Power System Analysis
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Lecture - 29
Load Flow Studies (Contd.)

Then we let us start with Gauss Seidel iterative method. So, for that we will consider a 4 bus power system right and this is bus 1, bus 2, bus 3, bus 4, this is general 1, general 2 and for the this your diagram.

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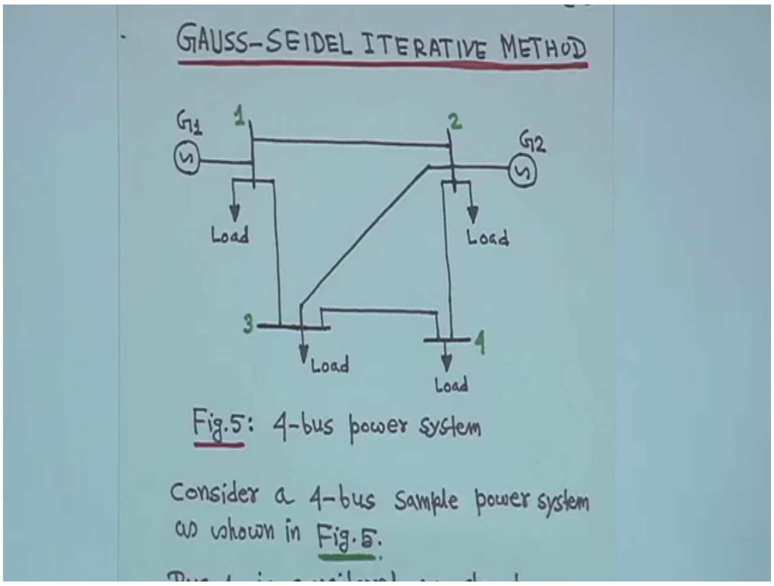

Generally for all generals have the transformers right after that consider your high voltage your what you called that transferred from the terminal voltage to the other side is high voltage side then transmission line, but we will lot. So, here transfer offer the timing right, but later the transfer modeling will go for that, but for the timing here we assume that this kind of diagram new transformer show showing here.

So, this general one and this general 2 and bus 1 2 3 4 right; so, in that in that case and loads are shown loads are shown right and transformers anyway that generators then we have a step up of transformer that connect to the line this is not considered here later I will give you the transformer modeling basically for transmission line transformer also can be represented by pi modeling.

So, we will see at the end right, but for this it is like a classroom exercise. So, take this and these are the loads are given and as and for P Q bus P V bus all will be explain later right. So, these are 4 bus power system, but we have to write you have to see that how you can solve using Gauss Seidel iterative method. So, this bus 1 you consider as a slack bus. So, s is equal to 1 is stands f or slack bus s is equal to 1 right and; that means, here gauss voltage and magnitude and its angles are known.

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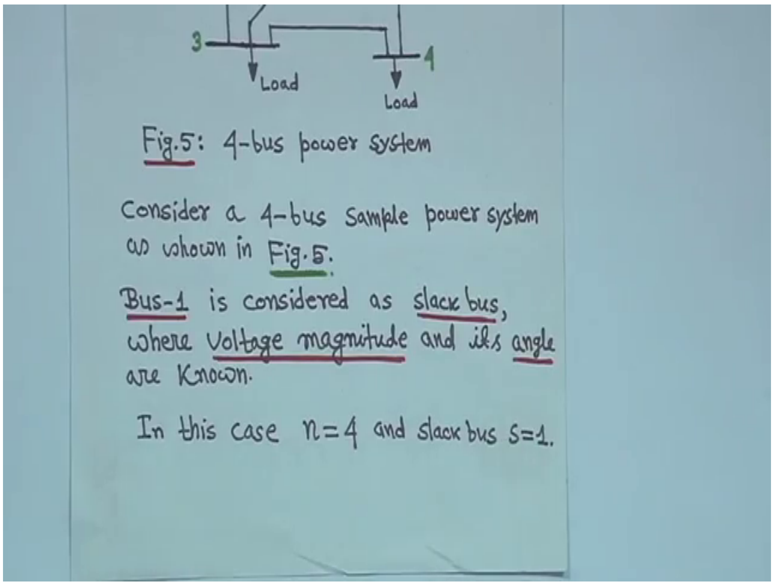

So, in this case there is a 4 bus problem 1 2 3 4. So, n is equal to 4 and slack bus we define s is equal to 1.

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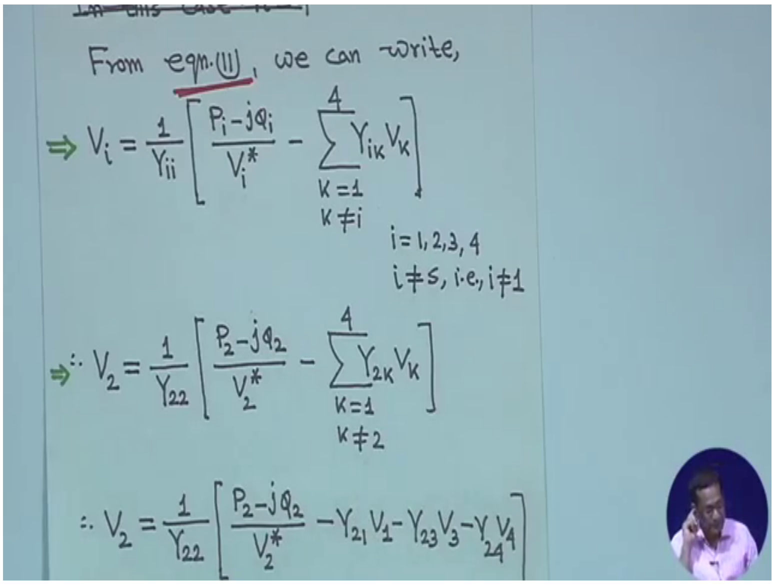

So, from equation 11 we can write like the equation 11 that is this; this one.

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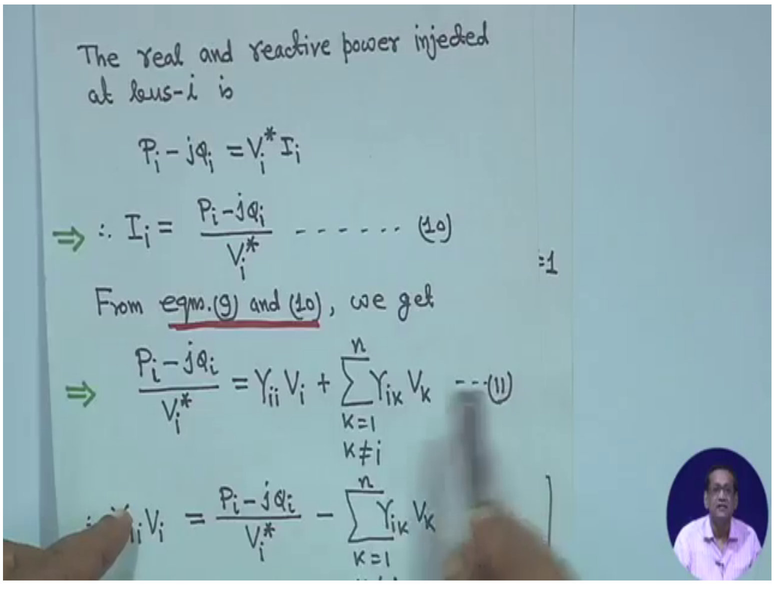

This is your equation 11 the same equation we are actually re writing this is actually equation 11; this is equation 11 right same equation we are rewriting that your V i is equal to 1 upon capital Y ii in bracket pi minus jqi upon V i conjugate minus Y i sigma Y ik V k k is equal to 1 to four, but k not is equal to I is equal to 1 2 3 4, but I not is equals to slack bus that is I is not equal to 1 because slack bus voltage actually known to you right voltage magnitude and angle both are known to you.

