Power System Analysis
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 25
Characteristic and Performance of Transmission Lines (Contd.)

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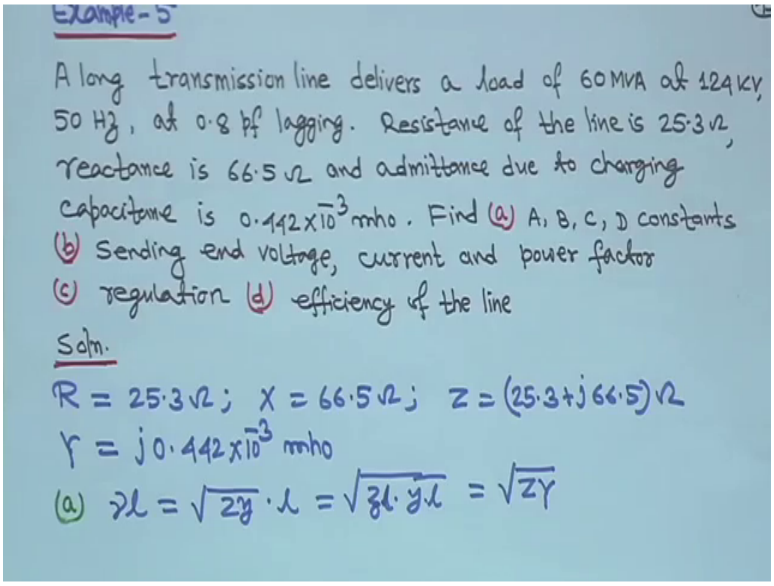

So that this continuation of that previous topic that is characteristic of your transmission line performance and characteristics so, will take another example right 2, let after this one more example and after that will go to some another regarding your voltage waves right. So, it is this one earlier we have seen that your short circuit line medium also with pi your pi model of the transmission line and now this one we will see that your what you call that exact one that is the long transmission line right and after this will take on short line medium line and long line and with the same parameters and will compare that will be the next example.

So, it is a long transmission line it is delivering a load of 60 MVA at 124 KV 50 hertz and 0.8 power factor lagging right resistance and reactance are given right and admittance due to charging capacitance is also given 0.442 in to 10 to the power minus 3 mho you have to find out a B C D constant sending end voltage current and power; power factor regulation and efficiency of the line. So, resistance is given 25.3 ohm X is 66.5 ohm and impudence and hence impudence is 25.3 plus j 66.5 ohm and gamma your sorry Y is equal to that charging admittance j 0.442 in to 10 to the power minus 3 mho this is the charging admittance and gamma L because we are going for a your what you call long line thing.

So, gamma L actually root over Z Y in this is a, this is actually small small z right. So, small z y in to L is equal to you can write root over it is actually inside if you bring L square. So, Z L in to Y L that is actually gamma L is equal to root over capital Z capital Y right because and Z and Z and Y both are known to you this is this is Z and this is Y both are known to you.

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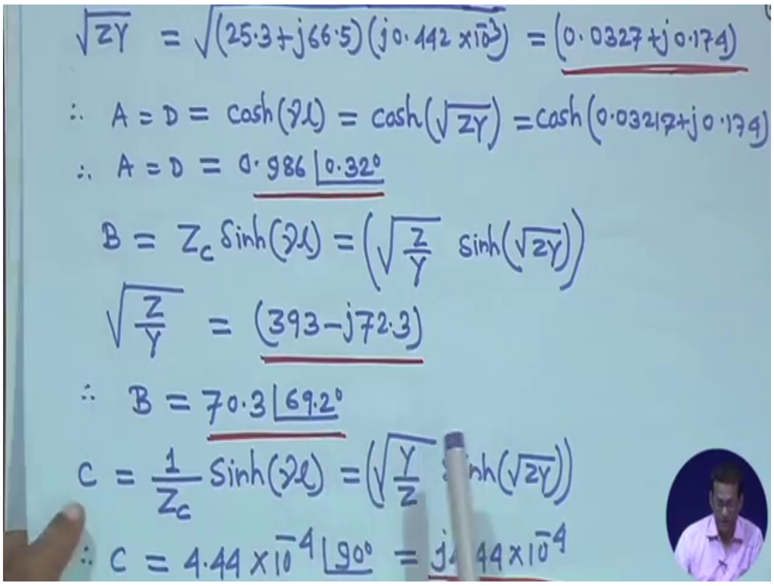

Therefore root over Z Y is equal to this is Z and this is Y you will get 0.0327 plus j 0.174. So, A is equal to D that is A B C D parameter for long line it is cos hyperbolic gamma L. So, cos hyperbolic root over Z Y is equal to cos hyperbolic this value substitute here root over Z Y you will get a is equal to D is equal to 0.986 angle 0.32 degree right and B is equal to Z C sin hyperbolic gamma L right and is equal to you can write root over Z C is equal to root over Z by Y capital Z up on capital Y earlier also we have all this things have been given all this term and definition. So, sin hyperbolic root over Z Y if you substitute this thing root over Z by Y if you compute it will be 393 minus j 72.3 and hence this B if you substitute here you will get 70.3 angle 69.2 degree.

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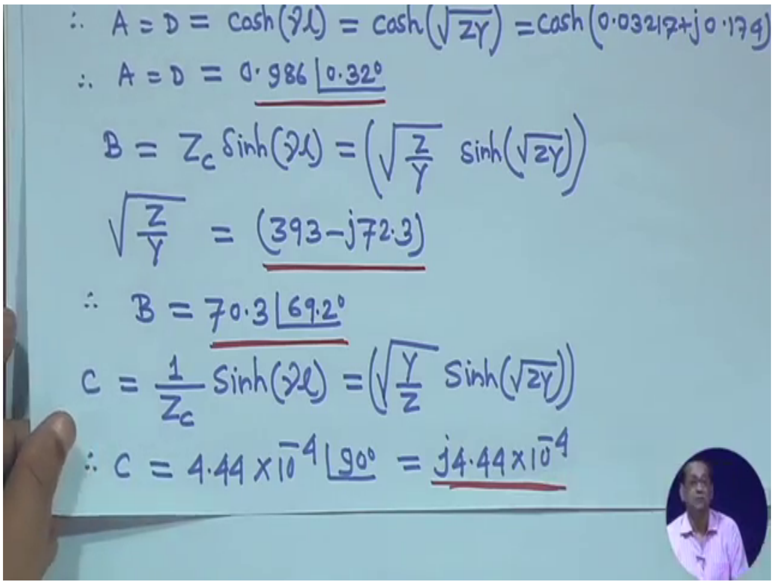

And C is equal to 1 up on Z C sin hyperbolic gamma l. So, it is 1 up on Z C. So, hence it is root over Y by Z capital Y by capital Z sin hyperbolic root over Z Y. So, if you substitute all this values you will get C is equal to j 0.44 10 to the power minus 4 right.

