Power System Analysis
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Lecture - 23
Characteristic and Performance of Transmission Lines (Contd.)

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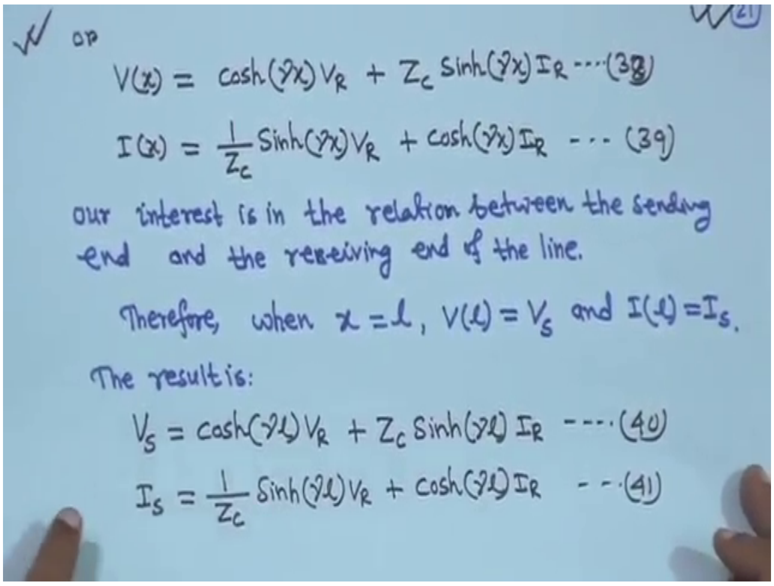

So we have seen that for the long line case. So, this is the relationship that sending end voltage and receiving end voltage and receiving end current and this is for the sending end current that relationship that receiving end and this receiving voltage and this is this receiving end current, these are all cos and sin hyperbolic terms. Now you that very one thing is very understandable to you that this is a b c d parameters. So, a actually is cos hyperbolic gamma l and this is your b is Z c sin hyperbolic gamma l and your c is 1 upon Z c sin hyperbolic gamma l and a is equal to d. So, here also a is d is equal to cos hyperbolic gamma l. This is the exact one. Now what we want actually we want to represent this that that your exact your calculation of a b c d parameter that distribute uniformly distributed parameters right in your pi representation. So, what will be the equivalent pi representation?

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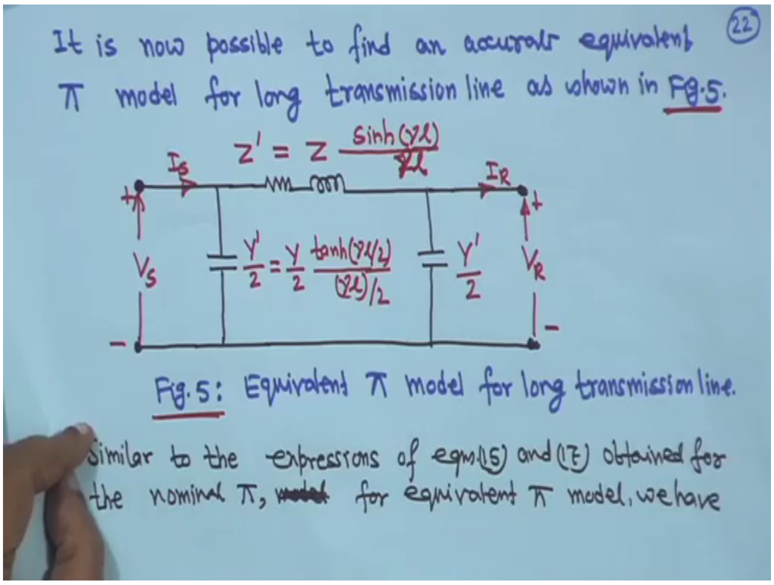

Now, I will show you how to do it right. So, in this case, this is the pi model how it is coming I will tell you. So, this is your equivalent pi model. So, it is Z dash is equal to Z sin hyperbolic gamma l upon gamma l this is Z dash what is Z dash I will come to that then Y dash upon 2 I mean it is instead of Z this has become your Z dash generally when you are doing in a pi model right when you are doing in a pi model that that time it was Z and Y by 2, Y by 2. But as you have taken the exact one that, so this Z actually will be multiplying the factors sin hyperbolic gamma l upon gamma l it is coming from that only I will come to that and instead of Y by 2 this side is Y dash by 2 this side is Y dash by 2.

And Y dash by 2 is equal to Y by 2 when you do did a b c d parameter per pi model it was only Y by 2, but Y by 2 is multiplied with tan hyperbolic gamma l by 2 divided by gamma l by 2. So, how this is the how things are coming I will come to that, but this is that what you call that pi representation of long line.

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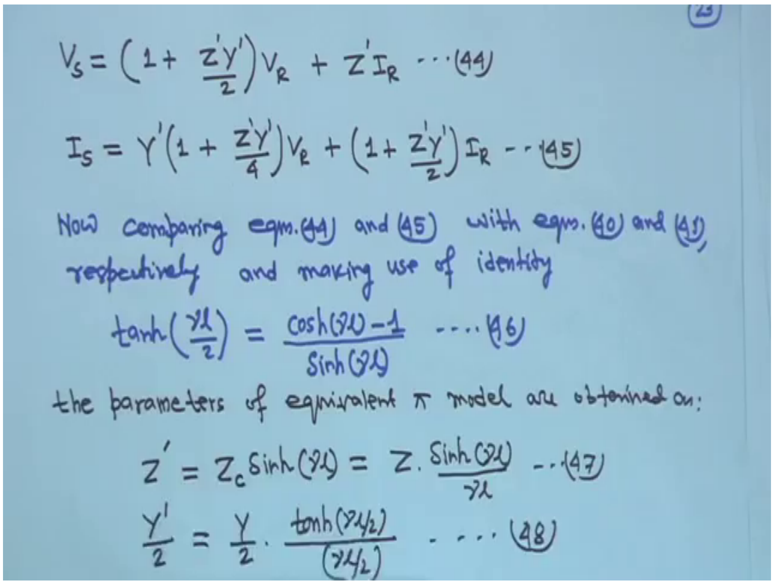

Now, if you see that for pi network that sending end and receiving end your relationship this is your what you call this is V S that a is equal to this is per equation 15 and 17 that you go back to that thing that pi method, pi model for a b c d parameters this is 1 plus Z Y upon 2 this is Z, this is Y in to 1 plus Z Y by 4 and this is 1 plus Z Y by 2 a is equal to d you know.

