Power System Analysis
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 30
Load Flow Studies (Contd.)

Next is that your computation of line flows and line losses right.

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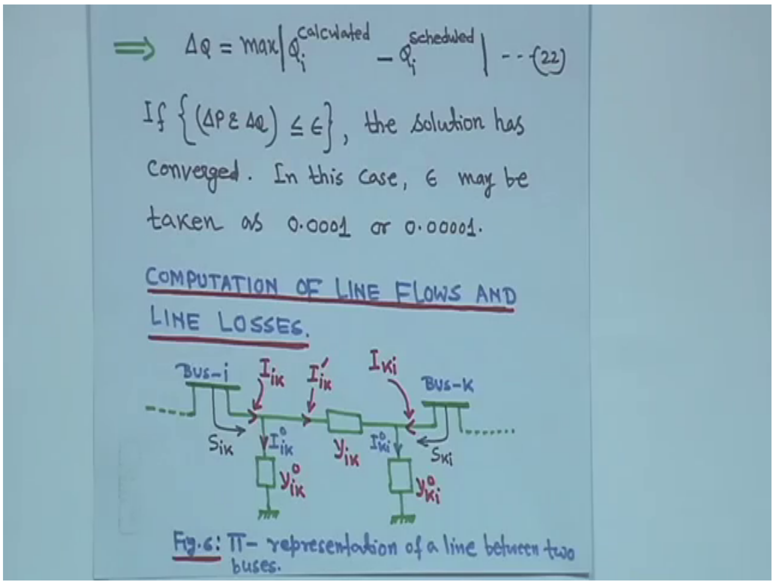

So, here also you will think that way right this you we have already studied that transmission line and pi representation you have seen know that that when you are making like this when you are making like this; this you are what you call this.

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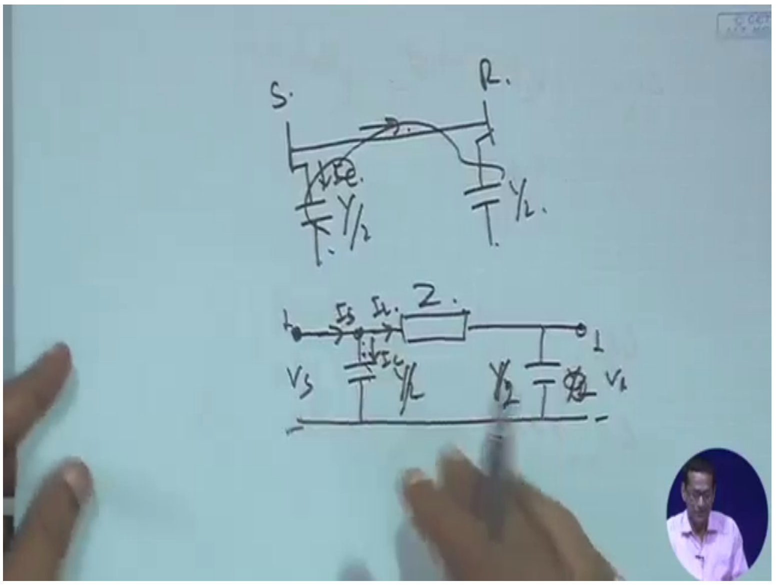

This is sending and right I am making in my way right say this is your sending a line and this is your receiving a line right. And this side you have your charging admittance right and this side also you have charging admittance in that case it was Y by 2 and Y by 2 if this is your sending end and this is your receiving end right.

So, this is the current and some current say IC is going to this and this is your the way we represent that I will I know this is your sending end and this is your line impedance that is z and this side is your you are as that Y by 2 and this side is your Y by 2 right this is Y by this is Y by 2 this is Y by 2 right Y by 2 and this is your plus minus this side is V s and this side is your V R right and this is your sending in current and if these current is your IC say for example, and this current is your line current. So, I is at this point if you apply KCL I s is equal to il plus ic you know.

So, if you know this then look at this it should not be just looking at these there is nothing to be this thing. So, suppose you consider bus I and bus k these dash line are shown this thing when line this bus I may be connected to some other buses bus k also connected to some other buses right. So, instead of line impedance we are making it Y ik that is line admittance small y small Y right and instead of sending end or receiving end those thing this is we are taking current I ik and this is I ik 0 going to the shunt admittance right shunt admittance.

And this side and this line current here it is I ik dash I ik dash right and it is Y ik and it is I ik 0 similarly and this power actually going to this we are making it like S ik I to k S ik similarly bus k actually you are making power at S ki k to I and current here it is I ik here it is I ki k to I right and here the charging admittance is there it is Y ik 0 it is Y ki 0 right it is I ki 0 here it is I ki 0 here it is ii I ik 0 right it capitalize suffix ik superscript 0 right.

So, and this is bus I bus k this pi representation of a line between 2 these are actually pi representation you have seen already, but this one you are representing by instead of impedance that admittance Y ik of the line and this side you have seen I ik this side you have seen iiki right that is the thing and in this case you have to write few equations, so, it very easy actually. So, if you if you make it like this and this is your figure 6. So, from figure 6 actually we can write I ik dash is equal to V i minus V k Y ik. So, this current line current is actually I ik I your I ik that is actually here this Y ik instead of drawing at the instead of making it from the bus bar right then it does not look good right, but. So, that is why maybe clear here actually this I ik dash the; it is actually it is V i, this I have made it here and here right.

But actually it is all this things lump together I will make it at the bus I only, but then I can I cannot show I ik or S ik. So, that is why made it like this. So, why you are I ik dash then this is current in the line Y ik actually B i minus B k that is B i, b i is the voltage of bus I and B k is the voltage of bus k. So, that is why I ik dash is equal to V i minus V k Y ilk, but I ik this I ik you apply at.

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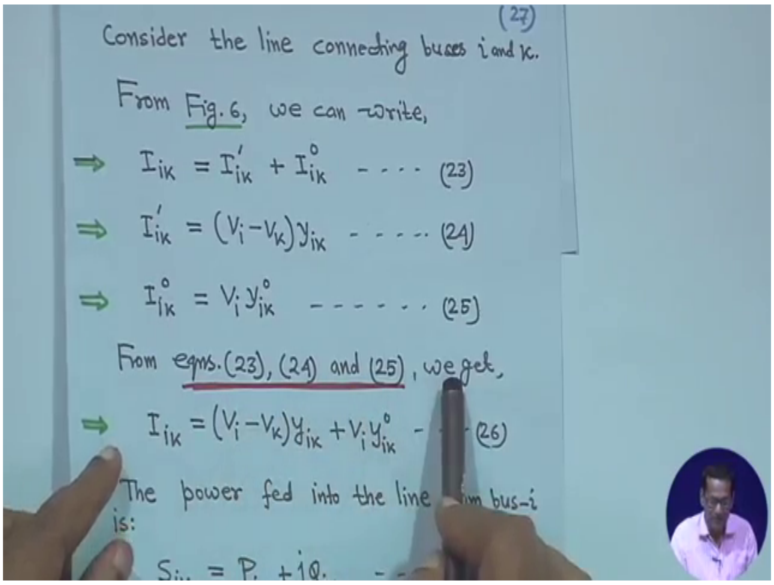

This node just now I told that Kirchhoff’s first law I ik is equal to I ik dash plus I ik 0 right that is why I ik is equal to I ik dash plus capital I right capital I suffix ik next plus capital I ik superscript 0 this is equation 23 and this I ik dash is equal to V i V k minus small y ik and I ik; I ik 0 this one this voltage is vi. So, it is actually Y ik 0 into V i right it is at ground potential, but well if I put it here and here everything it will look come very crumbsy very difficult to explain then that is why for clarity this why this had been drawn right.

So, I ik 0 is equal to from equation 23, 24 and 25 nothing is I ik dash you substitute here I ik you put 0 put it here.

