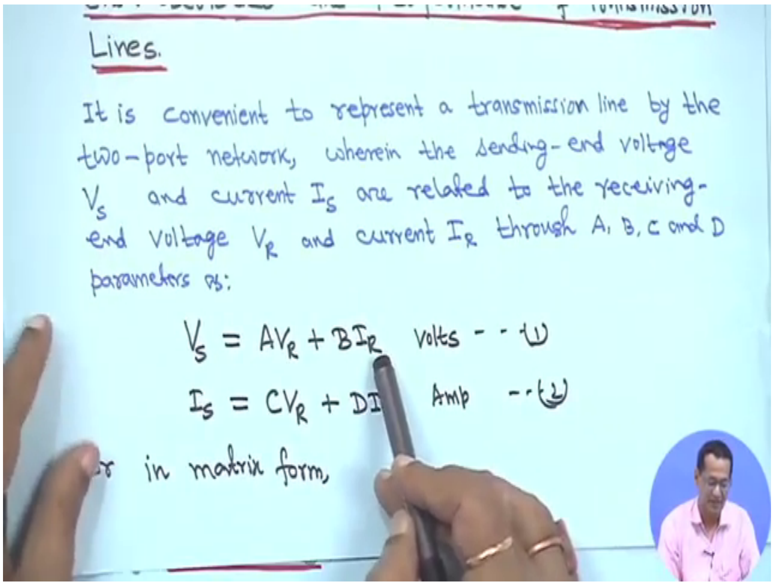

In equation 1say then you will get and at I R is equal to 0 V R is equal to V R no load. So, at this equation put I R is equal to 0 and here you put V R is equal to V R N L no load, then you will get V R N L is equal to V s upon a this is equation 8 right. Though this is understandable from this equation, you will put I R is equal to 0 and V R is equal to V R N L. So, you will get V R N L is equal to V R V s upon a this is equation 8 right therefore, this V R no load is equal to V s upon a you put in the previous equation; that means, this equation this equation 7 right; that means, in this equation you took V R N L is equal to your V s upon a right. So, once you took V R N L is equal to V s upon a. So, it will put and simplify little bit it will be magnitude V s this is V s minus magnitude a into V R F L divided by magnitude of a into V R F L this is equivalent to 100 right. So, this is equation 9; now for this short line you have seen that A is equal to 1. So, mode A is equal to 1, mode A is equal to 1 and for a short line V R full load is equal to V R right.

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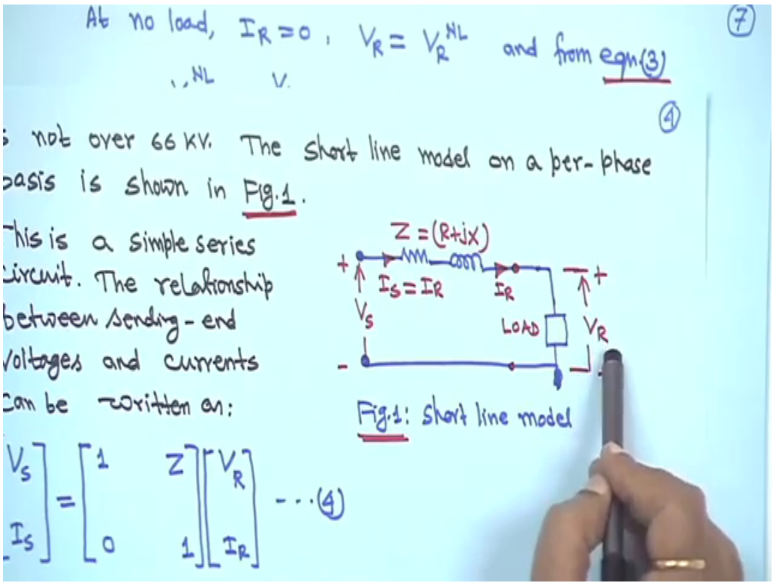

Then this diagram for a short line, V R full load is equal to V R. So, if it is so, then percentage voltage regulation can be written as that V s minus V R upon V R all are magnitude into 100 right. So, this is equation 10. Now using equation 10 and 6 you will get right that means, in this case that where we go back to equation 6; that means, this one just hold on this one.

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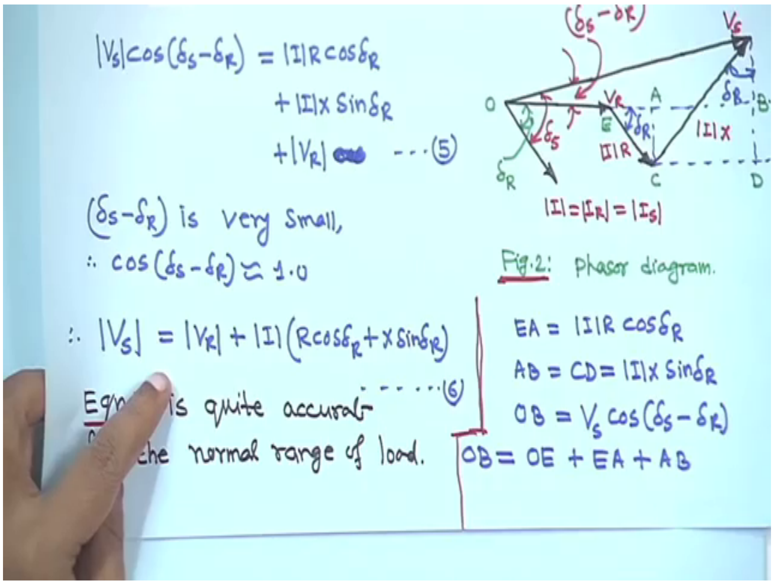

This one that is equation 6, it is magnitude V s is equal to this one V magnitude V R plus this term; that means, magnitude V s minus magnitude V R is equal to magnitude I into R cos delta plus x sin delta. So, in this expression equation 10 magnitude replace magnitude V s minus magnitude V R by this term that magnitude I into R cos delta plus x sin delta right. If you put it then it will this equation will become that percent volt regulation will become magnitude I R cos delta R plus x sin delta R upon magnitude V R into 100.