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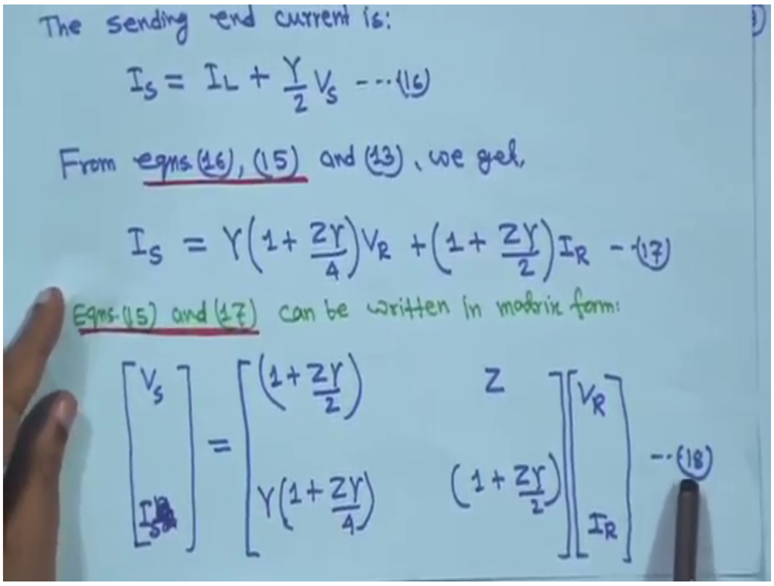

So, same this equation sending long lining pi representation what we are doing is we are instead of writing Z or Y we are replacing them by 1 plus Z dash Y dash by 2 R, V R right plus Z dash I R and I S is equal to Y dash 1 plus Z dash Y dash upon 4 in to V R plus 1 plus Z dash Y dash by 2 in to I R this is equation 44 and this is equation 45 right. So, what you have to do is that you have to; that means, this long line expression; that means, this expression that, this expression this is the exact expression right.

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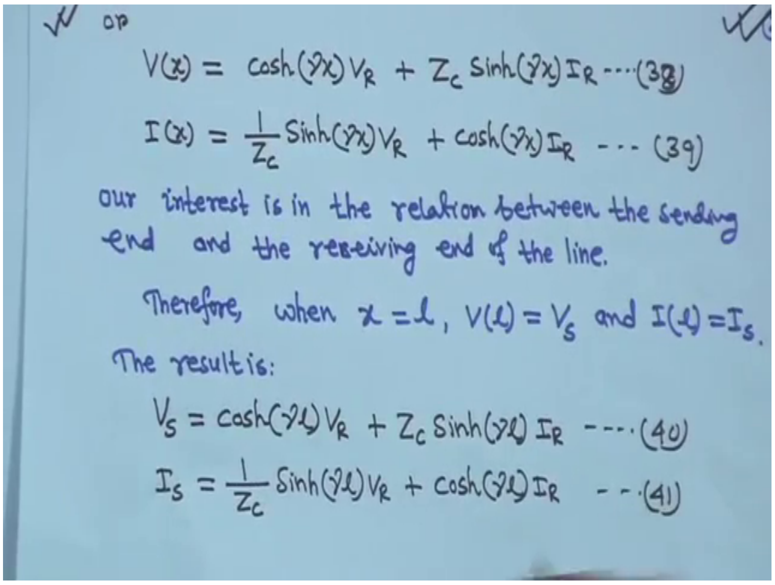

This, but we want instead of taking a b c d of this who want that we want to refresh this one by equivalent pi model.

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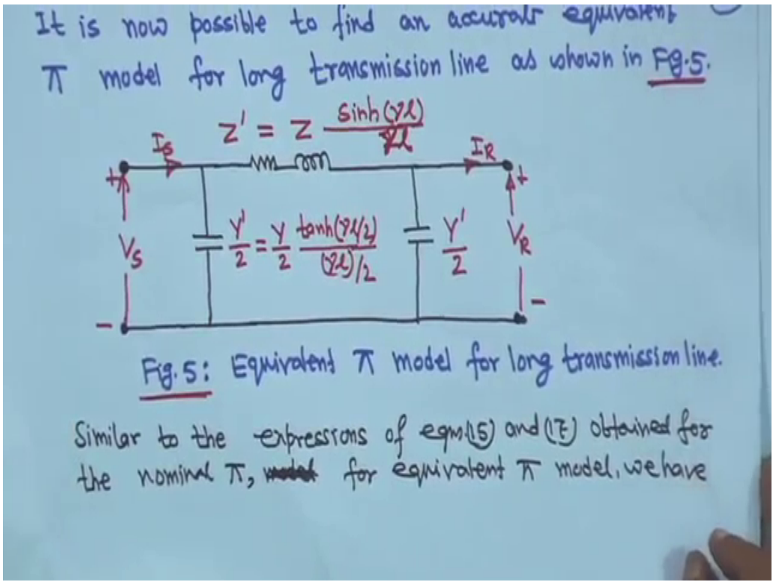

So, in that case what will happen that we will get this one how we get it, now I will show you. So, what you have to do is that we have taken that 1 plus Z dash Y dash upon 2. So, Z dash and Y dash we have to obtain. So, in that case what we could we can do is you compare equation 44 and 45 with equation 40 and 41 and make use of make making use of this identity right. So, what you have to do is that in that case in this case look that first you use this mathematical identity that tan hyperbolic gamma l by 2 is equal to cosine hyperbolic gamma l minus 1 divided by sin hyperbolic gamma l. This relationship will use this equation number I have made 46, but this equation will use.