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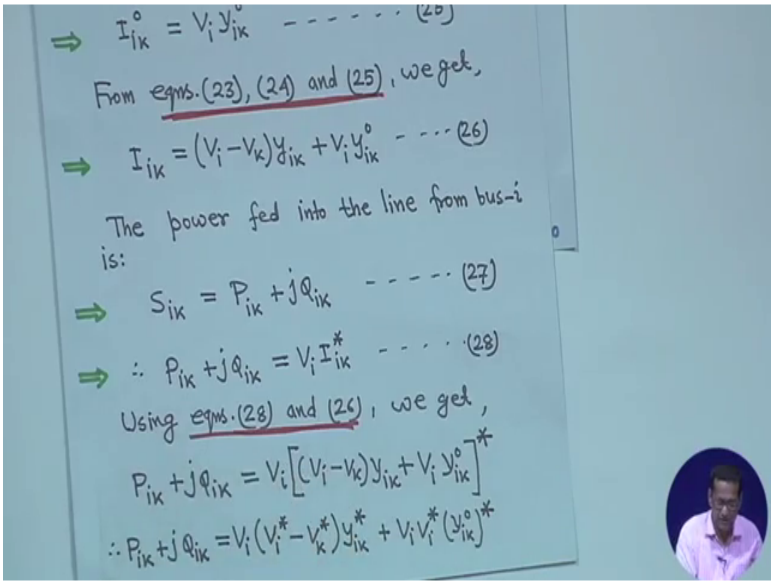

So, you will get I ik is equal to V i minus V k small y ik plus V i Y ik 0 this is equation 26, this is required because computation of line flows and line losses right now power fed into the line from bus I S ik is equal to P ik plus j Q ik this is S ik power fed into the bus. So, this is shown like this right.

So, power fed into the bus is your S ik is equal to P ik plus j Q ik this is equation 27. Now we know that P ik plus j Q ik is equal to V i I ik conjugate right therefore, this equation 28 and this 28 and 26. So, I ik is equal to this equation this equation is I ik right. So, you put it here. So, the I ik this equation of I ik you substitute here that is why you substitute toward that is why for using equation 28 and 26 you will get P ik plus j Q ik is equal to V i in bracket this is your I ik V i minus V k small y ik plus V i small y ik 0. So, superscripts is called conjugate; that means, P ik plus j Q ik is equal to V i whole conjugate you take. So, V i conjugate minus V k conjugate the small y ik conjugate plus your V i your this thing this multiply these by the V i right this is also multiplied by V i then V i into these V i conjugate V i into V i conjugate then Y ik 0 is conjugate right.

So, that mean you are this thing actually directly also you should have made it P ik minus j Q ik is V i conjugate I ik directly you should have also made it after the taking and I have been taking P ik minus j Q ik.

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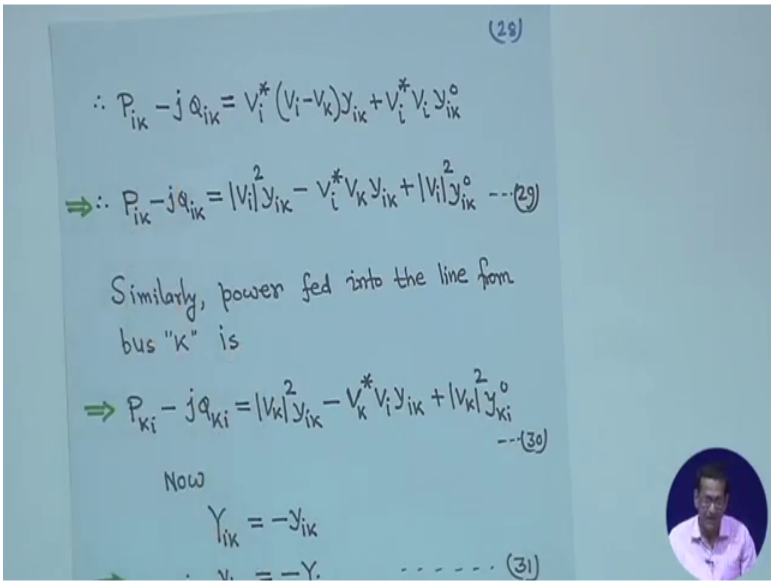

So, in that case P ik minus j Q ik if you take it will be basically V i conjugate V i minus V k Y ik plus V i conjugate V i Y ik 0 you take the conjugate of conjugate of this equation or these equation you can take P ik minus j Q ik is equal to V i I ik V i conjugate I ik this ik I ik directly you can substitute same thing whatever will get here right. So, we will get this one; that means, now P ik minus j Q ik multiply this V i conjugate V i is magnitude V i square Y ik minus V i conjugate V k small y ik plus V i conjugate V i means magnitude V i square Y ik 0 this is equation 29.

Similarly, power fed into the line from bus k; that means this equation their power fed from the line bus k that is S ki also their right. So, they would need not derive it what you can do is in this expression just in this expression equation 29 interchange k and I. So, because it is ik ki, so, just interchange. So, I am I have just interchange you will get the same expression just look how it that it will be it was ik it is ki it is ik it was minus j Q ki is equal to instead of I it is V k square that Y ik and Y ki same. So, small y ik right minus it is V i conjugate it is V k conjugate it is V k it will be V i and again small y ik because Y ik yk is same plus it is V i square magnitude it will be V k square it Y ik 0 it will be Y ki 0. So, similarly power fed into the line from bus k just interchanges k and I in equation 29.

Now, capital Y ik is equal to minus small y ik this we have seen earlier also you taken this; that means, small y ik is equal to minus capital Y ik right from here we can write like this; so in this 2 equation.

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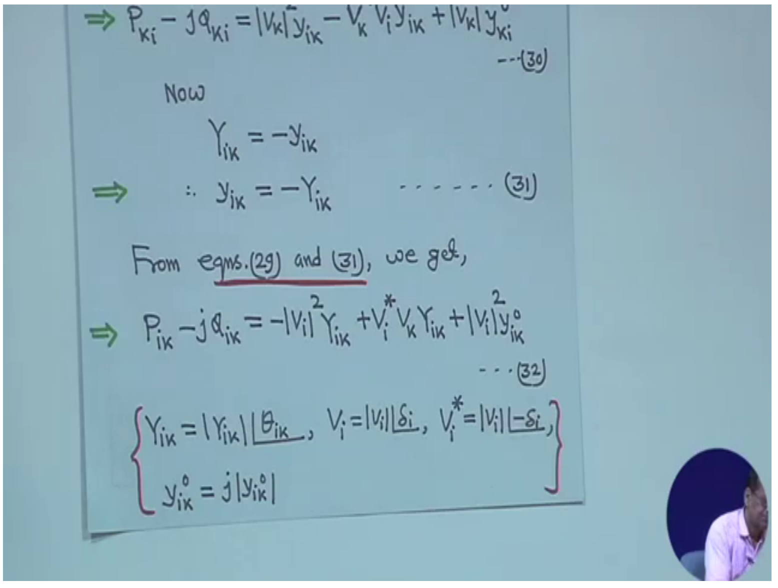

So, in this 2 equation 29 and 30 you replace small y ik by minus capital Y ik. So, that is why writing from equation 29 and 31 right if you do. So, you will get P ik minus j Q ik first see is minus magnitude V i square capital Y ik plus V i conjugate V k capital Y ik right plus V i square magnitude small y ik 0 because this charging admittance right.

Now, same as before you write capital Y ik is equal to magnitude Y ik angel theta ik then V i is equal to magnitude V i angle delta I V i conjugate is equal to magnitude V i minus angle delta I and it is a charging admittance. So, Y ik 0 j magnitude yk Y ik 0 it is got charging admittance. So, kappa that is that you that we have seen earlier also it will be j magnitude Y ik 0 all this thing you substitute in this equation then we will separate real and imaginary part right.