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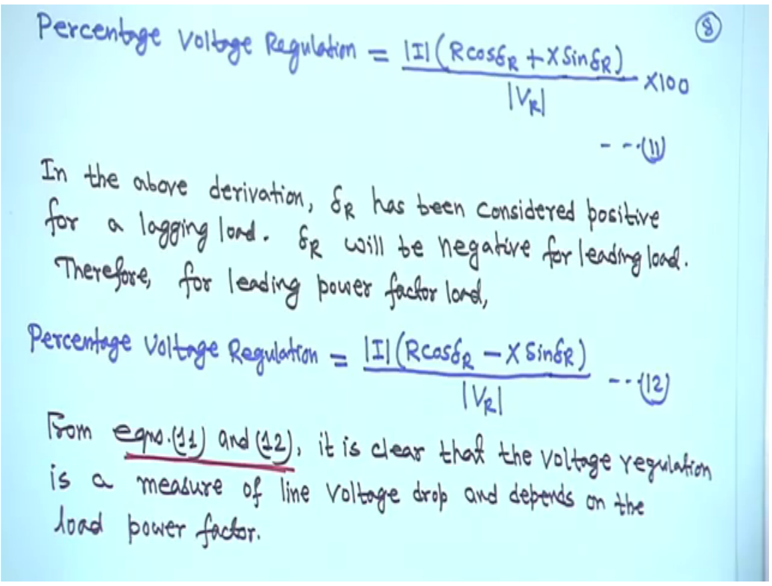

This is equation 11, in the above derivation delta has been considered delta R has been considered positive for a lagging load right. According to our phasor diagram, we have considered that your lagging load delta R is positive this is delta its positive right. So, and delta R will be negative for leading load therefore, for leading power factor load, this magnitude I R cos delta R minus x sin delta R because if delta R is negative then cos minus delta R cos delta R sin minus delta R minus sin delta. So, minus x sin delta of a magnitude V R this is equation 12 right. From equation 11 and 12 it is clear that the voltage regulation is a measure of line voltage drop, and depends on the load power factor because this is basically it is a line voltage drop either this term or this term this is basically line voltage drop, because magnitude V s minus magnitude V R is the line voltage drop is equal to magnitude I into this term right.

So, this is basically your what you call that is line voltage drop and it depends on the low power factor right of course, it depends on the low power factor because you have to this thing what you call because delta R, this delta R actually it is low current angle because this is your this is the current I this is basically current is going to the load right so; that means, this current is this current is going to the load. So, that is angle and its angle is delta R right this green color delta R; that means, this depends on the your load power factor right. So, basically its load angle and not current angle delta R, and the receiving voltage right; that means, it depends on this equation 12. So, this is for short line what will do? You will make all this mathematical expression long line, medium line and short line medium line and long line after that will see that numericals right.

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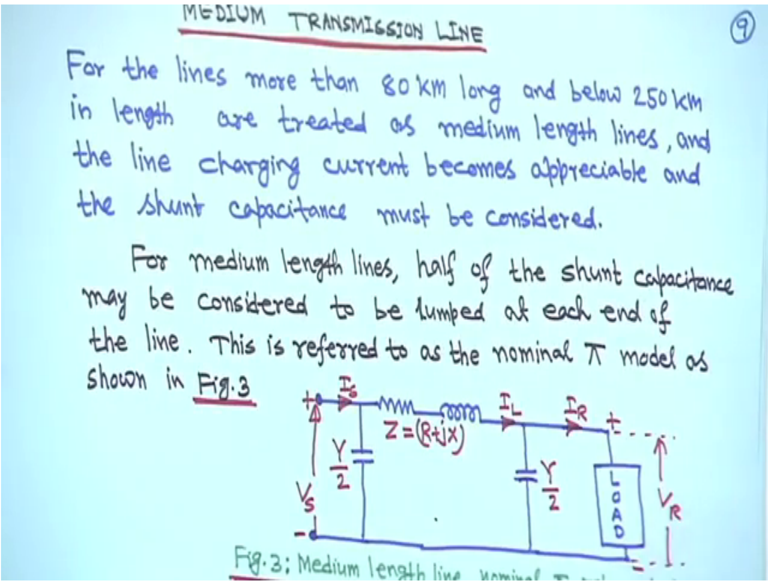

Next is the medium transmission line. Generally we call the 80 kilometer line, long more than 80 kilometer and below 250 kilometer right length is a basically we call medium length lines, and the line charging current becomes appreciable and the shunt capacitance must be considered for the such a long line right. So, in that case you have to consider the charging capacitance or shunt admitted is this thing you are admitted you have to consider.

So, now in this case suppose this is a medium long line and representation by a nominal pi representation right; that means, whatever your charging admittance are there total charging admittance are there, you lumped on both side half half; that means, this side Y by 2 and this side Y by 2. So, this is called a pi network and this is the transmission line Z is equal to R plus j x and this is the load right. So, sending end voltage is V s receiving end voltage is V R right and here what you call and this is your sending end current I s right here I C 1, I C 2 not shown right current through this is a I S this is I L and again this receiving current I R this is going to the load this is I L. I L stands for not load for line right and receiving in current I R this is actually load current right and I L stands for I line this is line, this are representation this is the representation this way you represent and this is the load and this is the voltage V R there is voltage across the load is V R right.

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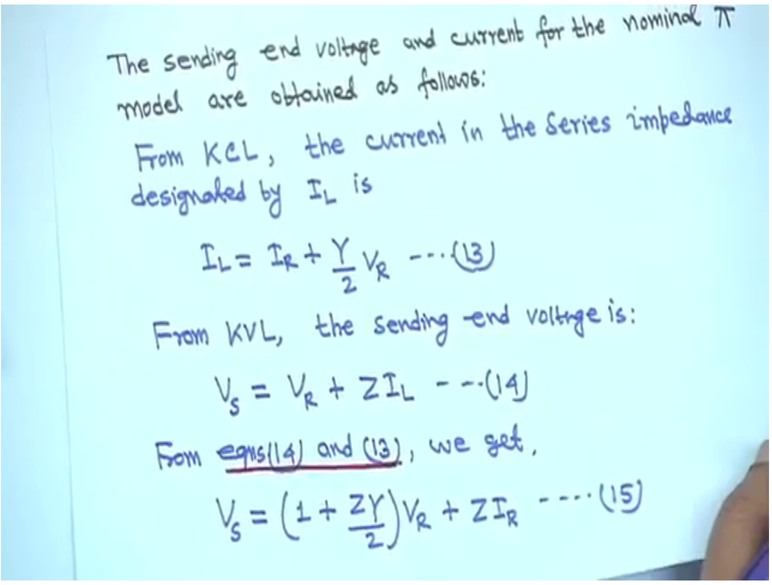

Now, if you now go for this what you call this sending end voltage now you have to find out sending end voltage and current for the nominal pi model, and in the form of a b c d parameters right.