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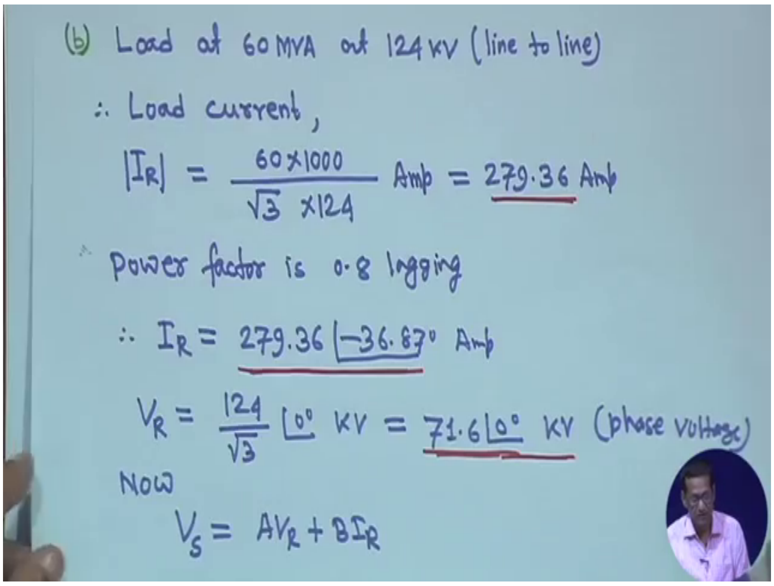

So, that is your all the A B C D parameter computation for long line this are over now next is at that your part B that you have to you have to find out the sending end voltage current and power factor that is the part b. So, load at 60 MVA 124 KV if nothing is mention means you have to assume this is line to line. So, this voltage is line to line right. So, load current same as before right root 3 your V I is equal to MVA. So, it is 60 into 1000 right. So, it is basically magnitude of the current is 279.36 ampere right. So, and this 1 power factor is 0.8 lagging right therefore, I R is equal to 279.36 angle minus 36.87 degree ampere because power factor is 0.8 is lagging right.

So, here your; what you call the V R is equal to you I told you that you have to take everything in line to neutral that is phase voltage. So, 124 up on root 3 angle 0 degree KV. So, it is 71.6 angle 0 degree kilovolt that is phase voltage, now you know that sending end voltage is equal to a V R plus B I R right. So this current is actually 220 your for 279.36 angle minus 36.87 degree; that means, this V S is equal to 0.986 angle 0.32 degree into 70 what that is a V R 71.6 angle 0.

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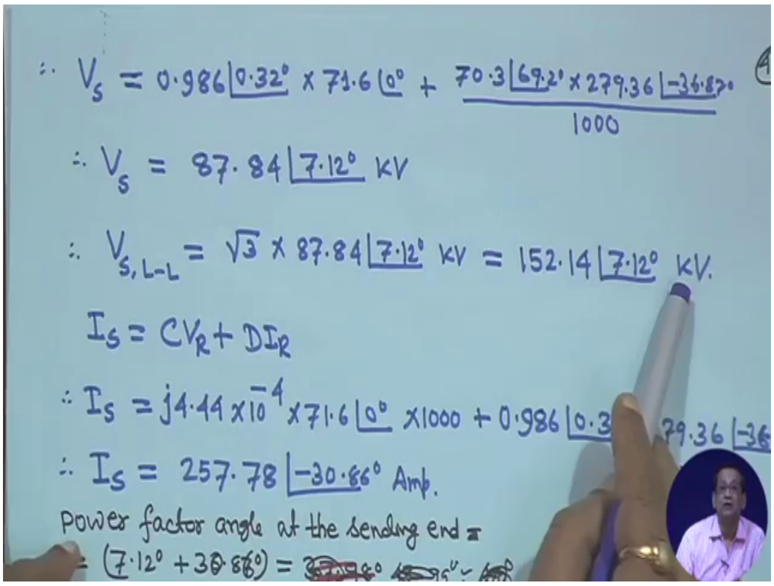

And this is your V value 70.3 angle 69.2 and this current is 279.36 ampere. So, divided by 1000 such that it will become this whole quantity will become in terms of voltage will be kilo volt right.

Angle minus 36.87 degree because this is kilo volt right, so, you will get V S is equal to 87.84 angle 7.1 degree kilo volt. So, sending end voltage L dash L means line to line the root 3 in to 87.84 angle 7.12 degree kilo volt that is 152.14 angle 7.12 degree kilo volt right.

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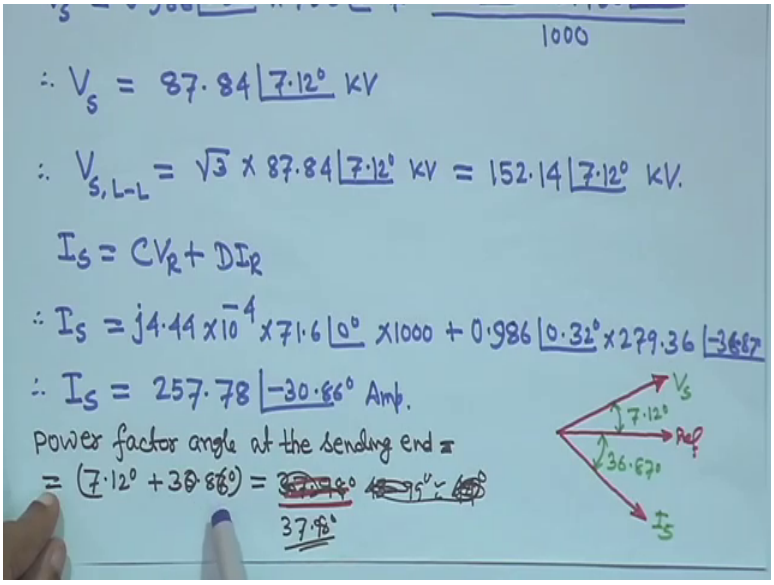

Now, sending end current I S is equal to C V R plus D I R C and D both have been computed right. So, and V R is known I R also known right. So, all this value you substitute here you substitute here right here you substitute and compute you will get you will get that I S is equal to 257.78 angle minus 30.86 degree ampere right therefore, this then to find you take a reference line this is your reference line your V S is 152.14 angle 7.12 degree; so, leading from this reference line. So, this is your V S 7.12 degree and current is minus 30.8786 degree; so, this side 36.87 degree. So, angle the power factor angle will be angle between this sending end voltage and this sending end current. So, it will be 7.12 plus 30.86 degree is equal to 37.98 degree right.

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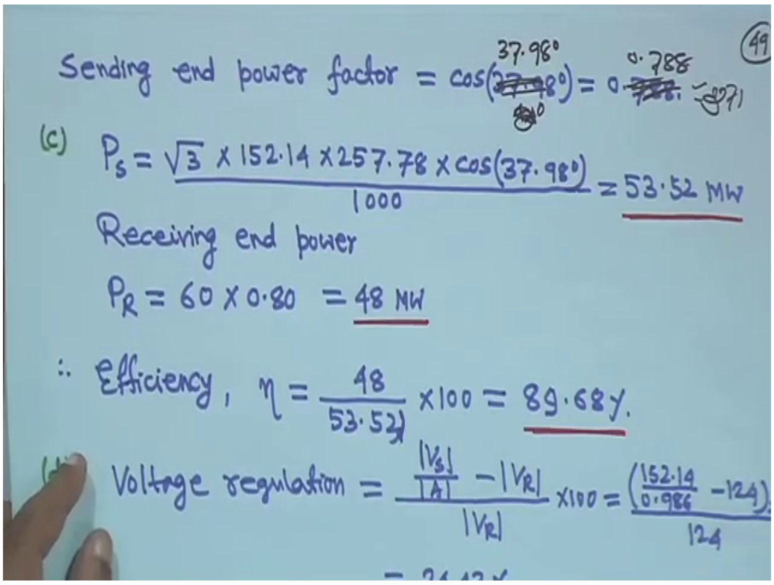

So, this is that your power factor angle. So, once you get this power factor angle. So, you take this you take this as a sending end power factor that is cosine of 37.98 degree that is equal to 0.788 right, this is the sending end power factor now third part is that you have to find out the regulation right. So, P S is equal to sending end power is equal to root 3 your V I this is your one line to line this is your 152.14 KV in to your 257.78 cosine, this 1 divide by 1000 because this is in ampere and that is why we have divide by 1000. So, and this is in kilo volt this will be kilo ampere. So, ultimately it will become 53.52 megawatt right and receiving end power 60 in to 0.8. So, 48 megawatt right this is your 3 phase power right and this is 3 phase sending end power. So, 53.52 receiving end is 48. So, efficiency is your what you call 48 up on that is P R up on P S this we know. So, 48 up on 53.52; 89.68 percent right with efficiency of the line it was.

