Power System Analysis
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Lecture - 41
Optimal system operation (Contd.)

So, next come to that your solution of that problem right. So, P loss is equal to this is given 0.001 into P g 2 minus 70 square say whole square.

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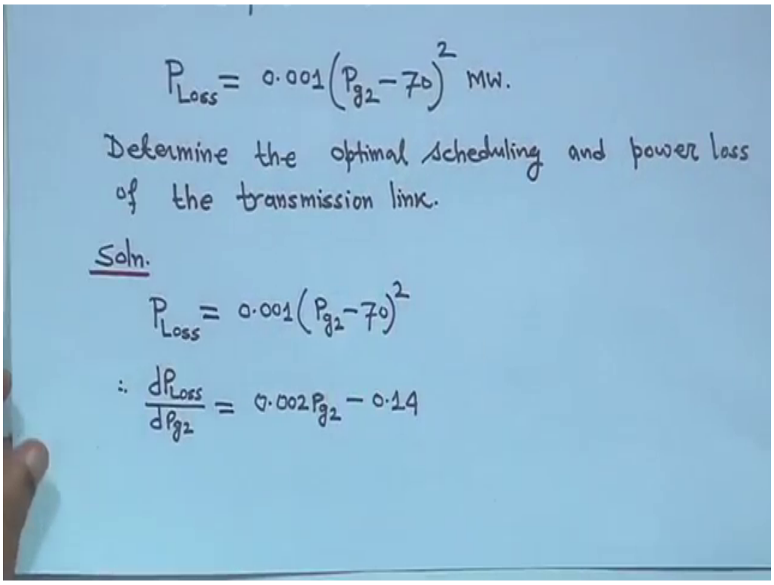

So, take the derivative you respect to p or what you call that P g 2, right. D P loss by d P g 2 is equal to 0.002 P g 2 minus 0.14, right. You take the derivative. Now the L 1 is given, right. L 1 is given that is 1 that is known not given actually L 1 is 1 it is known always right.

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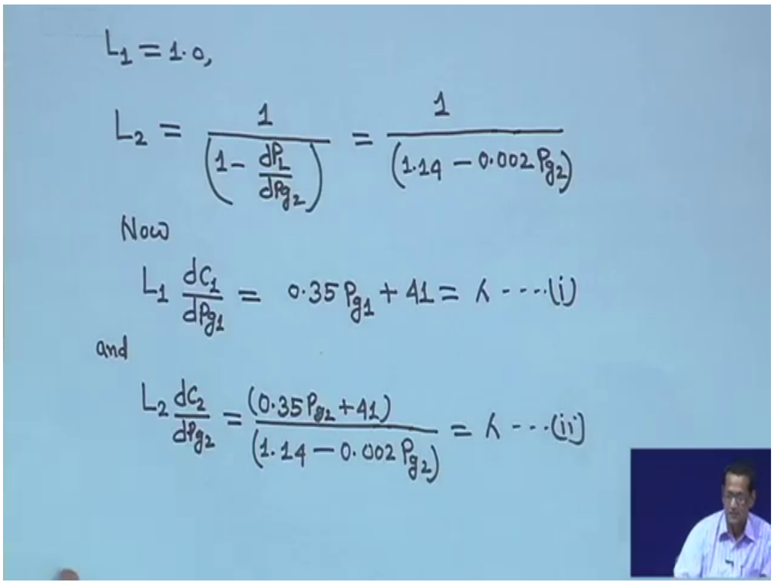

Now, L 2 is equal to 1, 1 by 1 minus d P L upon d P g 2 because, we have assume that L i is equal to 1 minus your d P L upon d P g d P L means P loss, this there should not be any confusion that it is loss actually, right. Because loss is a function of P g 2 P g 3 P g, I mean of course, it is only P g 2. So, this one so, it is 1 then, d P loss upon this expression, which d P loss by d P g 2 this expression right.

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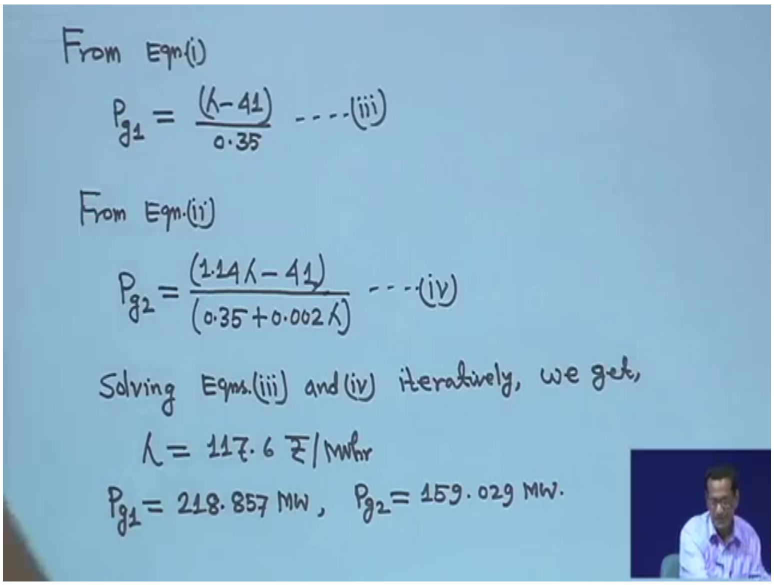

This expression you substitute here, you substitute or you will get 1 upon after that then 1 upon 1.14 minus 0.002 P g 2 right.

