Power System Analysis
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Lecture - 28
Load Flow Studies (Contd.)

So this we have seen, now from this equation V bus then can be I is equal to i bus equal to Y bus V bus right; Y bus V bus.

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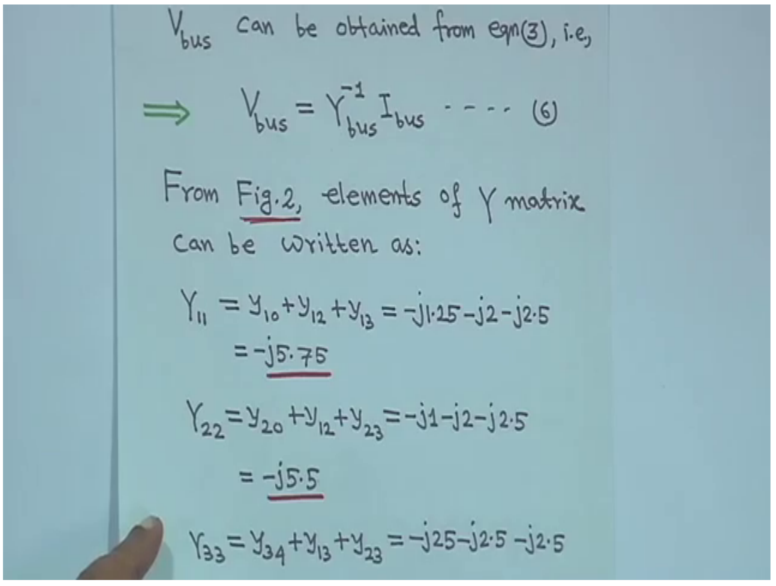

Therefore V bus can be made at that your Y bus inverse I bus right. So, now, from figure 2 elements of Y this figure 2 just hold on, just hold on, just hold on, where that figures 2.

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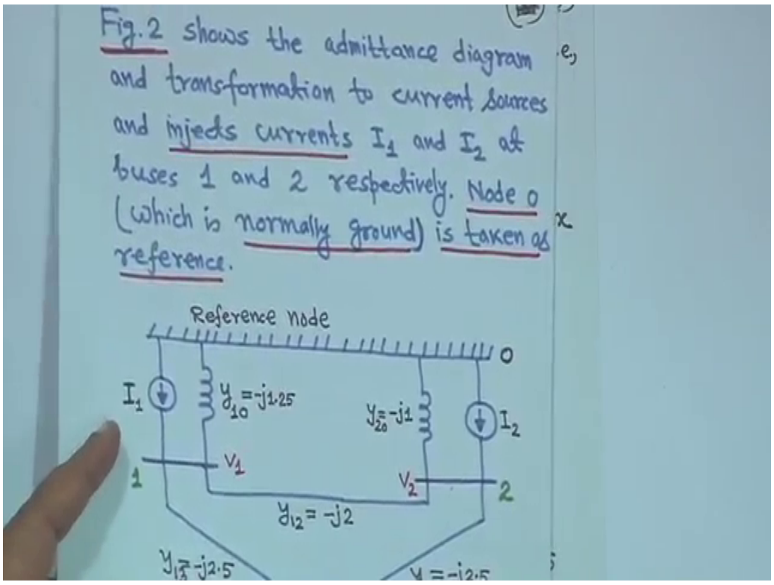

So, from figure 2, you can compute know Y 1 just now we have seen, Y 11 is equal to from here only from this figure only Y 11 it will capital Y 11 is small y 10 plus small y 12 plus small y 13 right.

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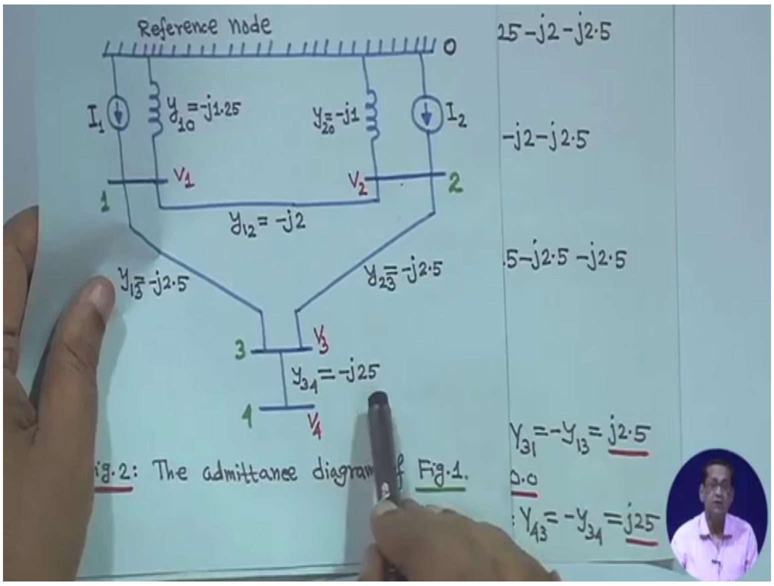

So, all this Y 10, Y 20, Y 12, small y 12, all small one. Small y 13 small y 23 and small y 34 all, these are known to you right. Therefore, you compute capital Y 11 is equal to Y 10 plus Y 12 plus Y 13. Add all these it will become minus j 5.75 right.