Now what we will do is that this is Z dash I R right. So, we have taken that; that means, this is your this is this is your what you call this is a b c d parameter right a b c d parameter and you know that b is equal to your Z right from this equation you that equation 18 for normal pi you know b is equal to Z right, but here we are taking that b here it is instead of Z we are writing this one your what you call Z dash this is Z dash. So, Z dash actually your basically this is your Z dash, Z dash is equal to Z c sin hyperbolic gamma l right therefore, this Z dash is equal to we are writing your sin Z c sin hyperbolic gamma l right, is equal to I will give you the next page I will give you that how it is coming is equal to we can write Z into sin hyperbolic gamma l divided by gamma l.

This is equation 47 and Y dash 2 is equal to Y by 2 in to tan hyperbolic gamma l by 2 by gamma l by 2 this 2 how it is coming I am coming to the next page right that using this I mean when you when you get this thing. So, you can represent that long line by an equivalent pi equivalent pi representation right for long transmission line this is I S this is I R this is sending end current this is receiving end current this is V S this is V R and this is Y dash by 2 and this is also Y dash by 2 and this is your Z dash is equal to Z into sin in hyperbolic gamma l upon gamma l and this original in pi, I mean when you considerably only pi model at that time this hyperbolic term was not attached. That means, Z is multiplied by this factor similarly Y by 2 is multiplied by this factor earlier it was only Z and Y by 2, but as long line the mode exact one this here and here these 2 factors are multiplied. Similarly here also not so is here also Y dash by 2 in this same thing right.

So, now, how this 2 things are coming - this Z dash is equal to this one and Y dash by 2 is this one right. Look I am coming to that, next that this one actually Z dash is equal to your Z c.

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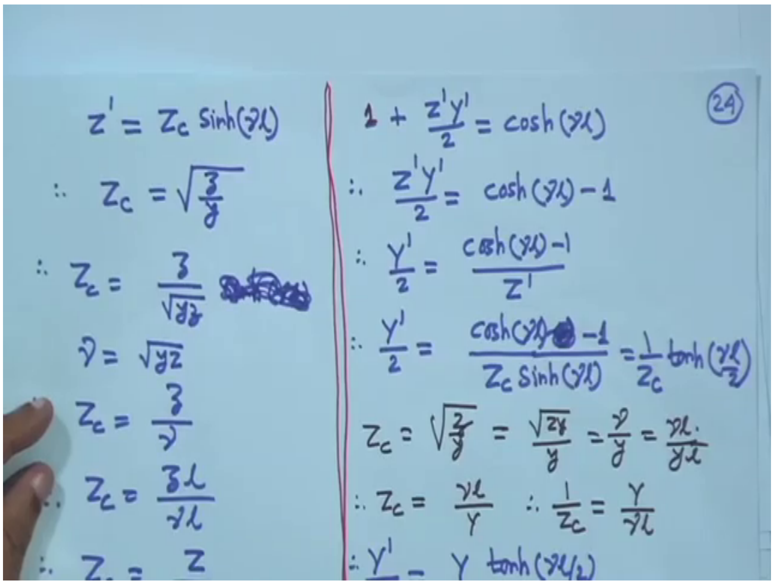

Look at this side right this side Z dash is equal to Z c sin hyperbolic gamma l. So, we know Z c your Z c is equal to you know root of small Z by small Y earlier we have defined. So, numerator and denominator this here you multiply by your this thing Z and here also Z, so Z square. So, Z will come out from the root square this thing it will be root over Y Z right, but we know propagation constant gamma is equal to root over Y Z that also we have seen earlier. That means Z c will be small Z by gamma.

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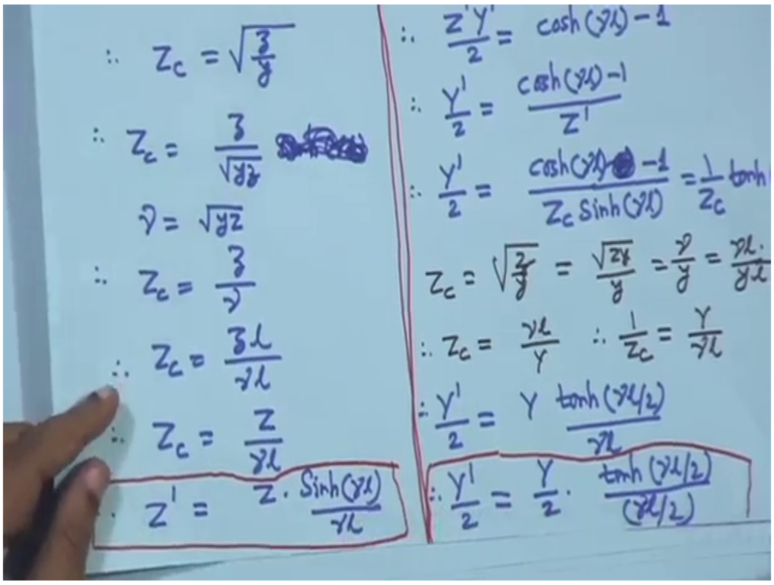

Now numerator and denominator you multiply by l that length of the line; that means, it is because Z is your impudence that line impudence per unit length. So, Z l upon gamma l; that means, capital Z is equal to small Z in to l earlier, we have seen that this are total impudence and this is Z c is equal to Z by your gamma l right. Therefore, if Z c is equal to Z by gamma l then that your that para within that equation you can submit you can put this one right, that your Z dash is equal to Z c sin hyperbolic gamma l. So, Z c is equal to Z by gamma l right. So, here you submit, here you substitute that Z c is equal to Z by gamma l. So, it will be Z in to sin hyperbolic gamma l by gamma l here you have put it. That is why Z dash will become Z in to sin hyperbolic gamma l upon gamma l this is the Z dash.