Now, L 1 into d C 1 d P g 1 is equal to pint 3 5 P g 1 plus 41 is equal to lambda this we know you have seen, just now right. So, anyway L 1 is equal to 1. And L 2 into dc 2 by d P g 2 is equal to 0.35 P g 2 plus 41 this is actually same ICs divided by your this thing, what you call what you call into, this L 2 into L 2 that is 1 upon this one.

So, it is 1.14 minus 0.002 P g 2 is equal to lambda this is equation 1 this is equation 2, right and from equation 1, right. From equation 1 you can you can get from equation 1 you can get P g 1 is equal to, right. P g 1 is equal to you will get lambda minus 41 by 0.35 and from equation 2 you will get P g 2 is equal to 1.14 lambda minus 41 by 0.35 plus 0.002 lambda.

So, P g 1 lambda this equation is linear L 1 was 1 right, but P g 2 is equal to lambda numerical lambda is there this is also lambda here this is a non-linear. So, direct solution you will not get you have to get, solution iteratively right. So, I have I have I have made something, for which lambda is equal to 100, 117.6 rupees per megawatt hour, this is more or less optimal value later we will go for gradient method other thing this particular example you will saw all what your own iteratively right.

Later when I will explain everything right. So, P g 1 you will get 218.857 megawatt and P g 2 you will get 159.029 megawatt right. So, that is iteratively has been solve you shall to start with some initial value of lambda then you solve it, right.

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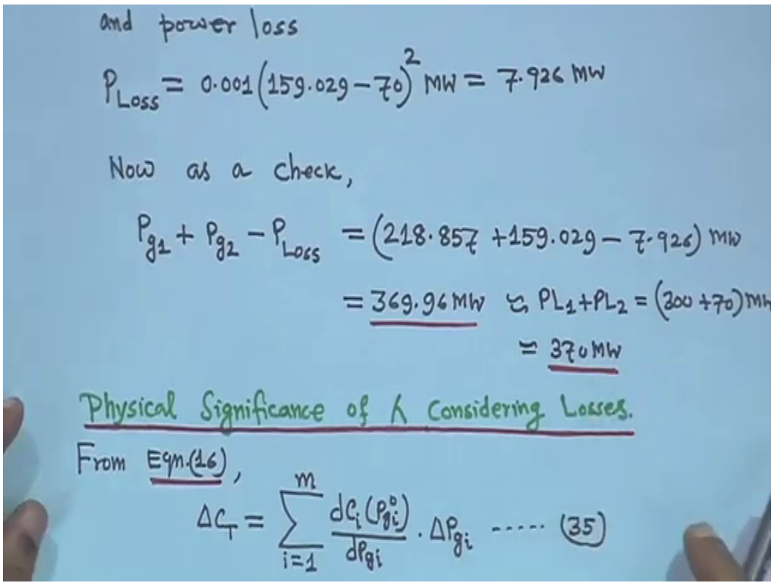

So, next is that power loss, that expression was given that P loss is equal to 0.001 P g 2 minus 70 whole square P g 2 is equal to 159.029 minus 70 substitute to your P g 2 value it is 7.926 megawatt now as I check P g 1 plus P g 2 minus P loss, right. How much it is coming it has to be coming loss. So, it is 218.857 you got plus 159.029 minus 7.926 megawatt.

So, this is coming 369.96 megawatt approximately is equal to PL 1 plus PL 2. PL 1 is 300 megawatt data it has given PL 2 70 megawatt given, so total 3 70 megawatt. So, that lambda is equal to your 117.6 is the correct solution. Now physical significance of lambda considering losses right. So, you will see the same physical significance whatever you have seen without losses right. So, from equation 16 we are seen that delta C T is equal to i is equal to 1 to m, d C i P gi 0 d P gi into delta P gi. So, rewrite in this equation now number in again 35, right.

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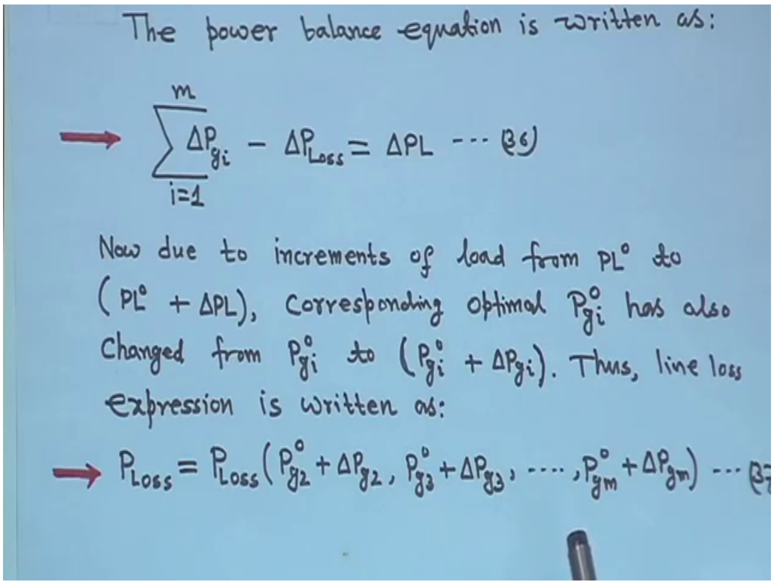

Next is, next is the power balance equation is written as sigma i is equal to and del P gi total generation because of small change upload this has happen change in generation change in loss, minus delta loss is equal to delta PL this is equations 36.