So, from KCL the current in the series impedance designated by I L is. So, I L is equal to right this is this apply it actually at this point where you have to apply R for law I L is equal to I R and current through this is this is that Y 2 that shunt admittance, Y 2 into V R that is why there from the K C L this line current I L is equal to I R plus Y 2 V R this is equation 13 right you are applying K C L here this point.

So, from K V L the sending end voltage is; from K V L the sending end voltage is in this loop to apply K V L right. So, your V s is equal to we become basically Z into I L plus V R. So, V R we writing fast this V R actually writing fast. So, V s is equal to V R plus Z into I L equation 14 right. Now this expression for I L you substitute here you will substitute here the expression for I L here. If you do so, you will get V s is equal to like writing from equation 14 and 13 again V s is equal to 1 plus Z Y upon 2 V R plus Z into I R this is equation 15. So, relationship the sending end voltage here with V R and I R we got this is equation 15.

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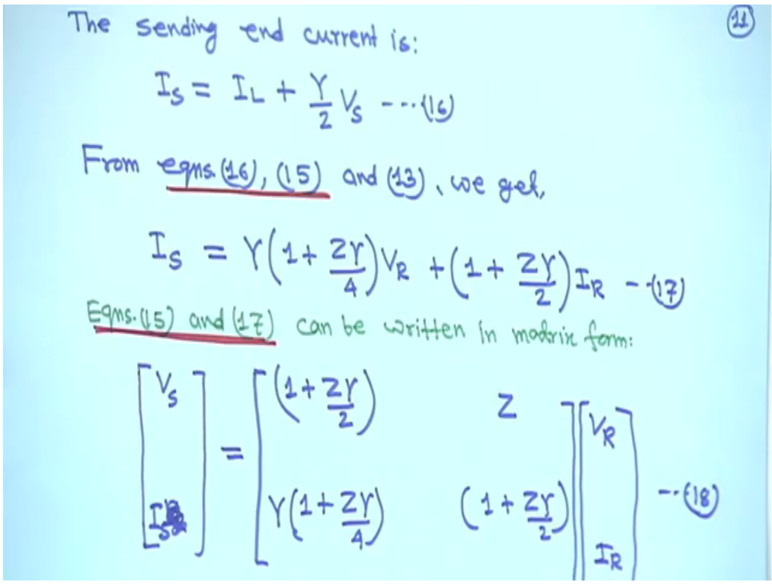

Similarly, the sending end current is I s is equal to I L plus Y into V s, where is the diagram this diagram. So, sending end current this apply your K C L here at this point curves of half we apply here. So, this is I s this branch current I L plus Y by 2 V s right Y by 2 into V s shunt admittance into this voltage V s right. So, in this case you will get I s is equal to I L plus Y by 2 V s. Now from equations 16, 15 and 13. So, I substitute here in this expression you substitute for in this equation 16, is a function of I L and V s you substitute I L is equal to this one in this equation and V s from this expression you substitute here if you substitute and simplify right you will get I s is equal to y into 1 plus Z Y upon 4 into V R plus 1 plus Z Y upon 2 I R this is 17.

So, V s I s (Refer Time: 11:24) if you got in terms of V R and I R. Now put in matrix form. So, V s I s 1 plus Z, Y by 2 Z Y into 1 plus Z Y upon 4, 1 plus Z Y upon 2 V R upon I R this is equation 18. So, this is a, this is b, this is c d, a is equal to your d right. So, this is that a b c d parameters for your what you call for medium transmission line right. So, next is.

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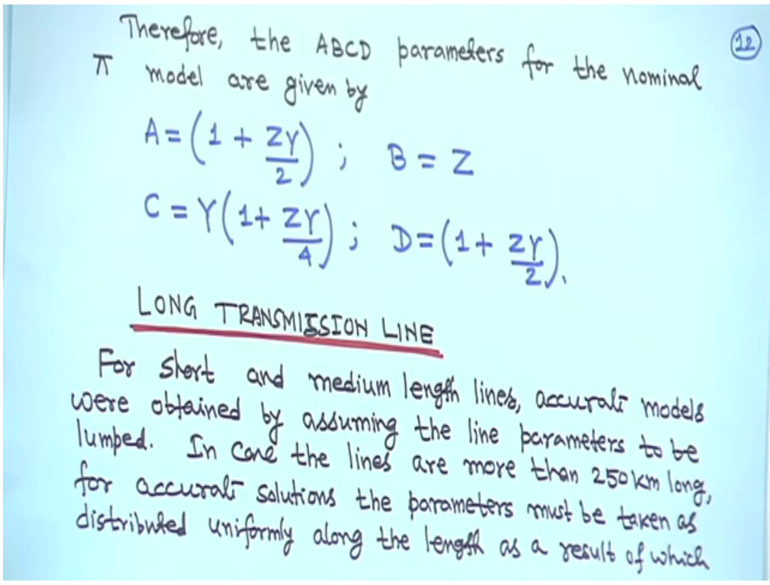

So, the a b c d parameters a is equal to 1 plus Z Y upon , b is equal to Z c is equal to y into 1 plus Z Y upon 4 and d is equal to again a, 1 plus Z Y by 2. So, we got a b c d parameters from the medium what you call transmission line, next is that long transmission line. So, for short and medium length lines accurate models were obtained by assuming that line parameters are to be lumped. For short line in any way we have not considered anything, for medium line we have consider that admittance are long both side half half, another thing for then using that pi method pi model. Only one thing I would like to tell we do not call use that t network. Suppose if you suppose I am just showing you suppose you have suppose you have transmission line right.

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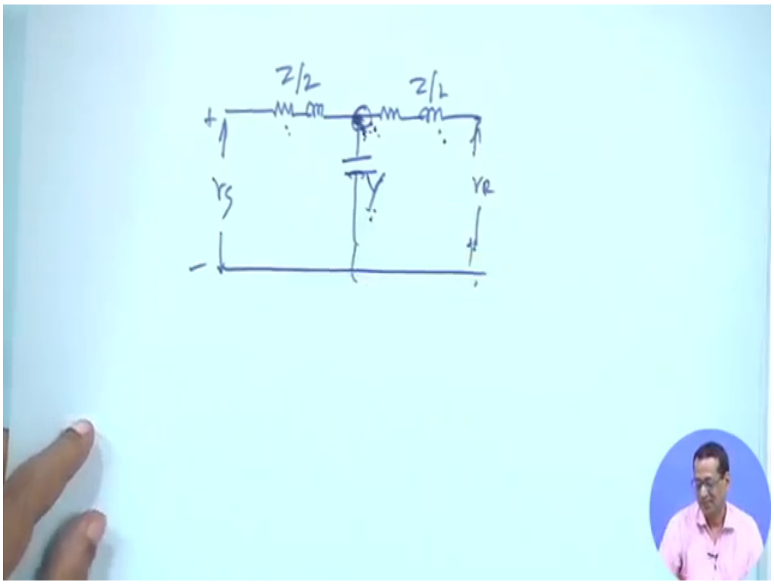

Suppose you have a transmission line and these side is Z by 2 and this side is Z by 2 and if you put y here and this side is your V s right plus minus.

