Power System Analysis
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Lecture - 40
Optimal system operation (Contd.)

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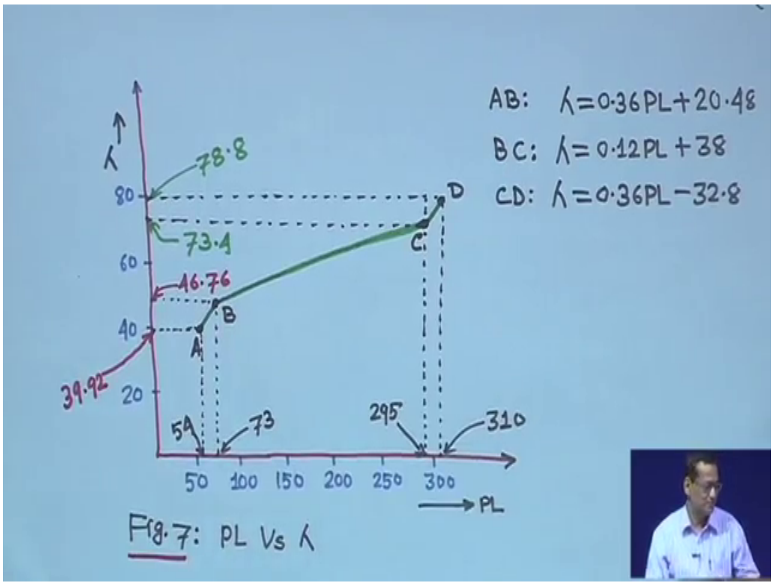

So, whatever we have seen for this problem, so we got know that for different say if lambda is equal to if this curve A to B shows that lambda the in between 39.92 and 46.76 load is in between 54 and 73, so this part is AB. So, this equation, this is a linear equation. So, this part is drawn then B C is lambda is equal to 0.2 PL plus 38 - 38 we have got this one also. Then from here it is start that 73 megawatt to 295 megawatt total. So, we get this straight line and then last one is lambda is equal to 0.36 PL minus 32.8. So, this is C to D curve, this is D to D that is 73.4 to 7 to 8.8 lambda. So, this is your PL versus your lambda graph. So, this is just a three different straight lines have been drawn with three different slopes and that you can easily make it. This side is the total load and this side is your lambda.

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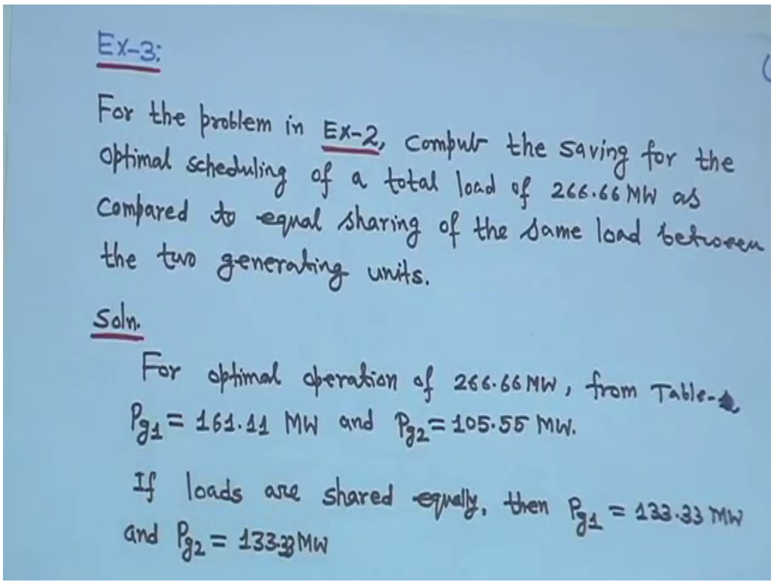

Next, based on this example only we will take another some different data and we will solve this one. For example, that for the example problem in example 2, this previous one you compute the saving for the optimum scheduling of a total load of 266.66 megawatt as compared to the equal sharing of the same load between the two generating units. Just hold on that just hold on that so tabular form. When you total the load is I mean equal share means 133.33 megawatt each - this one, and what you call optimum.

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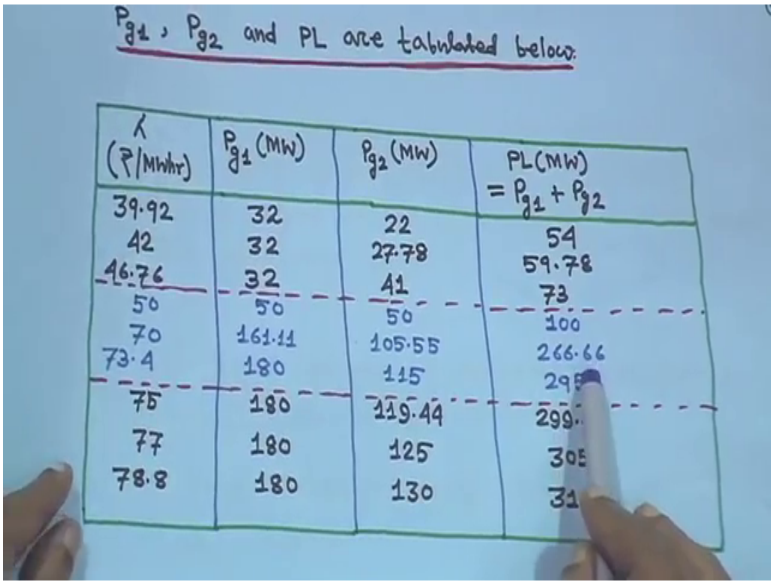

λ (₹/MWhr)	Pg1 (MW)	Pg2 (MW)	PL (MW) = Pg1 + Pg2
39.92	32	22	54
42	32	27.78	59.78
46.76	32	41	73
50	50	50	100
70	161.11	105.55	266.66
73.4	180	115	29
75	180	119.44	299.
77	180	125	305
78.8	180	130	31

If you take optimum one 266.66 we have given, then if you take this is the optimal one then in that case P g 1 is equal to 161.11 megawatt and this 108.55 megawatt, so that means, for optimal operation of 266.66 megawatt from table. That means, from this table only P g 1 is equal to 161.11 megawatt and pg 2 is equal to 105.5 megawatt. So, if and if loads are shared equally then both will be that essentially divided by 2. So, it will be P g 1 will be 133.33 megawatt and P g 2 is equal to 133.33 megawatt both are equal.

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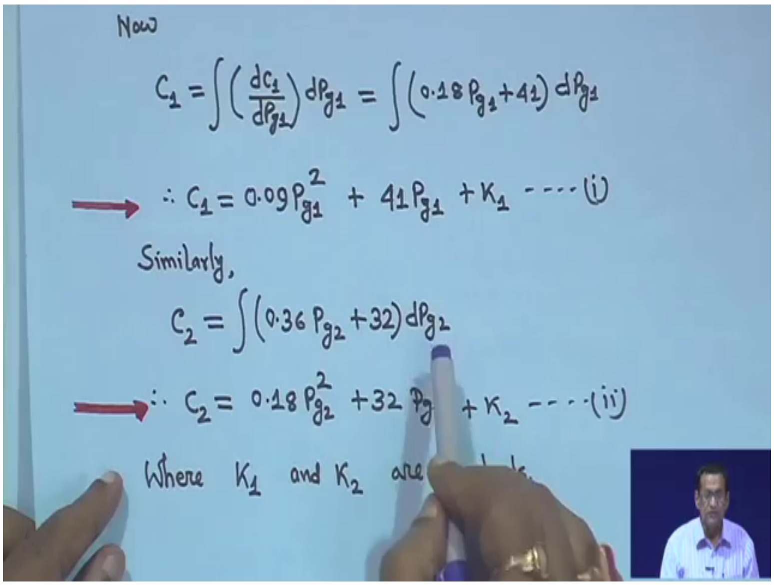

So, this now that C 1 look C 1 is your what you call just if because I c 1 is given to integrate that one C 1 is equal to dC 1 dP g 1 into dP g 1. So, integrate this one. So, dC 1 dP g 1 is 0.18 pg 1 plus 41 if you integrate this, it will become 0.09 P g 1 square plus 41 P g 1 plus K 1 K 1 is a constant. Similarly, C 2 also dC 2 dP 2 your dP g 2.