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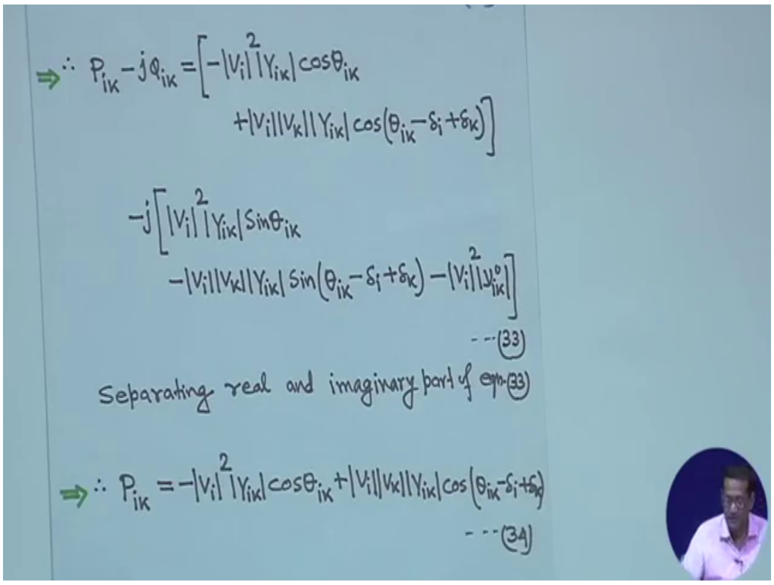

That means if you substitute and make it like this it will become P ik minus j Q ik real part will become minus magnitude V i square Y ik magnitude cos theta ik plus magnitude V i magnitude V k magnitude Y ik cosine theta ik minus delta I plus delta k this is the real part minus j in bracket magnitude V i square magnitude Y ik sin theta ik minus magnitude V i magnitude V k magnitude Y ik sin theta ik minus delta I plus delta k minus magnitude V i square Y ik 0. That means, these equation you substitute all in this equation and you just make real part and imaginary part right separately right.

So; that means, this is the equation now you separate real and imaginary part if you do. So, this is your real part is your P ik and imaginary part will be your Q ik. So, P ik will be this one right whatever it comes equation 34 it is and Q ik will be the imaginary part is this one; that means, this is the real part for P ik.

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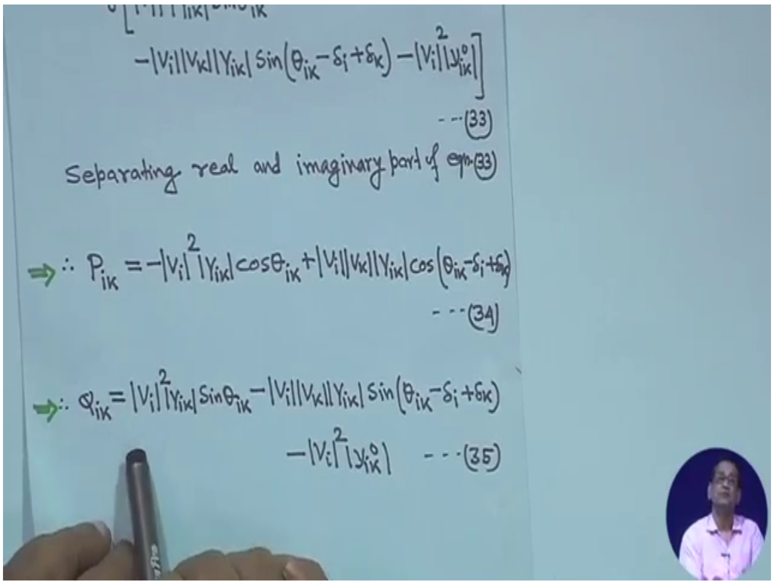

And this is the real reactive power flow from I to k; now before going to this thing suppose you have.

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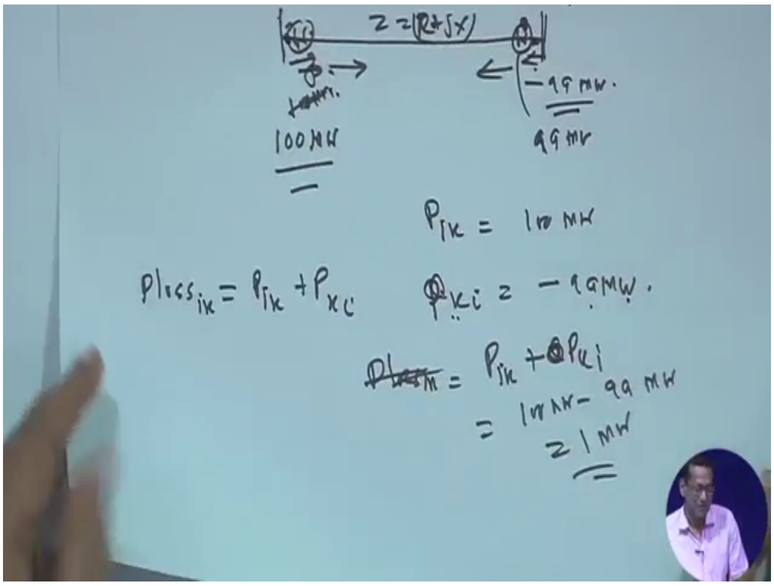

Suppose you have a bus bar suppose you have a bus bar this is have a i-th bus and this is k-th bus right suppose at this point suppose the line has the impedance z is equal to your R plus jx line has the impedance right. So, at this point if you suppose power flowing is these direction right. So, and this side you take this direction. So, at this point you measure the power right.

Suppose at this point if you measure the power for example, suppose you have a what meter here suppose you have right use power you using your what you called for transmission line for directly you cannot measure you need seat we where need ct pts everything suppose you measuring power here right you are measuring power here suppose you got this power is 100 megawatt right, but in between when will measure at this point say flowing in this direction right in this point if you measure because I square all loss is there. So, here your power is not 100 megawatt right it is not 100 megawatt right clear let me write clearly 100 mega it will be suppose flowing in this direction power flowing in this direction.

So, here is for example, here in this direction is here you measuring here your measuring right say here it is 99 megawatt in; that means, in this direction it will be minus 99 megawatt; that means you are I to k that means, power P to ik is equal to 100 megawatt you are measuring here and this side when you take Q ki that is your minus 99 megawatt because deduction is change this is the deduction, but here it is 99, but deduction is change. So, minus 99; that means, power loss of the line ik P loss ik is equal to P ik plus your what you sorry this is P ik this is P ki and this is your sorry this is your P ki. So, P ik is 100 megawatt and this is your 99 megawatt that is one megawatt power loss right.

That means the here you are measuring 100 megawatt here you are measuring 99 megawatt that, but direction is these way we will take this deduction it will minus 99 that is your P ik and this P ki is minus 99 right therefore, P loss let me write clearly P loss ik is equal to P ik plus P ki. So, P ik 100 P ki minus 90, so, it is 1 megawatt. So, that is why here that is why here P ik expression you have got Q ik expression you have got.

Now, k to I interchange in these 2 equation k to I that you replace to interchange just make instead of ik and k to I right replace I by k and k by I in these 2 equation then what you will get this P ki.

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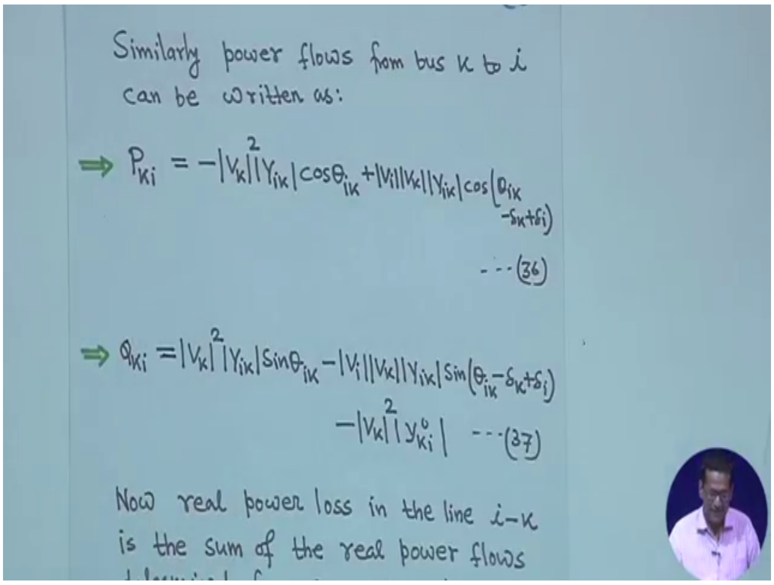

Expression will be your this one I mean I mean equation 34 and 35 just is just you interchange I and k then you will get the expression of P ki this is the expression of P ki and this is the expression of Q ki that is your this is equation 36 and this is 37 this is a power flows from k to I and reactive power the real power flows reactive power flows from k to I right.