This side load is of course there, but this side is a we are not showing the load just draw this and this is your Y, this is your Y and this is Z by 2 then what will happen this side if you put a 18 is (Refer Time: 13:33) to R, then what will happen you are creating one additional node right so; that means, your problem will become more complicated; and line impedance we have made half this side half this side total is Z, this side Z by 2 Z by 2 and if you put y then only problem is that that you are creating one more node; that means, in trans power system. So, many nodes bass bar are there right. So, so many lines are there, and for each line then one additional bass bar you will create and this will increase the dimension of the problem. That is why we do not use t in way to R for this carry or what you call for line modeling we will use pi method right. So that means, for long transmission is there or what you call in case the lines are more than 250 kilometer long, and for accurate solution the parameters might be taken as distributed uniformly along the line length right. So that means, at every point that voltage and current will vary from point to point right it will be distributed uniformly along the line length right.

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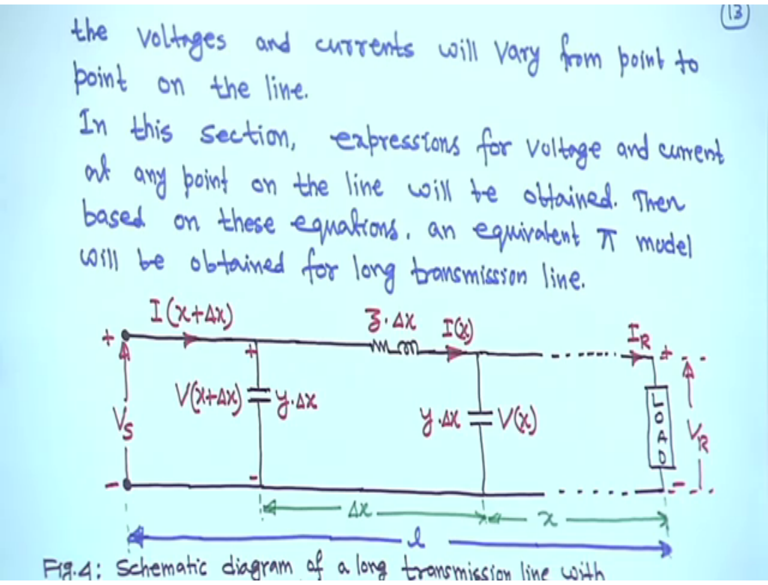

And in that case that voltage and currents will vary from point to point on the line. Now in this section what will do this expression and volt expression for voltage and current at any point on the line will be obtained, then based on the these based on these equation we try to find out a pi equivalent. First you will go for exact one, then the pi equivalent right. Now before going to this first you look at this diagram right this is a long transmission line, then this distance actually we are measuring from the receiving end to the sending end. This is sending end voltage V s plus minus where it is shown this is V R this dash dash means is a long line and from this side we are measuring the distance right now. So, what we are doing from the receiving end total line length from here to here is l total line line length is l. So, from receiving end at a distance x we are considering a small distance that is delta x; say small small in very small right. So, we are considering delta x. So, at a distance x this voltage that means, this is x and this is your x plus delta x right. So, in this case here, here we are writing the at a distance x voltage here is V x right and for this small length delta x that it is small y again, it is not total admittance it is per unit length initially when I was giving the nomenclature I told you small Z means it is per unit it is impudence per unit length and y is small y is impudence per unit length and delta x is the length in per meter right. So, this small y into delta x that means, this small increment we have taken pi model, and at a distance x the voltage is V x there is a pressure quantity.

And at a distance x plus delta x this voltage is V x plus delta x right this voltage and at a distance the current here is I x plus delta x. This up to this if it is understandable to you then everything is understandable right; that means, length if we measure from the receiving end, at a distance x we are considering this small instrumental length to delta x right and therefore, for this one that if you go for this small this incremental thing in pi representation for example,. So, this is small y into delta x at a distance x voltage is V x and x plus delta x is V x plus delta x, here the current here at a distance x here in this section small incremental section this I x right and this is small z into delta x small only for small this section this is impudence z is per unit in impudence per unit length multiplied by delta x, and this side is I x plus delta x right this current. So, and this is V s so, but these say anyway this is not the sending end, sending of this is small distance x at a distance x is small sending end is far away similarly this is dash; dash shown means this one is also far away right.

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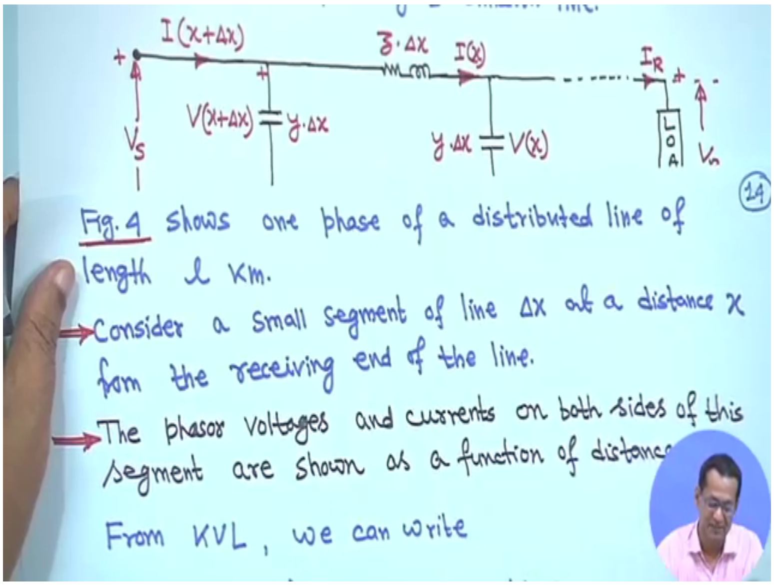

So, now, with this explanation; now, this is the figure 4, I told you everything right. So, this one phase of the length this is actually balance system right the phasor voltage and current on both side of this segment are shown as a function of distance right.