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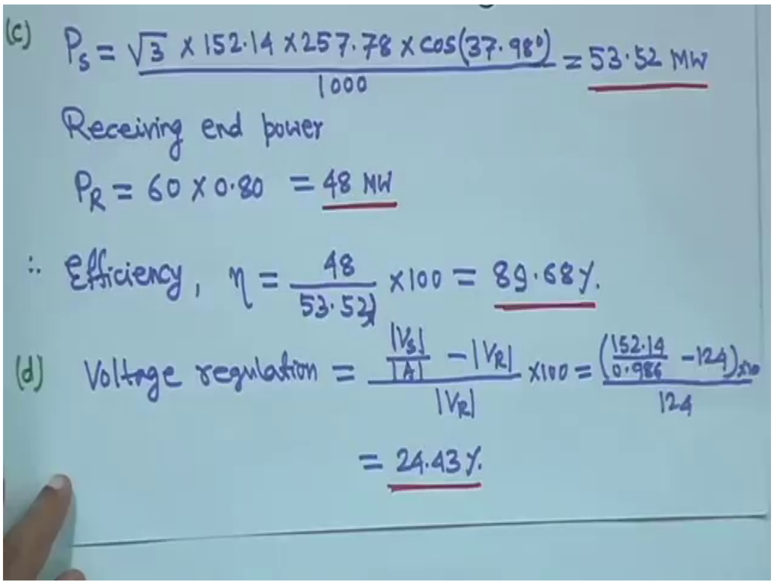

Then this D actually it was given in D efficiency of the line, but here itself we have computed right and voltage regulation is mode V S up on a minus mode V R divided by mode V R V S is known magnitude of V S is known magnitude of a is known magnitude of V R is all known you substitute here. So, if your voltage regulation is 24.43 percent right. So, this is this is actually if you have this example considering a long transmission line.

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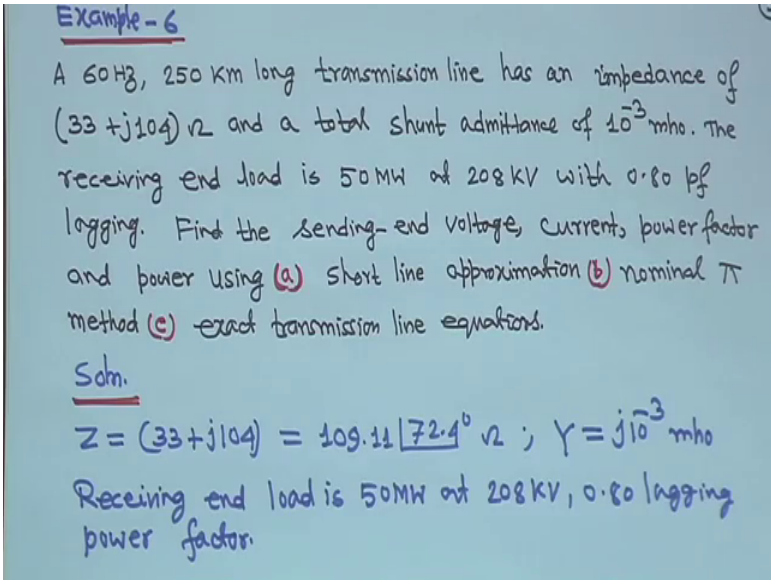

Right next one is for this for this long transmission line this is an example where same data have been taken and used for short line medium line and long line. So, a 6 a 60 hertz 250 kilo meter long transmission line has an impedance of 33 plus j 104 ohm and a total shunt admittance of 10 to the power minus 3 mho the receiving end load is 50 megawatt at 208 KV with a 0.8 power factor lagging. You have to find out the sending end find the sending end voltage current power factor and power using short line nominal pi method and exact transmission line and then compare although line is 250 kilo meter long, but you consider short line assumption then medium and then long line right.

So, will use the same data will use the same data. So, Z is given 33 plus j 104 is equal to 109.11 angle 72.4 degree ohm Y is equal to j 10 to the power minus 3 mho Siemens I am not writing I prefer to write mho right. So, writing mho receiving end load is 50 megawatt at 2 208 KV and 0.8 lagging power factor. So, first is you have to find out the your current.

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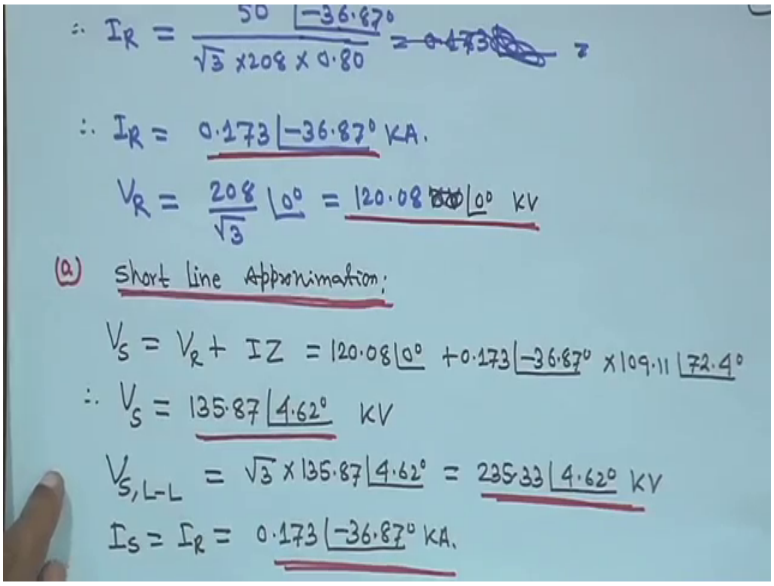

So, it is your; it is 50 that your receiving end load is 50 megawatt right. So, it is your root 3 V I cos phi is equal to 50, but current angle is also taken because power factor is 0.8 lagging. So, we are writing I R is equal to 50 by root 3 208 in to 0.8 and angle minus 36.87 degree. So, I R is equal to this one; this is your; what you call this 50 is a megawatt and this 208 is a kilo volt. So, that is why this current will be kilo ampere right.