Now, what we have to do is we have to go for that again that 2 term Taylor series expansion. Now due to increments of load from PL 0 to PL 0 plus delta PL correspondingly the optimal PL 0 also has all changed, right. From P gi 0 to P gi plus deltas P gi therefore, lineless expression also because it is a function of P g 2 P g 3 it up to P gm right. So, P loss is equal to P loss P g 2 plus delta P g 2 P g 3 0 plus delta P g 3 up to P gm 0 plus delta P gm that is equation 37 right.

So, because of load change generation and loss also will change, right.

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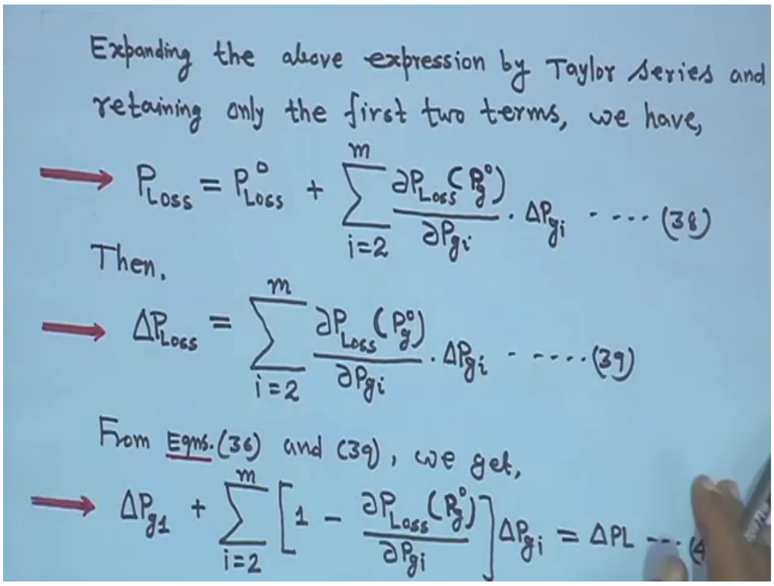

Now this expression that expanding the above expression by Taylor series and retaining only the first 2 terms. So, this one can be written as P loss is equal to P loss 0 plus i is equal to 2 to m del P loss del P gi, P gi P g 0 is function of it is your all the P g P g 2 0 P g 3 0 that is why you are making P g 0 in general into delta P gi, right. Therefore, delta P loss is equal to i is equal to 2 to m del PL a loss P g 0 del P gi into delta this is equation 39, so equation 36 and 39 from this thing we get, right.

This one that here add delta P g 1 subtract delta this thing equation 30 and 39 yes. What you can, what you, what we you can do is that this is your delta P loss is your P loss g delta P loss is this one, right. Now what is your equation 36 and 39 equation sorry, equation 36 is here right. So, then delta P loss you will substitute here, delta P loss you will substitute here right. So, from that you will get there be these equation it is writing 1 to m you can write this equation as I mean you can write it like this, you can, you can write like this your a delta this thing that hold on, hold on.

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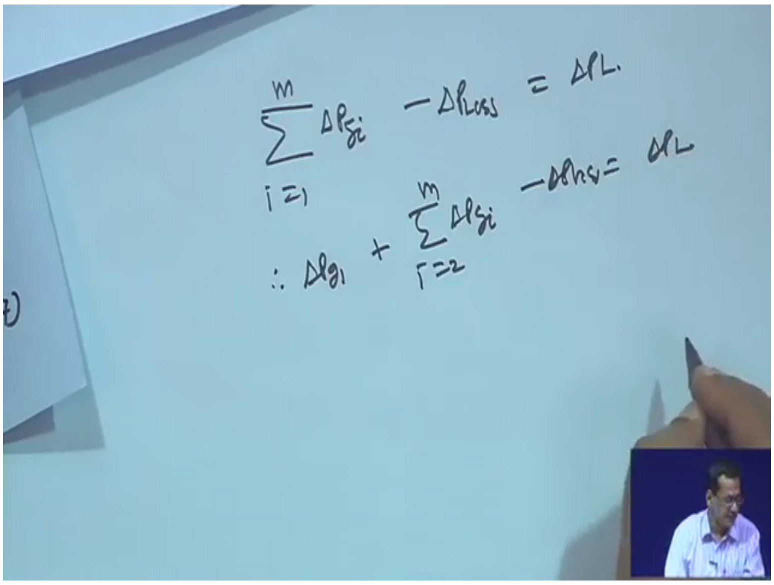

Then this equation i is equal to 1 to m delta P gi minus delta P loss is equal to your delta PL right; that means, these your what you call this equation you can write delta P g 1 plus sigma i is equal to 2, 2 this P g 1 is taken out . So, delta P gi minus delta P g loss is equal to delta PL that is the total load, right. Therefore, this your what you call and your delta P loss this delta P loss is equal to your this thing here it is delta P loss is equal to i is equal to 2 to m del P loss delta P gi into P gi, right. Therefore, this equation you can you can write that equation 36 and 39, right. Then equation 36 is here equation 36 is here right.