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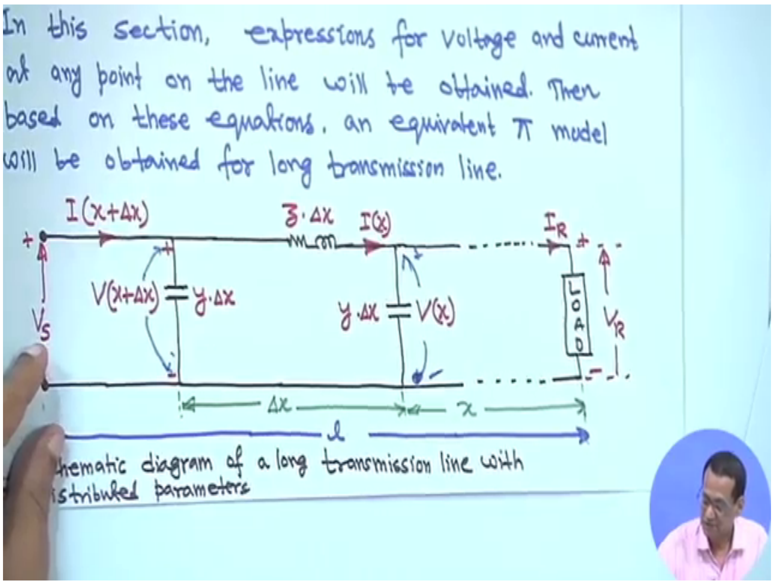

Now, in this case we apply K V L. If you apply K V L this thing, if you apply K V L then this voltage is V x plus delta x that is this voltage actually right. This is V x plus delta x right and this is actually V x right. So, if you apply K V L here, then V x plus delta x will be its small Z into delta x and plus your V x right. So that means, V x plus delta x is equal to small Z into delta x into I x plus V x.

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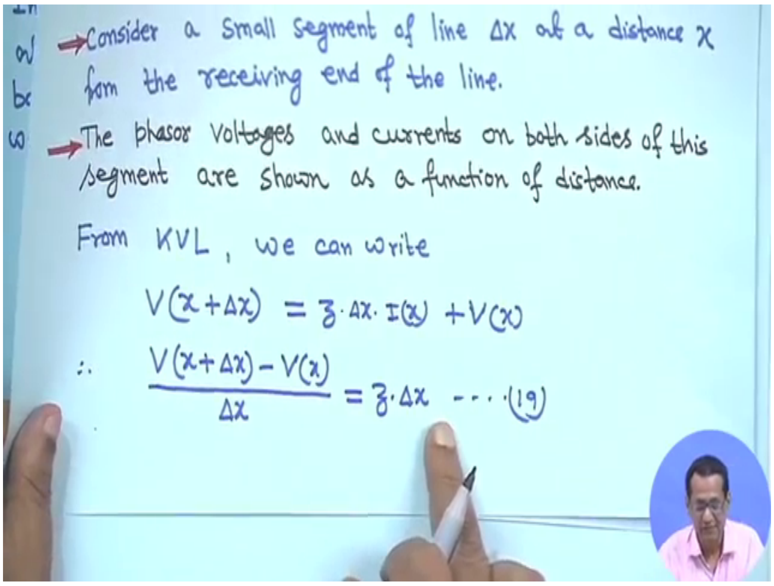

So; that means, V x plus I a V x plus delta x this is the voltage here, across this is equal to small Z into delta x into I x plus V x right. So, this equation is written; that means, V x plus delta x minus V x upon delta x right is equal to small z into delta x. This is equation nineteen right so that means, if delta x tends to 0 right.

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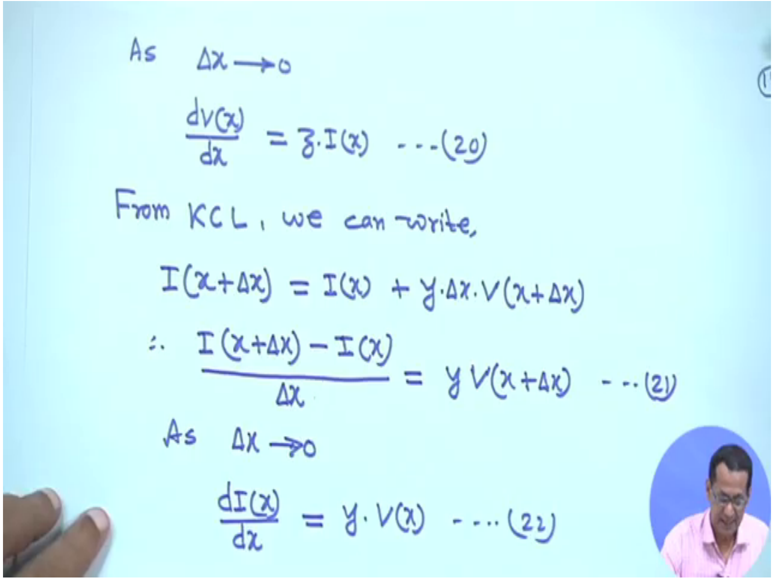

This equation if delta x tends to 0 this equation can be written as no d v x upon d x is equal to your small your what you call your this thing, here no I have missed one thing that is V x is a delta x actually it is I x, it is I x writing here it is small z into I x right. So that means, that d V x upon d x then is equal to small Z into I x this is equation twenty when delta x tends to 0 right similarly. So, this is equation 20, similarly if you apply K C L then I x plus delta x will be if you apply K C L, then I this is I x plus delta x this is the current here apply curves of fast flow here, here at this point. So, I x plus delta x will be is equal to your I x; that means, this current I x this branch current I x this segment current I x plus y into delta x into V x plus delta x.

So, plus Y into delta x into V x plus delta x. So, you are applying curves of your what you call fast flow here. So, I x plus delta x will be current through this is I x plus, this is shunt admittance y small y into delta x into V x plus delta x right. So, this is your current equation. Now then or you can write I x plus delta x minus I x, divided by delta x is equal to y into V x plus delta x. Now as delta x tends to 0, you can write d I x upon d x is equal to y into V x right. So, delta x tends to 0. So, this is equation 22 right. Now next you take the you differentiate equation 20 right.