So, I R is equal to 0.173 angle minus 36.87 degree kilo ampere. So, phase voltage 200 up on root 3. So, 120.08 angle 0 degree KV. Now first you consider this is short line approximation short line means V S is equal to V R plus I Z. So, substitute V R the I and then Z right you will get V S is equal to 135.87 angle 4.62 degree kilo volt right and. Therefore, sending end voltage line to line multiply this 1 by root 3 you will get 235.33 angle 4.62 degree KV right and for short line sending end current is equal to receiving end current is equal to this 1.173 angle minus 36.87 degree kilo ampere we for the short line right therefore, your therefore, sending end your sending end power factor I have told you that your current is minus 36.8 and voltage is your what you call angle is 4.6.

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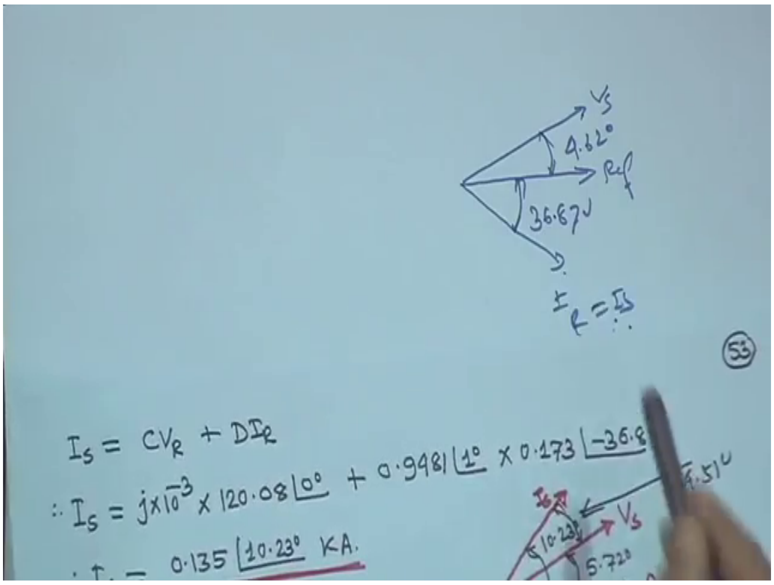

One second I making it suppose this is your reference line this is your reference line and your voltage is your 235.33 sending end voltage 4.62 degree. So, from this reference line this is your V S and this angle is your 4.62 degree right and current is your what you call minus 36.87. So, this is your this is your current right I R say this thing your what you call and I R is equal to I S V S I S for short line I R is equal to I S. So, this angle is equal to thirty six point your eight seven degree this sending end voltage sending end current, but let me tell you.

This I R is equal to actually I S sending end volt for short line right. So, power factor angle will be angle between this voltage and current. So, it will be 4.62 plus 36.87 this is what I have done here right so; that means, that power factor angle this thing you will be actually directly I am writing that power factor angle is 36.87 degree plus 4.62. So, basically it is cosine of this thing.

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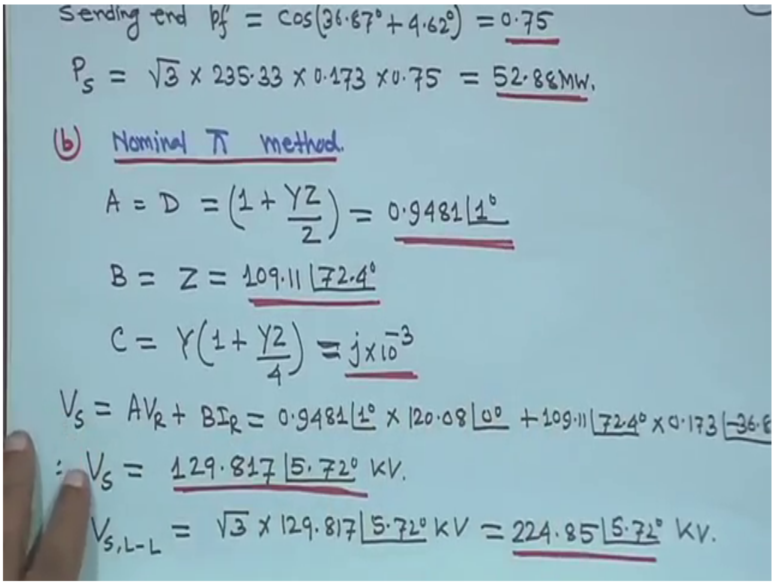

So, 0.75 and sending end power is equal to root 3 sending end voltage this line to line in to that current point this thing your 173 in to the power factor this 0.75 right. So, this is 52.88 megawatt this is for short line right after that will put in a table tabular form and you will see the difference the when the same with the same data when you consider the nominal pi method it is a is equal to D 1 plus Y Z by 2 if compute you will get 0.9481 angle just 1 degree right and B is equal to Z.

So, 109.11 angle 72.4 degree and C is equal to Y into 1 plus Y Z by 4 that will become approximately j in to 10 to the power minus 3 and you know V S is equal to a V R plus B I R you substitute here all the parameter A V R B and I R you substitute all you will get sending end voltage 129.817 angle 5.72 degree KV therefore, V S line to line it will be root 3 in to this 129.817 that is your 224.85 angle 5.7272 degree KV right.

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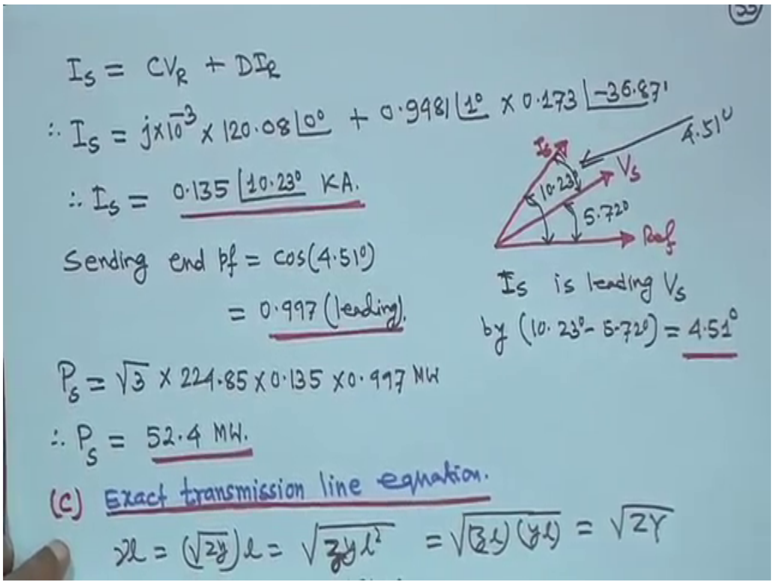

This is line to line voltage next is that I sending end current I S is equal to C V R plus D I R that is your put C D value V R; I R value here right all you put you will get I S is equal to 0.135 angle 10.2323 degree kilo ampere therefore, this is your this is your this is this one you take this reference line this reference line and V S line to line is given 5.72 degree. So, this is V S 5.72 degree and current angle I S is angle 10.23 degree. So, this is your I S from the reference line 10.23 degree. So, you have to get the angle between voltage and current V S and I S.

So, it will become your 10.23 minus 5.72 right; that means, this one; that means, your this one this angle you have to find out this angle right and this angle actually comes 4.51 degree right so; that means, the sending end power factor is cosine 4.51 degree right that is 0.997 and it is leading power factor because current I S actually leading the voltage right. Previous case you have seen that it is lagging current is lagging right from V S it is lagging right, but that is for short line and for this one you have seen that current sending end actually leading the your voltage right that that is why it is a leading power factor 0.997. So, and here I have written I S is leading V S and by 10.23 minus 5.72 degree that is 4.51 degree.