So, these equation you can write your this thing, this equation you can write the delta P g 1 plus i is equal to 1 to m 1 minus delta P loss delta P gi this thing, right. Delta p or delta P loss delta P gi then you take delta P gi common because delta P loss is equal to this one, right. ; that means, this equation actually you break it delta P g 1 plus i is equal to 2 to m delta P gi minus delta P loss is equal to delta PL, right. That delta P loss expression you substitute delta P loss expression you substitute and or then you can get this equation delta P g 1 plus i is equal 2 to m 1 minus del P loss del P gi again and again not telling that function of P g 2 and P g 3 and so on, into delta P gi this is delta PL, so need not do this small thing right. So, this one I have shown you this one, I have shown you, right. This there you here you put the delta P loss expression, right. So that means, your, that means, this is your this equation is mottos 40 this is equation 40, right.

So, this one actually that, this one actually you know all that L i is equal to 1 minus 1 upon 1 minus del P loss upon del P gi. So, it is 1 minus means it will be L i inverse that is the 1 upon L i this is actually 1 upon L i that means.

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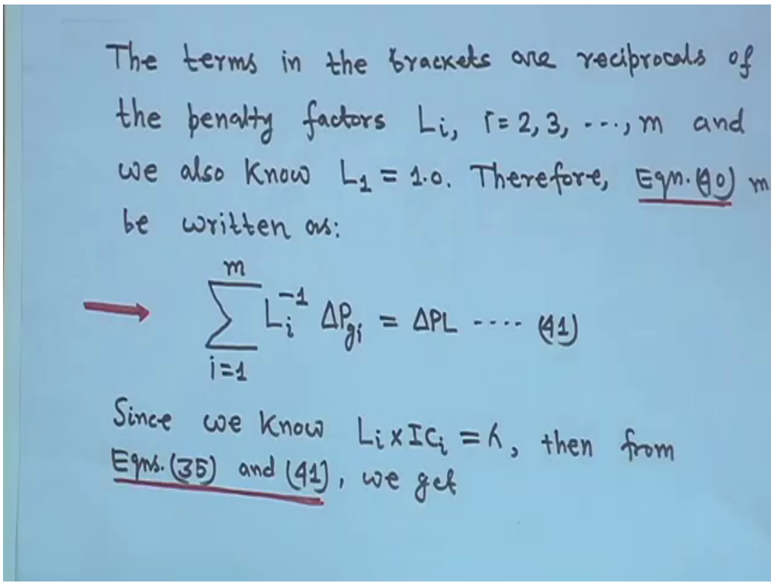

That means this equation the your i is equal L 1 is 1 you know that right. So, the terms in the bracket are reciprocal of the penalty factors L i because we have taken L i is equal to 1 upon 1 minus this one, so the reciprocal 1 minus delta P loss upon delta P gi reciprocal of L i, right.

And we also know that L 1 is equal to 1 therefore, equation 40 may be written as because here it is L 1 is equal to 1; that means, here it is it is a it is something like this I am writing above just one or I am writing one or 2 line for you . So, as that you will understand right. So, this equation that equation 40, 40 where we have got this equation 40, this also can be written as if for example, L 1 is equal to 1 then this equation also can be written as your L 1,

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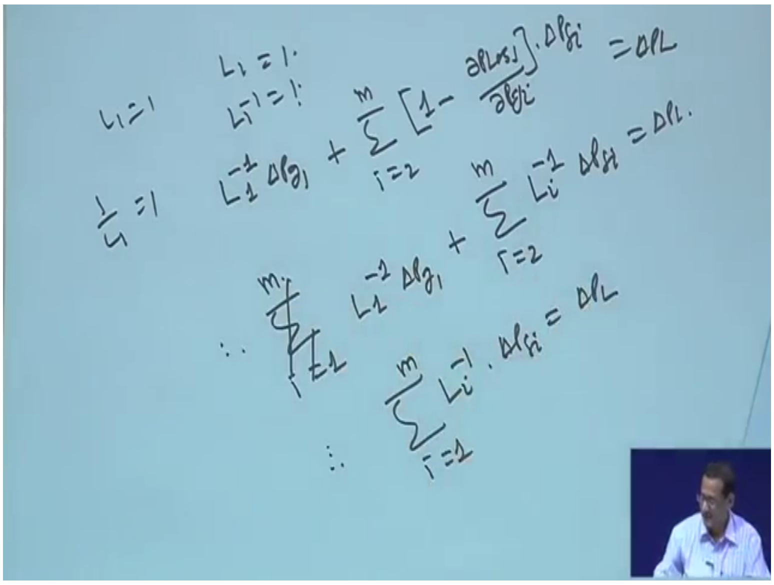

Then delta P g 1 because, L 1 is 1 anyway we are putting like this plus i is equal to 2 to m in bracket 1 minus delta P loss it is function of P g understandable, it is function of P g understandable delta P loss divided by this one, right. Bracket close delta P gi is equal to delta PL right.