So, voltage is known to you therefore, if you write V 2 equation because V 1 is not required because voltage magnitude and angles known. So, V 2 is equal to; that means, when I is equal to 2. So, V 2 is equal to 1; 1 upon capital Y 2 2 from this equation only P 2 minus j Q 2 upon V 2 conjugate right then minus sigma k is equal to 1 to 4 I is equal to 2. So, it is y 2 k y 2 k V k, because I is equal to 2 that is why P 2 minus jq 2 upon V 2 conjugate and I is equal to 2. So, capital Y 2 k V k k not is equal to 2 because k not is equal to I i is equal to 2. So, k not is equal to 2.

So, an if you expend this 1 V 2 will become 1 upon y 2 2 P Q minus j Q 2 upon V 2 conjugate minus y capital Y 2 1 V 1 minus capital Y 2 3 V 3 minus capital Y 2 4 V 4 similarly when I is equal to 3 in this equation.

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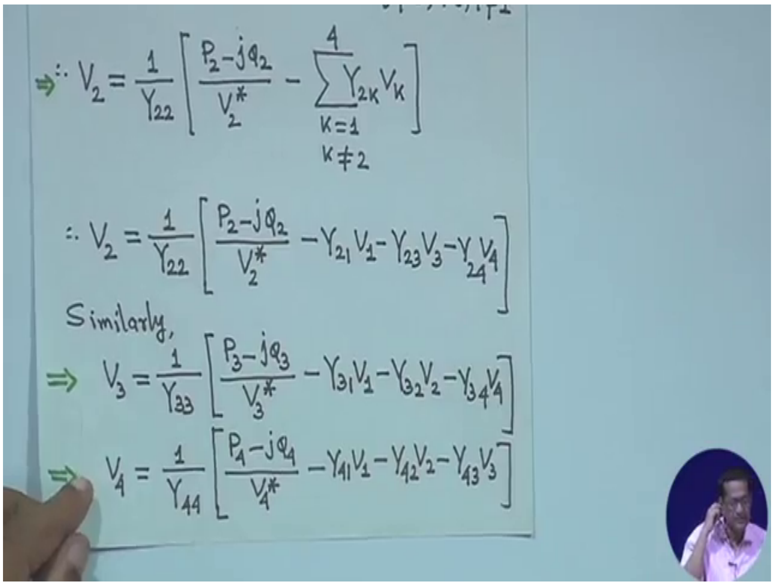

When I is equal to 2 4 and k not is equal to I means k not is equal to 3 right you will get V 3 is equal to 1 upon y 3 3 and P 3 minus jq 3 upon V 3 conjugate minus y 3 1 V 1 minus y 3 2 V 2 minus y 3 4 V 4 all capital again and again not uttering capital.

So, similarly V 4 is equal to 1 upon y 4 4 that is P 4 minus jq 4 upon V 4 conjugate minus capital Y 4 1 V 1 minus y 4 2 V 2 minus y 4 3 V 3 these are V 2 V 3 and V 4 3 questions, because 4 bus problem bus 1 is a slack bus right. So, how one can make it?

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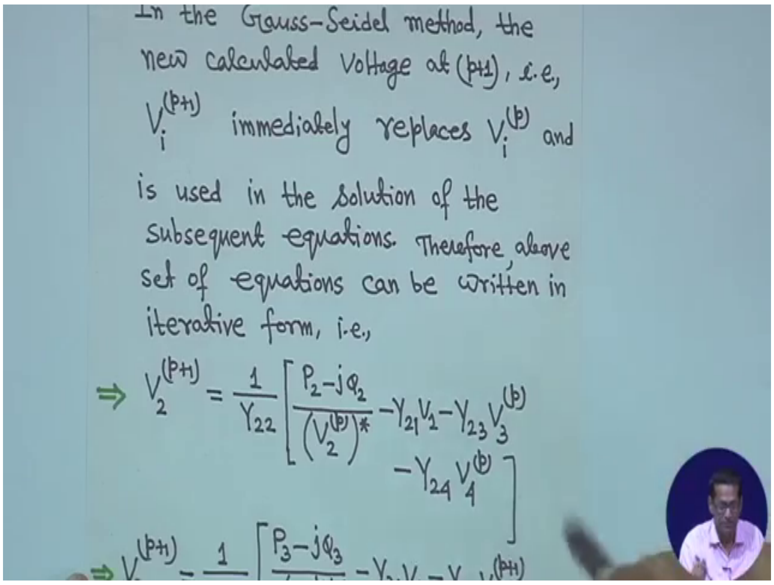

Now, in the Gauss Seidel method the new calculated voltage at P plus 1 iteration where P is the iteration count right P is a iteration count at P plus 1 that is V i P plus 1 immediately replaces V i P and is used in the solution of the subsequent equations; that means, what I want to mean before coming to this that suppose you have got this x equation V 2 V 2 V 3 V 4 these 3 equations you have got it right.

Now, when you are solving this suppose first you are computing V 2 right then you are solving V 3 then is V 4 suppose initially all the initial values of V 2 V 3 V 4 are known. So, one can do is like this V 2 is equal to put all the initial values you can get it V 3 is equal to all the initial values you put here get V 3 and V 4 all the initial values you put get 3, but in that case what will happen that computational tally will more and number of iterations will be more that is why what we do actually that first you calculate using all the initial values have of V 3 V 4 and P 2 Q 2 all this things are known right and then you get the V two, but whatever immediately whatever value of V 2 you get in this equation whatever V 2 you got in this equation put this value and other value are initial values you put right.

Whatever you got V 2 in this equation V 2 is advantage the initial values this value directly you put, but before initial value you have to take because V 4 is not computed here right and this in equation is also independent of V 3, because according to the connection that you are you are according to this line connectivity right now. Now V 2 computed V 3 is computed now in this equation in this equation V 2 instead of taking initial values of V 2 and V 3 V 2 have computed V 3 have computed both you use this.