Similarly from that relationship we know 1 plus Z dash Y dash by 2 is equal to cos hyperbolic gamma l right. So, if you look in to this expression your a b c d parameters a is equal to 1 equal is equal to 1 plus Z dash Y dash by 2 because you want to represent that equivalent pi and a is equal to cosine hyperbolic your sorry, here a is equal to cosine hyperbolic gamma l. Therefore, 1 plus Z dash Y dash by 2 that is your a actually or a dash rather right is equal to cosine hyperbolic gamma l, therefore, Z dash Y dash by 2 is equal to cos hyperbolic gamma l minus 1.

Now; that means, Y dash by 2 is equal to cos hyperbolic gamma l minus 1 divided by Z dash, therefore, this Z dash is equal to your Z in to sin hyperbolic gamma l upon gamma l. So, what you can do is that this here first that cos hyperbolic gamma l minus 1 divided by sin hyperbolic gamma l actually is equal to tan hyperbolic gamma l by 2; that means, this Y dash by 2 can be written as 1 upon Z c in to tan hyperbolic gamma l by 2. So, we have seen, we know that Z c is equal to here root over Z y. So, here numerator you multiply by numerator denominator multiply by Y. So, it will be root over Z Y upon Y and gamma is equal to root over Z Y this is actually small Z, this is actually small Z. So, this is gamma by small Y. Now numerator denominator you multiply by l, so gamma l by small Y in to l. So, Z c will be gamma l upon capital Y right therefore, 1 upon Z c is equal to Y upon gamma l. So, here 1 upon, this 1 upon Z c here you put here that 1 here right. So, 1 upon Z c is equal to you put Y upon gamma l then you will get Y dash by 2 is equal to Y tan hyperbolic gamma l by 2 divided by gamma l right; that means, this one and to make the you know this Y dash by 2 here also you make Y by 2. But this one you write tan hyperbolic gamma l by 2 by gamma l by 2 because 2 2 will cancel actually, but to make it Y by 2 in to tan hyperbolic gamma l by 2 gamma l by 2; that means, this Y by 2 is multiplied by this factor for a long line pi or pi representation.

That is why this your what you call this diagram pi equivalent it is your this kind of pi representation. So, Z multiplied by this term and Y by 2 multiplied by this term, but let me tell you that this term and this term are you will find approximately this sin hyperbolic gamma l by gamma l approximately your very close to unity. Here also the same thing this term right, but this is that more or less quite accurate model. So, up to this we have come for short line medium line and long line right. So, now, will see few examples after that will go to some more details of this characteristic of the transmission line. So, first you consider that example one.

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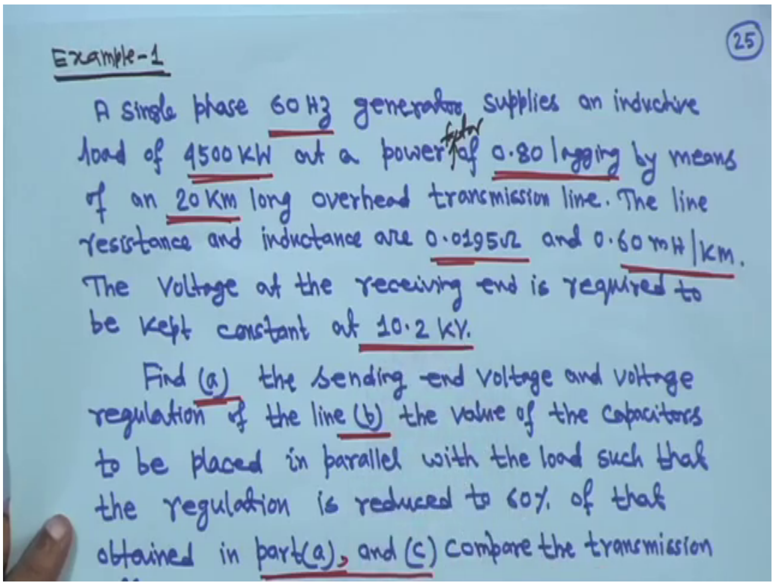

Only first one, first example will be a single phase line, but let me tell you one thing whenever you solve numericals even it is three phase you transform all the parameters first per phase basis, line to neutral after that you solve it. So, first you see this a single phase frequency 60 hertz generator supplies an inductive load of 4500 kilo watt at a power, at a it is power factor right it is at a your power factor I have missed this, power factor of 0.8 lagging by means of an 20 kilometer long overhead transmission line.

The line resistance and inductance are 0.0195 Ohm and 0.60 millihenry per kilometer> The voltage at the receiving end is required to be kept constant at 10.2 KV; that means, receiving end voltage is kept constant at 10.2 KV. We have to find out the first one, find a the sending end voltage and voltage regulation of the line.