Now, the real power loss in the line ik is the sum of the real power flows determined from equation 34 and 36 right I told you know just now that that is.

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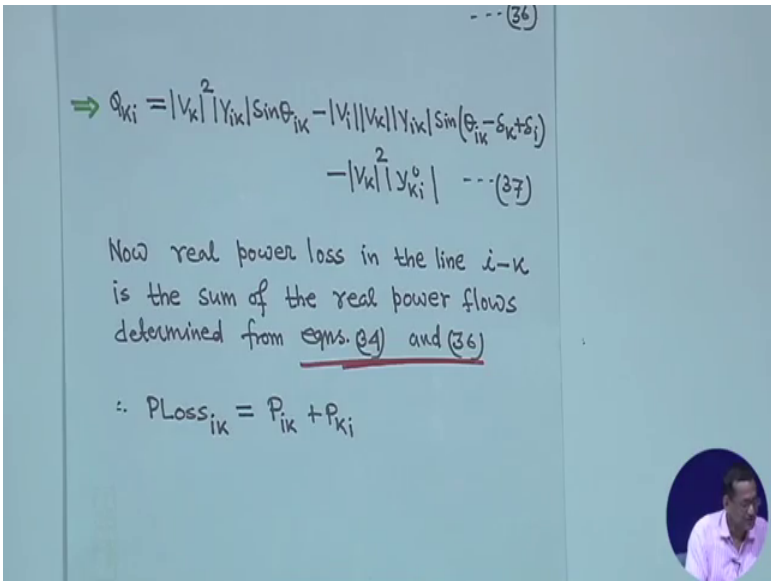

Now that I to k it will be P ik plus P ki that how things are happening right; that means, P loss ik will be your P ik plus P ki right now this P ik and P ki these 2 expressions are known to you. So, what you will do these 2 expression you P ik is also known and P ki is also known that equation 34 and 36 now do add these 2.

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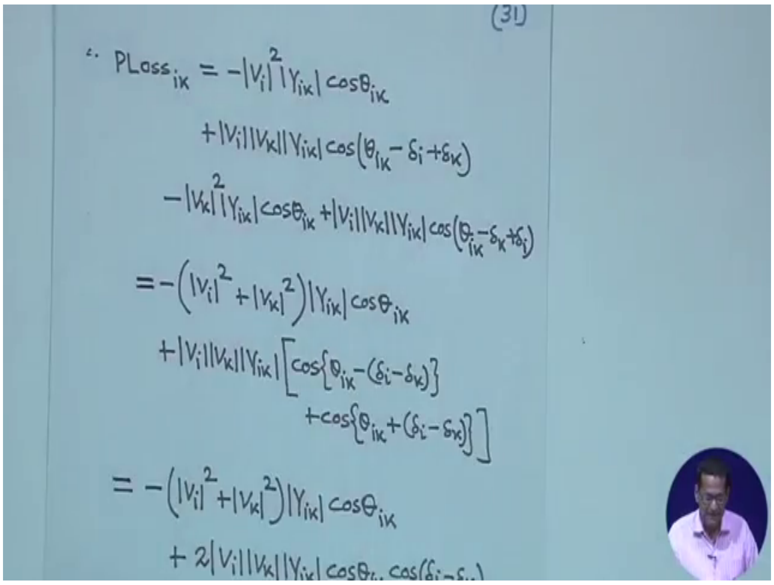

If you add these 2 the P loss ik expression will become minus magnitude V i square yi magnitude Y ik cos theta ik then I am not telling again and again magnitude V i V k Y ik cos theta ik minus delta I plus delta k minus V k square Y ik cos theta ik plus V i V k Y ik cos theta ik minus delta k plus delta I these equation is equal to it can be written as minus in bracket the magnitude of course, voltage magnitude V i square plus V k square bracket close then Y ik again magnitude of admittance line Y ik that is your capital Y ik into cos theta ik plus magnitude V i V k Y ik in bracket these equation you can write that cos theta ik minus delta I minus delta k right.

So, these 1 cos theta ik minus got a common delta I minus delta k and this 1 cos theta ik it is plus delta I minus delta k these way it is written after that what you do this one you expanded as a cos a cos b plus sin a sin b.

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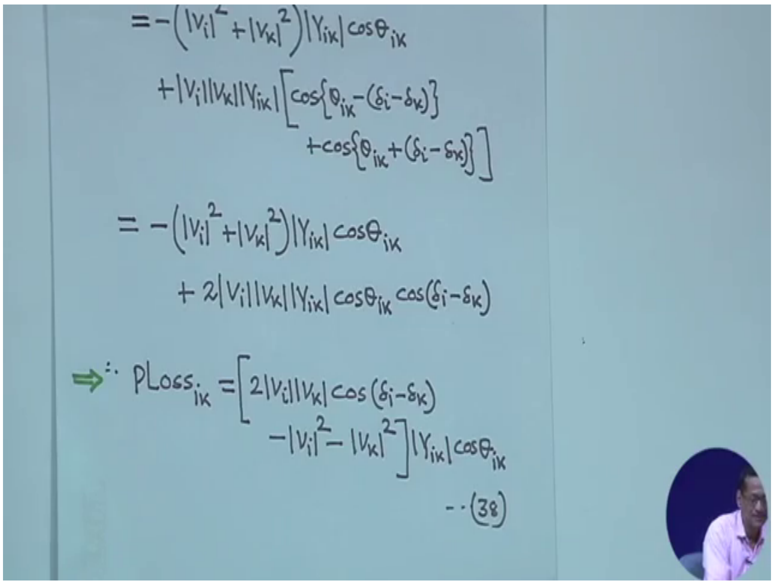

And this 1 cos a cos b minus sin a sin b and simplify if you do. So, you will get minus magnitude V i square plus magnitude V k vk square bracket close into Y ik cos theta ik and plus if you when you will when you will when you will expand it some you will find that you are theta cos theta term will be cancel right this thing. So, you are what you call these you are that sin thing will be cancel because this cos a cos b plus sin a sin b this we make it and cos a cos b minus sin a sin b you will find sin thing will be cancel.

So, it will become. So, would basically 2 because of there is it is 2 magnitude V i V k magnitude Y ik cosine theta ik cosine delta I minus delta k this expressly will get; that means, that mean these equation you can write is equal to P loss ik is equal to 2 magnitude V i V k cos theta I delta I minus delta k minus magnitude V i square minus magnitude V k square bracket close magnitude Y ik cos theta ik equation 38 this simple form of these equation you will get right after simplifying all this.

So, these are state forward you can do of your own also everything I have told write now further this one further it can be simplify right this equation further can be simplify you assume let capital Y ik is equal to G ik plus j B ik.