So, P S is equal to root 3 224.85 and this is the current in kilo ampere in to the power factor 0.997. So, P S is equal to 52.4 megawatt that is the sending end power right.

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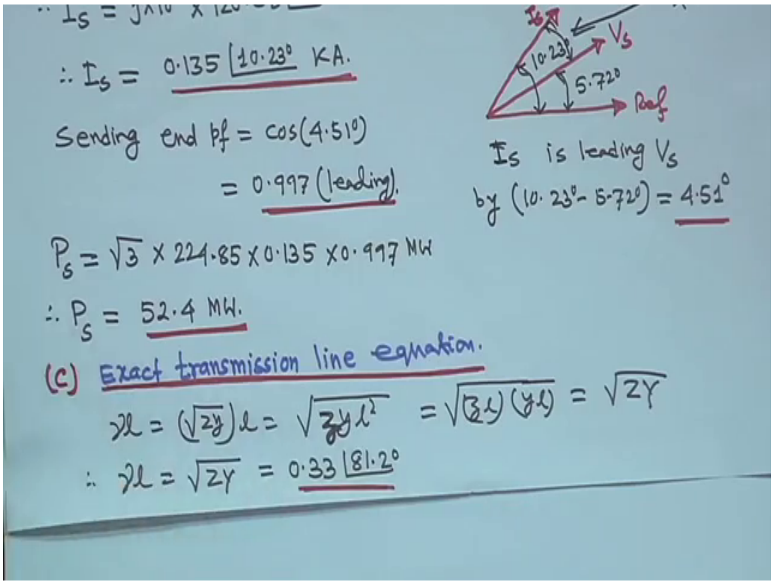

Then last one is that exact transmission line equation that is your tan hyperbolic right those things whatever you have got cosine sin tan everything. So, first you find out the gamma L is equal to root over Z Y L we have seen that the gamma L actually just in the previous example you have seen that gamma L root over Z Y capital Z capital Y you substitute all capital Z capital Y values because all are known to you will get 0.33 angle 81.2 degree right.

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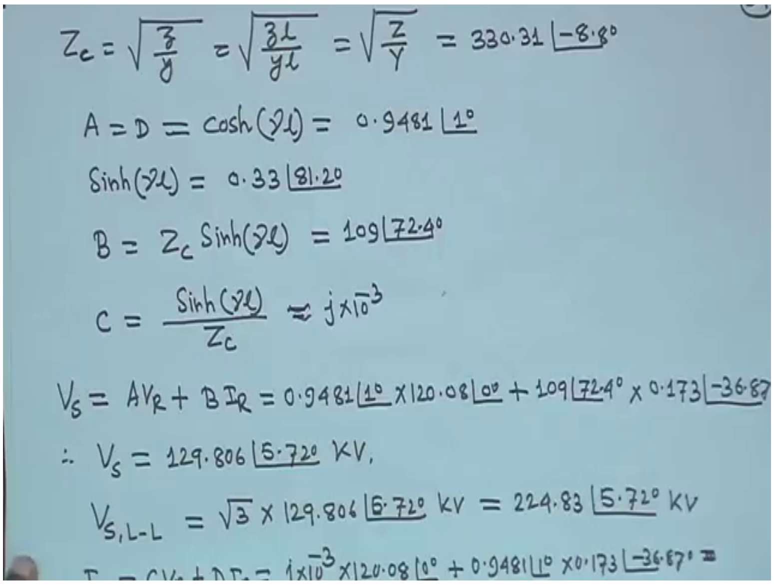

So, Z C is equal to root over Z Y. So, small z y is equal to root over capital Z Y this one is also you have seen you put Z and Y values you will get 330.31 angle minus 8.8 degree. Now a is equal to D is equal to sod hyperbolic gamma L put all this value you will get it 0.9481 angle 1 degree right sin hyperbolic gamma L is equal to you will get 0.33 angle 81.2 degree right and B is equal to Z C sin hyperbolic gamma L that will get hundred nine angle 72.4 and C is equal to sin hyperbolic gamma L up on Z C you substitute all the values you will almost get the same value j in to your j in to 10 to the power minus 3, now sending end voltage V S is equal to A V R plus B I R. So, A V R B and I R you substitute here you substitute here all these values have been substituted right.

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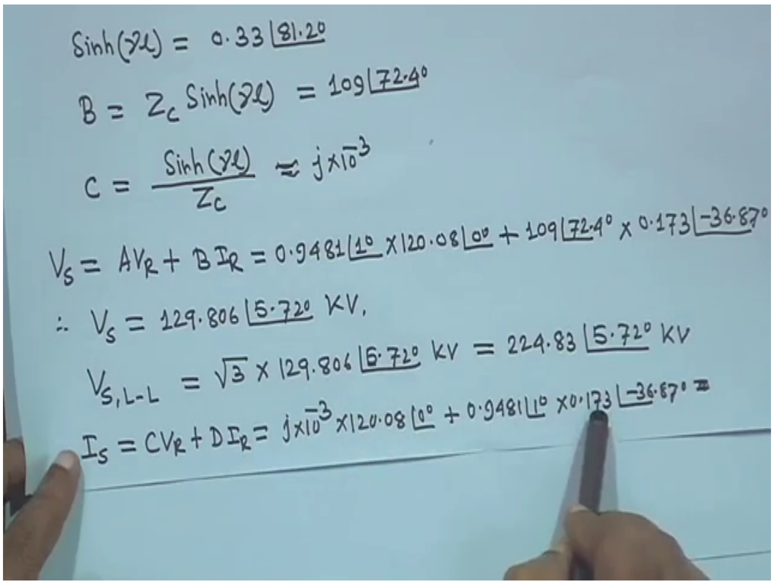

And you will get it V S is equal to 129.806 angle 5.72 degree kilo volt right therefore, sending end voltage line to line multiply by root 3 in to this one 224.83 angle 5.72 degree kilo volt now similarly sending end current I S is equal to C V R plus D I R C is known D is known V R I R all known you substitute here you substitute this they have not to repeating again, but you substitute all these values only thing is this is in KV. So, this is in kilo ampere. So, this drop will be this; this part will be in terms of kilo volt right. So, in that case if you compute right.

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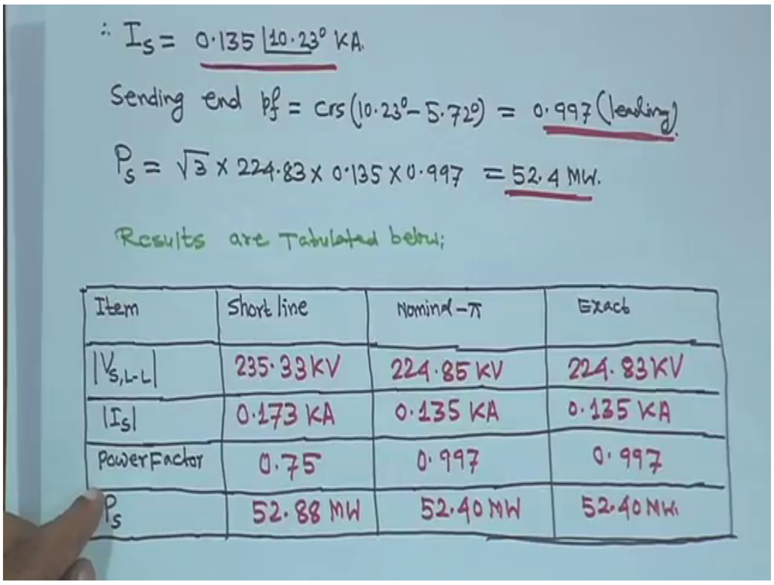

Using your calculator you will get that your what you call this I S is equal to 0.135 angle 10.23 degree 2 3 degree kilo ampere right and in this case also in this case also that V S is V S is angle is 5.72 degree right and current is 10.2323 degree from a reference line if you take the same as before. So, current actually is leading the sending end voltage here also. So, power factor will be at cosine 10.23 minus 5.72 degree same as before that is 0.997 leading. So, sending end power is equal to root 3 in to V R in to this current magnitude in to 0.997 power factor that is 52.4 megawatt right.