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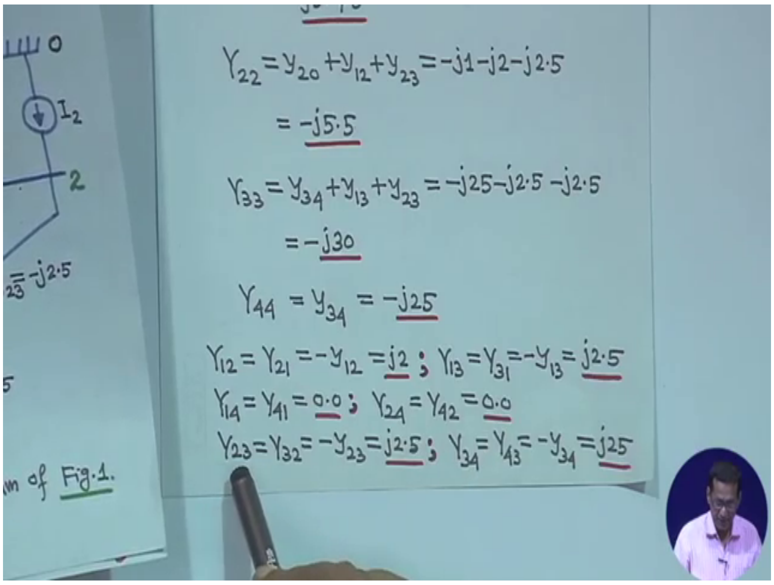

Similarly, diagonal elements Y 22 is equal to small y 20 plus small y 12 plus Y 23 that is minus j 1 minus j 2 minus j 2.5. So, capital Y 22 is equal to minus j 5.5. Similarly capital Y 33 is equal to small y 34, plus small y 13, plus small y 23. That is minus j 25, minus j 2.5 minus j 2.5 that is minus j 30. Similarly capital Y 44, is equal to small y 34 is equal to minus j 25 right, because this small y 34 is minus j 25. Now capital Y 12 is equal to capital Y 21 is equal to minus small y 12 that is j 2. Similarly capital Y 13 is equal to capital Y 31 is equal to minus small y 13. So, it is j 2.5 because Y 13 is equal to minus j 2.5; so minus Y 13 j 2.5 right.

Y 14 is equal to capital Y 14 is equal to Y 41 is equal to 0, because line 1 to 4 there is no connection. So, Y capital Y 14 is equal to Y 4 is 0, similarly 2 to 4 there is no connection right, no line. So, capital Y 24 is equal to capital Y 42 is equal to 0.0 right. And capital Y 23 is equal to capital Y 32 is equal to minus small y 23.

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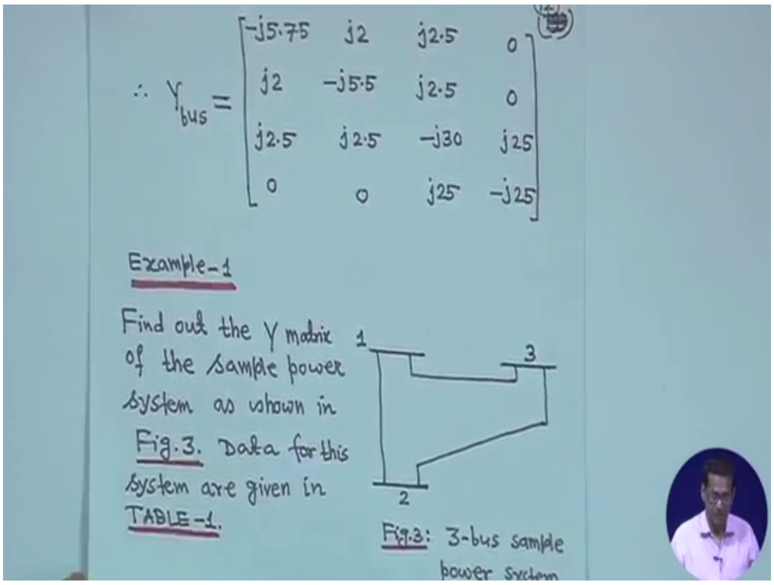

That is actually j 2.5, and capital Y 34 is equal to capital Y 43 is equal to minus Y 34 is equal to j 2 point sorry, j 25 right. So, with this, with this if you form that your what you call from the bus admittance matrix. So, this is your Y bus matrix right. And it is a symmetric matrix in general Y ij is equal to Y ji. So, for example, you 1 Y 12, Y 21, Y 13 is equal to Y 31 right. And Y 14 is equal to Y 41 right. Similarly all that is a symmetric matrix right and this is your Y bus matrix, but here resistance is not considered right.

So, next you take a small example, small example because in as a classroom study we cannot go beyond 3 bus problem right. Because then lot of computation will be involved and it is not possible, but 3 bus problem is sufficient and I will not tell all this thing only fundamentals of this load flow studies right. How actually one can work out? So, you have to find out the Y matrix of the sample power system as shown in figure 3 data for this system are given in table one.

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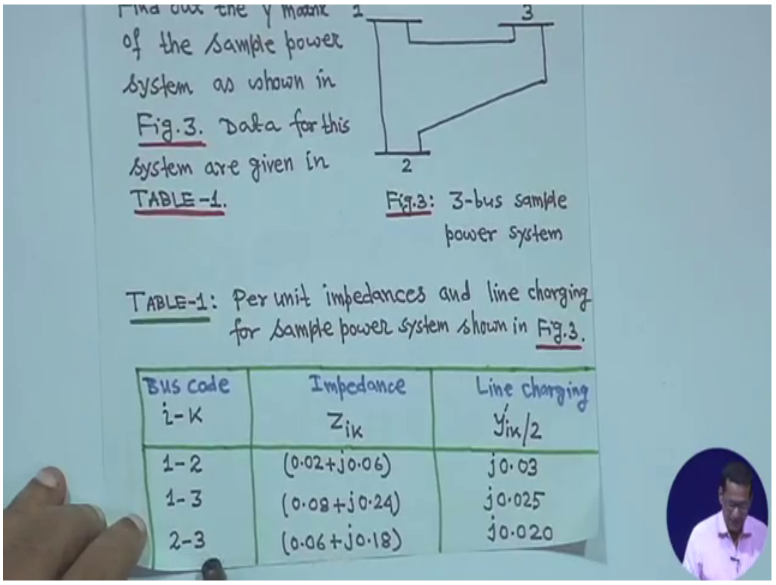

So, this is 1 2 3, the 3 bus problem and generator are thing nothing shown only 1 2 to 3 all this things are given right. So now, the charging admittance we have taken an how charging admittance will included I will tell you right. So, this is the bus code they write bus code that is I mean current sending and receiving and bus i to bus k right. So is so, it is just one end to another end one bus to another bus.