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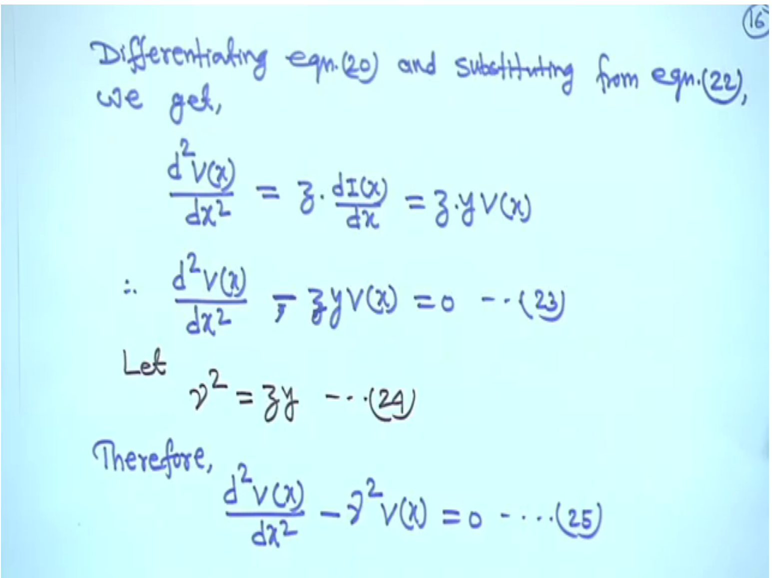

If you take if you differentiate equation 20, just hold on if I where I kept the page number here what you call 13, 14 anyway does not matter right. If you take the differentiate equation 20, right and substituting from equation 22 we get d square V x upon d x square is equal to z small z into d I x upon d x. And this d I x upon d x right you take the derivative of this equation d square V x d x square is equal to z into d I x upon d x and whatever d I x upon d x is here you substitute in this expression right. If you do so, you will get that z small z small y, V x right; that means, d square V x upon d square minus small z small y into V x equal to 0 this is equation 23 right.

Next you assume gamma square is equal to z y small z small y that is gamma square, this is equation 24 right. Therefore, this instead of z y in this equation you replace equation z y by gamma square. If you do so, d square v x upon d x square minus gamma square, v x is equal to 0 this is equation 25 right. So, this is secondary differentially equation right now general solution of this equation this secondary differentially equation. So, general solution of this equation right is equal to you can write in general V x is equal to C 1 e to the power gamma x plus C 2 e to the power minus gamma x, this is equation 26. C 1, C 2 all can be obtained by putting initial conditions we will see just later right.

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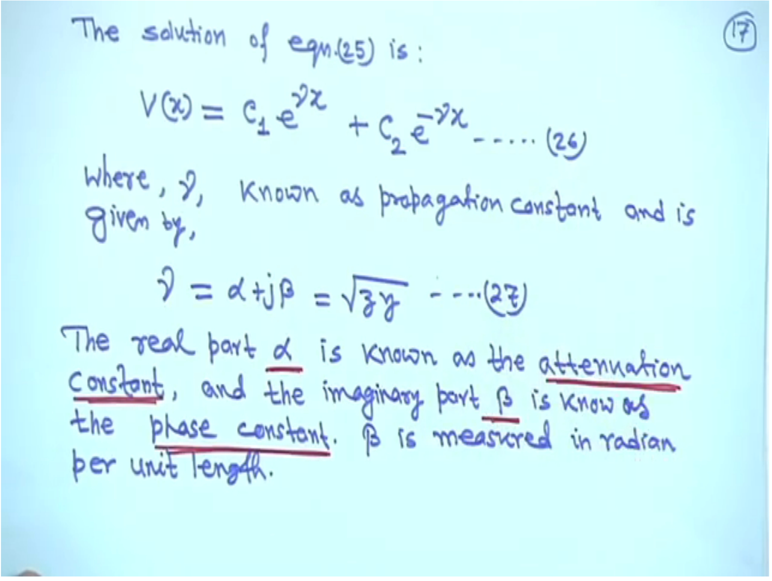

So, this is equation 26. So, gamma actually known as the propagation constant and is given by gamma is equal to alpha plus j beta is equal to root over j y; because gamma square we have taken z y. So, gamma is equal to root over j y sorry z y; that means, your gamma is equal to alpha plus j beta is equal to root over z y. This is equation 27, it is called propagation constant. Now real part alpha is known as attenuation constant and the imaginary beta is known as the phase constant, and beta is measured in radian per unit length right will see later on alpha beta if numerical comes at that time will see that right. So, this is next is that your

So, now you have to find out the C 1 and C 2 this constant right constant you have to find out, now from equation 20 that means, from this equation just hold on from equation 20 means from this equation that d V x upon d x is equal to z into I x from this equation.

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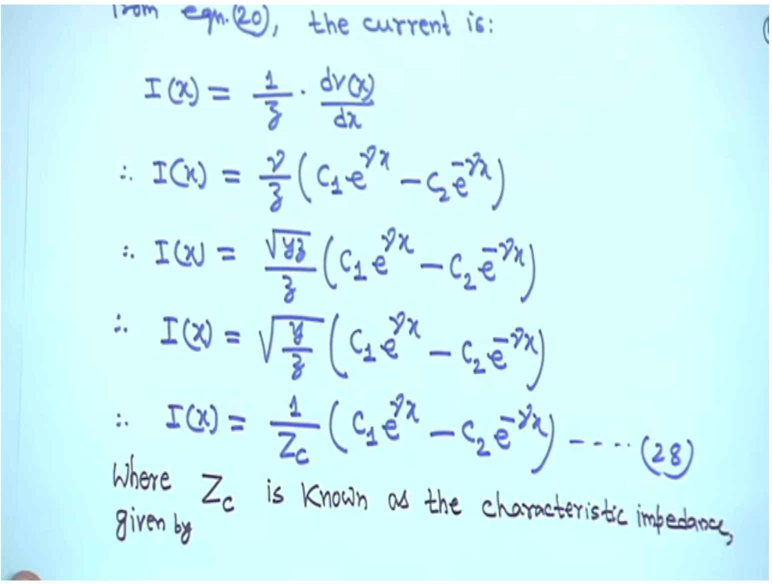

We can write I x is equal to 1 upon small z into d V x upon d x right therefore, this one your what you call that d V x upon d x that V x expression we have got expression for V x we have you have got this one you have got. So, you take the derivative of this equation d V x upon d x, you can take the derivative of this and substitute here and you take the derivative of this one with respect to x, and you and you substitute here right. So, I am not showing derivatives they are understandable very simple thing.