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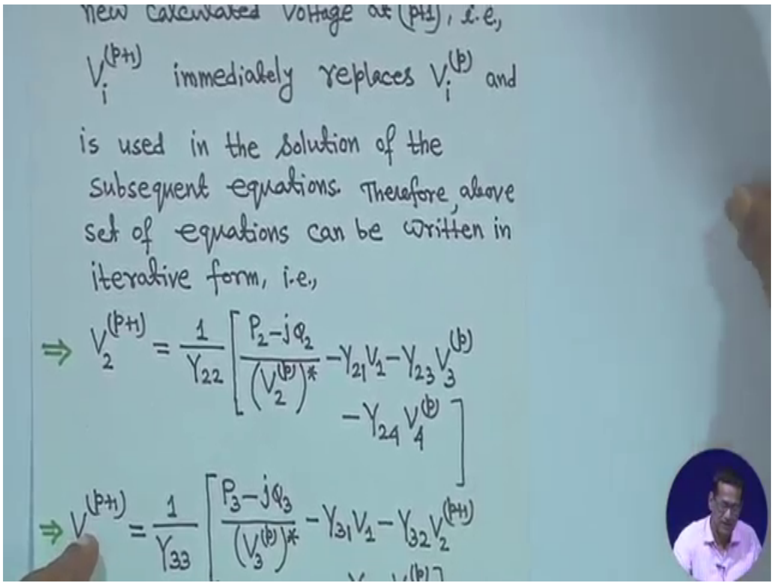

Such that your excellence your convergence excellence faster right that is why that mean is that you that immediately replaces V i P and is used in the solution of the subsequent equation such that your conversation will be faster right.

So; that means, that means in terms of iteration P is the iteration count I can write this equation I can write I mean these equation this V 2 equation I can write in terms of P right; that means, you are V 2 P plus 1 is equal to 1 upon capital these are parameter.

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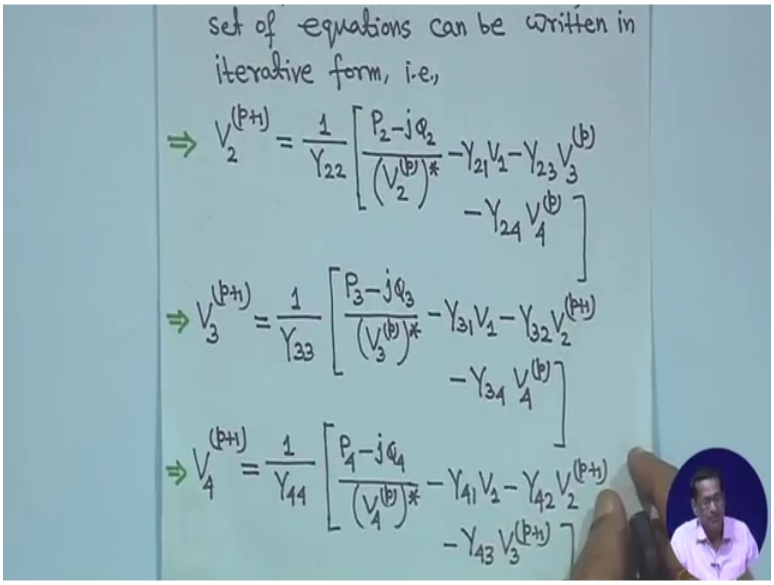

So, one upon capital Y 2 two P 2 minus j Q 2 these are known right and here I am writing V 2 P conjugate right because here whenever first whenever I leave this I have to use the initial values right initial values that is why instead of P plus 1 I am writing V 2 P conjugate right then minus capital Y 2 1 V 1 V 1 actually known 1 plus j 0 generally angle 0 1 angle 0. So, do not put P here no need right minus capital Y 2 3 V 3 P minus capital Y 2 4 V 4 P, P is the iteration count here you have take the initial what is the next iteration in the next iteration that V 3 P plus 1 y upon capital Y 3 3 P 3 minus j Q 3 it will be V 3 P conjugate as I told like V 2 right it will like that minus y capital Y 3 one V 1 minus y 3 2 do not write here V 2 P because this V 2 P plus 1 already computed from here you make it here right.

So, it will be V 2 P plus 1. Similarly V 4 P plus 1 equal to 1 upon capital Y 4 four P 4 minus j Q 4 V 4 P conjugate this thing and here V 2 P plus 1 V 3 P plus 1 both have been computed. So, here you make minus y 4 2 V 2 P plus 1 minus capital Y 4 3 V 3 P plus 1 such that here convergence rate to will convergence will be faster and number of iteration will be lays.

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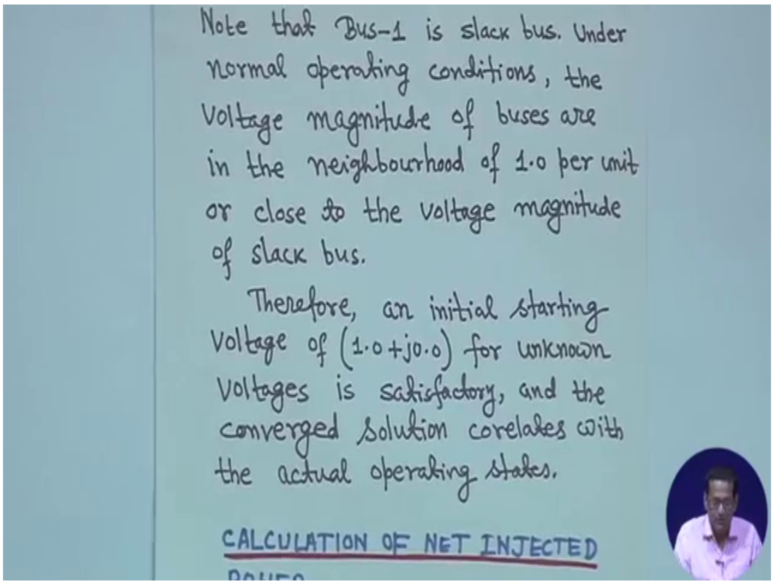

So, that is why this way one can solve now algorithm on the things will come later right now you note that bus 1 is a slack bus right. So, under normal now starting values how you will take that we have to discuss. So, under normal operating condition the voltage magnitude of buses are in the neighborhood of your one per unit right that is 1.0 per unit and or close to the voltage magnitude of the slack bus right whenever you solve it.

So, an initial starting voltage 1 plus j 0 for unknown voltage is satisfactory of course, right and the converged solution correlates with the actual operating states. So, these are the thing now calculation of the net injected power.