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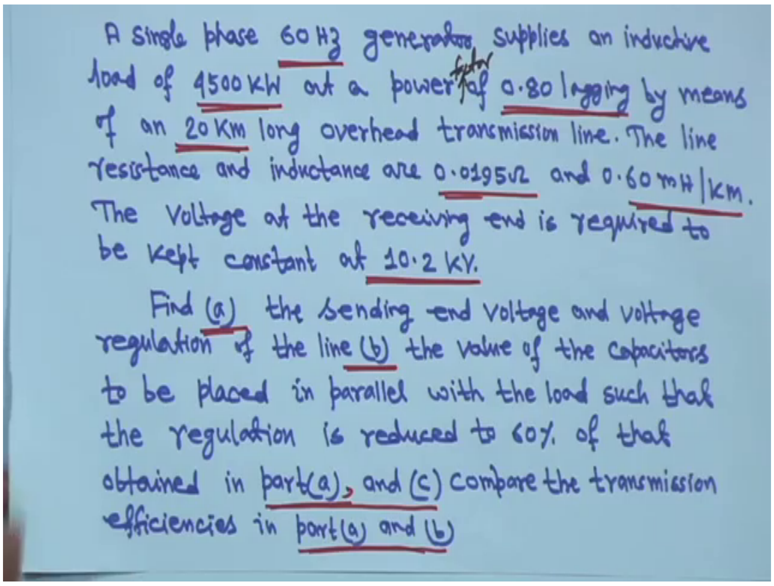

Now part b, the value of the capacitors to be placed in parallel with the load such that the regulation is reduced to 60 percent of that obtained in part a and c compared the transmission line efficiencies in part a and part b. So, these are the things we have to find out. So, you have to find out that your what you call that your sending end voltage and voltage regulation, at the same time that the value of the capacitor placed your in that receiving end that is across the road in parallel right such that whatever regulation you got in the first case it should be 60 percent of that. That means, you have to improve the power factor and you have to find out what is the value of c and that is the problem, but this is the single phase diagram this is this problem is a single phase single phase line.

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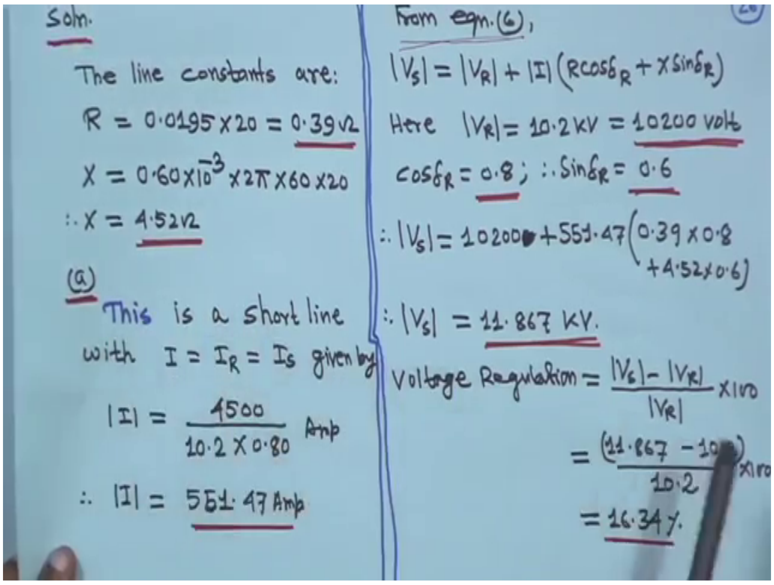

Now, how to do this the look at this, this side only the line constants are given R is equal to 0.0195 that 20 kilometer length because it is ohm per kilometer. So, it is 20 kilometer length multiplied by 20. So, 0.39 ohm right and x is equal to for your given millihenry per kilometer. So, that is your l is l is given that millihenry per kilometer. So, 0.6 in to minus 3 in to 2 pi 60 has system in to 20 kilometer the length of the line. So, it is 4.52 ohm, now come to the part a.

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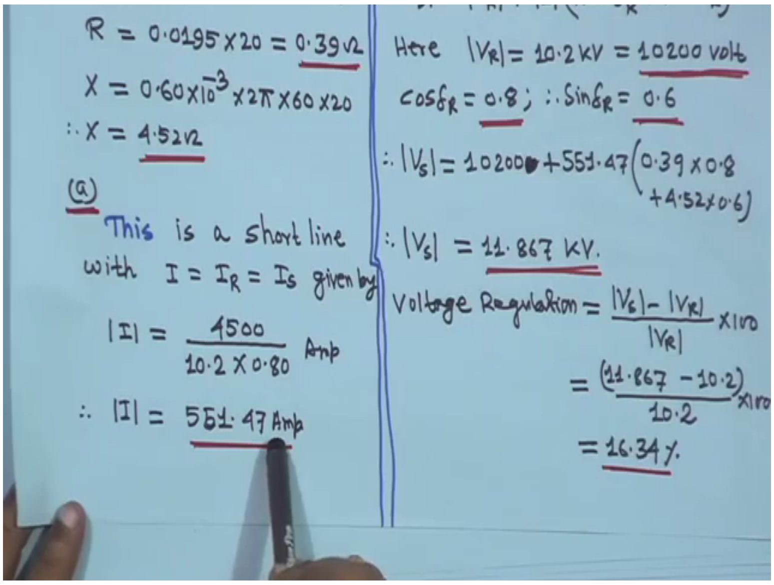

This is short line right. So, for short line no question of considering charging admittance. So, I is equal to I R is equal to I S because short line as no shunt capacitance there, there is a charging capacitor such that I mean that is that charging capacitance. So, I is equal to I R is equal to I S. So, first you find out that magnitude of the current it is load is given at kilo watt and this is kilo volt of course. So, 4500 divided by 10.2 and power factor is 0.8 that is V I cos pi actually V I cos pi is equal to power. So, mode I is equal to is writing like this is a ampere. So, magnitude of the current is 551.47 ampere first you compute the current. Now from equation 6 we have seen know magnitude of V S is equal to magnitude of V R plus magnitude of I in to R R cosine delta plus x sin delta. So, receiving end voltage is kept fixed at 10.2 kilo volt k v. So, is equal to you write 10200 volt all unit we are transferring in to volt here and power factor is given. So, cosine delta R is equal to 0.8. So, sin delta R is equal to 0.6.