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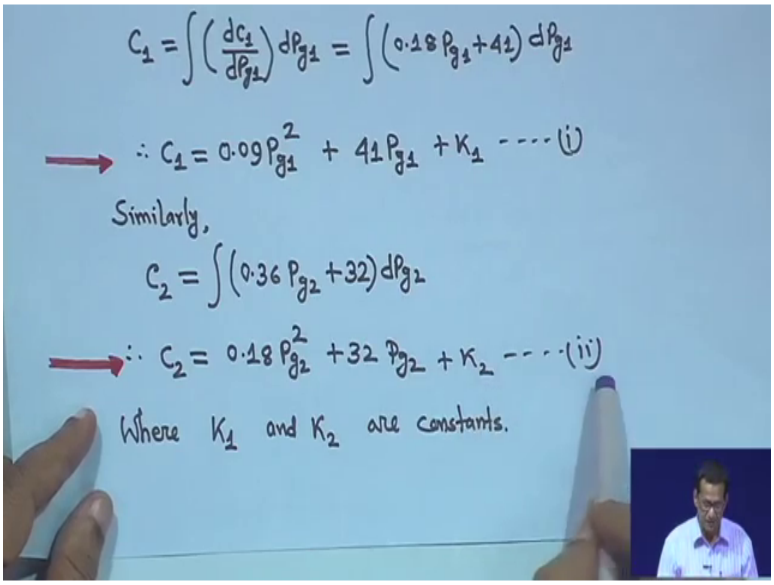

So, if directly you substitute 0.36 P g 2 plus 32 dP g 2 that is C 2 will be 0.18 P g 2 square you integrate this one plus 32 P g 2 plus K 2 K 2 is another constant. This is two where K 1 and K 2 are constants. So, their ICs are given you have to integrate them to get the C 1 and C 2.

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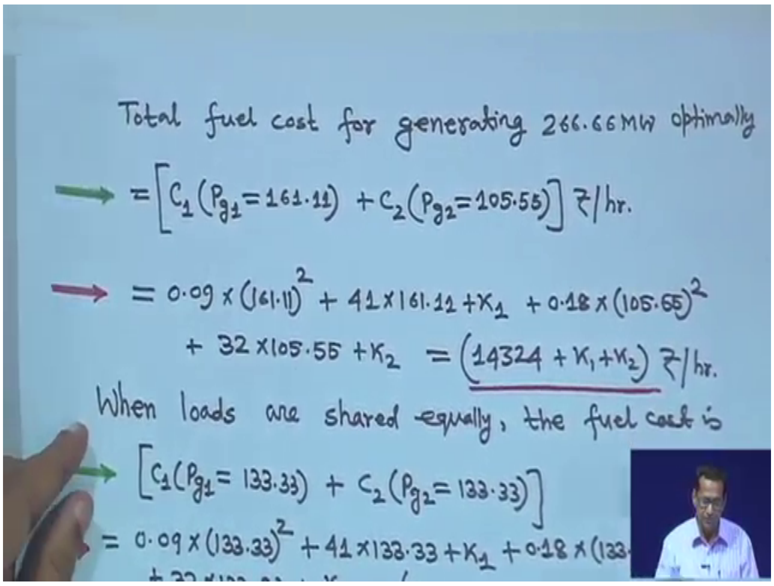

So, now if these two are given, now if you take the optimal one that optimal one means the fuel cost for generating 266.66 optimally. So, we solve that P g 1 will be 161.11 and C 2 will be 105.55 rupees per hour. So, in that in that characteristic of a P g 1 and C 1 and C 2 you substitute, so it will become 0.09 into 161.11 square plus 41 into 161.11 plus K 1 plus 0.18 into 105.55 square plus 32 into 105.55 plus K 2 that is equal to your 14324 plus K 1 plus K 2 rupees per hour. So, K 1 K 2 constants are here.

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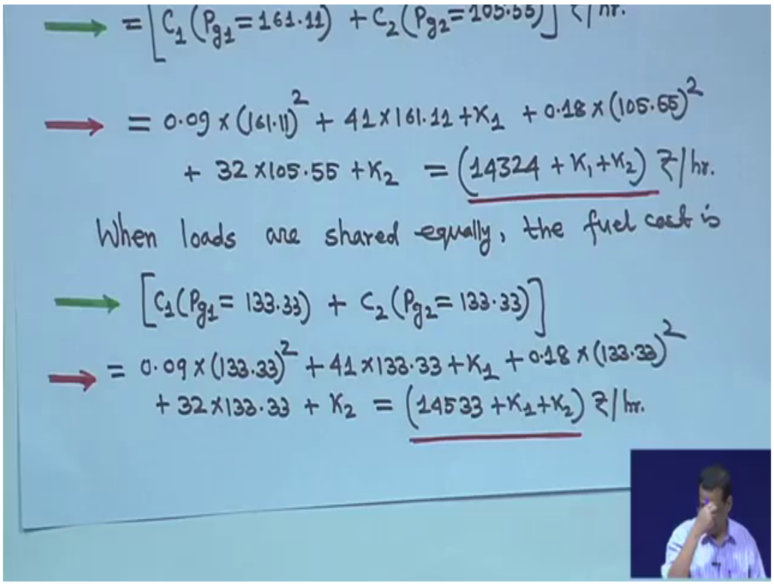

Now, when loads are shared equally the fuel cost will be P g 1 also 133.33 P g 2 also 133.33 . So, in that case it is 0.09 into 133.33 square plus 41 into 133.33 plus K 1 plus 0.8 into 133.33 square plus 32 into 133.33 plus K 2 is equal to 14,533 plus K 1 plus K 2 rupees per hour. This is also K 1, K 2 is here, here also K 1 K 2 now. Therefore, net saving for optimal your scheduling operation. So, from that you can make out actually this is higher because if it is not optimally shared.