So, if you compare this 3 cases short line nominal pi method and exact method for the short line case that your sending end line to line voltage is 235.33 KV, but for nominal pi method sending end voltage is 224.85 KV and for exact method also 224.83. So, nominal pi and exact both are more or less same, but for short line your this is actually 235.33 KV right for the short line. Now for sending end magnitude of current this is 0.173 kilo ampere, but here it is 0.135 kilo ampere here it is 0.135 kilo ampere this 2 are almost same this 2 are same, but here it is slightly more. So, what happens actually when you have your considering the shunt capacitor right sorry that charging capacitor right. So, basically it injects your what you call in the line that your reactive current right.

So, in that case what happen that to some extent that current has 2 component right your this thing that is one in real component another is your reactive component. So, when you are in that capacitance charging capacitance actually injecting reactive current. So, the reactive component of the current will be slightly less right. So, and in that case when you take the magnitude of the current. So, that is why this current will be less than this one because here no charging no charging admittance is consider for short line, but for this one or this one those things have been considered that is why current is slightly less than this one this is in a bitable right and power factor when you are not considering anything right for short line no charging and it is 0.75.

But it in, but if you consider nominal pi or exact method the power factor is much improve 0.997 and both are same this is again the same philosophy this is happening due to consideration of charging admittance. So, for long line right you cannot ignore the charging admittance and for the short line case that your sending end power 52.88 megawatt, but in the case of nominal pi your exact method it is slightly less 52.4 here also 52.4 because of that reactive current injection I told you the current magnitude is less. So, this current is slightly less right and this is ok.

So, if you think this way; that means, that means line loss will be for the nominal pi or exact will be slightly less compared to the short line your what you call consideration right. So, this is you know example I gave short line nominal pi and exact method, but one thing you can see nominal pi almost act as accurate as exact method. So, that is why you will find for all transmission line load flow studies charging admittance they are considering basically nominal pi method right if line is too long. So, this is per overhead transmission line next is.

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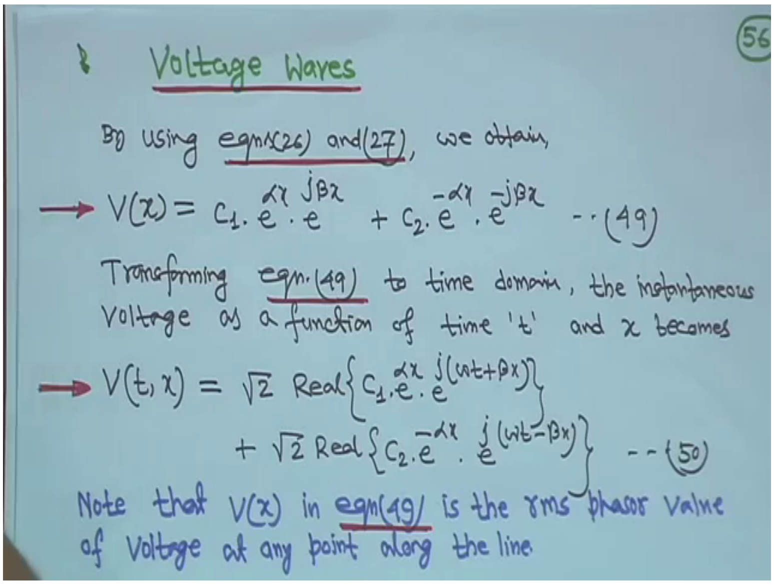

So, for this transmission line some six different examples I have given to you right and this will this will clear your ideas that different type of problem right now next your next is that your voltage waves right. So, by using equation 26 and 27 right, so, I will show you 26 and 27 just few second right this if you recall this is your equation 26 this is your equation 26 V X is equal to C 1 e to the power gamma X plus C 2 e to the power minus gamma X right.

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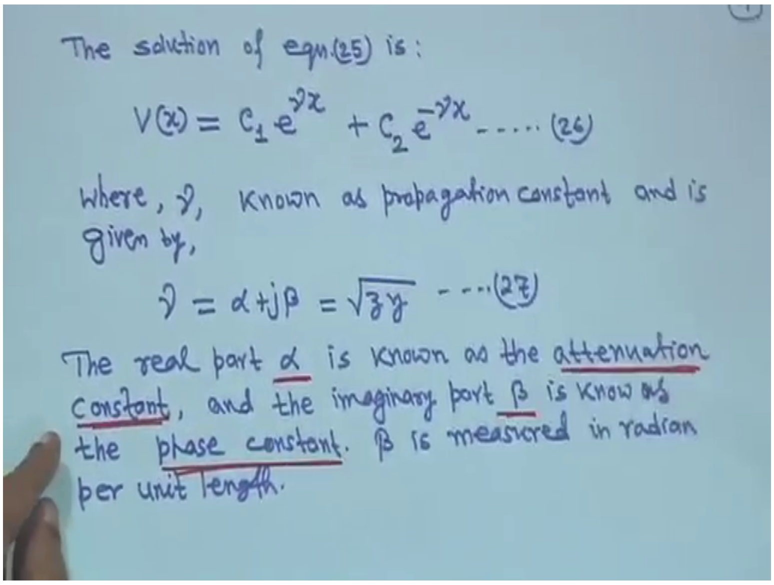

This is equation 26 and gamma that your propagation constant we have taken alpha plus j beta right. So, using 26 and 27 means you put this gamma value here you put the gamma value here if you if you do. So, then it will that 26 and 27 it will become V X is equal to C or C 1 e to the power alpha X plus in to e to the power j beta X because it is e to the power actually alpha plus j beta X. So, I have we are separating it like e to the power alpha X in to e to the power j beta X plus C 2 e to the power minus alpha X in to e to the power minus j beta X right this is equation forty nine now this equation forty nine you have to transform to time domain because this value actually its say R m S value this V X value actually it is a R M S value.

So, if you transform it to a time domain right the instantaneous voltage as a function of time T and X you can give it you can write V function of it is T X T coma X that is V is a function of time and the distance X right is equal to if this is R M S value it will be root 2 in to real part of this one C 1 e to the power alpha X then e to the power j omega T plus beta X because we are considering in time domain that is why writing e to the power j omega T plus beta X right and this is R m S value.

So, same as before whatever you have studied for your a C a C circuit theory right philosophy same right plus root 2 real part of C 2 e to the power minus X in to e to the power j omega T minus beta X right. So, this is that try your try with this equation we are transforming in to your time domain right. So, at this is equation 50, but after that I have whatever I said I wrote here that note that V X in equation 49 is the R M S phasor value of voltage at any point along the line; that means, this one we are transforming in to time domain right and X is the distant that is measured from the receiving end.