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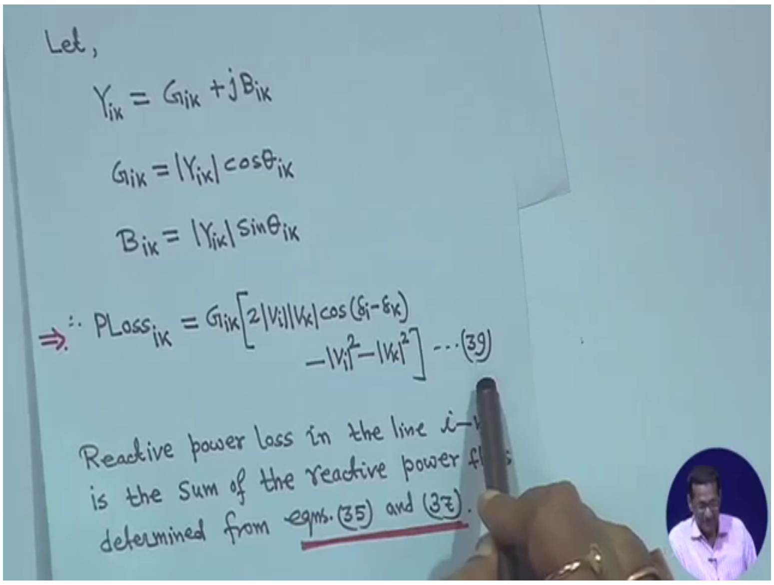

So, this; so, you define conductance acceptance right G ik this. So, you take the G ik plus j B ik and then you can if it is like this then G ik will be Y ik cos theta ik and B ik will be Y ik sin theta ik if you define like this right; that means, this your what you call this Y ik cos theta ik you can make it G ik therefore, here in this expression Y ik magnitude Y ik cos theta ik will be replace by G ik that is equation 38.

So, if you do. So, then P loss ik will become this put in pass G ik in bracket multiplied by 2 V i magnitude V i magnitude V k cosine delta I minus delta k minus magnitude V i square minus magnitude V k square this is equation 39 this is the loss formula for your what you call line I and k.

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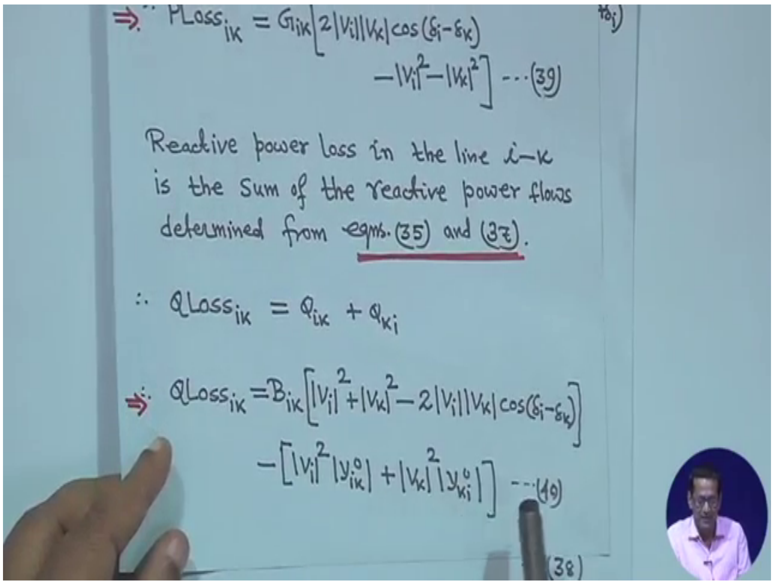

So, in terms of bus voltage magnitude and its bus voltage angle also right. So, this is the and that your what you call G ik that a capital G ik the conductance right. So, this is the formula for the line loss of course, standard formula you know that is simple thing is I square R right, but this is require for many places right later time permits at the end I will try to tell you right.

Similarly, if you make Q loss ik similarly Q ik plus Q ki Q ik expression is known to you Q ki also known to you add this 2 and simplify already in this case the charging admittance term will be there because that is reactive component right. So, in this case it will become Q loss ik will be B ik right it will be magnitude V i square plus magnitude V k square minus 2 magnitude V i V k cosine delta I minus delta k minus minus then in then in bracket magnitude V i square Y ik 0 magnitude again plus V k square Y ki 0 right this is equation forty then be this is the expression of your Q loss.

So, from this what you have seen we have found that your loss formula use for the line I and k at the same this thing what you call your and we previous we saw power injection pi qi right. So, this is the loss formula for this is your reactive power loss formula in that case charging admittance. So, will come if it is not there then it is 0 right if it is not there then this term will vanish, but for all transmission line it will be there.

Another thing is that for the further simplification I am just telling it that generally if you assume the delta I minus delta k the difference is very small if you assume it is very small then cos cosine delta I minus delta Q will be 1. So, in that case it will be 2 magnitude V i upon V k minus V i square magnitude V i square I mean it is something like this I mean it is something.

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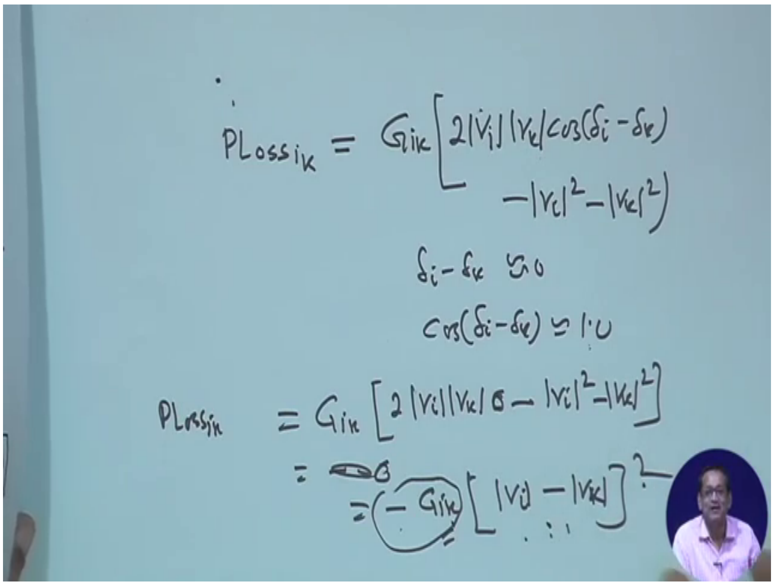

Like this if this P loss ik P loss ik is equal to G ik 2 V i 2 V k right then cosine delta I minus delta k minus V i square minus V k square.

Now, if you take delta I minus delta k approximately 0 then cosine delta I minus delta k it is 1 right. So, if it is so; that means, this P loss ik it can be written as G ik that 2 V i V k then minus V i square minus V k square this one you can write this one you can write minus G ik actually magnitude V i right minus magnitude V k whole square this is minus minus you take minus common. So, magnitude V i minus magnitude V k whole square right then as you have taken capital Y ik is equal to your minus small y ik basically the G ik will become negative right when you solve numerical G ik will become negative; that means, it this term actually will become positive was a right.

So, this thing; so, you are this V i minus V k whole square this way you can take right and this is that simplest version of course, reality it is not true right for transmission system delta I minus delta k if similarly for Q loss ik also that for 3 terms you can made it like this right similarly here also if you assume this is delta I minus delta cosine delta I minus delta k approximately one it will B ik then bracket V i minus V k whole square minus this term right this will be there. So, this is your what you call that your regarding

Gauss Seidel where loss formula further this thing and P ik Q ik all this things are known.

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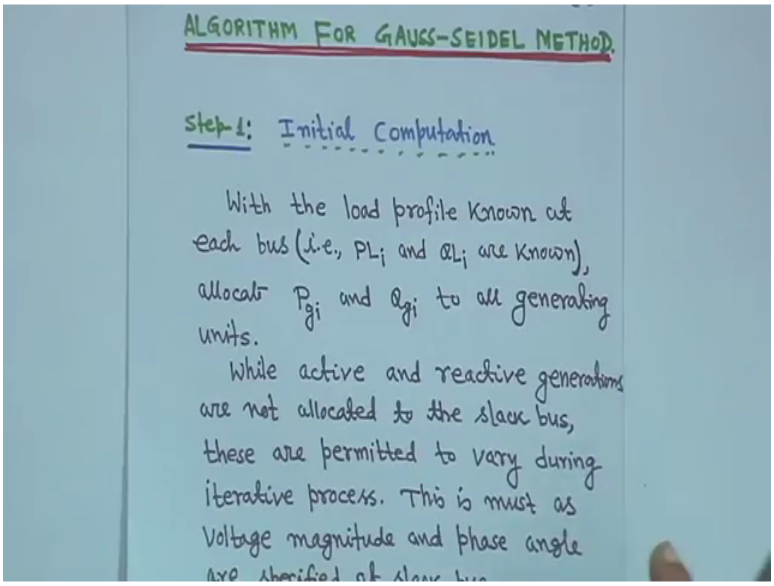

Now, algorithm for Gauss Seidel method first is the step one instead of flowchart I prepare that algorithm right initially with the load profile known at each bus that mean at each bus P li this is step one right step one initial computation. So, that is P li and Qli are known right you have to allocate P gi and Q gi to all generating units right if it is for P Q bus if it is a P V bus then pg will be known Q g will be unknown right. So, while active and reactive generations are not allocated to the slack bus slack bus voltage and its angles known right and these are permitted to this thing or vary during iterative process that is active and reactive generations at the slack bus right.