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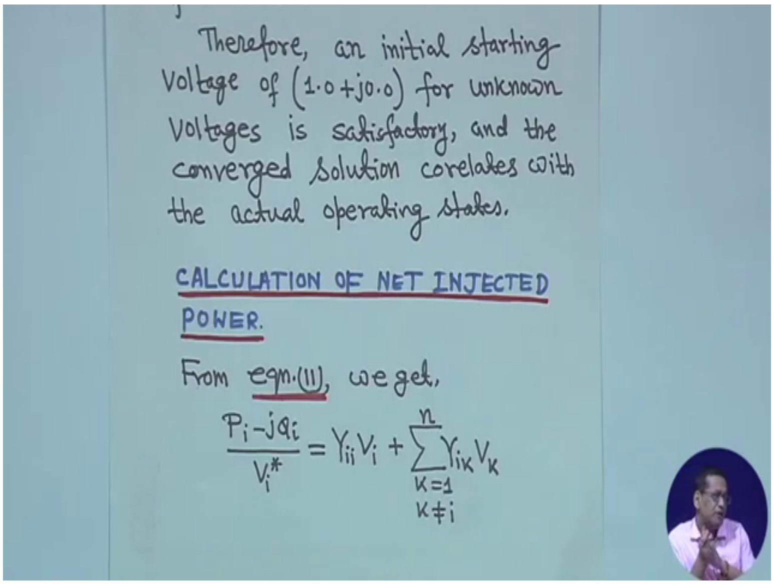

So, from equation 11 again we are rewriting equation 11 right. So, in this case just hold on. So, your this you have to find out the expression of P i and Q i right the calculation of the net injected power because when you will solve the lot flow this calculation P i and Q i separate expression is needed right. So, because in lot flow that the injection P i and Q i injection for P Q bus a that it will be is that schedule value will be given right, but this pi and Q i you have to calculate also iteratively and you have to make go for checking the mismatch.

So, that is why this P i and Q i both you have to find out its expression. So, that is why equation 11 we are rewriting pi minus j Q i upon V i conjugate is equal to capital Y ii V i plus your k is equal to 1 to n y ik V k k not is equal to I right. So, these equation; that means, P i minus that means, from these equation only that P i minus j Q i you can write V i conjugate right capital Y ii V i plus k is equal to 1 to n capital Y ik V k k not is equal to i right. So, this is equation 13.

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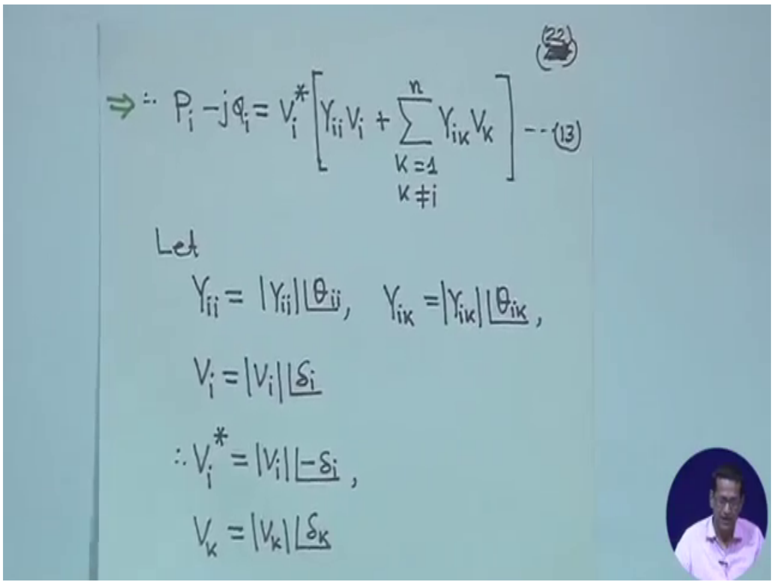

Now, define this y matrix admittance matrix admission any way all the admittance elements in terms of its magnitude and angle therefore, the capital Y ii you write a more magnitude of y ii and l theta ii and capital Y ik magnitude your y ik angle theta ik right.

Similarly, V i is equal to magnitude V i angle delta i therefore, V i conjugate will be magnitude V i angle minus delta I similarly V k is equal to magnitude your what you call V k angle delta k right; that means, all these value.

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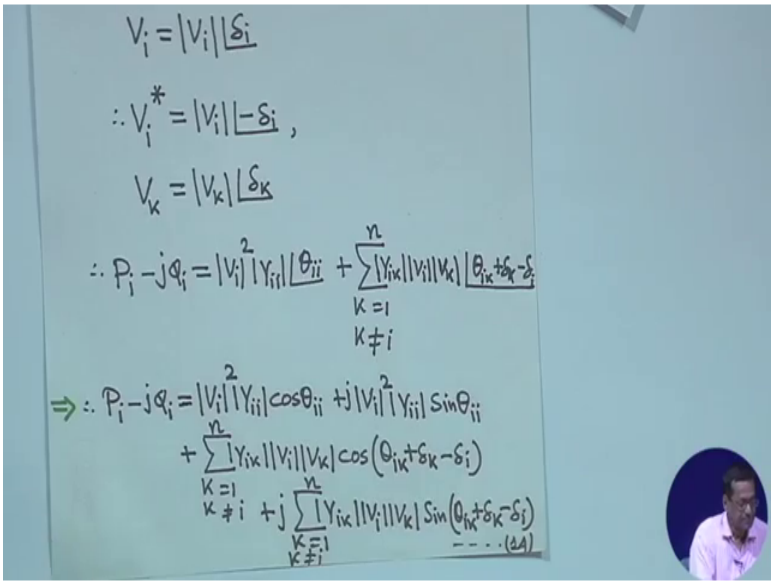

All these value this y ii V i V k all these thing you can substitute here right if you substitute here all these things that V i is equal to magnitude conjugate is equal to your magnitude V i minus delta i V i is equal to magnitude V i angle delta i and simplified you please simplify this one right.

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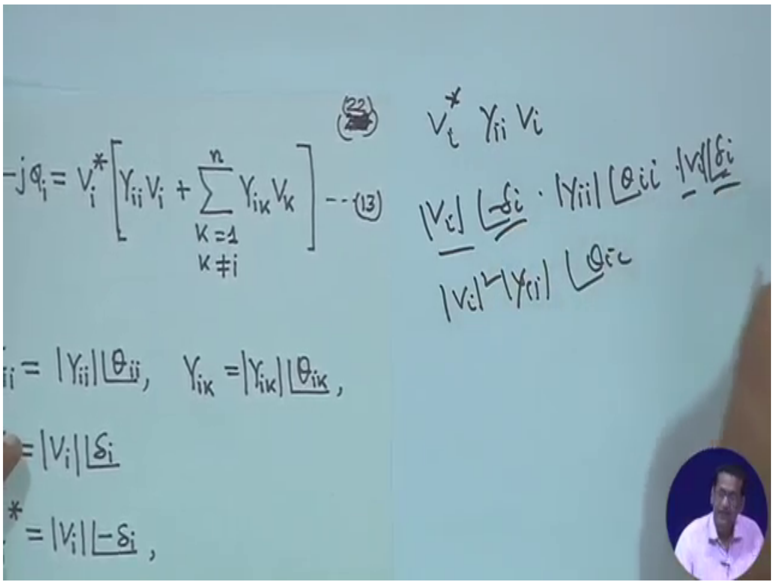

What you will get pi minus j Q j Q i right and first one I am showing it that this first term that is your V i conjugate then y ii and V i.