That means this equation is also it can be written as like this that your sigma i is equal to say one to m right, so then in bracket you can, right. This one also you can add this is actually L 1; that means, L 1 inverse also one that is reciprocal of L 1 is 1 right. So, this actually this equation 1 minus this one first [meek/make] make it like this, right. This is you write L 1 inverse or the L 1 is 1 reciprocal of 1 in this inverse right; that means, L 1 inverse delta P g 1 plus this one you can write i is equal to 2 to m L i inverse delta P gi is equal to delta PL. In general you can write this one that i is equal to 1 to m, right. L i inverse delta P gi is equal to delta PL, right. L 1 is 1; that means, L 1 is 1. So, 1 by L 1 is also 1 right.

So, there is write L 1 inverse and this is, this is your this is your L i inverse. So that means, in general that this equation this equation if you make it together then it will become your this thing. L i inverse delta P gi, but note that L 1 is L 1 is equal to 1 right. So, since we know that L i into IC i is equal to lambda because this we are seen earlier, right. Then from equation 35 and 41 you will get this one right. So, this we know L i IC i is equal to lambda L i IC i is equal to actually lambda.

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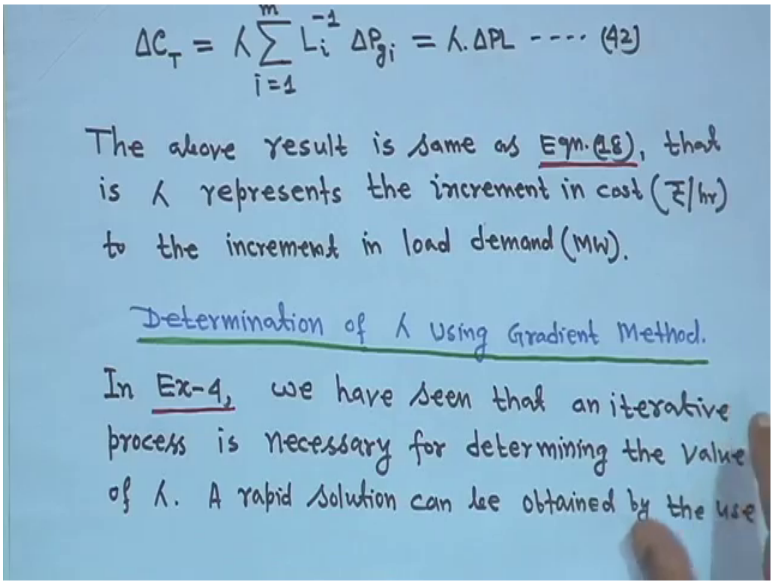

Then from 35 and 41 just you will get it the delta C T will be actually is equal to delta C T will be lambda i is equal to 1 to m L i inverse delta P gi is equal to lambda into delta PL. This is very simple actually go 35 and 36 I just port it in you will get it there right; that means, what does it mean ultimately the whole thing actually is become in delta C T is equal to C T is equal to lambda into delta PL same as before, that mean the above result is the same as equation 18, right. And that is lambda represent the increment in cost rupees per hour to the increment in the load demand this thing same.

Whatever we without loss or with loss the meaning is same. So, same physical significance, right. So, next one is, So up to this that we have seen the penalty factor that lambda sorry, L i that in terms of this thing here what you call that, derivative of the power loss with respect to generation except your general one, right. And L 1 is always 1 and you are otherwise same philosophy L i I L i IC i must be is equal to equal lambda that is equal lambda basis, right. And physical significance of change in load also same as [with/without] without considering the losses, right. Only thing is that the derivation actually whatever little bit mathematics is very, very simple just little bit practice you do and you will find things are very simple and problems are also not the difficult as per as class room exercise is concerned, right. Next we have to go for some proceed here, right. That how to determine the lambda we will go for with using gradient method, right.

So, for example, in example 4 you have seen that an iterative process is necessary for determining the value of lambda previous example we have taken I mean saw that P g 2 is a function of lambda, but relationship is non-linear right; that means, you have to follow some iterative process for finding out the lambda right. So, what you do a rapid solution can be obtained by the use of that gradient method, right. Gradient method when losses are considered right.