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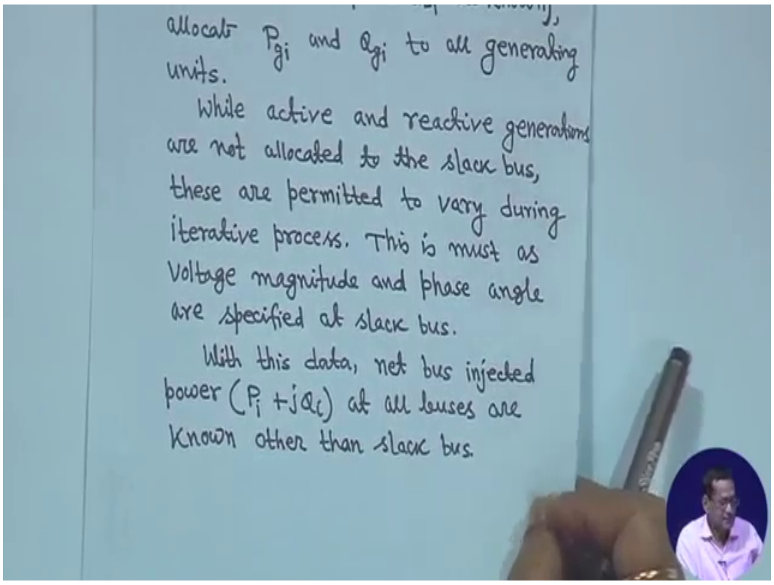

And this is must as voltage magnitude and phase angle are specified at slack bus this is known with this data net bus injected power Pi plus J qi at all buses are known after the other than the slack bus if it is I mean if all buses are P Q buses right. So, this is the first step that all the data you have to prepare.

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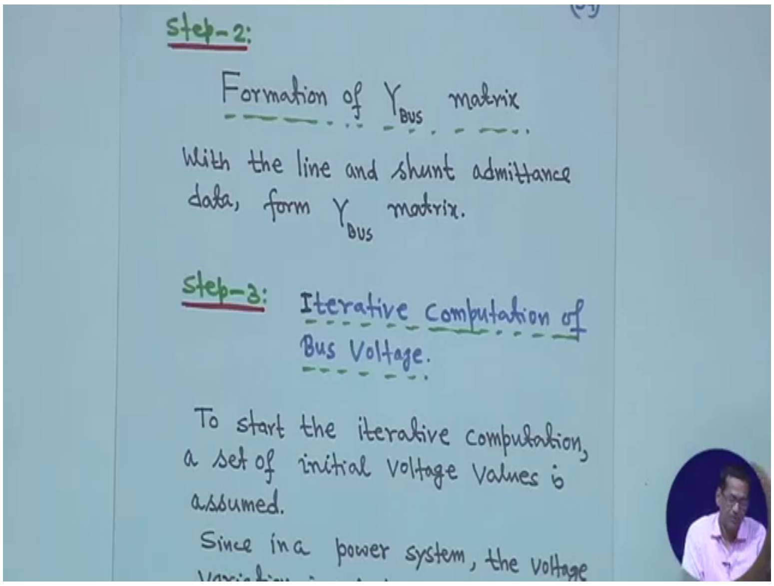

Second step is form the Y bus matrix that is line parameters are given. So, second step is that you have to form the Y bus matrix right then third step is the iterative computation of bus voltage right to start the iterative process you have go for a flat voltage start suppose slack bus voltage is given if it is 1 plus j 0.

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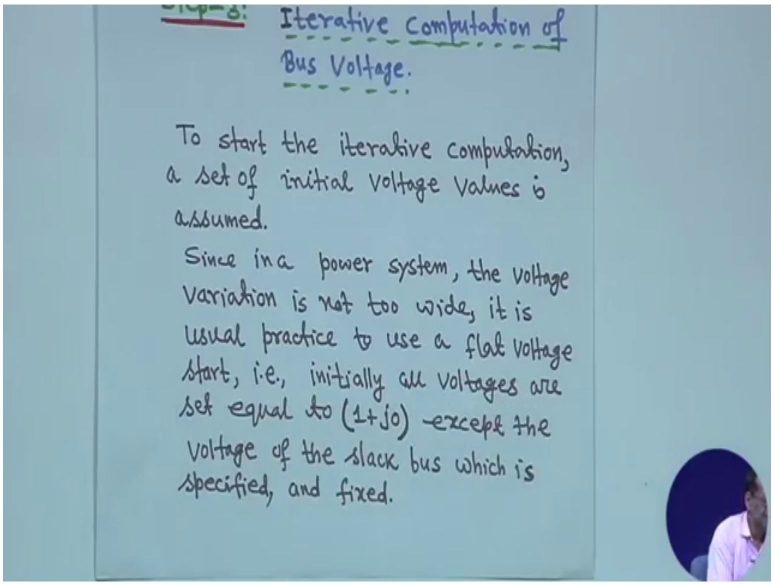

Then initial values of all bus voltage is except P V bus because there voltage magnitude is specified you consider that voltage initial voltage of all if you as all the P Q buses all the initial your what you call voltages should be equal to the slack bus flat voltage start right. So, if the slack bus voltage is equal to 1 plus j 0 then all other initial values of all other bus voltage is P Q buses right it should be 1 plus j 0 right behind slack bus voltage is known right. So, this is the third step.

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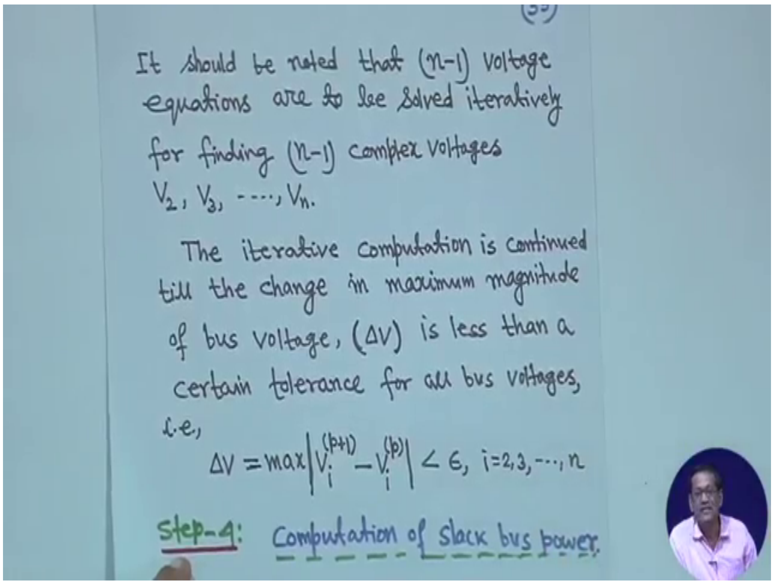

Now, fourth step will be sorry third step and here also in the third step itself right. So, in the in this can then it should be noted that n minus 1 voltage equation are to be solved that equation that all the equation that if bus one is a slack bus then V 2 V 3 V 4 all the equations you have to make it that you have seen right all these all these thing you have n minus 1 number of complex voltage equation V 2 V 3 up to V n first you have to make it then you have to solve it right because bus one is a slack bus.