So, this V i conjugate actually it is V i angle minus delta and y ii actually magnitude y ii angle theta ii right into your magnitude V i and its angle is delta I. So, magnitude V i magnitude V i means magnitude V i square then magnitude y ii and this angle theta ii because these delta I delta I minus delta I plus delta I will be cancelled right. That means, similarly your second term in that second term in that you substitute all these values you substitute all these values you will get this first term will be coming V magnitude V i square magnitude y ii and l theta ii second term k is equal to 1 to n k not is equal to I magnitude y ik then magnitude V i mangle V V k then angle it will be theta ik plus your delta k minus delta I this angle right.

Now, this angle theta ik plus delta k minus delta I you write cosine theta ik plus delta ik minus delta I plus j sin theta ik plus delta k minus delta i. So, if you write this one you will get pi minus j Q i is equal to magnitude V i square y ii cosine theta ii plus j magnitude V i square y ii sin theta ii right this is the this is the first term this 2 terms and second term also one real 1 in yor complex part similarly second term also on the complex part.

So, k is equal to 1 to n k not is equal to I y ik V i V k these are all magnitude not telling again and again cosine theta ik plus delta k minus delta I plus this is j k is equal to 1 to n k not is equal to I then y ik V i V k all magnitude sin theta ik plus delta k minus delta I this is equation 14.

Now, from this you separate real and imaginary part from which will get pi is equal to and Q i is equal to separate right.

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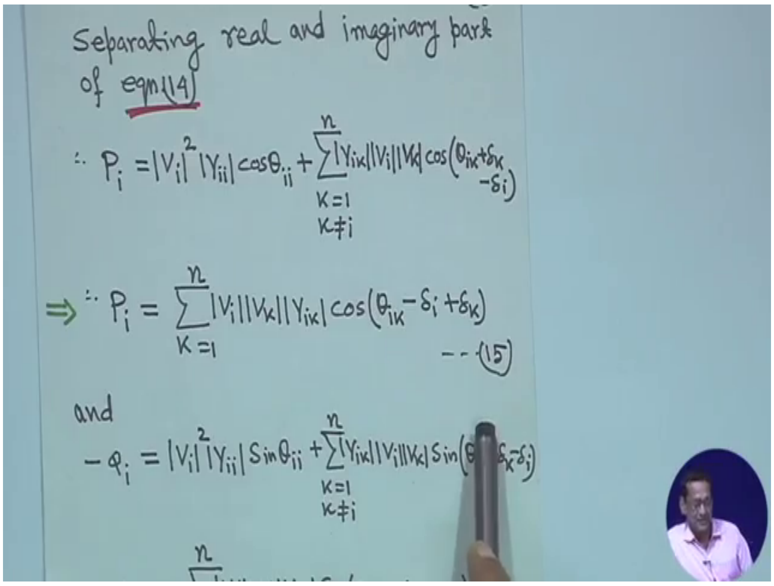

Then you separating real and imaginary part pi you will get that magnitude V i square magnitude y ii cos theta ii plus k is equal one to n, but k is not equal to I magnitude y ik magnitude V i magnitude v k cosine theta ik plus delta k minus delta i.

Similarly, this one right look this term is out of this is a i term it is out of this sigma right summation, but k also k not is equal to I in a if you want include this term in this right in this term that will be k is equal to 1 to n v i v k y ik cos theta ik minus delta I plus this term is taken inside that although it is cos theta i, but when it is your when your; what you call when k is equal to I when k is equal to i. So, this delta I delta I will be cancelled right in cos theta ii you will get when k is equal to I and when k is equal to I that V i square and y k is equal to y i will be there.

So, v i square y ii cos theta I when you take into that it will be k is equal to 1 to n right. So, take this term inside this sigma and remove this right.

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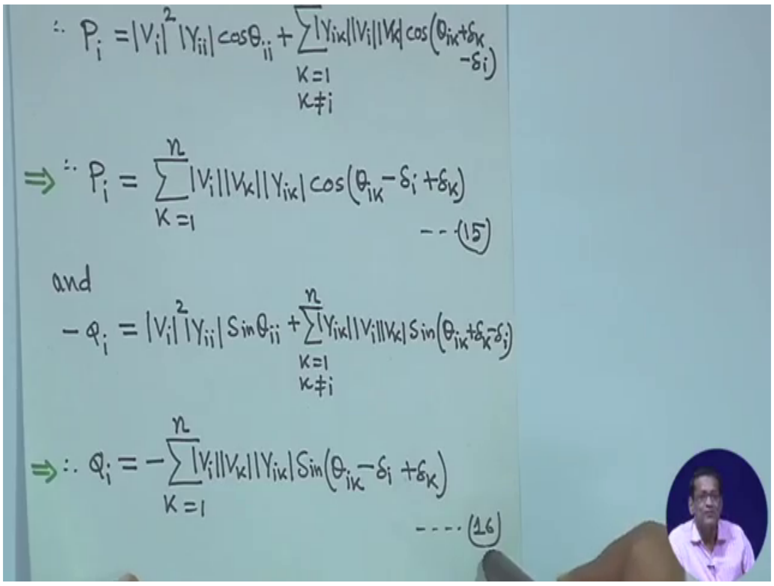

Similarly, minus q i is equal to this term that imaginary term minus your minus k is equal to V i square y ii sin theta ii plus that y ik v i v k sin theta ik plus delta k minus delta ik k not is equal to similar way this term you take into this summation, because this is the I term you take into the summation.

So, in that case q i will be minus k is equal to 1 to n magnitude V i magnitude v k magnitude y ik then sin theta ik minus delta I plus delta k this is equation 16 this 15th equation 15 and 16 you have to recall again and again this; that means, expression for power injection pi is equation 15 very easy to remember k is equal to 1 to n V i V k y ik cos theta ik minus delta I plus delta k and Q i is equal to minus minus Q is equal to 1 to n V i V k your y ik sin theta ik minus delta I plus delta k. So, this is 15 and this is equation 16.

Now, I hope this is; this you have understood right, the simple thing only little bit a practice right.