If you take the derivative and substitute, then you will get I x is equal to gamma upon Z C 1 e to the power gamma x minus C 2 e to the power minus gamma x right. Now that gamma is equal to root over y z we have just seen therefore, I x is equal to root over y upon z right because it is z means root z into root z 1 root z we cancel. So, root over y upon z, C 1 e to the power gamma x minus C 2 e to the power minus gamma x right. Now or we define I x is equal to 1 upon Z c C 1 e to the power gamma x, minus C 2 e to the power minus gamma x where Z c is known as the characteristic impudence right and given by this is root over y upon z small. So, Z c is given as your root over your what you call Z by y because reciprocal is taken it is understandable right.

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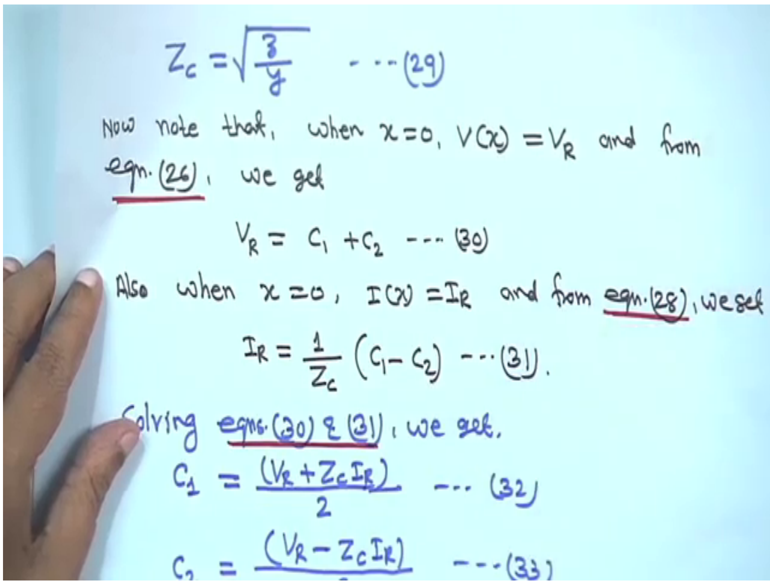

So that means, just before moving next that Z c actually root over. So, small z by y is equal to root over z multiply numerator and denominator by l length of the line, is equal to basically capital Z root over capital Z by capital Y.

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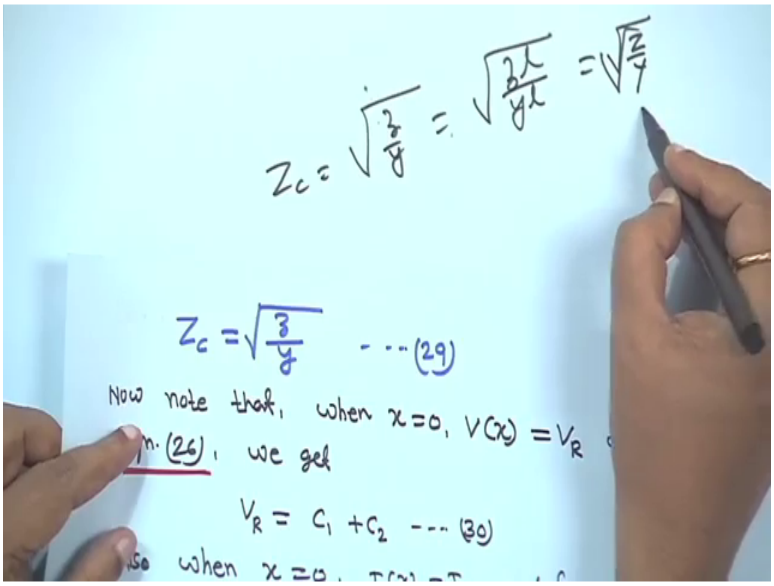

Because it is per unit length multiply l l it will become capital Z by capital Y right anyway. Now this is called characteristic impudence of the transmission line Z c, next is now note that when x is equal to 0 V x is equal to V R look at this diagram. This distance is measured from receiving end right. So, when x is equal to 0, so V x will be V R that is

V 0 is equal to V R that is here therefore, when x is equal to 0, V x is equal to V R and from equation 26 we get V R is equal to C 1 plus C 2 right. So, this equation where x is equal to 0 right when x is equal to 0 your V x is equal to V R that is C 1 plus C 2 is equal to V R. So, C 1 plus C 2 is equal to V R this is equation 30. Now also when x is equal to 0, I x is equal to I R. So, when x is equal to 0. So, this is the receiving end current. So, I x is equal to I R therefore, your equation 28 right. So, if then equation 28 in this equation when x is equal to 0, I x is equal to I R you put it. If you do so, then you will get I R is equal to 1 upon Z c, C 1 minus C 2 right. So, solve equation 30 and 31 this for C 1 and C 2.

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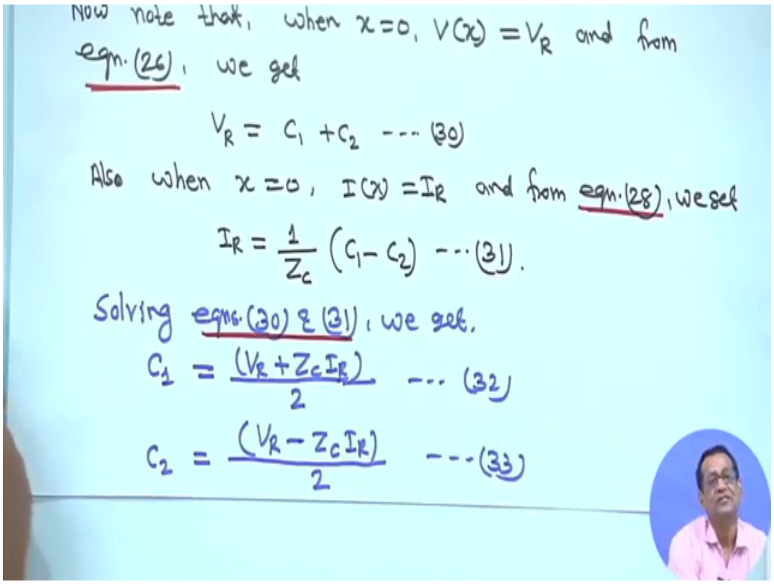

If you solve it, you will get C 1 is equal to V R plus Z c into I R by 2 this is equation 32 and C 2 is equal to V R minus Z c into I R by 2 this is equation 33. So, C 1 and C 2 both the constants are evaluated right.