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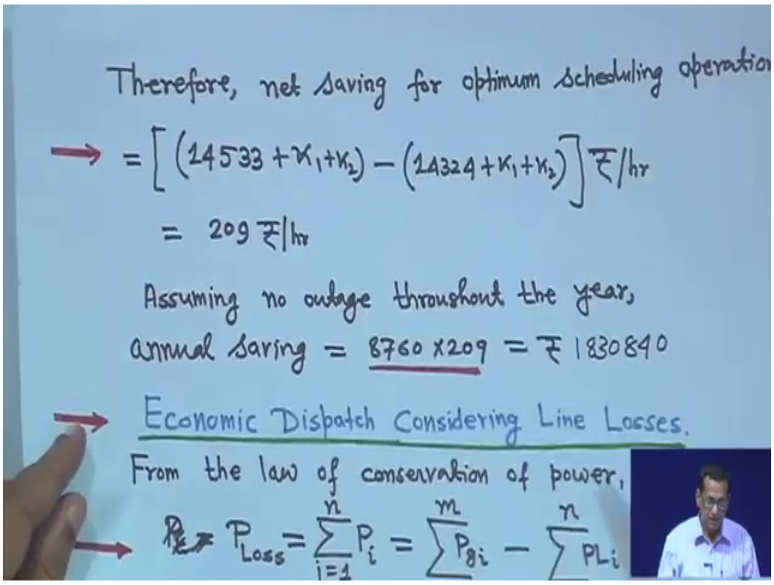

So, therefore, the net saving for optimum scheduling operation, so this is equal sharing 14,533 plus K 1 plus K 2 minus 14,324 plus K 1 plus K 2 rupees per hour that is 200 rupees per hour if you do not the K 1 K 2 will be cancel. So, if you do not operate optimally, you will be looser by 209 rupees per hour I mean this is simple example because if equal sharing means this much plus K 1, K 2 constant and optimally means this much. So, naturally one should go for optimal operation. So, assuming no change no outage throughout the year, so annual saving will be 8760 because in year total number of hours 8760 into 209, so it will be rupees 1830840, so this much rupees.

So, now this is whatever little bit you have done all these thing that is without your what you call without losses. Now, what we will do we will study the economic dispatch considering line losses. Now, you have to consider the line losses now one thing as lower cost studies we have already lower cost studies we have already done.

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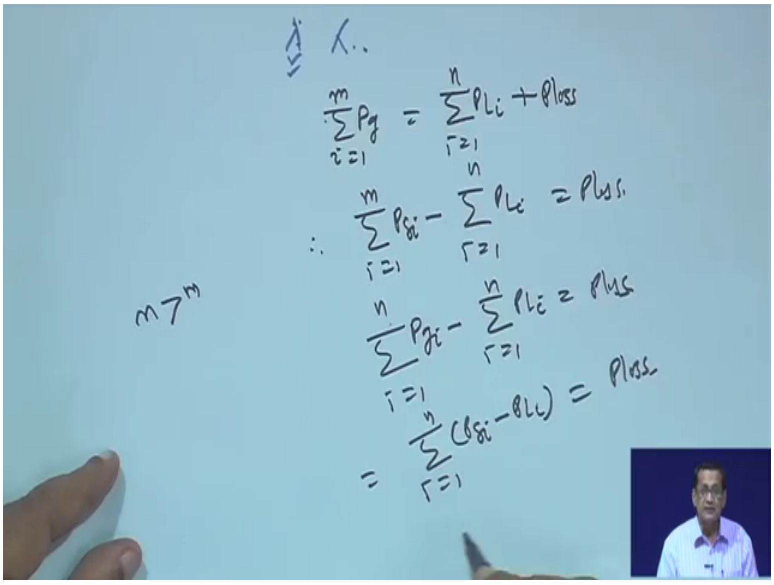

So, naturally as that injected power in anywhere that loss is equal to that look generation is equal to total generation say i is equal to your what you call m total generation is equal to total load say P L i plus P loss generation is equal to load plus loss. That means your i is equal to 1 to m P g i minus i is equal to 1 to n P L i is equal to P loss, this is the thing. Now, question is that that instead of now we have n number of buses and m number of generators. So, instead of that these equation we can write like this i is equal to 1 to n P g i minus i is equal to 1 to n P L i is equal to P loss that means, where your this thing n is greater than m . So, where generators are there that we consider P g i if it is not there P g i is zero, so that is why taking this one. So, this one you can write i is equal to 1 to n P g i minus P L i is equal to P loss. So, this is the thing

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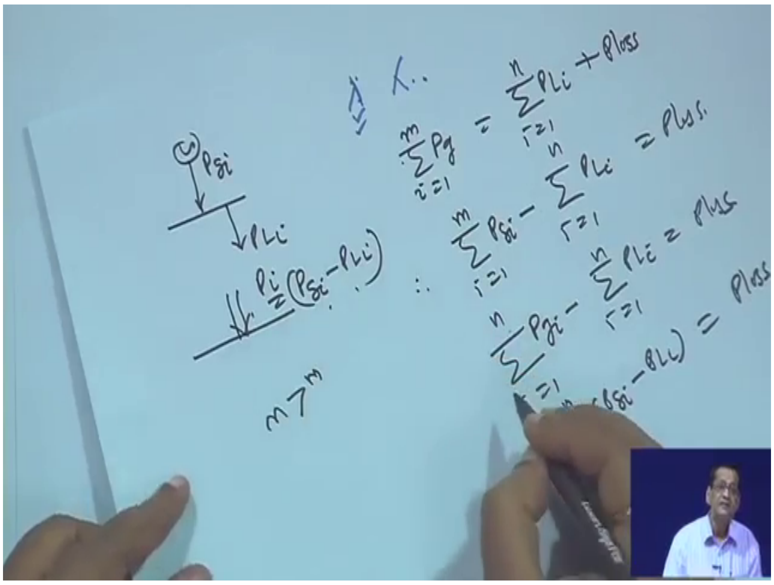

Now, net injected power we do not know this was your this was your generator say for P g i only, and this is your load P L i only the real power you have considering that means, net injected power this is your P i that is you will come your P g i minus your P L i. That is your net injected power P g i minus P L i that means these equation it can be written as that sigma i equal to 1 to n pi is equal to P loss that means, that some of injected power at all buses will give you the power loss. This I wanted to mean because this thing I will use. So, some of injected power at all the buses will give you the power loss.

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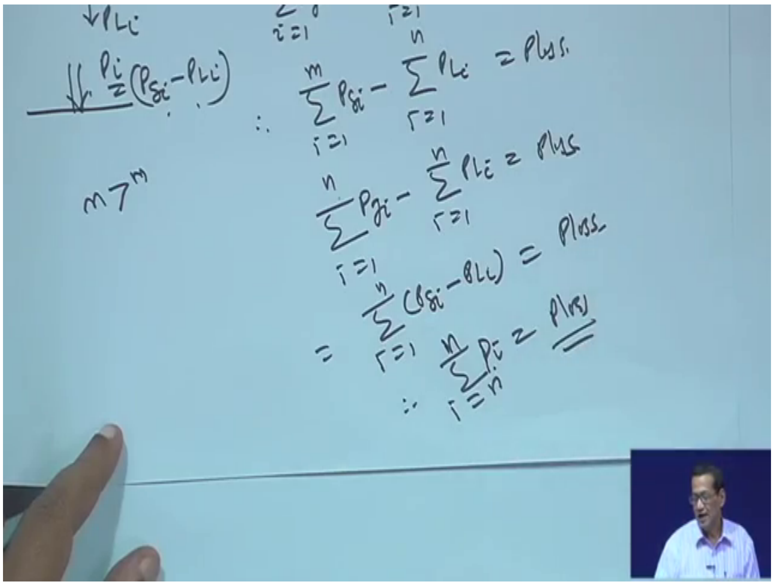

That is why this n I took instead of m n greater than m while generators are there you consider and while generators are not there it will be 0 minus P L i 0 minus say in general 0 minus PL that is why to show you that i is equal to 1 to n we took this.