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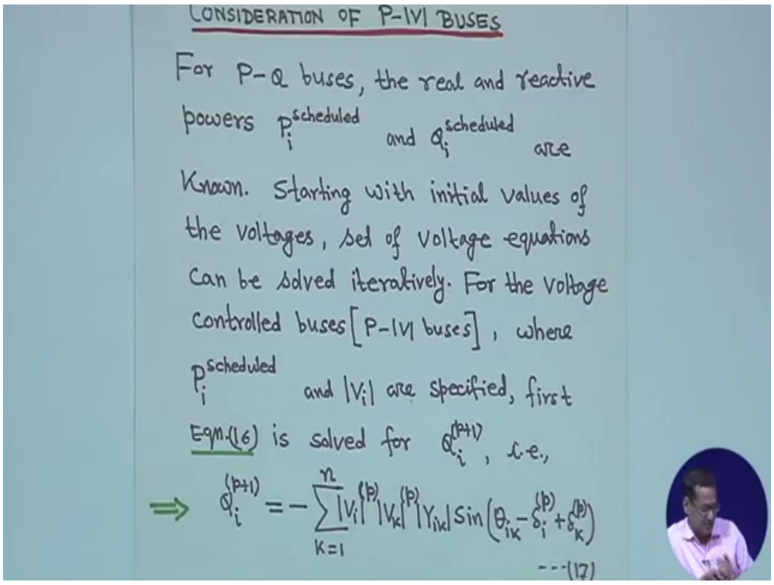

Now, consideration of P Q buses now in Gauss Seidel method and Newton Raphson method Newton Raphson is some later. So, Gauss Seidel your this thing this P V buses for P Q buses the real and reactive powers pi schedule and Q i schedule both are known for P Q buses later we will see how to compute right and starting with the initial values of the voltages set of voltage equation can be solved iteratively using this Gauss Seidel method we are talking about Gauss Seidel method or for the voltage controlled bus the P V bus right where pi schedule and V i are specified; that means, P and magnitude voltage magnitude are specified first; that means, that first are specified.

So, first equation 16 is solved for Q i P plus 1 iteration; that means, this equation because at the P V buses that your P and voltage magnitude both are known and Q and angle voltage angles are not known. So, so in that case you this equation this equation that is equation 16 this equation when you are considering a P V buses in Gauss Seidel method this equation you can write in P plus 1 iteration.

So, in that case you can write Q i P plus 1 that is P is iteration count is equal to minus k is equal to 1 to n. So, instead of V i were putting magnitude V i P p iteration magnitude V k P y ik is the constant admittance matrix. So, no question of P anywhere the sin theta ik is the angle of y ik sin element constant, but delta will vary. So, minus delta ip plus delta kp this is equation seventeen and then this is required actually for P V bus consideration and Gauss Seidel method competition of this one you call fast right.

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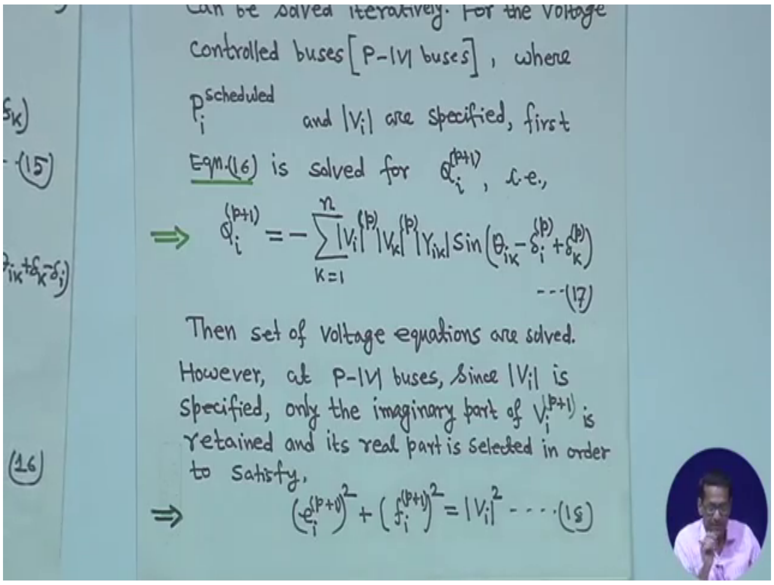

That means then set of voltage equation are solved after getting this you this; the set of voltage equations are solved.

So; however, P V buses sin V i is specified right in the P V bus write only the imaginary part of V i P plus 1 is retained and real part is selected in order to satisfy this thing actually suppose if I bus is P V bus right my objective is to make the voltage magnitude remain constant. Therefore, this in general you general that your this thing that V is equal to in general you can write e plus j p right that e p is is suppose I in the case of I bus you can write V i is equal to e i plus it is your j your this thing j jp right sorry jf right e plus jf right; that means, e plus your jfi right. So, this way you can write and for I and for any iterations say any iteration say your; these thing you can put in that iteration numbers.

So, V i is equal to ei plus jfi generally ei is much much higher they fi; fi is very small because voltage angles in transmission system is quite small right compare to is this thing your this; this fi is quite this thing your small compare to ei right. So, in this case; that means, I keep my voltage magnitude remain constant; that means, V i is equal to root over ei square plus fi square.

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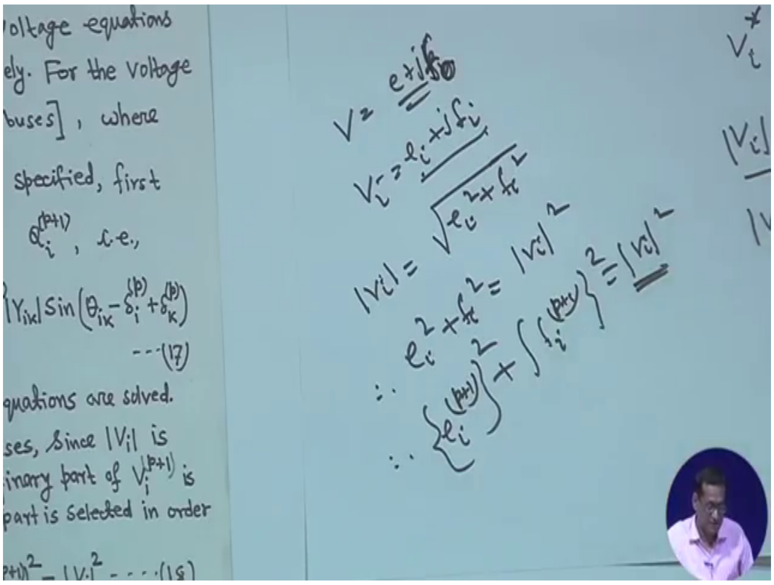

That means I can write in a other way that ei square plus fi square is equal to magnitude V i square this one I can write; that means, and; that means, at P plus 1 theta iteration we can write this one whole square plus fi at P plus 1 iteration whole square is equal to magnitude V i square, but magnitude V i remains constant no need to put P here because P V buses voltage magnitude remain constant right that is why this that is why you can write this your P V bus case that ei P plus 1 square P plus 1 is that iteration count at P plus 1 iteration plus fi P plus 1 this also square is equal to magnitude V i square, but this voltage magnitude remain constant right.

So, in that case what will happen that when you will do this we ultimately this one whatever we vary that is different thing, but this voltage magnitude has to be make call your what you call the voltage magnitude has to be remain constant otherwise in the Gauss Seidel method do you want to increase the P V bus. So, this voltage magnitude constant what will do that you in this; whatever we get only the imaginary part is retained and real part is selected in order to satisfy; that means, from this equation from this equation.