So, 1 to 2 bus impedance is given Z ik it is 0.02 plus j 0.06 as you all are in per unit a line charging in bit admittance is given Y ik dash by 2 half. When you, because you for transmission line you have to make pi nominal method Y by 2 Y by 2. So, it is given Y dash ik by 2 by chance if it is given Y dash ik, then you have to make it half, half right. Similarly 1 to 3 0.08 plus j 0.24 and this is j 0.025, that is Y dash your basically 13 ik. So, Y dash 13 by 2 this is given. Similarly line 2 to 3 that is j 23 is given 0.06 plus j 0 0.18 and here it is one 2 to 3. So, Y dash 2 to 3 actually 0.020 right.

Now, question is that before proceeding that, how will take that line charging admittance? Suppose you have a line, suppose you have a you have a line.

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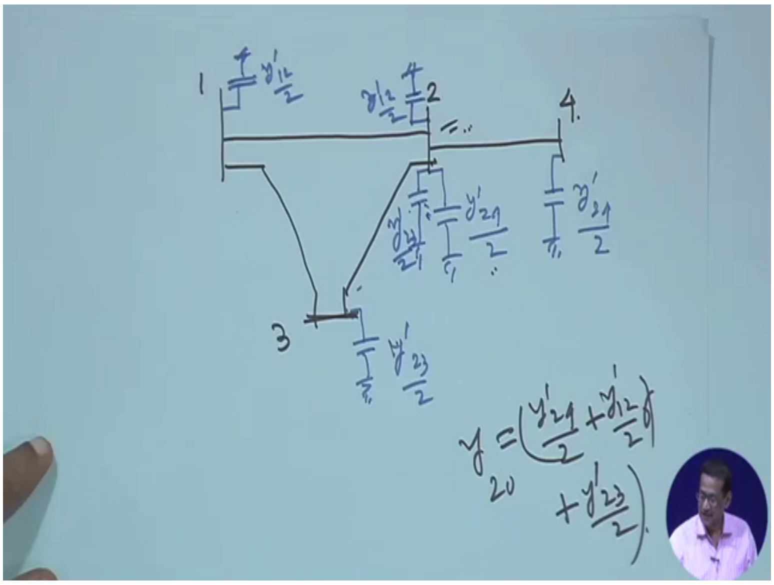

You have a suppose you have a this kind of line say, transmission line right. You have a transmission line say this one right. You have more line it does not matter may be another line right. Suppose this is 1, this is 2, this is 3, and this is 4. So, these are 4, 1 2 3 4 transmission lines are there. So, in that case what you can what you can this thing that, for this one this line has charging admittance. So, this side, this side if you take.

Say this is 2 to 4. 2 to 4 we can put say Y dash 2 to 4 by 2 this side, and this is also Y dash your 2 to 4 by 2 this is 1 right. Similarly say line 3 to 4 also charging admittance. So, for this line also 3 to 4 sorry, 2 to 3, 2 to 3; 2 to 3 then here this side also will be I am putting like this side will be half that your Y dash, your 23 by 2 and similarly this side also. So, this is also your Y dash 2 to 3 by 2 this side also right. Similarly 2 to 1 you have charging admittance 2 to 1; that means, this is actually here also your what you call here also you have this one, and here also this side also 1 to 2, this side also you have charging admittance right.

So, this should be Y dash 1 2 by 2 and here also this one also Y dash 1 2 by 2; that means, as bus 2 because of line 2 for Y 2 dash for Y 2 dash 24 by 2 and for line 2 to 3 Y dash 23 by 2 and because of line 1 to 2 Y dash 1 2 by 2; that means, at this point, at this point I mean at bus by 2 say that total charging admittance we can put Y 20 is equal to Y dash 24 by 2 right. Y dash 24 by 2 plus right. Then Y dash 1 2 by 2 plus this Y dash 23 by 2. This will be the total charging admittance this 3 terms you have to add at bus 2 right. At bus 2, this way you have to consider the charging admittance right; that means, half, half you make this then sum up all the charging admittance right. So, this way you have to consider the charging admittance for the transmission line.

So, that is why this your what you call this Y dash ik by 2 everywhere it is given; that means, if you consider this your what you call this example.

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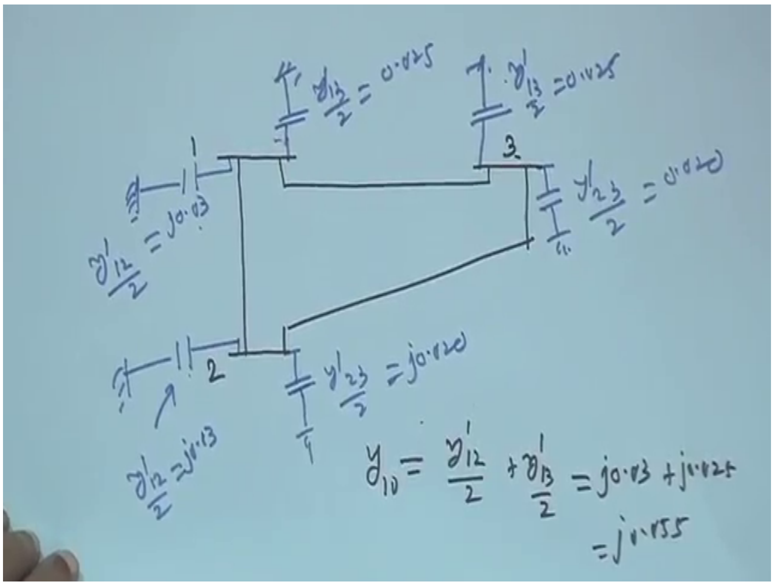

This example, I am making it for you that this is just for your understanding, say 3 bus problems it is this is 1 and this is your bus 2, and this is your bus 3. So, this is one then this is your bus 2 and this is your bus 3. Now for this case it is given that your line 1 to 2 it is given Y dash ik by 2 which is given there is half is already given; that means, your this one what you call this 1 to 2, that is 1 to 2; that means, this side I make it like this that this side right. This is your j 0.03, this is actually Y dash 1 2 by 2, this is given right.