After this you have to see that your what is what is the voltage you have got then you have to check the convergence. So, this already we have discussed that take this convergence criteria delta V is equal to max of absolute V ip plus 1 minus V ip, P is the iteration count is less than epsilon.

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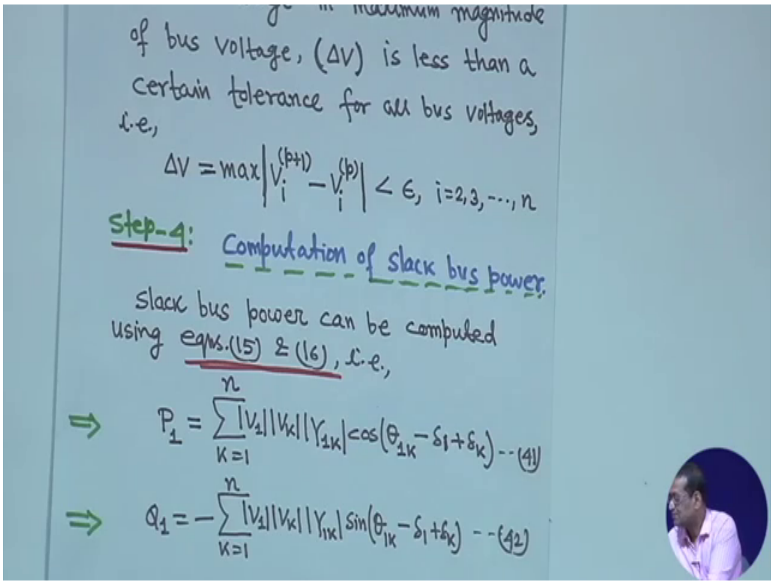

So, check epsilon you have to specify in the; you have data file. So, epsilon you make 10 to the power minus 4 or minus 5 right and see and check the these thing whether solution has converge or not this you have to check if it is not iteration will start we will take one example and will show up to 2 iteration how 2 or 3 iterations how Gauss Seidel is doing right.

After this computation of slack bus power slack bus power can be computed using equation 15 and 16 let me search equation 15 or 16 right just hold on if I if I get it I will show you because papers are mixed up right just hold on that is the question 15 here it 16 just hold on just for just hold on I will just hold on I will show you 15 and 16 if I get it quickly I will tell you this is 15.

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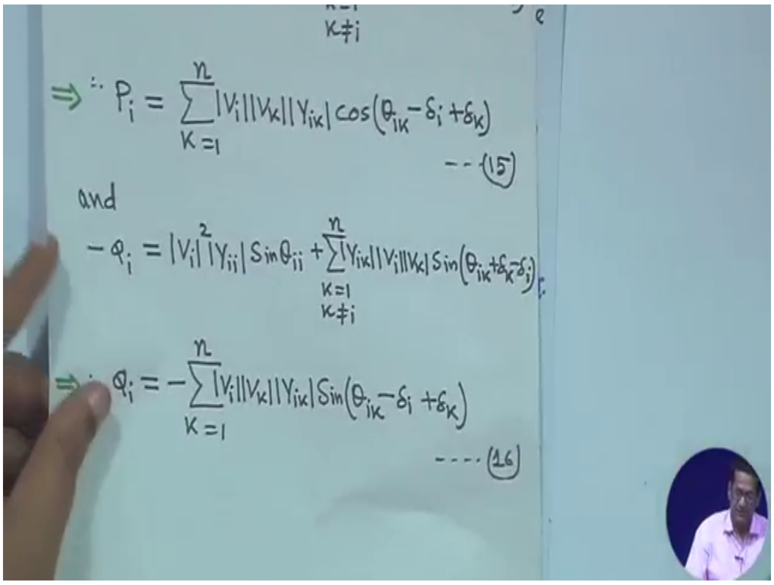

This equation 15 and this is 16 in this equation you put I is equal to 1 because slack bus is one. So, k is equal to 1 to n V 1 then you make V k Y 1 k cos theta one k minus delta 1 plus delta k same equation we are write here k is equal to 1 to n magnitude V 1 V k Y 1 k cos theta 1 k minus delta 1 plus delta k this is equation 41 we are marking.

Similarly, for Q 1 is equal to minus here you put I is equal to 1 minus k is equal to 1 to n this is magnitude V 1 V k magnitude Y capital Y 1 k sin theta one k minus delta 1 plus delta k this equation we are writing minus Q 1 is equal to minus k is equal to 1 to n and the same thing we are writing putting I is equal to 1 in this equation 16 and here it is equation 15 write once you do the this slack bus power computation you have you have to make it using this equation when I is equal to 1 in equation 15 and 16 you put I is equal to 1 right once you have done it right then this computation of line flows.

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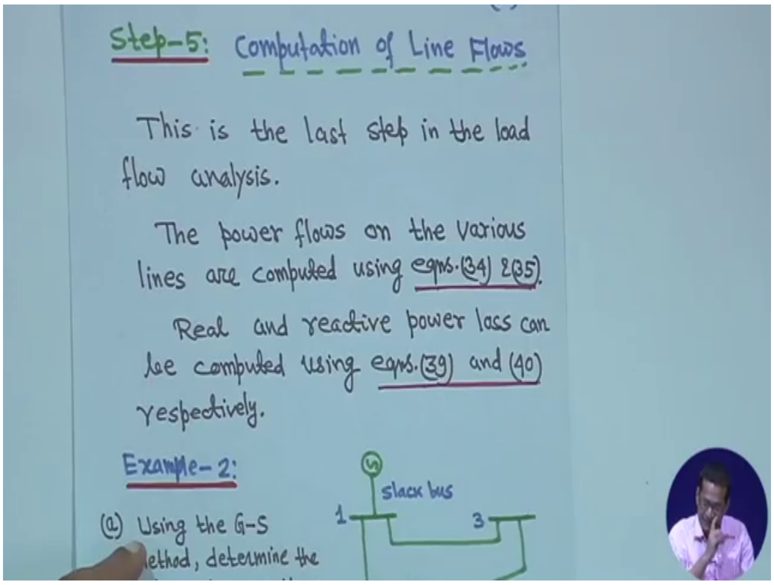

So, in this in this case this is the last step right. So, line flows equation power flows all you have using equation 34 and 35 right. So, P ik Q ik all these equations are those are your what you call if solution has converged right then equation 34 and 35 use all you have you know P ik and your Q ik. So, that that way you can find out what is the power flows those expression and put those values.

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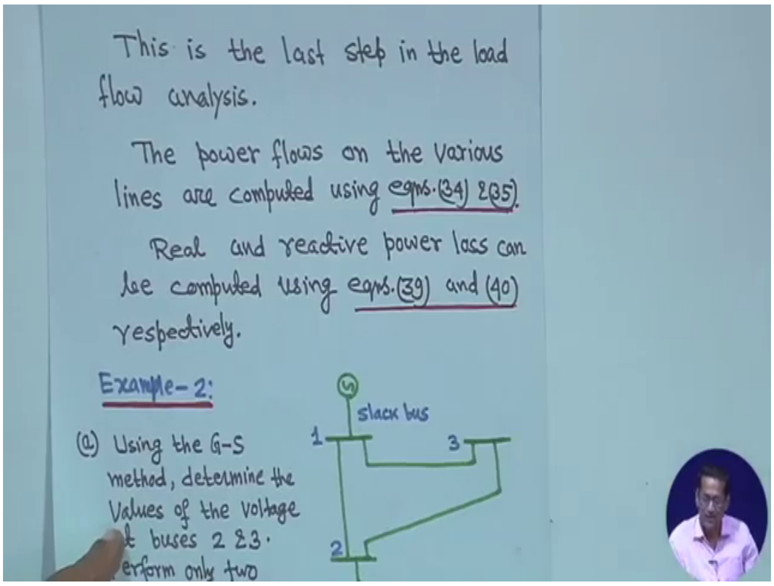

So, this is equation 34 and 35 and real and reactive power loss for the line can be computed using equation 39 and 40 that also we have shown know P loss ik and Q loss ik equation 39 and 40. So, that way also you can you can solve it right.