And R and X values are known here. So, substitute everything here and I is also known magnitude of the current 551.47 you substitute all right and put it here. So, you will get sending end voltage is equal to 11867 kilo volt right. So, voltage regulation formula you know for your what you call for short line right it is magnitude V S minus magnitude V R upon magnitude V R in to 100. So, substitute here V S is equal to how much V R is equal to how much and here also the regulation will become 16.34 percent right. So, this is that voltage regulation with respect to their basically you compete with respect to the receiving end voltage.

Now, in the part b showing that you have to improve that voltage regulation; that means, in that first part regulation was 16.3. But in the second part we want to make it is 60 percent of this voltage regulation by connecting a sound capacitor across the load. So, this is a short line.

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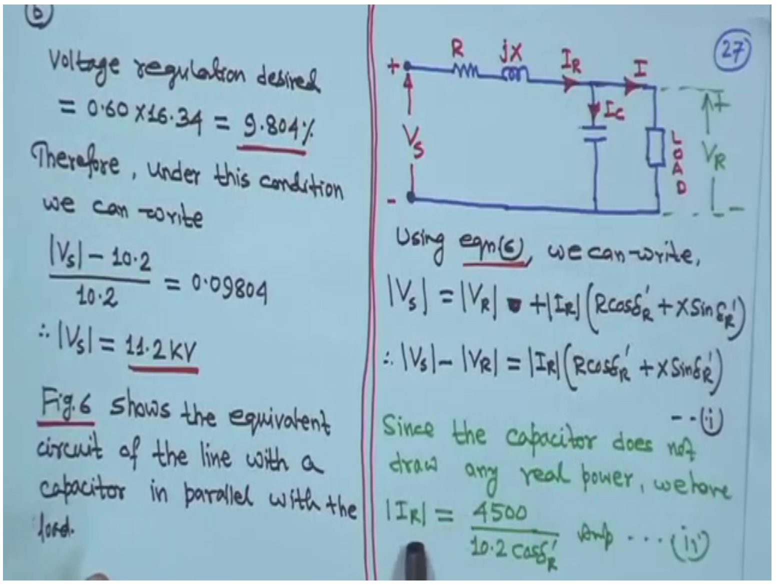

So, this is resistance R this is x and current flowing through this is I R this one capacitor is connected across the load. So, this is the I c that is current going to your capacitor and this is the current I going to the load. So, it is a simple series parallel circuit right, the whole you have whatever you have done. In your basic circuit problem also to improve the power factor you might have taken many your what you call such kind of circuit problem, but here it is power system problem right

So, therefore, under this condition regulation we want that it should be, but receiving end voltage is kept fixed at 10.2 KV it is unchanged therefore, if regulation is known this is 9.804 percent you know the regulation formula. So, mode V S minus mode V R upon mode V r, so magnitude V R is 10.2. So, 10.2 is equal to this sending end voltage is 11.2 KV right. So, with this regulation if you want to keep this receiving end voltage fixed at 10.2 KV then sending end voltage magnitude will be 11.2 KV. So, this is that equivalent circuit diagram when you are connected a capacitor.

Now again if we using equation 6, using equation 6 we can write magnitude of V S is equal to magnitude V R plus magnitude I R because this is the current please do not take this current you have to take the receiving end current right this take this current - one part is going to capacitor another part going to I whatever you will write this equation please do not take this I then there will be mistake here you have to take receiving end current because voltage it is parallel.

So, voltage across the load or across the capacitor is same right and this load is known and this load is known that equivalent your what you call equivalent load or equivalent impudence can be obtained. So, do not take I, you have to take this I R this is I R right. So, be careful about that. Sometimes a common silly mistake means current is going to load; that means, this is I R no this is incorrect if capacitor is connected. So, this is your I R. So, magnitude V S is equal to magnitude V R plus I R then as the capacitor is connected. So, delta R will change. So, R cosine delta dash we are taking plus x sin delta dash.

So, magnitude V S minus V R is equal to I R magnitude of course, in to that R cos R cosine delta R dash plus x sin delta R dash. Now actually capacitor we assume this is I mean capacitor has no your what you call not consuming any real power, but little bit is there, but we neglect that, we will assume the capacitor consume only reactive power right, but no real power. So, since the capacitor does not draw any real power, as it is not drawing we have magnitude of I R we are writing is equal to this is the load power 4500 kilo watt and receiving end voltage is kept fix at 10.2 this is cosine delta R dash I am putting the unit here ampere because this is numerator this is in kilo watt I did not change it to watt and denominator also 10.2 kilo volt. So, no need to change it again right this no need to multiply by 1000 or this is 1000 understandable in to cosine delta R dash. So, this is that your what will be this that I R because capacitor does not any your what you call that your any real component of that your what you call does power right therefore, that combination of I R will be the same thing 4500 10.2 cosine delta R dash ampere. So, from that you, but you have to find out your cosine your what you call this is equation 2 I have marked and this V S minus V R I have marked equation 1 right this is 1 and this is 2. So, I hope you have understood this, very simple thing.

Next is that V S is given eleven point 2 KV and V R also given 10.2 KV this all this things are known to you right. So, you have to subtract. What you will do is that you from equation 1 and 2 and these 2 are known I R is also known R and x value is also known you have to find out delta R dash. So, from equation 1 and 2, you will substitute here it is V S is equal to 11.2 and V R is equal to 10.2 you will substitute here, I R also this one you will substitute here and R values and x values also known R and X you substitute here and you will simplify little bit right I did not right all this step, but this are simple things you will substitute.