Similarly your this side also because of this thing, this side also that this is also your this one that is also Y dash 2 by is equal to j 0.3 this side is given similarly, 123 also given j 0.025 it is 1 2 3; that means, this side actually making it here making it here that this is actually, Y dash 13 Y dash 13 by 2 is given that is 0.025. And similarly this side also you have to make it half, half right. And this is also Y dash 13 by 2 is equal to 0.25, this is also given. And Y 23 is also given that is 2 to 3; that means, here that Y dash 23 by 2 it is given, that is j 0.20, this is given and similarly, here also you have to make it here also you have to make it this is Y dash your 23 by 2 is equal to 0.20 right. So, this thing it is half is given suppose, it 2 is not given then you have to make it divided by 2 right.

So that means, your Y 1 this is your Y this is bus 1 So that means, your Y, your 10 this Y 10 this notation we write, it is Y dash 21 Y dash 1 2 by 2; that means, this line this side is half, and for this line Y dash 13 by 2 right. This is actually this is actually j 0.03 plus j 0.025 this you have to add right. That will become j 0.55. So, this things this 2 things you have to add. This way you have to consider the charging admittance right. I mean all the line you have to consider half, half Y 2 Y dash 2 and then you sum it up all, because that charging admittance So that means, that charging admittance of that example right, this example that Y, Y 1 Y 10.

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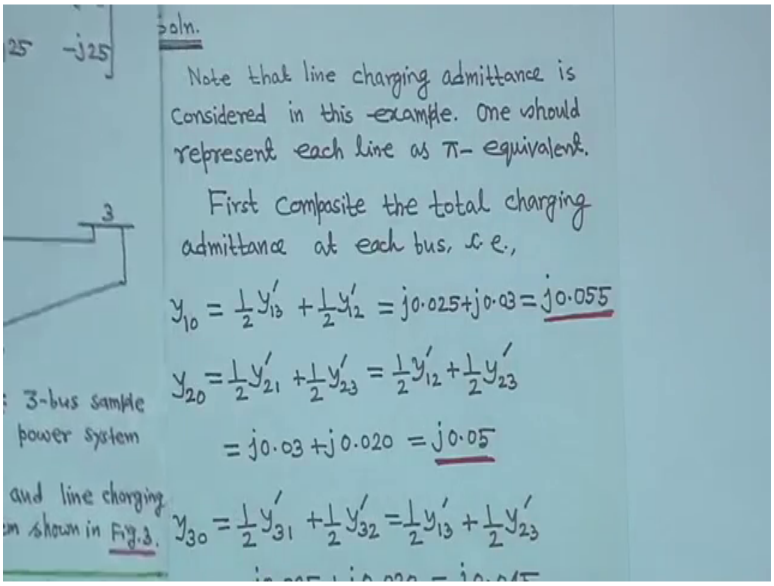

So, now you have understood this first one I told you, that Y 10 is half Y 13 dash plus half Y 12 dash, is equal to j 0.25 plus j 0.3 that is j 0.055 right. Similarly that Y 2 Y 20 Y 20 will be your half Y dash 21 actually Y 1 Y dash 21 or Y dash 1 2 they are same right. So, it is half Y dash 21 that is 221 they are same actually right. Plus half Y dash 23, 23 and 32 they are same. So, it is that is why I am writing is equal to half Y dash 1 2 plus half Y dash 23 right. That is it is j 0.03 is given, as a look already it is Y dash ik by 2 given, this is actually half, half values. So, do not divide it by 2 further do not do this right. So, this way you will get Y 20 is equal to j 0.5, 0.05 right

Similarly, for this bus also Y 3 0 is equal to half Y dash 31 plus half Y dash 32 is equal to they are same 31, 13 and 32, 23 actually same. So, half Y dash 13 plus half Y dash 23 is equal to j 0.025 plus j 0.20, 20 that is j 0.045 right. So, this is Y 10 Y 20 Y 3 0 charging admittance computed right. Now next you compute small y 12, because Z 1 2 Z 1 2 is 0.02 plus j 0.06 is given Z 13 0.0 8 plus j 0.24 is given and Z 23 0.06 plus j 0 0.18 is given right.

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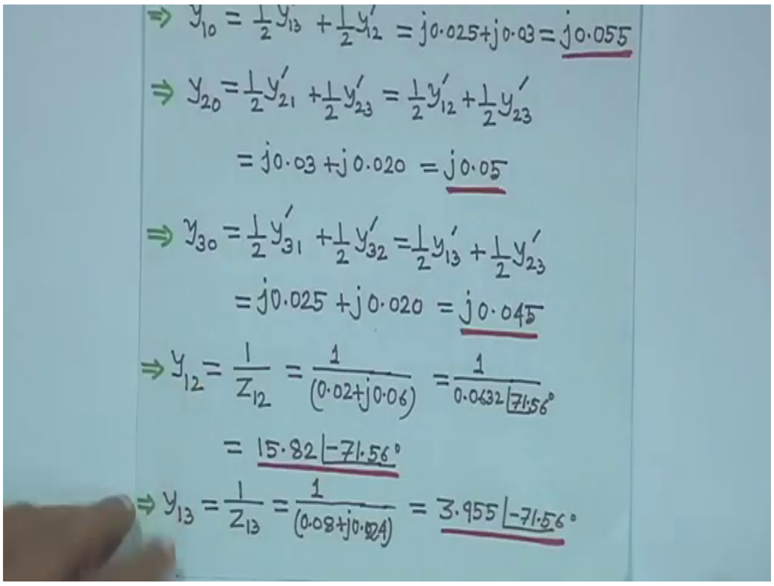

Therefore this Y 12 is equal to 1 up on Z 1 2 that is 1 up on 0.02 plus j 0.06. So, it will become actually 15.82 angle minus 71.57 degree that is Y small y 12. Now small y 13 is 1 up on Z 1 3. So, Z 13 is given 0.08 plus j 0.24. It will become 3.955 angle minus 71.56 degree right. Similarly, actually data taken in such a fashion, such that r Y x or x Y x ratio for this line parameters takes same actually ratio is same only multiplied by some other factor right.