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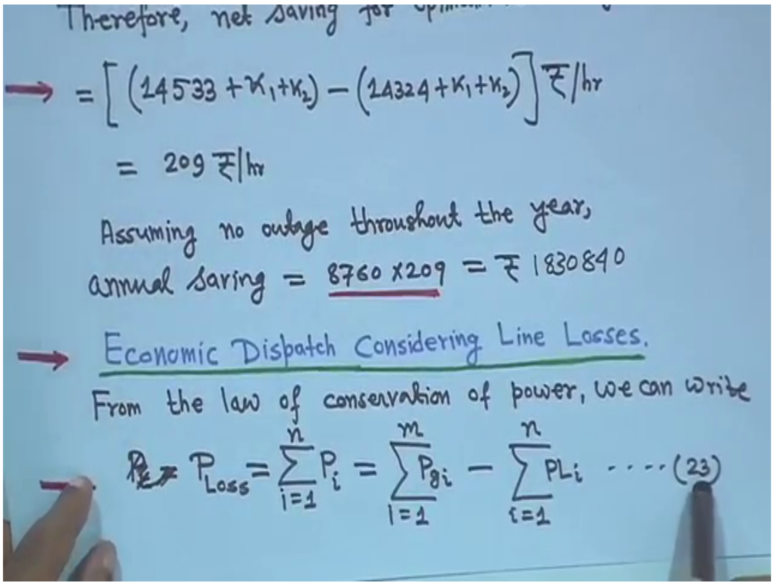

So, same thing same logic we will use here that from the law of conservation of power we can write P loss is equal to i is equal to when sigma pi that is that your total your this thing is equal to the power loss just now we solved the total power injection. So, among that you have m number of generator. So, i is equal to 1 to m P g i minus i is equal to 1 to n P L i this is equation 23. So, this is that means, loss is equal to actually is equal to the total injection of power some of the injection of power at all the (Refer Time: 10:01), this is understandable to you.

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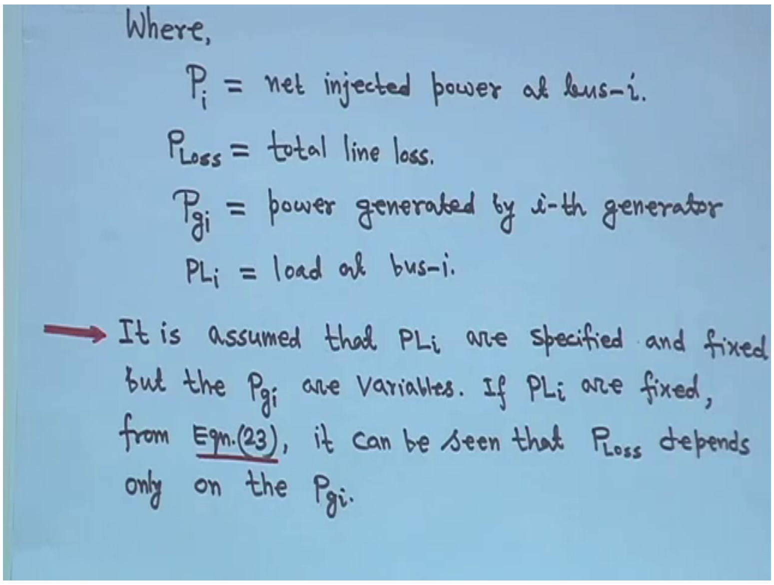

Now, P i is equal to net injected at bus-i just I showed I have given the nomenclature P loss is equal to total line loss then P g i power generated by i th generator this is understandable to and P L i is equal to load at bus-i. Now, it is assumed that P L i are specified and fixed. So, we will know that load is known to you load is always fixed and fixed, but the P g i are variables. So, if P L i are fixed then from equation three, it can be it can seen that P loss depends on P g i that means if load is fixed then P loss is function of P g i depends on P g i.

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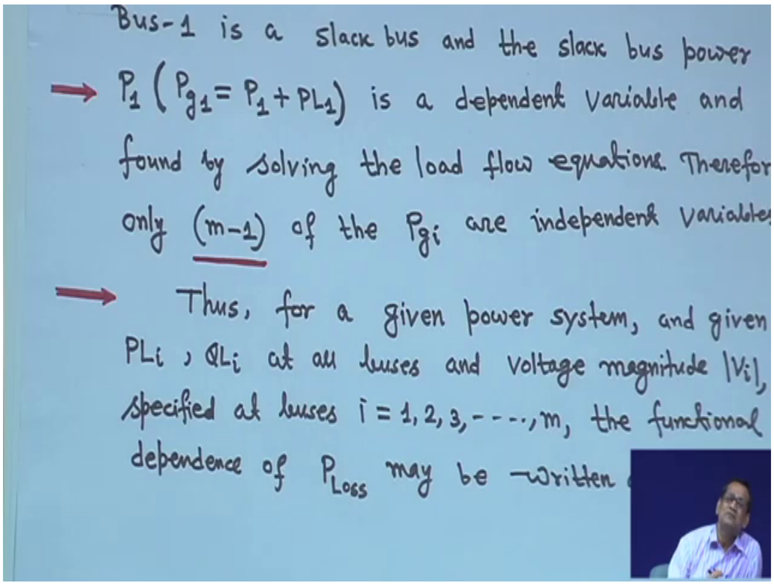

So, that means so bus one is a slack bus and the slack bus power P 1 P g 1 is equal to P 1 plus PL 1. If any load is connected at slack bus, I told you that during load iterative process if any load is connected to the slack bus that nothing know the you need not consider that will iterative process, because it say dummy variable actually you need not. But once you get the lower cost studies at that time that P 1 that P g 1 should be P 1 plus PL one is a dependent variable and found by solving the load flow equations therefore, only m minus one of the P g i are independent variables

So; that means, thus for a given power system and given P L i and Q L i at all buses and voltage magnitude V i specified at buses i is equal to 1, 2, 3 up to m the functional dependence of P loss may be written as that means, a bus one is slack bus. That means, all are the if you assume your bus total 1 to m buses and if bus one is a slack bus the loss actually is a function of P g 2, P g 3 up to P g m that means, this that means, we can write that loss actually.

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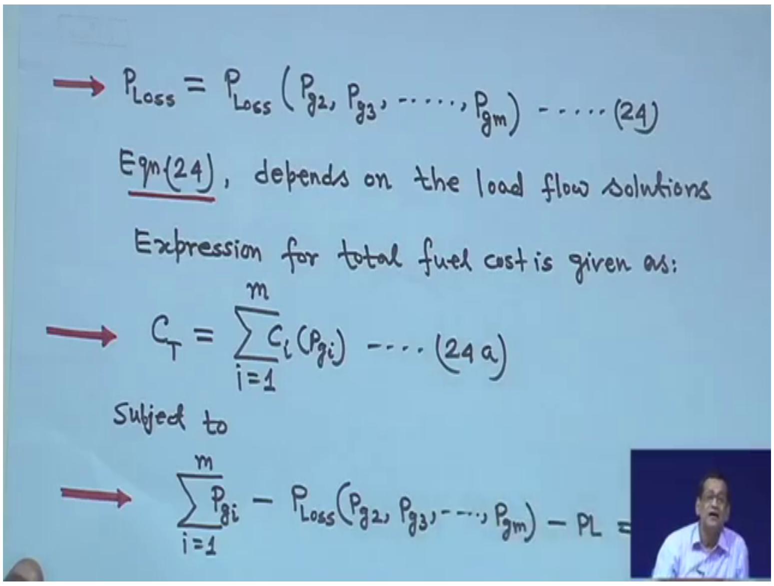

Just hold on loss actually is a function of this your what you call P g 2 P g 3 up to P g m because you have seen that P loss is equal to sigma i is equal to 1 to m your P g i minus your this thing sigma P L i i is equal to 1 to m. But if bus one is a slack bus like and your P g 1 is equal to P 1 plus your PL 1 if any load is there at slack bus; that means, this loss actually function of P g 2 P g 3 P g m. Now, this loss formula in terms of this generation thing, we will derive at the end of this topic, because if I try to derive this P loss is a function of all these things then whole continuation of this thing will be lost because this derivation is little bit lengthy.