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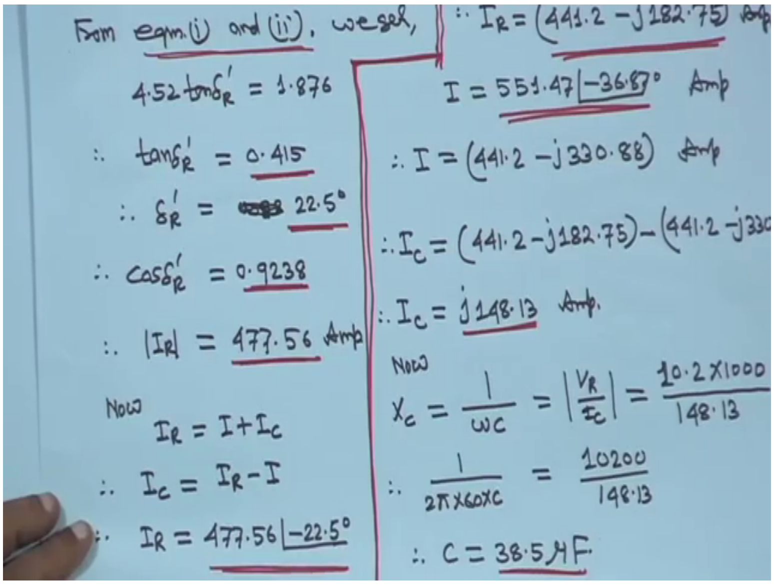

If you do so then you will get that is why I am writing from equation 1 and 2 from equation 1 and 2. After substituting a little bit simplification you do, what will get you will get 4.52 then tan delta R dash is equal to 1.876 right; that means, tan delta R dash is equal to 0.415 hence delta R dash is equal to actually 22.5 degree. So, cosine delta R dash is equal to 0.9238 this is the your improve power factor. Earlier it was 0.8, now it is 0.9238 because you want to maintain the regulation to 60 percent of that without any your capacitor across the load right and as cos delta R dash is 0.9238 now from here you substitute now cos delta R dash 0.9238. So, if you do so then magnitude of I R that current magnitude you will get 477.56 ampere this is the current.

Now, from this circuit now from this circuit I R is equal to I c plus I right. So, I R is equal to I i I your I plus I c right. So, I c is equal to I R minus I. So, I R your; what you have to do is that you have to now I R is 477.56 this is the magnitude of that current and this delta R dash is 22.5 be lagging. So, I R is equal to 477.56 minus 22.5 degree this is your I R. And I that is that is the current going to the load we have obtained know 551.47, this is that this is that current going to the load 551.47 and power factor is your 0.8 it is lagging; that means, your I is equal to o mean I mean this one right I mean this one, this I this I it is going to the it for first case this was going to this load, no this current at that time it was I R because at that time shunt capacitor was not there in the first case.

So, I is equal to 551.47 angle minus 36.86 degree 87 degree, and this is your I R that is the after putting that shunt capacitor. So, now, that means, your I c will be your I R minus I. So, I R minus I and you will get I c is equal to j 1 148.13 ampere. So, now, this is your current is leading that is capacitor. So, j is there current is leading. So, you can take more decimal places I will explain it little bit later. So, x c is equal to 1 upon omega c is equal to V R upon I c this magnitude. So, V R is 10.2 in to 1000 we are made it to bold because current here is ampere. So, divided by 148.13

So; that means, omega is equal to 2 pi we have taken 60 as systems 2 pi 60 in to C is equal to 10200 by 148.13 therefore, if you calculate c from here c will become 38.5 micro farad this is the capacitor, capacitor value. So, what we have observed that earlier it was minus 36.87 without capacitor now when you put the capacitor it is minus 22.5 degree that current your lagging earlier it was minus 36.8, but we have got the capacitor C 38.5, but how much how much reactive power actually supplied by the your capacitor. So, that also you have to, you have to get it.

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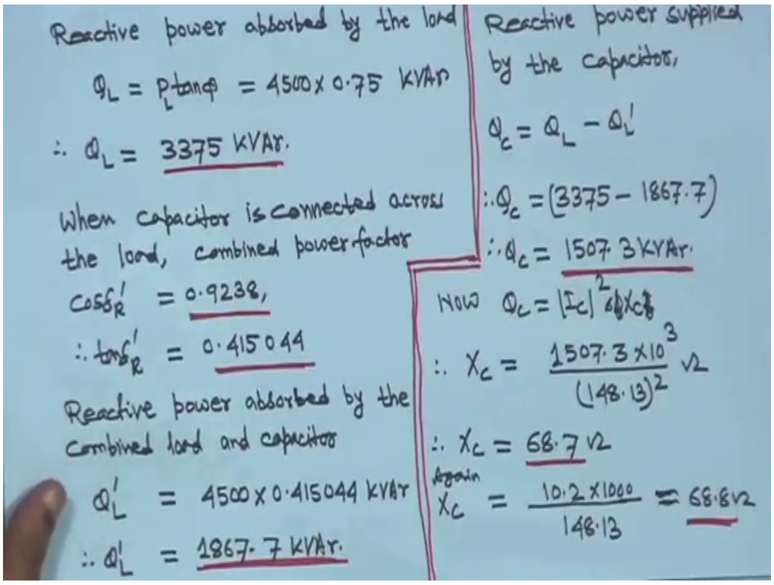

So in this case first thing is when there was no capacitor that reactive power absorbed by the load is Q L is equal to your I can put like this Q L is equal to P L tan phi.