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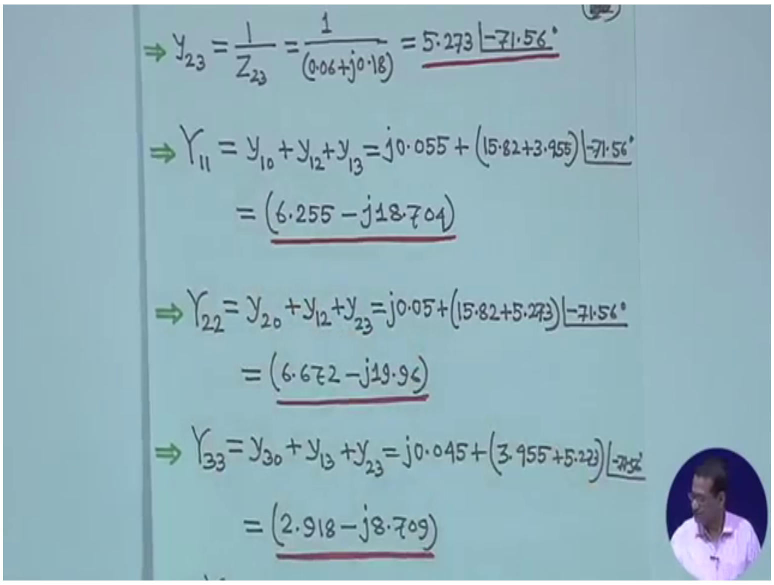

So, similarly Y 23 is equal to 1 up on Z 231 up on 0.06 plus j 0.1 it is equal to 5.273 angle minus 71.57 degree right. Therefore, Y 11 look at this diagram therefore, capital Y 11 is equal to Y 10 will be there, and this line connecting 2 and bus 2 and bus 3 bus 1 is connecting bus 2 and bus 3 therefore, Y capital Y 11 is equal to Y 10 plus Y 12 plus Y 13. So, Y 10 we have computed and Y 12 and what you call this Y angle is same, so Y 12 plus small y 12 plus small y 13. So, it is 15.82 plus 3.955 angle minus 71.56 degree, because all the cases angle same.

So, this 2 are in bracket and multiplying by this angle minus this one right, because basically this 15.82 angle minus 71.56 degree and this Y 13 is 3.955 angle minus 71.56 degree. So, this comes 6.255 minus j 18.704 right. Similarly capital Y 22, that is 22 means Y 220 plus Y 21 plus Y 23, Y 21 means Y 12 small y 21 small y 1 2. So, it is actually same as before j 0, j 0.05 plus this again 15.82 plus, those angles same for all the your this admittance line admittances. So, 5.273 angle minus 71.56 degree. So, this also come 6.672 minus j 19.96 right

Similarly, Y 33 is equal to Y 3 0 plus Y 31 plus Y 3 2. So, it is Y 3 0 small y 3 0 plus Y 13 plus Y 23 is equal to Y 3 0 small y 3 0 is we have computed j 0.45 plus these 2 term Y 13 plus Y 23 angle minus 71.56 these are all cases Y (Refer Time: 17:43) angle remains same right. So that means, 2.918 minus j 8.709 right

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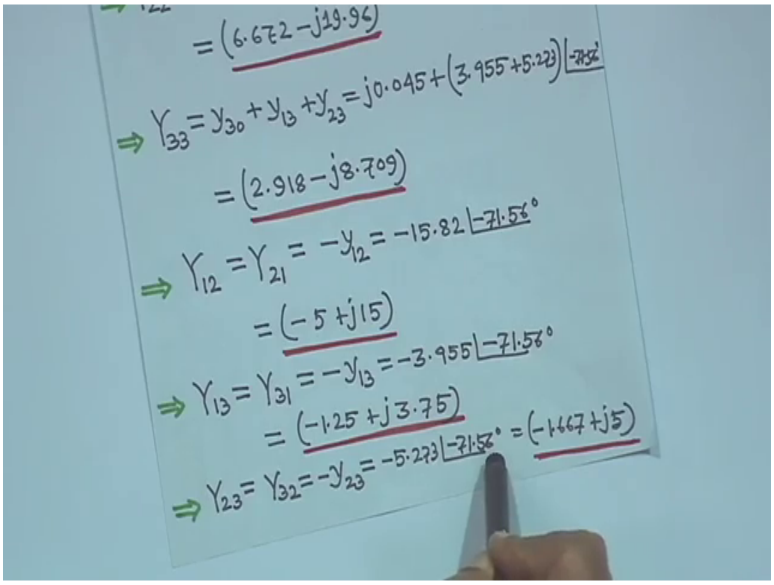

This is all diagonal elements, now of diagonal elements capital Y 12 is equal to capital Y 21 is equal to minus small y 12, it will be minus 15.82 angle minus 71.56 degree. That is actually minus 5 plus j 15 right. Similarly capital Y 13 is equal to capital Y 31 is equal to minus small y 13 it will be minus 3.955 angle minus 71.56 degree. This will become minus 0.25 plus j 3.75. Similarly capital Y 23 is equal to capital Y 32 is equal to minus Y 23 that will become minus 5.273 angle minus 71.56 degree, this will be minus 1.667 plus j 5 right.