So, at the end, we will make it at that time we will assume that P loss is function of P g 2, P g 3, P g m accordingly we will proceed, but at the end I will give you the loss formula, right now you do not need this. So, we will assume that. But if I try to do this then all this continuation will be I mean that will be disturbed actually. So, equation 24 depends on the load flow solutions that is true. So, now expression for total fuel cost is given by C T is equal to i is equal to 1 to m C i P g i this is actually 24, I have made this one actually 24 a say. So, this is known total fuel cost already earlier you have seen this one.

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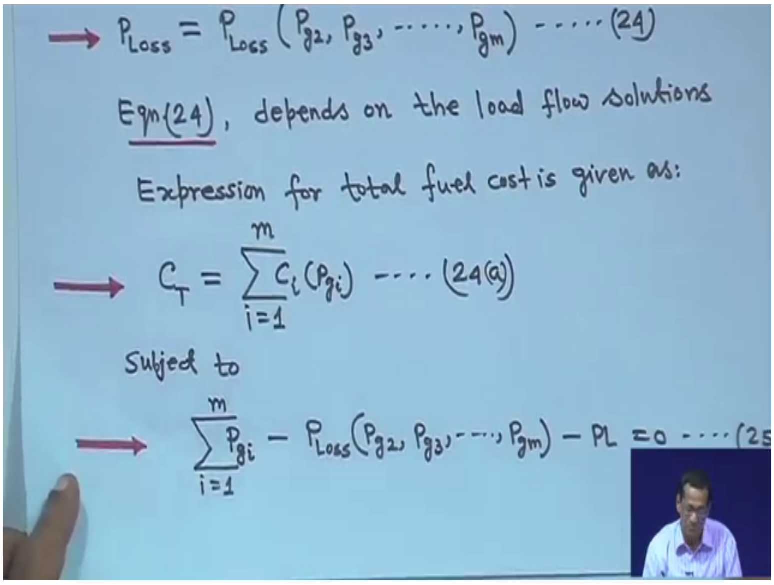

Now, subject to your now loss is considered because total generation is equal to load plus loss. So, that is why this equation I have writing i is equal to until P g i minus P loss which is function of P g 2, P g 3 up to P g m minus PL the total load is equal to 0 . So, generation minus loss minus load this is a power balance equation. So, this is equation 25.

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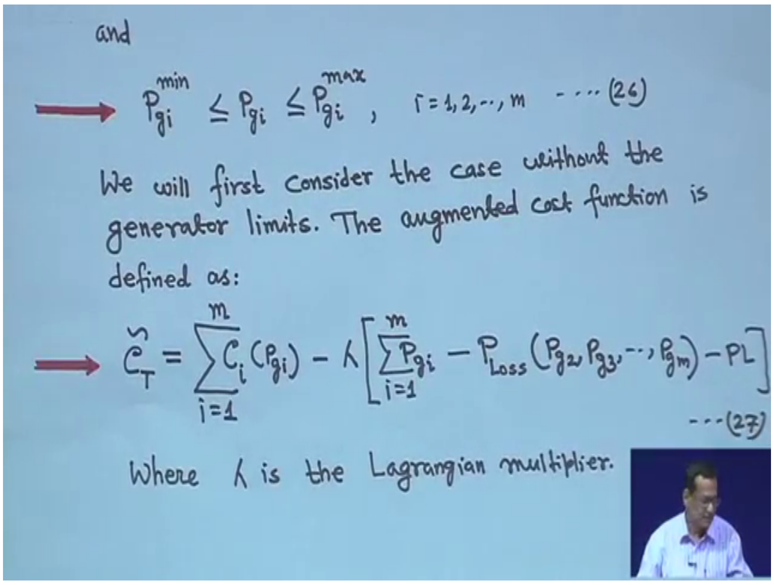

So, and P g limit is given P g i min in lying P g i lying in between P g i min and P g i max. So, now, what we will do then again you will follow the same thing that Lagrangian multiplier. So, we will first consider the case without the generator limits for example, generator limit that we will see later. The augmented cost function is defined as C T tilde is equal to i is equal to 1 to m C i P g i minus lambda into in the bracket sigma i to 1 to m P g i minus P loss function of P g 2, P g 3 up to P g m minus PL this is equation 27, where lambda is the Lagrangian multiplier.

If I take that derivative with respect to lambda and is equal to 0. So, if you take the derivative 0, so in general the this equation this power balance equation actually it will become 0, that means, look if you take derivative with respect to lambda there will be minus sign here, but is equal to 0. So, no need to consider minus before that is equal to 0.

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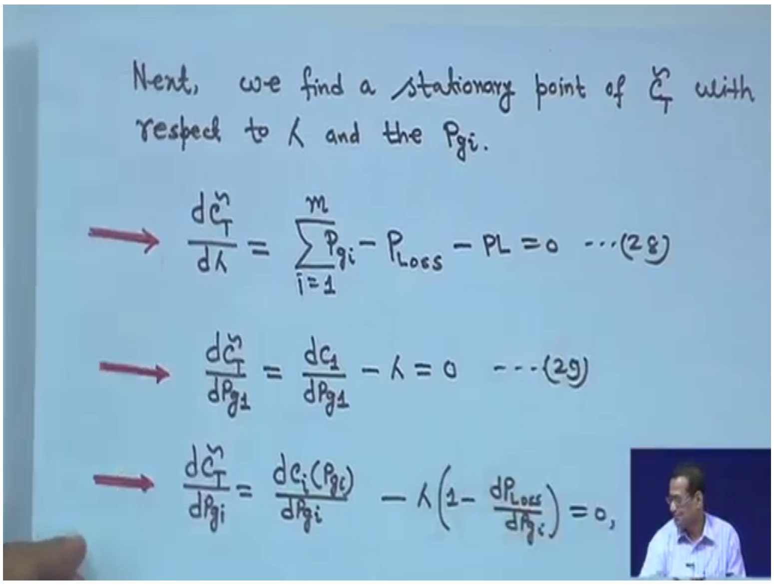

So, that is why that d C T tilde upon d lambda is equal to i is equal to 1 to m P g i that is sum of the all the generation minus P loss look. P loss is a function of P g 2, P g 3, P g m, but again and again not writing in the function of now it will understandable that P loss is function of the generator power except slack bus minus PL is equal to 0, this is equation 28. Now, in this equation again you take derivative with respect to P g 1 that is dC 1 upon dP g 1 will be you have a this thing I mean take the derivative with respect to P g 1 I mean if you to do so it will be something like this.