When you will solving Gauss Seidel method for the P V bus case when we will come to the numerical you will know from the P V bus case first we will find out suppose for example, bus 2 is a P V bus right in that case the way without considering your what we call P V bus the P V bus solving you solve for b 2 right you are solving, but we know this condition we actually has to be satisfied voltage magnitude or V 2 remain constant if it I is equal to 2 V 2 remain constant, but after getting V 2 the imaginary part will be your what your call imaginary part will be retained and new values of this one will be computed right such that voltage magnitude will be will be constant this way will follow.

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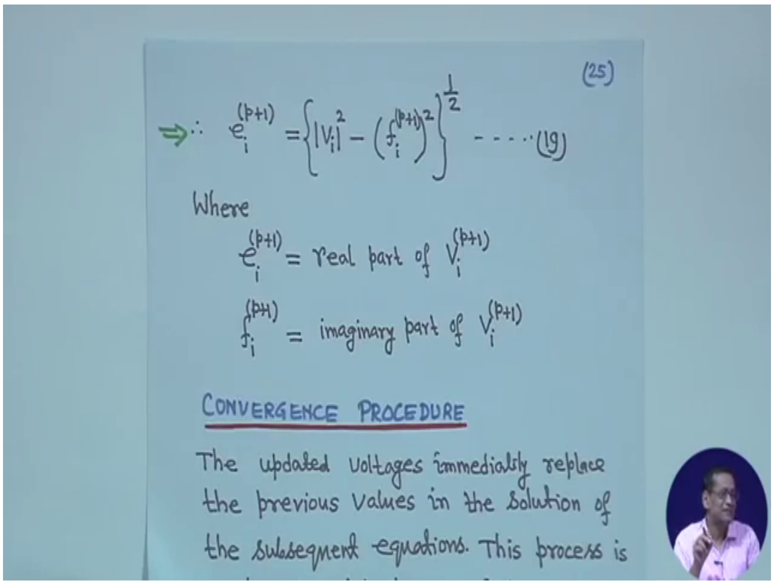

So; that means, this; that means, that your ei P plus 1 is equal to voltage magnitude V i square minus fi P plus 1 to the power half that is square root of this one that is equation nineteen. So, ei P plus 1 that is P plus 1 iteration real part of V i P plus 1 and fi P plus 1 imaginary part of V i P plus 1. So, in that case when I will solve this one a numerical then this thing will be cleared. That means, in generally you will find that what is the V two, but that is not exact V 2 because we have to maintain the voltage magnitude that say in that case that imaginary part will be retained and a real part of the voltage will be computed using this equation and that voltage will be taken such that magnitude will remain constant.

So, this is the idea for the P V buses, so, when I will take one example later first the theories and next will take the example right. So, I will when I will solve those things all that I will recall this thing and everything I will I will tell you in between if I tried to solve this thing then that your or continuity will be lost.

Next is the convergence procedure convergence procedure is something like this suppose.

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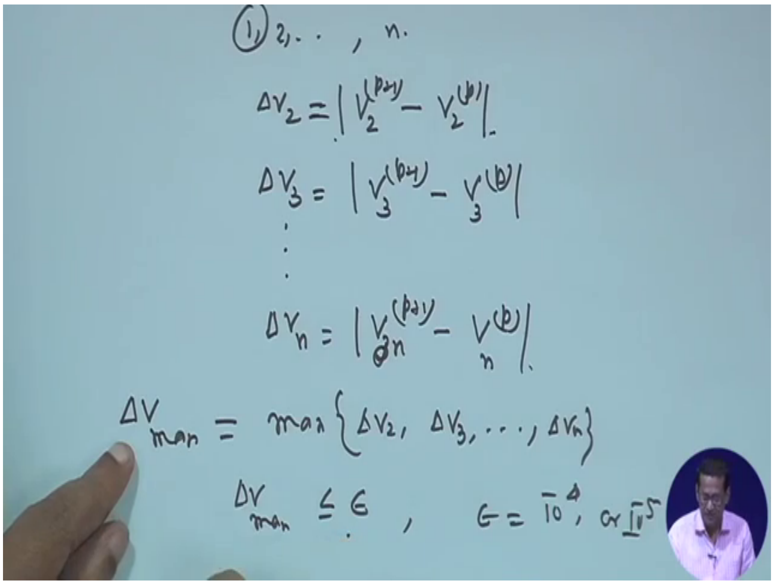

Suppose you have a suppose you have your n bus problem suppose 1 2 up to you have n number of bus suppose your bus 1 is a slack bus right that what you will do that you calculate that voltage this 1 delta V 2 in the Gauss Seidel method there are different ways of convergence different ways of convergence. So, delta V 2 you can write now that correct iteration value V 2 P plus 1 minus V 2 P this one you made.

Similarly, delta V 3 is equal to you make V 3 P plus 1 minus V 3 this is P this why a up to nth bus you make delta V n is equal to mod that V 3 sorry V n P pus 1 minus V n your P these are all magnitude right V 2 P plus 1 V 2 P these are all complex then you take the magnitude of this one up to differences all the magnitude you have taken there are there all real quantities.

So, once you these then after that what you do you take delta V max that is maximum that is equal to max of delta V 2 delta V 3 then up to delta V n right; that means, all these you have computed then maximum of all these values you take and if this delta V max if delta V max sum less than or equal to epsilon then solution is converge you assume epsilon may b 10 to the power minus 4 or 10 to the power minus five that depends on your up to the accuracy.

That means what you are doing that we found that difference of delta in every iteration you have to see this difference the current value and the previous iteration value V 2 V 3 up to V n and magnitude you take and out of which you take max of this one if the max of this one is less than epsilon out of these you find out what is the maximum of this and if that is less than epsilon the solution has converged, because if max of these less than epsilon means all other differences also less than I means this other thing all are less than epsilon.

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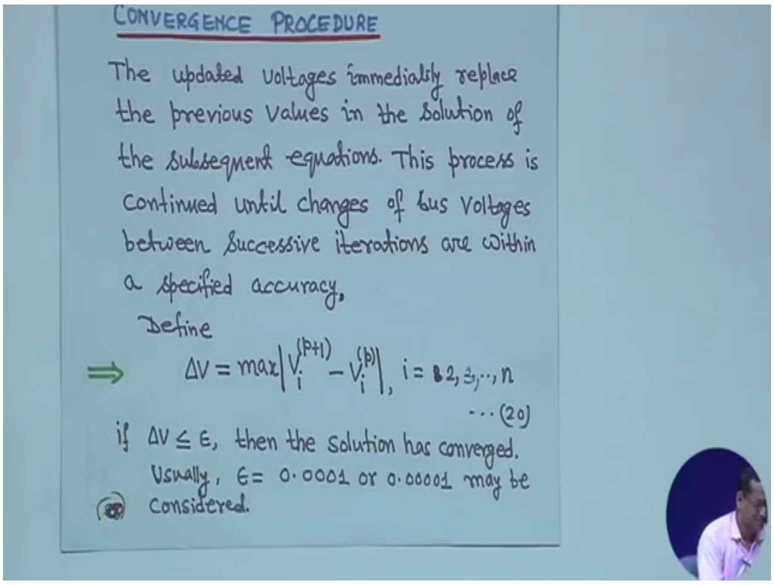

So, that is the convergence criteria then that is why that is why this; this in general I have written that delta V is equal to max of V i P plus 1 minus V i P mod and I is equal to 2 3 up to n because bus 1 we will take as a slack bus right. So, and if delta V less than epsilon equal to less than equal to epsilon then solution has converged you can take epsilon is equal to 10 to the power minus 4 that is your 3 naught 1 or 10 to the power minus 1 may be considered right.