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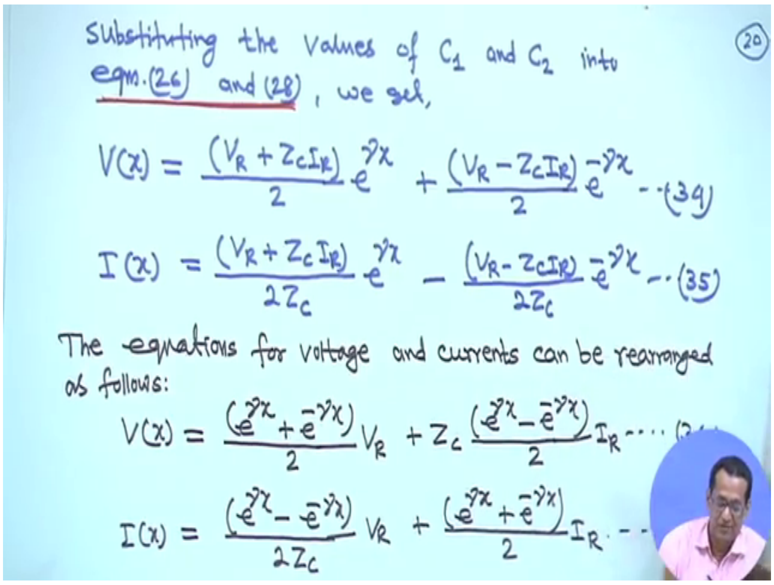

Next this values of C 1 and C 2 substitute equation 26 and 28 right because then only you will get the expression of V x and I x. Equation 26 the expression per V x and equation 28 expression per I x. So, we are substituting C 1 and C 2 value. So, this is equation 34 and this is equation 35, here you are substituting C 1 C 2 here also C 1 C 2 right. So, then this 2 equation you rearrange this equation for voltage and currents can be rearranged. So, first you rearrange the equation of V x. So, V x can be made it like this that e to the power gamma x plus e to the power minus gamma x by 2 into V R plus Z c into e to the power gamma x minus e to the power minus gamma x by 2 into I R right.

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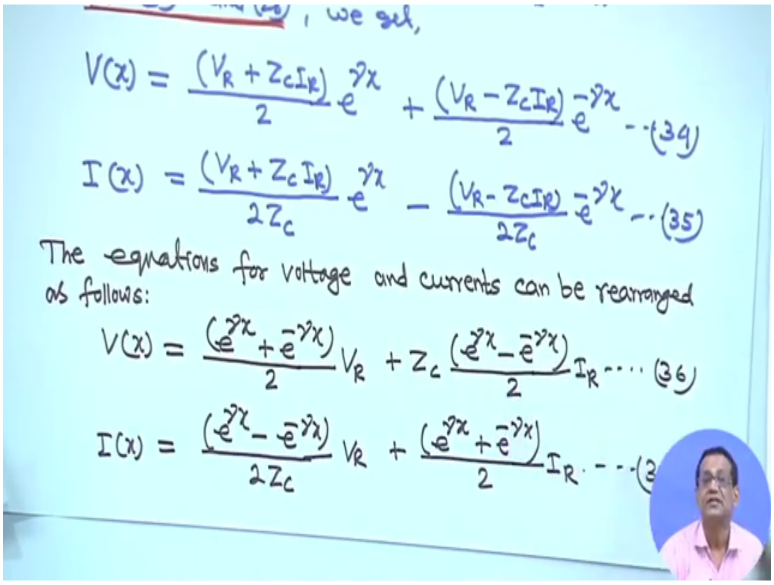

Similarly, I x also can be rearranged, this equation can be rearranged if you do so, I x is equal to e to the power gamma x, minus e to the power minus gamma x by 2 Z c into V R plus e to the power gamma x, plus e to the power minus gamma x by 2 into I R this is equation 37. That means, V x and I x you got in terms of V R I R right by making all this initial putting all the initial conditions right and we got this expression you know V x and I x in terms of V R and I R with some constant right therefore, this term actually cos hyperbolic and this term actually sin hyperbolic.

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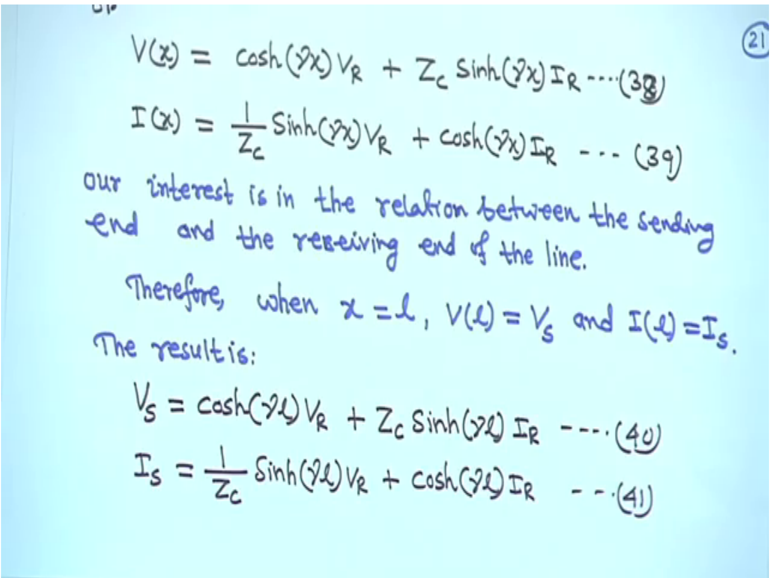

That means this thing can be written as V x is equal to this term actually cos hyperbolic x right gamma x, and this is sin hyperbolic gamma x right. So, this equation can be written as V x is equal to cos hyperbolic gamma x into V R, plus Z c sin hyperbolic gamma x into I R this is equation 38. Similarly I x is equal to 1 upon Z c sin hyperbolic gamma x into V R plus cos hyperbolic gamma x into I R this is equation 39 right. But our interest is the relation between the sending end and receiving end of the line right therefore, but when x is equal to l, V l is equal to V s and I L is equal to I s.

So, distance is measured from here this is the total line length l. So, distance is measured from the receiving end to sending end right here it is when x is equal to l, your V l is equal to V s right. That means, when x is equal to l, V l is equal to V s and this side also when x is equal to l at this point your I L will be is equal to I s.

So, I L is equal to will be I s. So, here we write this equation when x is equal to l, V x is equal to V s and I L is equal to I s. So, V s is equal to then instead of gamma x it will be cos hyperbolic gamma l into V R, plus Z c sin hyperbolic gamma l into I R this is equation 40. Similarly I s is equal to 1 upon Z c sin hyperbolic gamma l into V R, plus cos hyperbolic gamma l into I R this is equation 41 right so.

Thank you for this.