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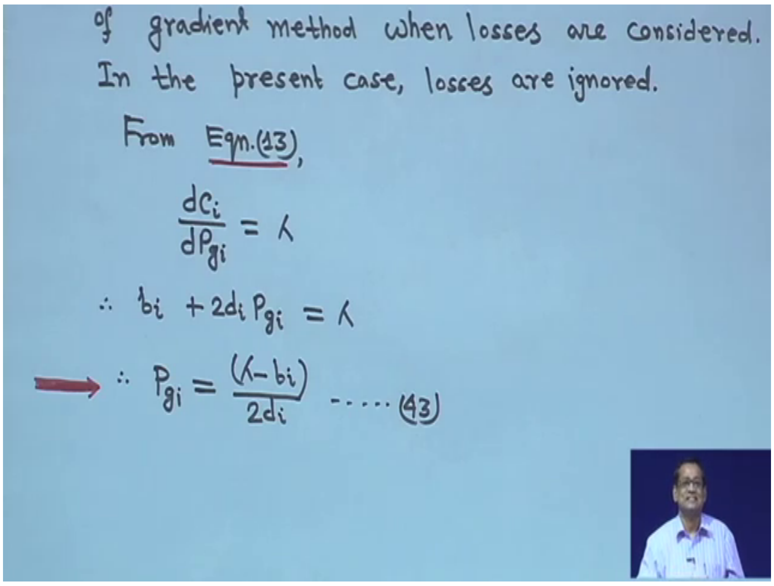

So, what we will do we will do we will solve both the cases. First we will consider that without any loss and that will consider the losses right. So, equation from equation 33, 13 we know that d C i upon dP gi is equal to lambda right. So, this is this is this we have seen from equation 13 right. So, your; that means, that C i actually is equal to a i plus b i P gi plus dP gi square . So, when you take this b i plus 2 d i P gi is equal to lambda right; that means, P gi is equal to lambda minus b i upon 2 d i this is equation 43 now we are trying to find out lambda using gradient method, right.

So, what one can do is, just look things are very interesting and very simple right. So, actually we will loss is not considered and loss is not considered just hold down, and just hold down, when loss is not considered that total generation is equal to total load this is from equation 10, this is from equation 10 right.

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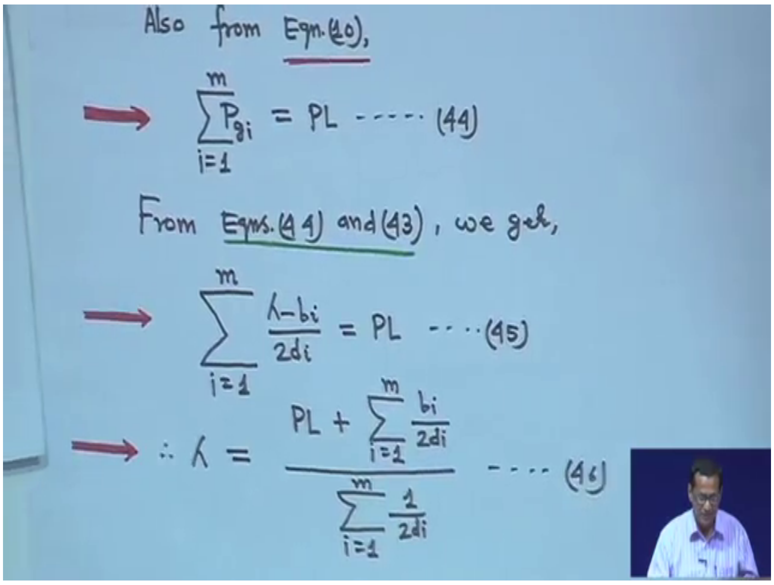

That is the thing; that means, from equation 44 and 44, right. That you can write that is P gi is equal to lambda minus b i upon 2 d i.

So, what you can do is that you take you, your these thing both side here what you call that sigma P gi is equal to your to your PL right.

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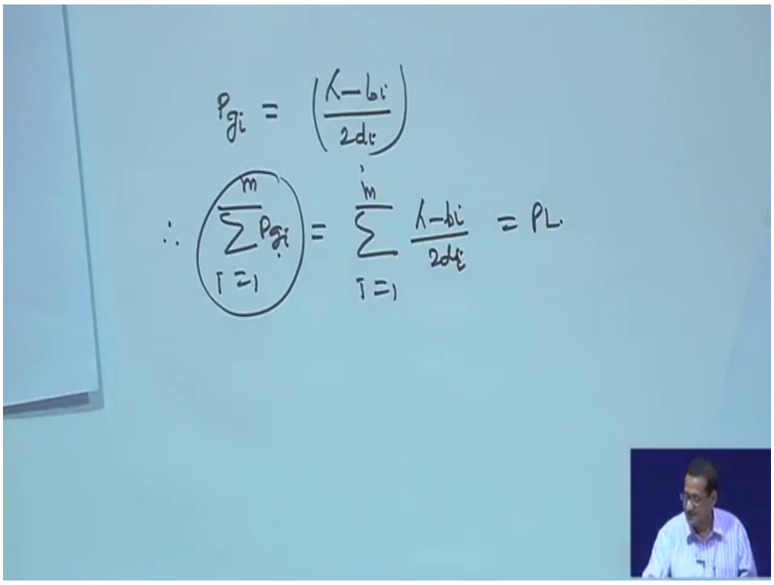

That means if you take this equation, I am rewriting for you if the P gi P gi is equal to lambda minus b i upon 2 d i, this is the equation. You take sigma on both side, right. Summation that i is equal to 1 to m P gi is equal to sigma i is equal to 1 to m lambda minus b i divided by 2 d i, right. If you take the your summation both the side, right. And then total generation that is equal to total load because this is the total generation that is equal to total load; that means, total load is equal to actually i is equal to 1 to m lambda minus b i upon 2 d i the i is the suffices, right. i is the suffices .