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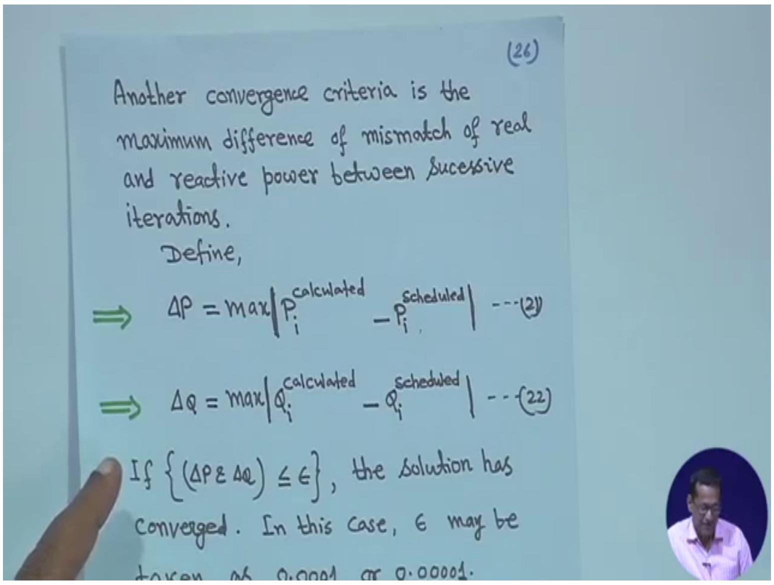

So, in that case you can get the your what you call that solution thing this is one we way of convergence criteria right this is one version then another one is and another one is that you take del P is equal to max of pi calculated my minus pi schedule there be this pi schedule value this one actually known to you I going to take the numerical.

So, will know right this is equation 21 and pi calculated value you have to calculate using equation 15 that express me got know for pi later will come right that equation 50 and take the absolute of this one right absolute value and take max of that. That means, for all the buses except slack bus like delta P 2 delta P 3 up to delta P n and you have to take the max of that right and your it is something like this just one I am showing that delta P is a.

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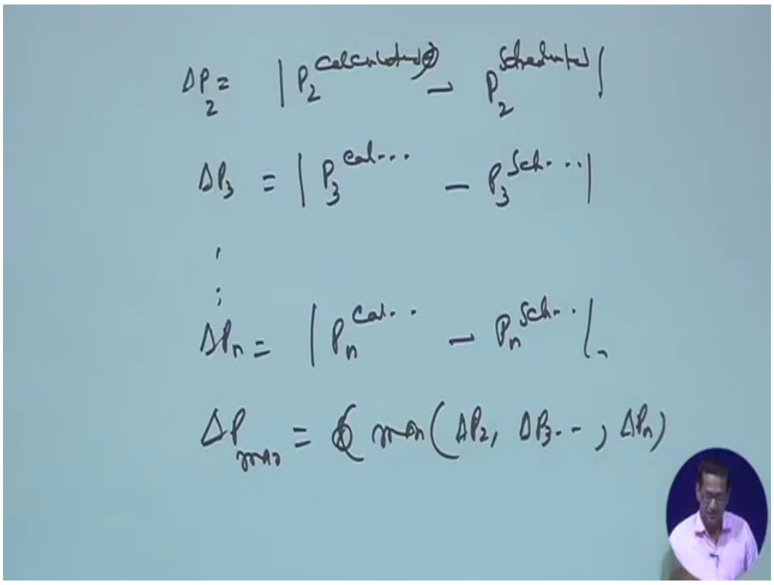

Suppose delta P 2 is equal to absolute P 2 calculated P 2 calculated right minus that P 2 schedule value.

Similarly, delta P 3 is equal to take again P 3 calculated minus P 3 not writing schedule, but making dash, dash, dash right. So, similarly this why you will take delta P n is equal to P n that calculated value minus P n plus schedule value right. So, out of this again you do same as before the why it is the voltage delta P max is equal to max of delta P 2 delta P 3 up to delta P n you take max of that right. So, similarly your delta Q also so; that means, this is the meaning of this equation.

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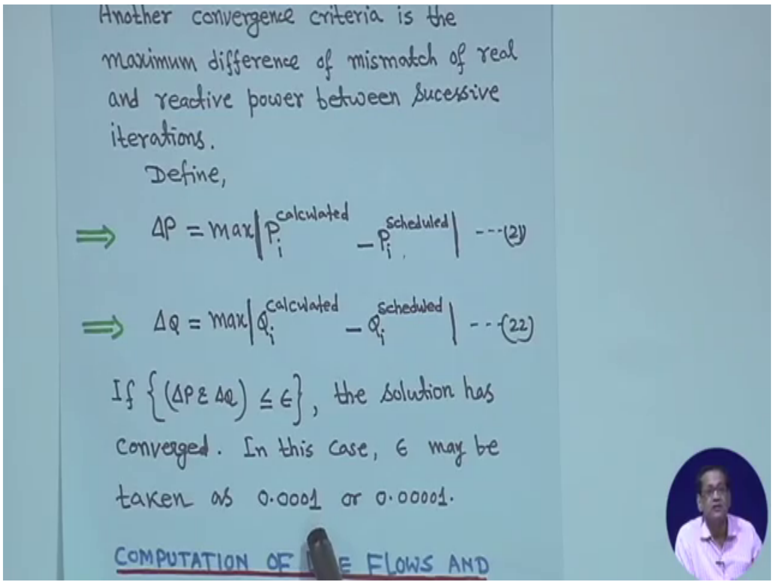

This is the meaning of this equation right same is applicable for delta Q same way. And then you take this 1 minus this 1 and if both delta P and delta Q together if they go for below your less than equal to epsilon then you take the solution has converged both delta P thing and delta Q both together should be less than solution has converged right both together. In this case epsilon may be taken again as 0.3 naught 1 10 to the power minus 4 or 0.4 naught 1 10 to the power minus 5 depends on their accuracy right. So, this is how one can these 2 ways of convergence characteristic I told you right.

Thank you, next we will come.