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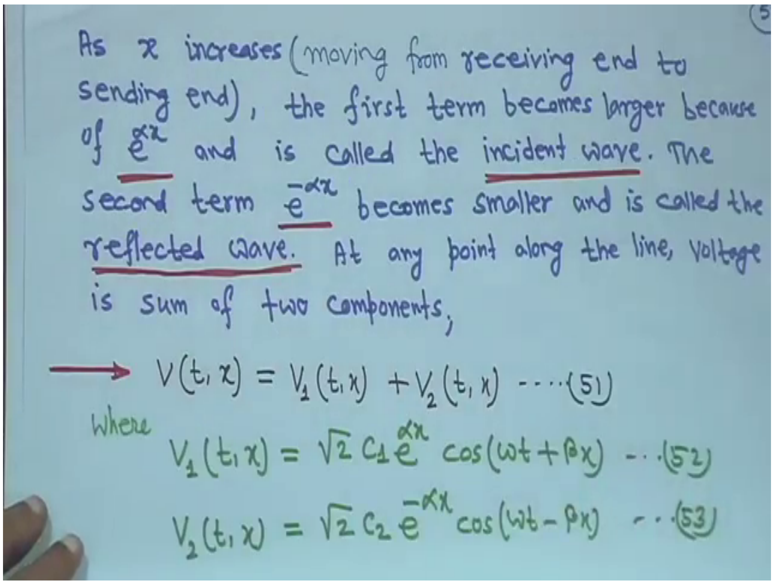

So, X is X increases that is moving from the receiving end to sending end the first term becomes larger right. So, if we if the as the length is increasing because when we are deriving the expression for long line I have considered that from the your distance is considered from receiving end to sending end; that means, when you are moving from receiving end to sending end X is increasing. So, when X increases e to the e to the power alpha X right. So, this term your will increase right. So, that is why that moving that is why the first term becomes larger because of e to the power alpha X and is called the incident wave right and the second term e to the power minus alpha X becomes smaller as X is increasing because it is e to the power minus alpha X and this is called the reflected wave.

So, one is incident wave another is called the reflected wave right. So, at any point along that line voltage is the sum of 2 components; that means, this term we are defining as V 1 T X the first term and the second term we are defining as V 2 T X right so; that means, that means V 1 T X that is a function V that is V 1 is a function of T X root 2 C 1 e to the power alpha X and if you take this cosine real that your real part of this one it will be cosine omega T plus beta X. So, this part is cosine omega T plus beta X this is 52 and second term V 2 T X will be root 2 C 2 e to the power minus alpha X it will be cosine omega T minus beta X because here e to the power j omega T minus beta X it would be the real part. So, it will be C 2 e to the power minus alpha X cosine of omega T minus beta X right this is equation 53. So, this is the expression for V 1 and this is the expression for V 2 right now as we move along the line. So, if you if you move along the line.

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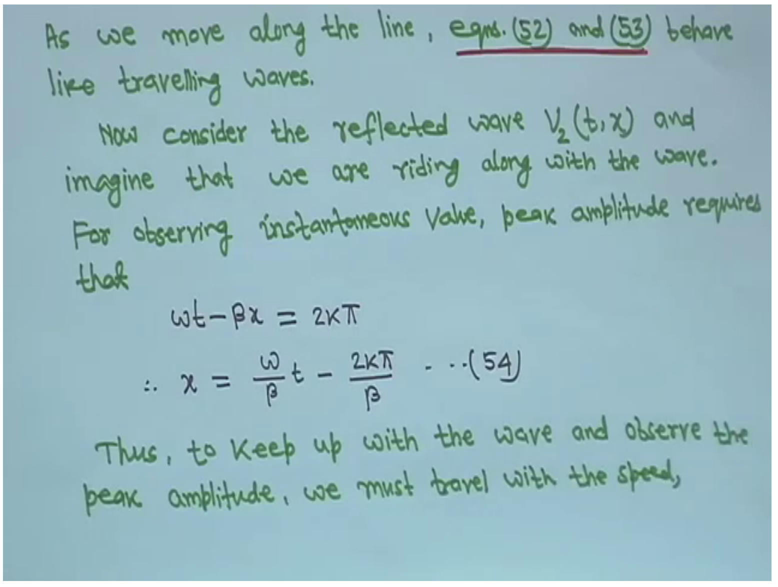

Then what will happen that your that equation 52 and 53 its behave like a travelling wave right because one is incident wave and another is reflected one now you consider the reflected wave will consider this reflected wave part this part this part right reflected wave. So, that is V 2 T X and imagine that we are riding along with the wave and imagine that we are riding along with the waves; that means, if you want to see the peak amplitude of the travelling wave the wave that that wave speed the what will be the wave speed and if you also move along with the same speed then only you can observe the peak.

So, imagine that that your what you call we are that we are riding along with the wave right we are considering the reflected wave for observing instantaneous value peak amplitude requires that omega T minus beta X is equal to 2 k pi; that means, this one; that means, your this one; this one that you want that cos if you want to this is the peak value if you want root 2 C 2 e to the power minus alpha X whatever will be the peak value that cosine omega 2 minus beta X is equal to 1 right; that means, your cosine your omega T minus beta X equal to 1 right.

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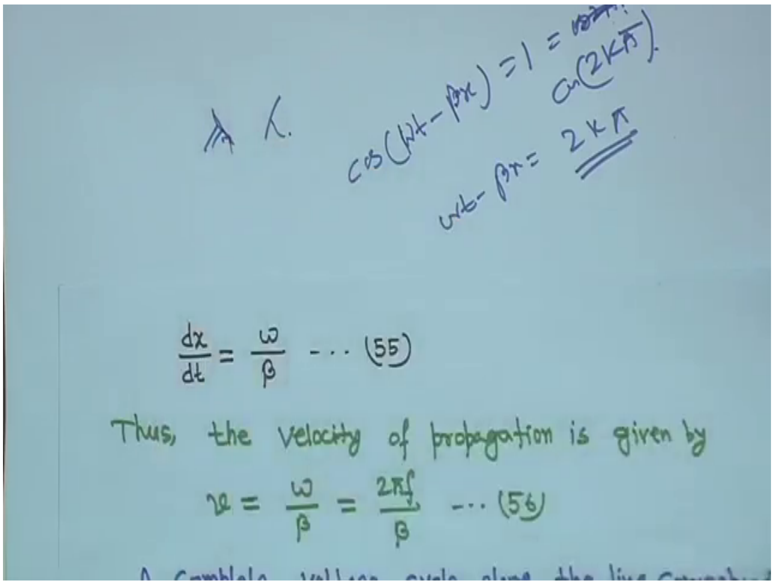

Is equal to you can write cosine 2 k pi 2 pi k or 2 k you can write is equal to 2 k pi I put like this right your cosine of 2 k pi right so; that means, omega 2 minus beta X is equal to 2 k pi it general thing right if k is equal to 0 cos 0 1 k is equal to 1 cos 2 pi also 360 degree 1 and so on. So, omega T minus beta X will be 2 k pi so; that means, for observing the instantaneous your what you call that value peak amplitude requires that that omega T minus beta X this should be one; that means, omega T minus beta X must be equal to 2 k pi just I told you right; that means, my X is equal to omega by beta T minus 2 k pi up on beta this is equation 4 right. Now to keep up with the wave and observe the peak amplitude we must travel with the wave speed right; that means, whatever is the speed right if you want to observe that peak then you have to travel with that.