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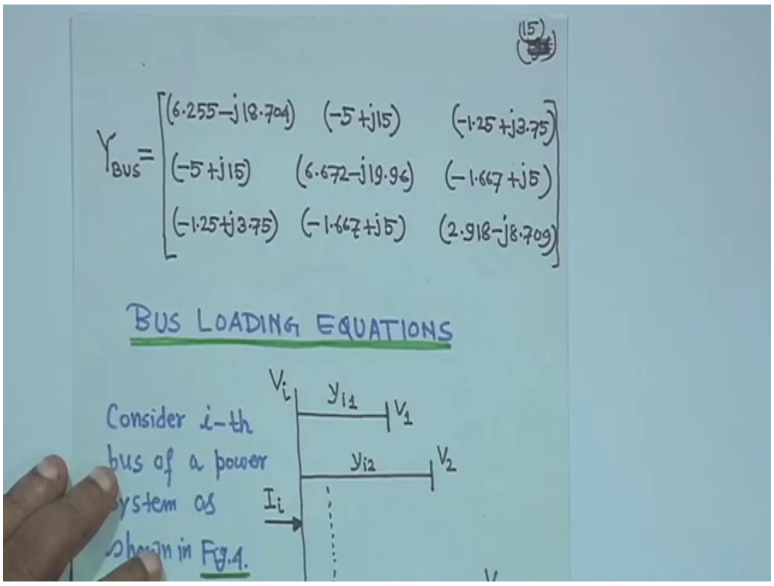

So, all these then matrix will be then why matrix will be, your this one 6.255 minus j 18.704 then minus 5 plus j 15 then minus 0.25 plus j 3.75 right. Similarly this one, now you can you can I am telling you can check this one. Now if you add if you add all this elements I will in a particular row if you add all the elements, you will find that except charging admittance other thing will not be there.

That mean in this first one your that is your Y 10 that charging admittance was just now, just now you have computed know, it has gone just Y 10 this one right. Y 10 is equal to j 0.055 right Y 10. So, if you add if you add this, all this thing first thing this is 6.255 right. And this is your if you add all this thing that minus 5 your this 6.255 or this thing, whatever it has gone that Y 11, Y 11 is equal to 6 point if you add all these things right. Then you will see that what you call that only charging admittance will left all other things will be cancel.

Similarly your here also similarly here also right, all other things will be cancel. Only thing is that when you calculating it is check the this will plusses correctly right. Then will find only in that case will assume that your Y bus calculation is correct.

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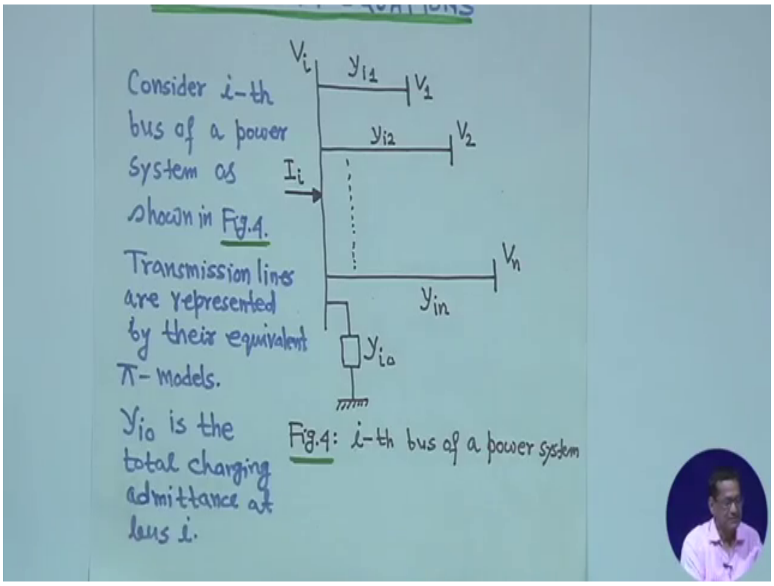

So, this is actually small example for your Y bus computation right. Next is the bus loading equation now this is that your (Refer Time: 20:36) more important for your, what you call that load flow studies right. Suppose now I have told you that that how to, how to add that charge consider the charging admittance.

Now you consider a bus bar this is your bus bar right. And voltage of this complex voltage of this bus is V i right. So, i th bus is connected to bus 1 bus 2, and up bus n. In general you want to make a general generalize thing. So, one 2 n is connected. So, admittance of this all this lines it is Y i 1 small y i 1 Y i2 and up to Y in right. And current injection at this bus is capital I i that capital I suffix I i, i the current injection right. And this is that bus and here the total charging at here the total charging Y i0 is that total charging admittance.

That means for this for this line also you have charging admittance this side half, this side half, this side half, this side half. Whatever numbers of lines connect to a bus bar take the pi model and make it half, half and club together sum it up. And that is actually that represent Y i0. So, this is the charging admittance of the of this bus bar right. So, it you can taken pi model that is why Y i0, I am writing is the total charging admittance at bus i; that means, all the lines taken pi representation and whatever come to this side half, half of this half of this all, sum it up all and that is total that is Y i0 for this bus right.

Now next injector current I i in to in to this bus i you can be you can write now this is at ground right. So, Y this is voltage V i, so before showing it. So, Y i0 V i right. Plus your Y i 1 V i minus V 1 plus Y i2 V i minus V 2 and so on up to V i minus V n in to Y in right.