So, example will take later will take later right example we take later , but before that I am giving you one 1 interesting problem at the beginning of this lecture when you take that your structure of the power system and few other aspect at the end I gave you something for example.

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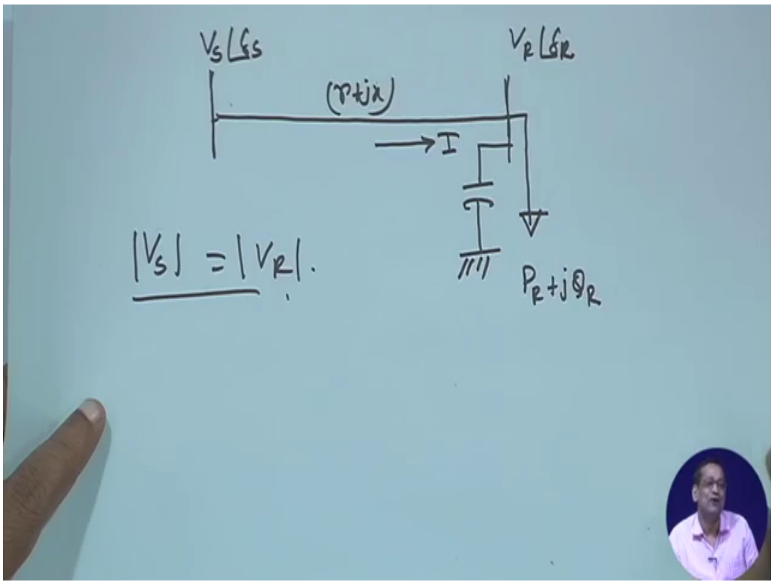

This one you will do of your own for example, this is sending and voltage and this is your receiving end this is sending end this side voltage you say V S and its angle is delta s right this your this thing and this side is V R and angle is delta R right and this line impedance say I am putting this small R j exist or total impedance of the line the resistance the reactor right and this side load is there load is there at the receiving end this is P R plus j QR right.

Now, this current actually flowing from sending end to this thing is say this is the current I right charging admittance you do not consider charging admittance you do not consider right then are what then what will be the relationship between sending end and receiving end voltage this is given right and this is this expression is given now question is that I have to find out that receiving end voltage in terms of sending end voltage R value X value P R and Q R this you have to find out and another objective is if I want to maintain this voltage magnitude that is the sending end voltage magnitude is equal to the receiving end voltage magnitude then how much capacity compensation I need here I need here right this is the thing.

So, before that before that right this is the second case first case is I have to obtain a relationship between V (Refer Time: 00:00), V S and P R that whatever we have seen if I recall correctly then magnitude V S is equal to magnitude V R plus magnitude I in bracket that is R cosine delta R plus X sin delta for a short line we have seen that right, but in this case if you want to make then how you will do it. So, I am giving you this thing.

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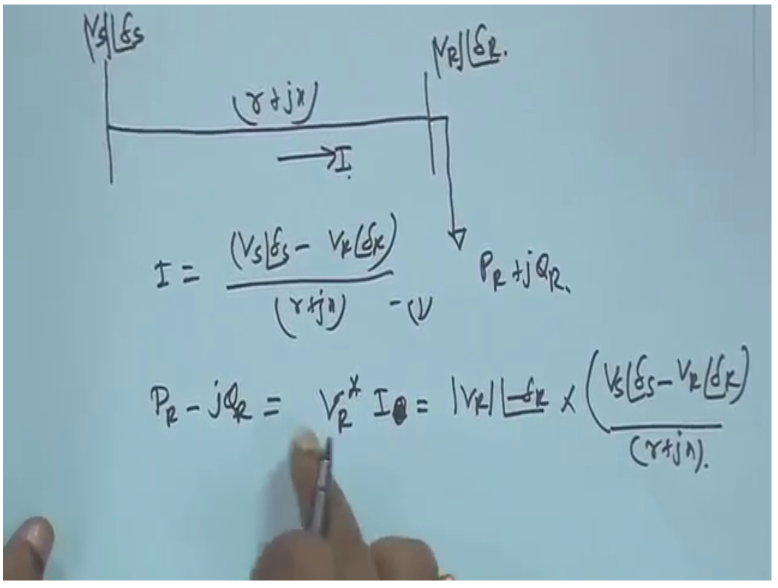

Suppose this is first do not say this is your sending end and this is your receiving end. So, this is your V S angle delta s this is your V R angle delta R right and this one is your load P R plus j QR this is the current I this is an exercise for you nothing to relate with the load flow studies this is an exercise for you example Gauss Seidel will take next thing right.

So, what you can do is that one equation I equal to right V S angle delta s minus V R angle delta R divided by R plus jx this is one equation. Equation one another equation is P R minus j QR is equal to V R conjugate that is ir V R conjugate means it is actually this is magnitude V S right this is magnitude actually right. So, previous diagram also previous diagram also it will be magnitude it will be magnitude right.

So, it is V R conjugate that is your V R angle you are minus delta R because it is conjugate into this I this is your it is I am breaking I. So, I right you can take I is equal to I are also no problem I. So, into your V S angle delta s minus V R angle delta R divided by R plus jx right this way this equation will come P R minus j QR now question is in this equation you will separate real part an imaginary part after that you will try to eliminate this delta that is delta R and delta s right and I am giving you one expression that is an exercise that when you will do this; this you will find out right. So, separate real; the part and you have you have to you have what you call you have to your eliminate delta right.

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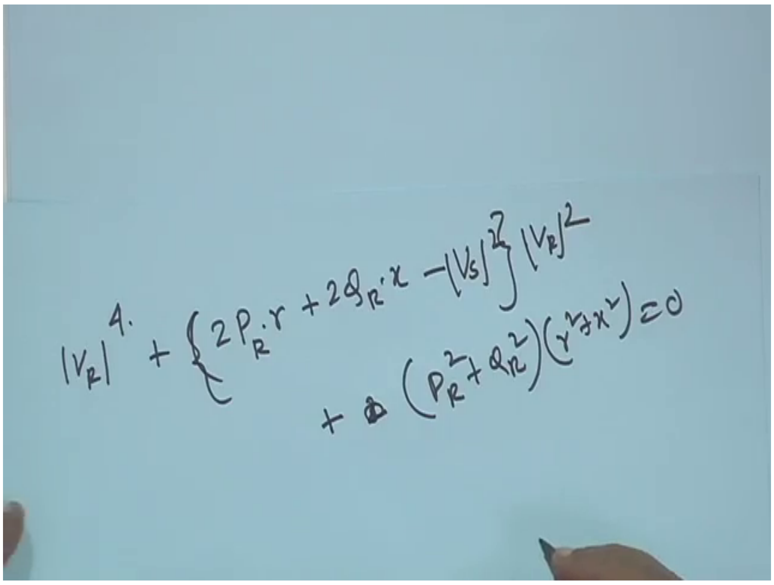

If you do so that equation will be that receiving end voltage there is a expression will become V R to the power 4 plus in bracket 2 P R right into R plus 2 your Q R into X minus that you are I can put like these minus your these voltage sending end voltage magnitude square bracket close into V R square right plus 2 no not this plus your P R square plus Q R square into R square plus X square is equal to 0 right. So, these equations whatever I have showed you this is an exercise for you nothing to related load flow Gauss Seidel. Will come next thing then exercise for you that these 2 equation you write and using this expression. You will make it by eliminating delta the relationship between your; what you call receiving end voltage and sending end voltage this is the relationship.

I hope this equation from my memory I have written I thing I have written correctly right. So, this one you will make it and solution. Now the next lecture will see the solution of that is how to make the numerical using at this thing for load flow studies using Gauss Seidel method.

Thank you.