That means these equation, that that is why writing from equation from equation 44 and 43 that i is equal to 1 to m lambda minus b i upon d i is equal to total load right; that means, this equation you take summations both side and that is equal to summation of generation load that is why these equation this equation your writing here this is equal to PL that is equation 45, right. From this equation you write lambda is equal to this expression because this one, this one this equation I am writing for you right. So, this equation that sigma i this equation only you can this, this equation only we can write like this that i equal to 1 to m right

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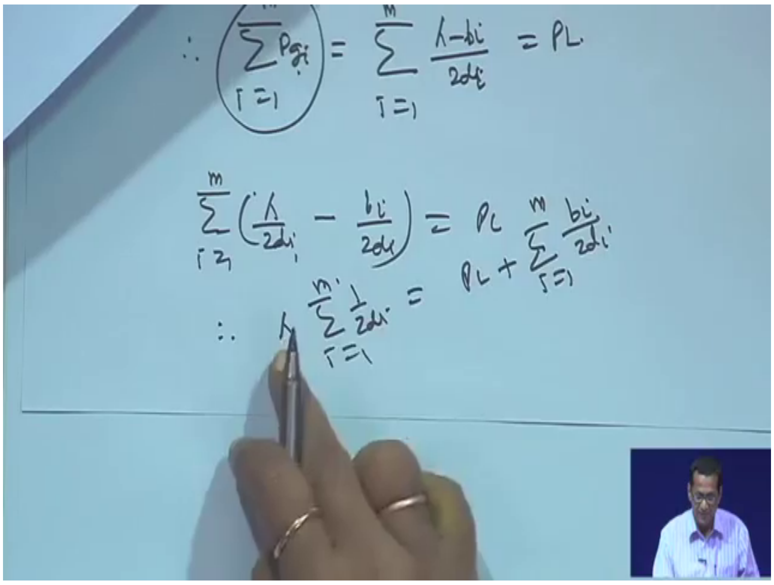

This is your lambda by 2 d i minus b i by 2 d i is equal to PL, right. This is the thing therefore, this equation this your what you call this one, you can write lambda i is equal to 1 to m, right. 1 upon 2 d i is equal to PL plus sigma i is equal to 1 to m b i upon 2 d i.

So, lambda is equal to divided by sigma i is equal to 1 to m to your to your this thing 2 d i. That is why this equation is writing lambda is equal to PL plus i is equal to 1 to m b i upon 2 d i divided by i is equal to 1 to m 1 upon 2 d i, this is equation 46, right. Now this one now this part you have to understand now this is actually lambda is a PL, lambda is equal to this is your PL plus all these constant I mean lambda is a function of PL right.

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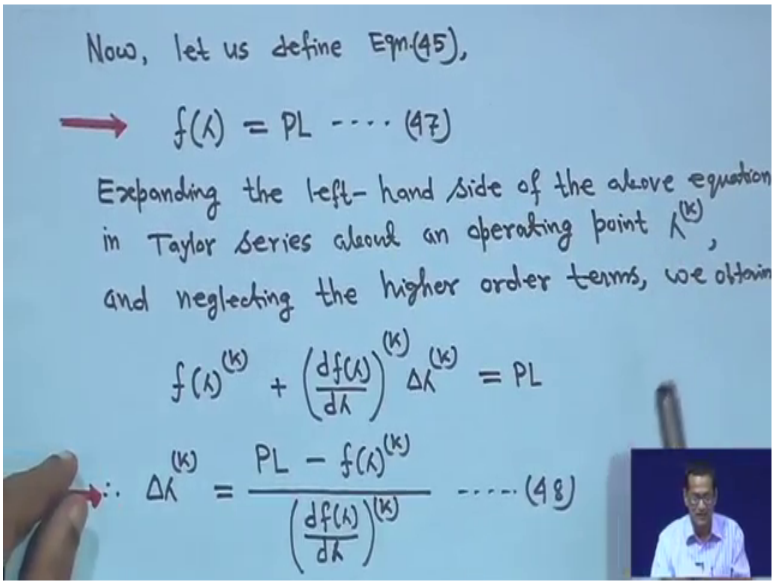

Now, in this, that means, this equation let us define equation 45 you define this equation 45, right. This one you define this equation and apply your because this PL is a function of lambda, right. Or lambda is a function of PL.

So, lambda is a function of lambda you define f lambda is equal to PL, but not only that this one also you have to see that your PL is equal to this is your PL, right. PL is equal to i is equal to 1 to m lambda minus b i divided by 2 d i right.