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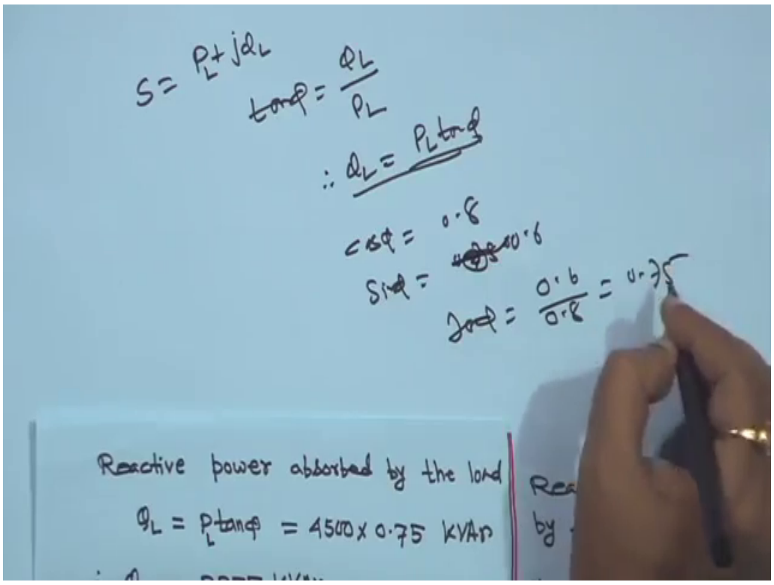

So, that means, when you right the power p I am putting in l j Q L that is load power right this is that your S we write; that means, your tan phi is equal to Q L upon P L right; that means, Q L is equal to P L tan phi, that is why we are writing that reactive your reactive power absorbed by the load Q L is equal to P L tan phi the simple thing. So, P L is equal to 4500 kilo watt and your cos phi is 0.8 your cos phi is 0.8 then sin phi is equal to 0.6, sin phi is equal to 0.6 therefore, tan phi is equal to 0.6 by 0.8 is equal to 0.75 that is why that is your 4500 in to 0.7 kilo volt that is Q L is equal to 3375 kilo volt. This is that your what you call load reactive power.

Now when capacitor is connected across the load that combined power factor you have got cos delta R dash is equal to 0.9238 this one you have got it cosine delta R dash is equal to 0.9238 this one we have got it therefore, tan delta R dash is equal to 0.415 I have taken, but I also add three more digit 0 4 4 you can go up to 3 digit only. So, this is that cosine delta R dash is this much therefore, tan delta dash will become this much. Therefore, reactive power absorbed by the combined load and capacitor right that Q L dash I made it is equal to 4500 because load power is real power and capacitor is not drawing any real power only reactive power. So, we can write 4500 in to this tan delta R here it was P L tan phi and here it is your what you call this thing this phi actually there should not be any confusion that phi actually you can take your delta R.

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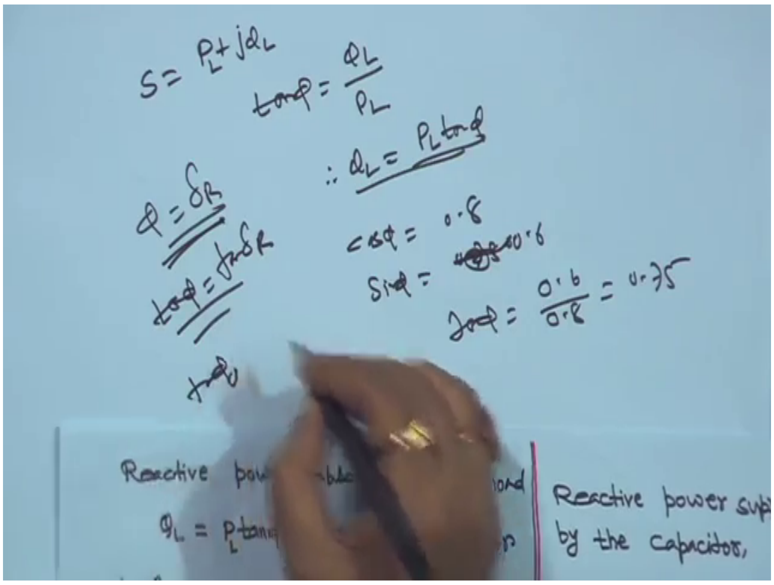

Generally sometimes you write phi only, but phi is equal to delta R, same thing. So, in this case what you can do is that it is Q L dash is equal to 4500 as capacitor does not any real powers. So, real reactive real power will remain same in to this tan delta this thing, your delta R dash, there should not be any confusion right. So, tan phi actually is equal to tan delta R right therefore, we can if you want to put that your this thing, what you call this Q L dash is equal to this 1 kilo volt multiplied by this tan value. So, Q L dash is equal to 1867.7 kilo hour right therefore, reactive power supply this is a combine one then reactive power supplied by the capacitor will be Q c is equal to Q L minus Q L dash because this was Q L and combine one was this one eighteen sixty seven point seven then how much supplied by the capacitor that Q L minus Q L dash. That is equal to 1507.3 kilo hour.

Now Q c is equal to I c square in to X c. Now we have to verify the result right that whether we are correct or not. Now Q c is equal to I c square X c if you make because current going through this capacitor is I c and X c is the reactance therefore, that X c is equal to Q c upon your what you call I c square it is kilo volt. So, 1507.3 10 to the power 3 and this is 148.13 is that current right. So, I c is that 148.13 magnitude. So, X c is equal to 68.7 ohm. Again X c is equal to you can write right V R upon I c, this is V R upon I c this is 68.8 ohm the difference is here it is 0.7 0.8 ohm.

Thank you I am coming to next.