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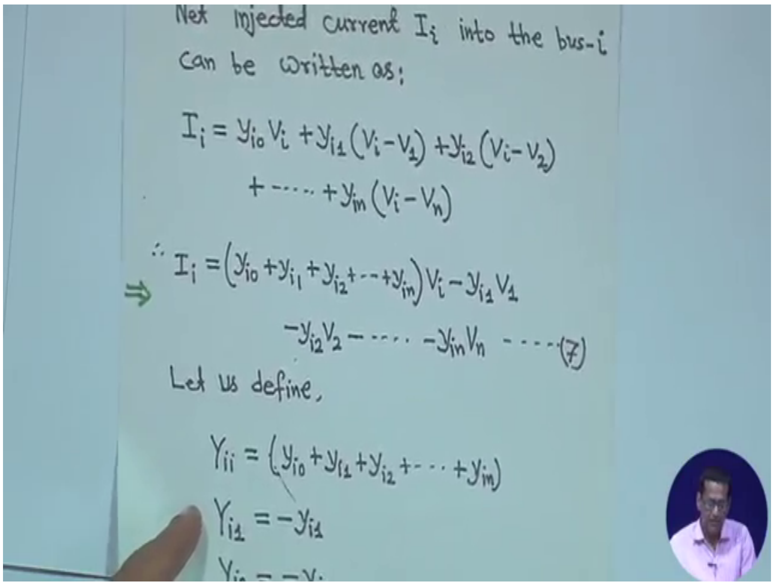

So that means the net current injection at this at this bus bar I right. You can write now then I i is equal to Y i0 V i plus all the small y right. This small y i 1 V i minus V 1 this small y i2 V i minus V 2 plus up to n th term Y in V i minus V n right. Then you collect all the terms corresponding to your V i and V 1 V 2 V n like this, then I i is equal to Y I Y i0 plus Y i 1 plus Y i2 up to Y in, and this is your V i minus Y i 1 small y i 1 V 1 small y i2 V 2 minus up to minus Y in V n this is equation 7.

So, from this equation from sorry, from this figure now you have learnt that how to write the current injection right; very simple actually current injection.

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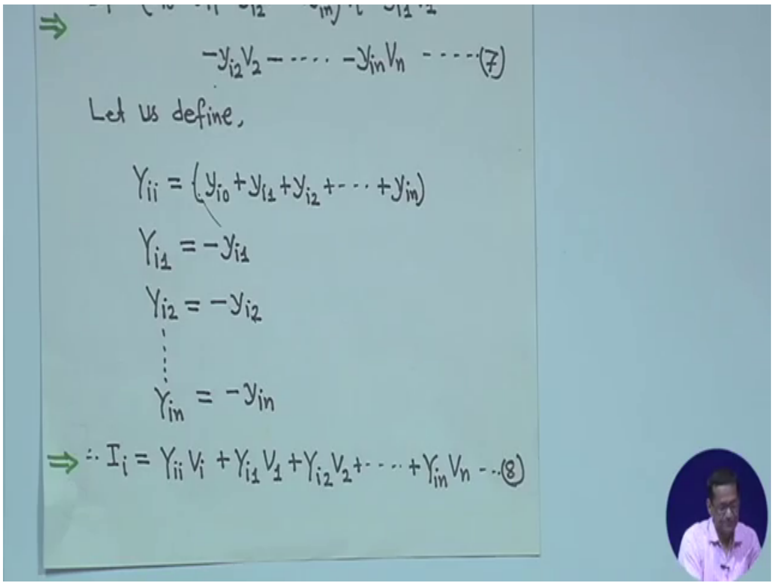

So that means, now will define, now let us define that capital Y I i is equal to Y i0 plus Y i 1 plus Y i2 up to Y in right. So, this one you define and then capital Y i 1 you define minus small y i.

Capital Y i2 you define minus small y i2 and so on. Up to capital Y in is equal to minus small y in, you define. After that you can write I i is equal to Y I i V i plus Y i 1 V 1 plus Y i2 V 2 plus up to Y in V n this is equation 8; that means, this I i I capital I i I we are writing by assuming this all this thing we are writing this equation this equation in this form right. In terms of capital Y and V this is equation 8. Next; that means, this equation, this equation you can write that this equation you can write right, that I i is equal to capital I i is equal to your capital Y I i V i plus sigma k is equal to one to n Y ik V k, k not is equal to i.

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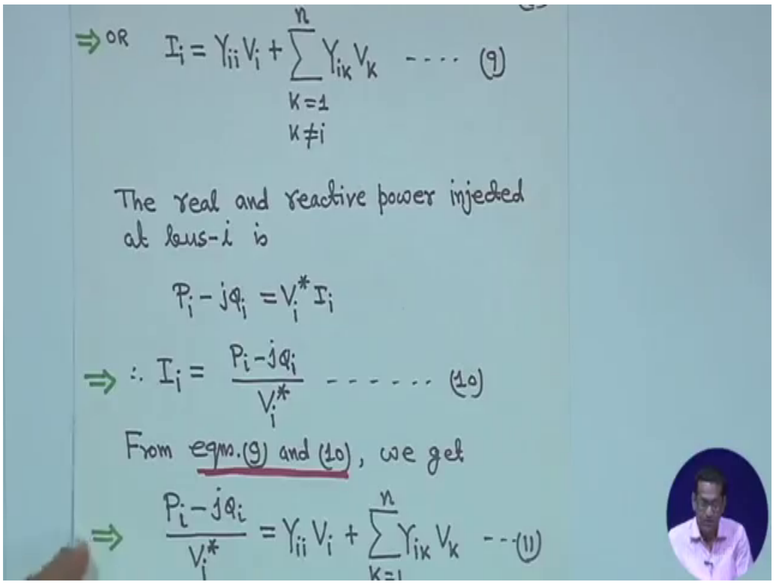

Because this Y I i V i V I term we have we have written outside of the summation. So, that is why k not is equal to i, Y ik V k this is equation 9 right. Now that when we are talking about that your transmission line characteristic other thing, at that time I showed you that p minus j q is equal to V conjugate I and I have told you also. I have told you also that conjugate is it to capture the power factor angle and when voltage and currents are purely sinusoidal right.

So, in that case the real and reactive power injected at bus I. So, we can write at a particular by bus i real and (Refer Time: 25:45) injected that is that, what is real power injection reactive? Power injection, we will come later right. Because generation will be there load will be there so, at that time will discuss right. So, P i minus jQ i that is a real that is the real and reactive power injection is equal to you can write V i conjugate I i right, at bus i.

That means in this diagram, in this in this diagram that if it is I am I am making it for you right. For the time being I am making it for you.