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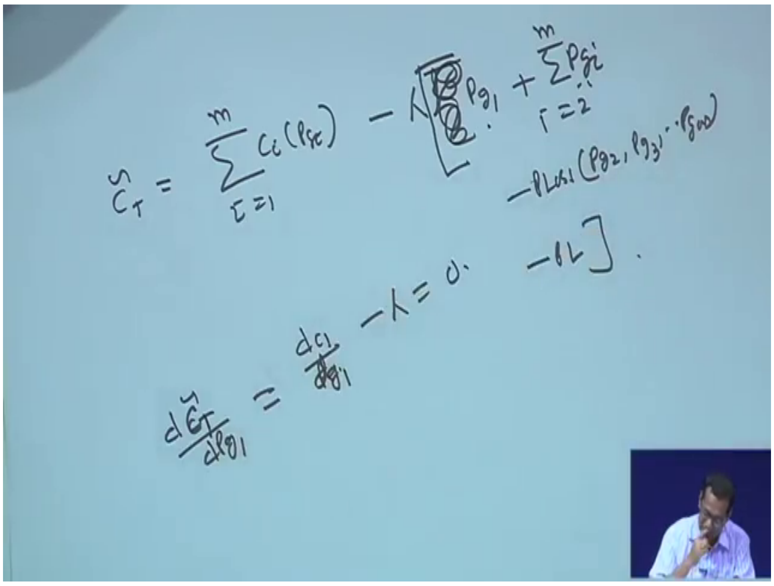

For example I am just writing for your just write you can understand this. C T tilde actually this C T tilde I am writing for you, C T tilde is equal to sigma i is equal to your 1 to m C i P g i then minus your what you call this your lambda this lambda it is i is equal to 1 to m. So, we can write in bracket that take P g 1 out take P g 1 out then this one you can write then i is equal to 2 to m then P g i then minus your this thing what you call minus this P loss function top the all these things. So, minus your P loss it is function of P g 2, P g 3 up to P g m then minus PL. So, this is the thing and this is your bracket close.

So, this is your, now if you take derivative with respect to P g 1 then d C T tilde then dP g 1 for these one will be dC 1 upon dP g 1 sorry dP g 1 it will be there. And then there it will be again minus lambda is equal to you said 0, because you should say this is sorry this sigma is not there here sigma is taken, so only P g 1. So, if you take the derivative with respect to P g 1, the lambda will be there, but this is function of P g 2 and this thing this will be 0, this is also function of P g 2 P g 3 will be 0, and this is a load is constant all this derivative will be 0. So, it will be your minus lambda into your what you call this thing dC 1 upon dP g 1 minus lambda is equal to 0 that means, d C 1 upon dP g 1 is equal to lambda.

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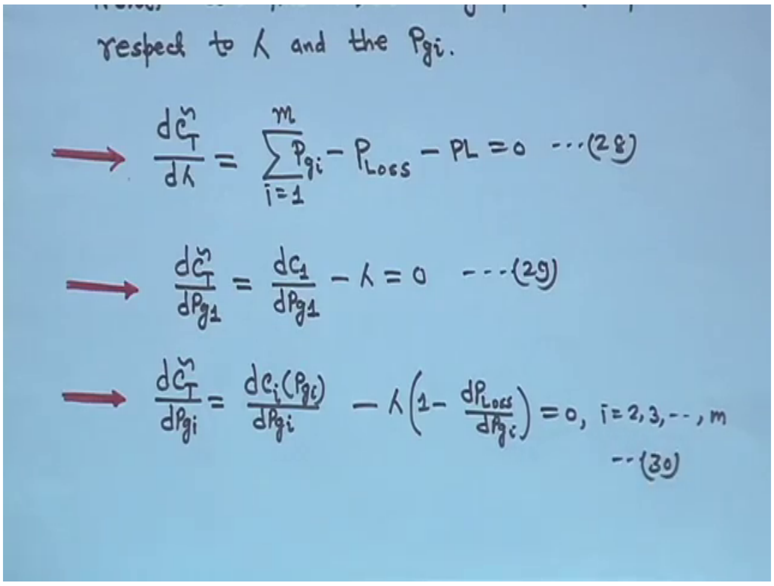

So, that is why this equation, when you take the derivative with respect to P g 1 dct tilde upon dP g 1 is equal to dC 1 dP g 1 minus lambda is equal to 0 this is equation 29. Now, another thing is that; that means, d CT tilde dP g i is equal to d C i upon d P g i minus lambda when i is equal to 2, 3 m that means, just now I your this thing just hold on. Just now, I explain you have know for dC 1 d C T tilde dP g 1 is dC 1 dP g 1 minus lambda. Now, next actually when you take derivative d C T tilde upon d P g i for i is equal to 2, 3 m, but here i is equal to 1 already derivative is taken here when i is equal to 2, 3, m at that time you take.

So, at that time dc tilde upon d P g or d C T tilde upon d P g i is equal to d C i upon d P g i that is fine then minus lambda your what you call minus lambda in bracket. It will be your one minus delta P loss upon delta P g i that is equal to your this thing is equal to 0, when i is equal to 2, 3, m. I mean if you take that derivative with respect to P g i that ith that means, this one you are this one you are taking i is equal to 2 to m for ith one, so it will be your this thing will be just your what you call this thing will be d C i upon d P g i this one will be d C i upon d P g i and minus lambda you are taking derivative with respect to ith one P g i pg that is P g i. That means, here it will be only one will be there that is why this one is here minus this loss is a function of P g 2, P g 3 up to P g m. So, it will be dP loss upon d P g i that is why minus lambda in bracket 1 minus dP loss by d P g i.

That means whenever taking for i is equal to 2 to m. So, as your under graduate student mostly I understand that you have understood this. You take that derivate of this you take the derivative of this equation with respect to ith one ith P g i when i is equal to 2 to m that is why it because of that we are taking with respect to P g i that is why only ith term. So, only one will be there, so minus lambda into one and this is function of P g 2, P g 3. So, dP loss d P g i. So, dP loss d P g i that is for i is equal to 2, 3, m hope you have understood this one nothing no difficulty actual. So, this is that, so 28, 29, 30 these three equations we got from this derivative.

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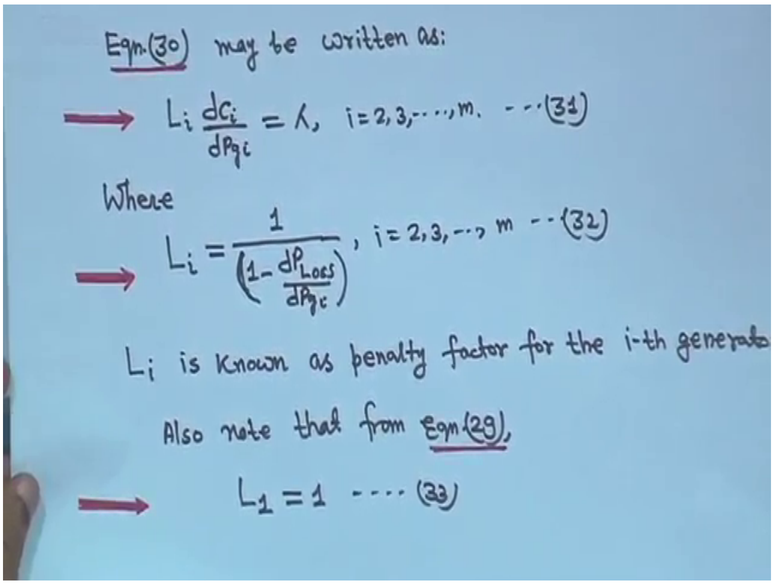

Next is your 30 may be written as this equation 30. I mean these equation 30 we are writing in this form look equation 30 may be written as that L i d C i d P g i is equal to lambda. What we are doing actually that your we defined that L i into d C i d P g i is equal to lambda, where L i is equal to 1 upon 1 minus dP less your upon d P g i. That means, these equation for your understanding rewriting these equation your these equation we can write.