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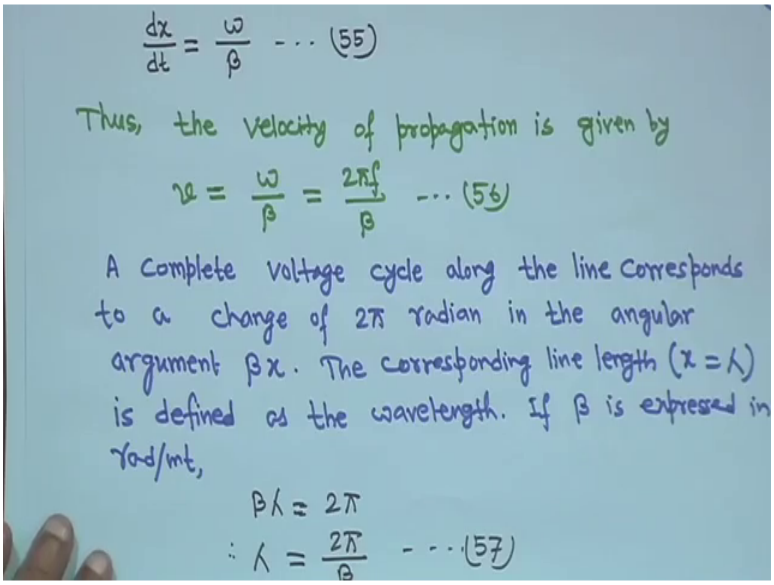

So, in that case what will happen that you take the; you take the derivative with respect to time that D for equation for equation 54. So, D X up on D T you find out right from the D X up on D T means basically it will be omega by beta right therefore, D X by D T will be omega up on beta right this is equation 55 thus the velocity of propagation is given by V is equal to omega by beta that is D X up on D T omega is equal to 2 pi f divided by beta this is equation 56 right.

So, this is the velocity of propagation now a complete voltage cycle along the line I have written here correspond to a change of 2 pi radian in the angular argument beta X right the corresponding line length X is equal to lambda right is defined as the wave length if beta you expressed in radian per meter beta is expressed in expressed in radian, but the meaning is that a complete voltage cycle along the correspond along the line corresponds to a change of 2 pi radian in the angular argument beta X right. So, in that case the corresponding line length X is equal to lambda is defined as the wave length; that means, that if beta X is equal to 2 pi that is X is equal to actually lambda right. So, that is why here we are putting that when X is equal to lambda beta lambda is equal to 2 pi. So, if lambda is equal to 2 pi by beta only one thing I will I will tell you.

Generally actually lambda this side become my habit I write like this, but actually it is like this right actually it is like this, but. So, you forgive me. So, this has become my habit. So, I write like this right. So, so beta lambda is equal to 2 pi. So, lambda is equal to your 2 pi by beta right. So, this is actually your; what you call that wave length right now some more important thing.

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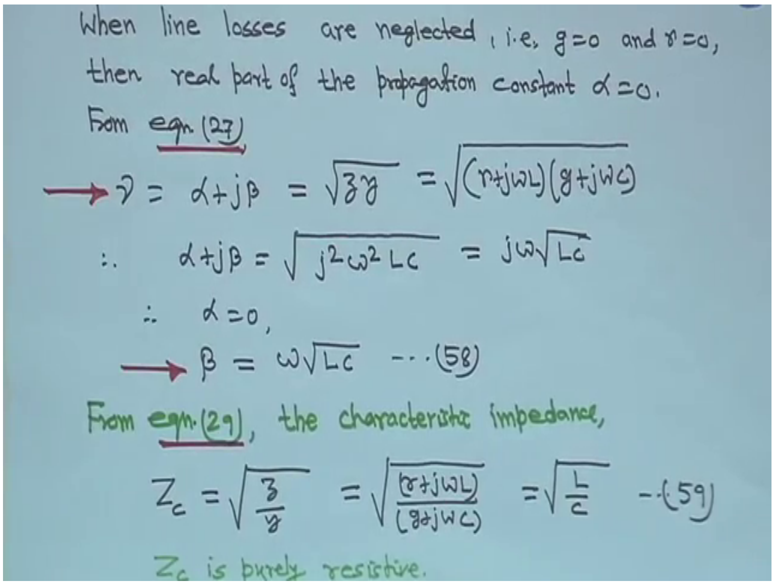

So, while line losses a neglected; that means, g should be 0 and R should be 0 right. So, in that case if line losses are neglected then the real part of the propagation constant alpha will become 0 right; that means, from equation 27 gamma is equal to alpha plus j beta that is equal to root over small z small y. So, small z all nomenclature was given in the beginning of this topic, so, R plus j omega L in to g plus j omega j omega C right. So, as R is 0 g is 0 you substitute R 0 g 0 here you will get square root j square omega square L C that will be j omega root over L C. So, no real part; that means, alpha is equal to 0 and beta is equal to omega root over L C this is happening when line is a loss less we are assuming that a loss less line.

Therefore equation twenty nine the characteristic impedance we know the Z C is equal to root over small z up on small Y and small z is equal to root over R plus j omega L divided by g plus j omega c, but R is 0 g is 0. So, if you put R is 0 g is 0 j omega j omega will be cancel and ultimately it will become Z C will simply become root over L by C right. So, Z C actually is a pure resistive it is purely resistive right there is no complex part here root over L by C is a purely your resistive.

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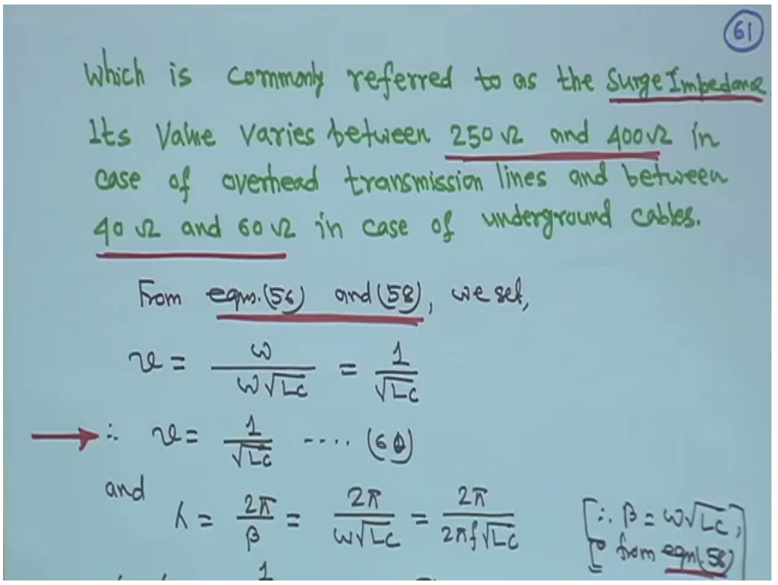

Then; that means, which is commonly referred to as the surge impedance right. So, Z C actually common to referred to as the surge impedance its value varies between 250 ohm and 400 ohm in the case of overhead transmission line and between 40 and 60 ohm in case of underground cables right.

Thank you then again will come back.