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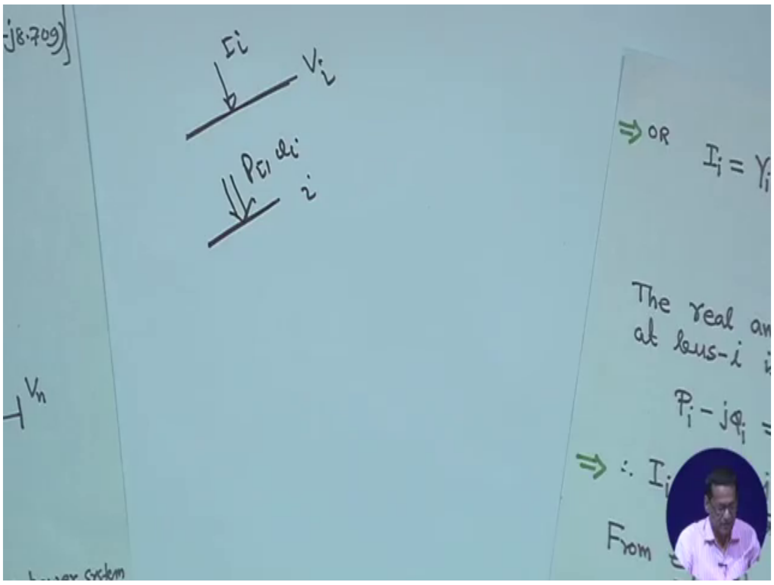

Suppose this is my, this is my bus bar this is my bus bar, this is the voltage V i right. We are taking this is the current injection actually I i right. So, this way you are taking basically any bus bar if you take any bus bar if you take i th bus bar right. So, I am putting like this that power injection is I put like this real power is P i and k power injection is Q i in this bus right.

So, basically that thing your what you call this thing we can write that P i minus jQ i is equal to V i conjugate I i right. That way just to I told you that to capture the power factor angle you have to write it like this; that means, I i is equal to P i minus jQ i upon V i conjugate right. This is equation 10. So, from equation 9 and 10 from equation this one and this one equation 9 is I i expression and 10; that means, you substitute this I i is equal to this thing your this, I mean this I i is equal to equate with this one.

So, what we are doing is this one writing first P i minus jQ i upon V i conjugate is equal to I i, this equation you are writing that is Y I i V i plus k is equal to 1 to n capital Y ik V k this is equation 11 and k not is equal to i right.

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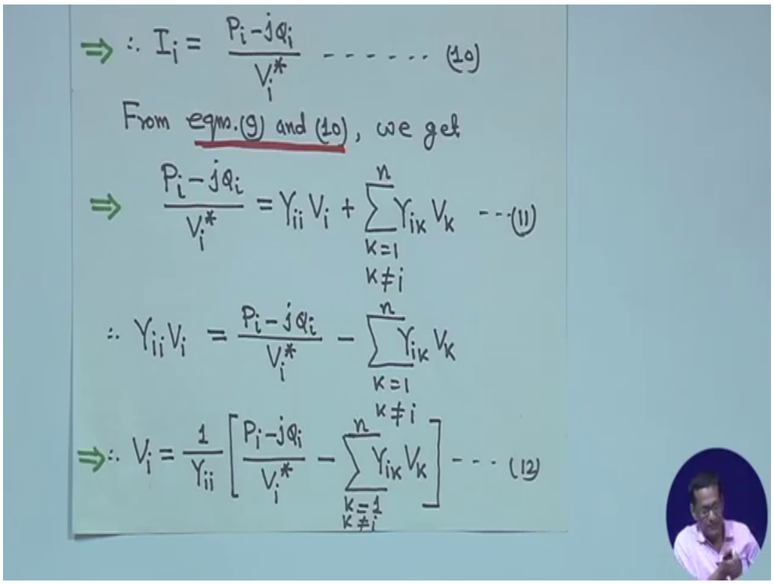

That means, we can write Y I i V i is equal to i means this term is equal to this term first minus this term. So, P i minus jQ i upon V i conjugate minus k is equal to 1 2 n capital Y ik V k, k not is equal to i right. So that means, V i is equal to V i is equal to 1 up on capital Y I i in bracket, P i minus jQ i divided by V i conjugate right. Minus k is equal to 1 2 n capital Y ik V k, k not is equal to i, this is equation 12 right.

That means, that means, we got voltage equation for bus i in terms of power injection P i and your Q i, that is real power injection and reactive power injection, and also that your in terms of your Y ik V k, but here also V i here also V i is present So that means, it is your V i conjugate this one, but V i also present here. So, we are writing V i V I is equal to 1 upon Y I i P i minus jQ i upon V i conjugate minus capital Y ik V k is equal to one 2 n k not is equal to i V i also present here, but it conjugate right. So that means, this is a non-linear equations right. All load flow or power flow equation they are non-linear equation. So, if we I mean one to solve fast this kind of equation we have to follow that iterative technique; that is how to solve this.

So, generally load flow studies of course, that Newton-Raphson method is very common right, but we will start first with Gauss-Seidel then will go for Newton-Raphson, such that you have you will have a good ideas about this load flow studies right. So, although Gauss-Seidel method is you know it takes computational time is more than the Newton-

Raphson method. Gauss-Seidel method actually is convergence characteristic is linear. Whereas, Newton-Raphson method is convergence characteristic is quadratic.

So, it is convergence is much faster than Gauss-Seidel method. And Gauss-Seidel Newton-Raphson method the convergence your what you, call that computational time right. In the almost it is independent of that your number of beta resistance of solving a Newton-Raphson method, is almost independent of the dimension of the problem. But in the case of Gauss-Seidel method, that if dimension of the problem increases then naturally your what you call, that number of beta resistance also much more.

But anyway later will see that, but this convergence characteristic like Gauss-Seidel is linear and Newton-Raphson is quadratic right. That is beyond the scope for this thing right. So, but next will move to the Gauss-Seidel method, but before coming to that (Refer Time: 30:32).

Thank you.