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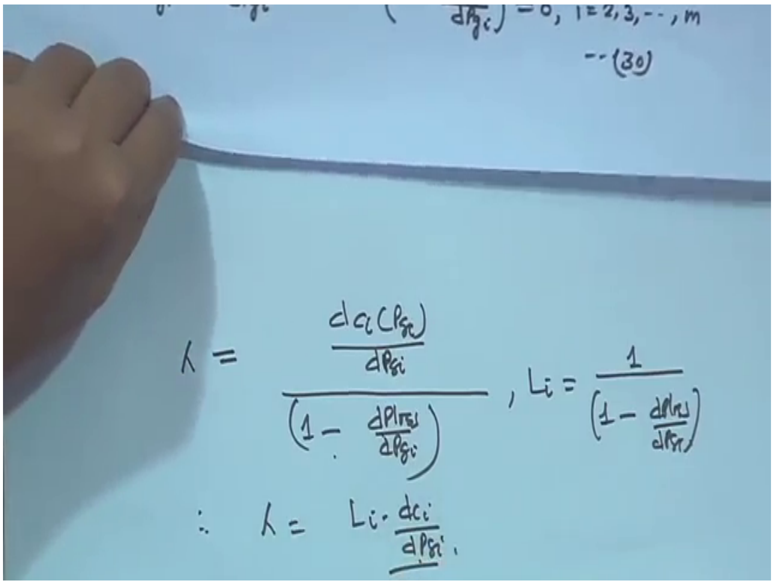

These equation we can write that your lambda is equal to d C i P g i then d P g i divided by 1 minus dP loss by d P g i these one we can write. And we are defining that L i is equal to 1 upon 1 minus dP loss upon d P g i this one we are defining that means, lambda equal to L i into d C i upon your what you call d P g i d C i d P g i this is lambda. So, that is why that I means these equation only that is why you are writing these equation you are these equation as your L i d C i is equal to d P g i that is your what you call is equal to lambda for i is equal to 2, 3 m. That means L i is equal to this is given one minus d P g that is i is equal to 2, 3. Now, Li is known as penalty factor for the ith generator and all the from these equation if you take L i d C i P g i is equal to lambda.

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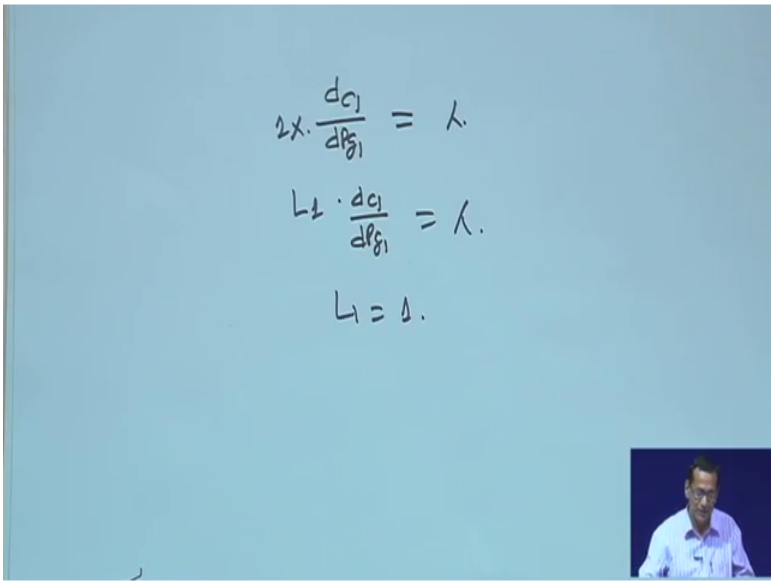

That means, from equation 29 that is equation 29 that is your these equation that dC 1 upon dC 1 dP g 1 is equal to lambda. If you multiply this is actually one into dC 1 dP g 1 that is your L 1 into dC 1 dP g 1 is equal to lambda; that means, your L 1 is equal to 1 so but L 2, L 3 are not one because they are related with this one. So, but L 1 is equal to 1 that means, that is why I have written here that your L 1 L i is known as penalty factor for the ith generator also note that from equation 29 L 1 is equal to one. So that means, in general, so this is your if you include here also L 1, L 1 it is given i is equal to 2, 3 m, but in this equation if you include L 1 also that is L 1 is equal to 1.

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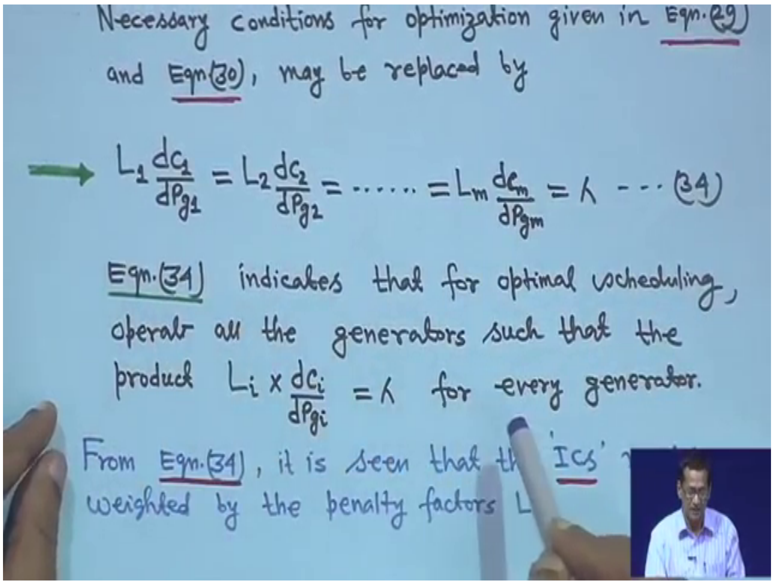

Then this necessary condition for optimization given in 29 and the equation 30, it is L 1 dC 1 upon dP g 1 is equal to L 2 dC 2 dP g 2 up to L m d C m d P g m that is equal to lambda this is equation 34, but L 1 is equal to 1. So, equation 34 actually it indicates that for optimal scheduling that operate all the generators such that the product L i into d C i d must be lambda for every generator. When loss term was not there, this L 1 loss term was not there L 1, L 2, L m it was not there because at that time loss was not consider. But as soon as you cause consider the loss L 1 is 1 it is ok, but L 2 to lm you have to calculate; that means, you have to know that will from the loss formula and the derivative that you have to compute L 2 up to L m.

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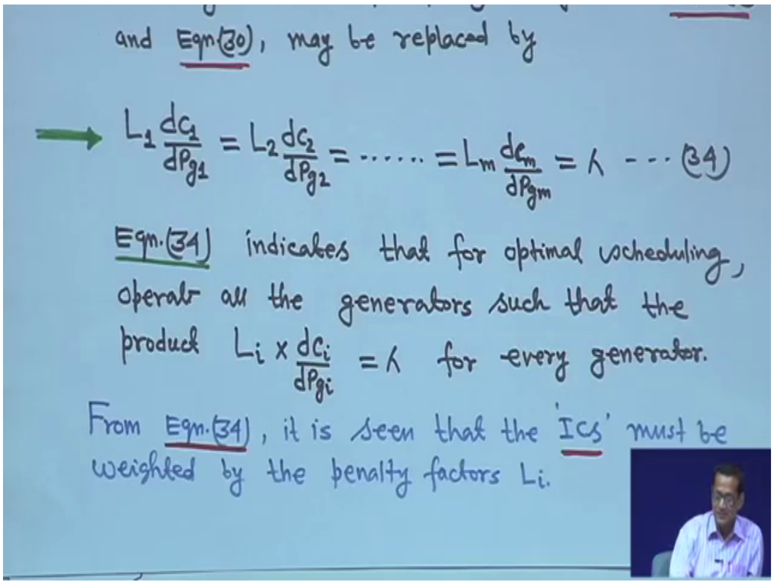

So, that means, what this dC 1 dP g 1 dC 2 dP g 2 all these thing they are ICs that is incremental cost and their weighted by their penalty term penalty factor we call penalty factor L i that is L 2, L 2 up to L m. So, this is the condition. So, what we will do we will take one I mean every theories whatever we will do you will see that one example is taken.

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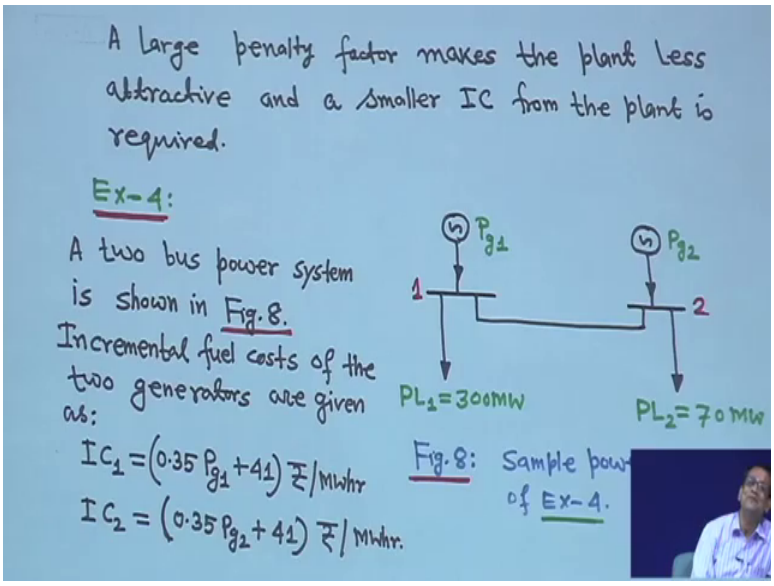

Actually, when you teach in the class we exchanges so many conversation with the student. So, here you are not in front of me, so I cannot go for conversation . So, anyway.

So, but anyway this hope you are understanding all these. So, in large penalty factors makes the plant less attractive that is true and a smaller incremental cost from the plant is required. So, naturally then your fuel cost will be minimum so that is why, but in large penalty factors makes the plant less attractive.

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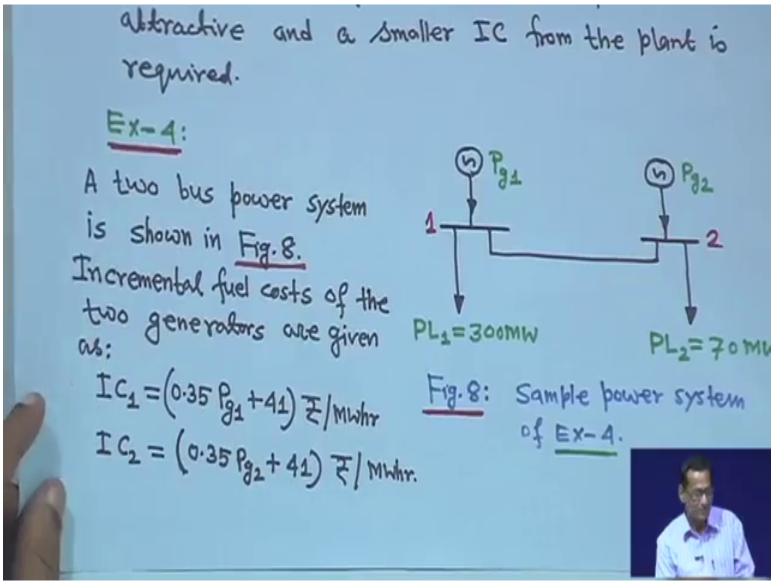

So, now say an example 4, this example is taken. I have few examples such that you can understand there are two bus two bus problem bus one bus two this is generator P g 1 P g 2 our objective is to real power generation we are not considering the active power anything, this is bus one. PL one is equal to 300 megawatt and PL 2 is equal to 70 megawatt, so sample power system, so this much. And incremental cost of these two generating units it is given IC 1 is given 0.35 P g 1 plus 41 rupees per megawatt hour and IC 2 also same 0.35 P g 2 plus 41 rupees per mega 2 ICs IC 1 IC 2 there are same identical.

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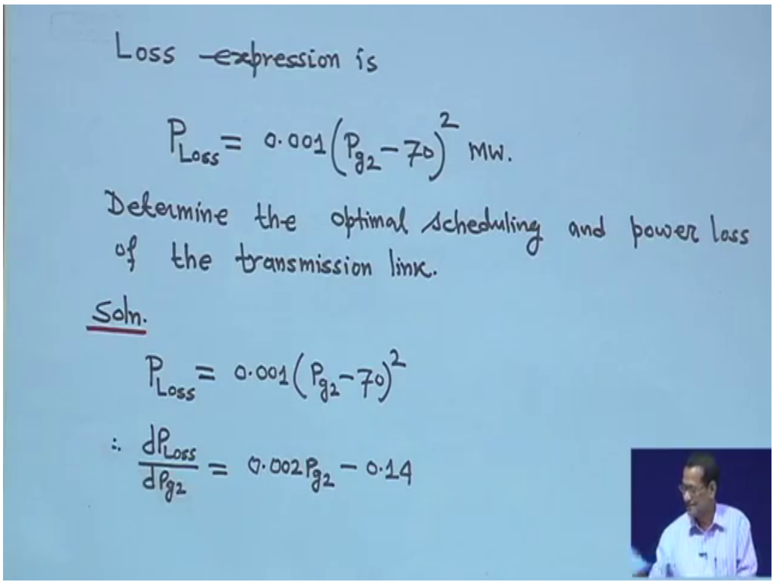

So, this is given and at another thing is a loss expression is given. That a loss expression power loss expression is given that is your P loss is equal to 0.001 P g 2 minus seventy whole square megawatt this expression is given. So, you have to determine the optimal scheduling and power loss of the transmission link, transmission link means that this line this line lost transmission link. So, question is that that when we will this problem it is non-linear equations will come directly cannot be solved. So, for this example I will take some trial and error value whatever I have got, and I will give you the how to solve it. Later stage when we will take the different type of problem this problem iteratively I will not give you the solution, I will give you the solution for other problem, but I will give you final equation how to do it, but this will be an exercise for you and that do iteratively. So, I will come to that.

Thank you.