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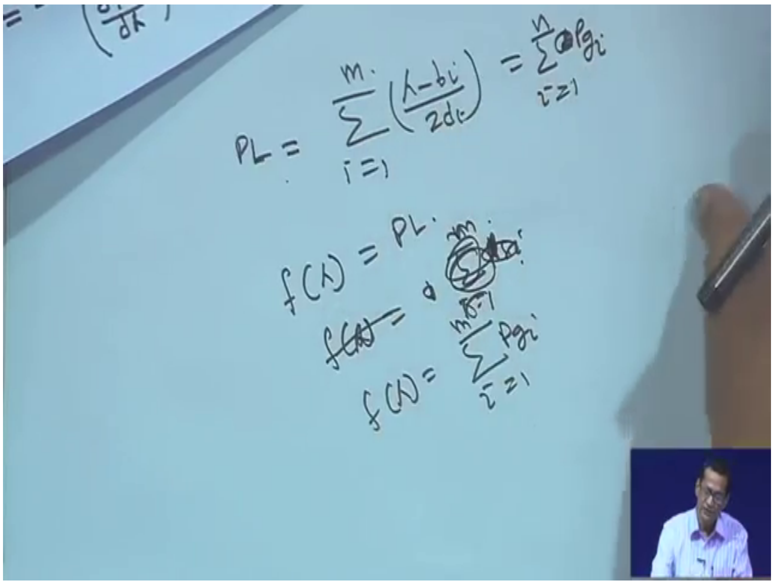

This is the thing and this is actually nothing but your total sorry, total i is equal to 1 to m. PL is equal to total generation; that means, if we take that it is PL is a function of lambda; that means, we are writing f lambda is equal to PL then you see also can write because this summation of generation also is a function of lambda right; that means, we can also write f lambda is equal to your, what you call? Your this thing that sigma i one to m that is also function of your this thing generation, right. Because generation is also function of lambda PL is also function of lambda.

So, you can write f lambda is equal to PL also you can write f lambda is equal to i is equal to 1 to m it is P gi this one also you can write, right. So, this is the actually p sorry, let me write in clearly. F lambda I equal to 1 to m P gi, right. Because this is also function of lambda this is also load is equal to generation loss is neglected here right; that means, this is now what we can do is expanding the left hand side of the equation. This equation in Taylor series about it is operating point lambda K say at some iteration K, right. You expand it in the Taylor series and only consider the past 2 terms, right. Some operating point, right and neglect the higher order term.

Then what we will do we will get f lambda, K means K (Refer Time: 22:39) K th some value plus derivative of it df lambda d lambda at K th iteration. So, again and again I am not altering K, right. Understandable into only this time delta lambda is equal to PL this is the see your what you call the Taylors series expansion, for the past 2 terms for any operating point; that means, from this equation delta lambda at K th iteration that PL minus f lambda K upon df lambda by K df lambda d lambda at K th iteration this is equation your 48 ; that means, this my f lambda, right. Now once again your this thing now that means.

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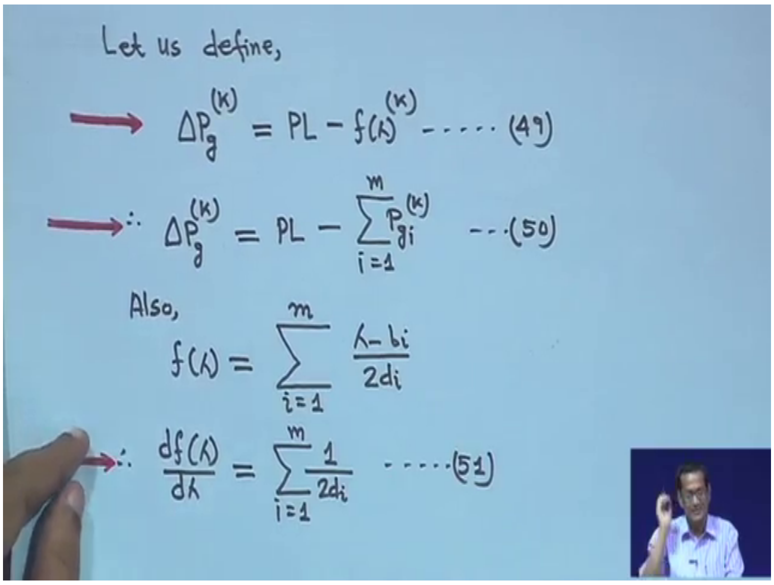

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Delta P g K, right. Just hold on; that means, delta P g K this delta P g K can be written as the total load minus f lambda K, right. Because PL load is fixed idea is this one I have just told you that idea is where I have told that one idea is that f lambda just hold on here, that that through the iterative process that load is not changing PL remains same right, but f lambda that f lambda also can be written as function of generation i is equal to 1 to m. So, P gi is changing in every iteration, but load is fixed, but f lambda P gi is changing in every iteration because this is also the lambda is changing in every iteration right.

So that means, this one you can make it that your what you call that delta P g K is equal to you can write PL minus f lambda K because every iteration you have to find out this f lambda value, right. That means, delta P g K is equal to PL minus i is equal to 1 to m P gi K because f lambda, f lambda, this f lambda I told you this f lambda I told you this one that f lambda is equal to i told you that i is equal to 1 to m P gi therefore, at K th iteration if you put K th iteration then this is also K right.

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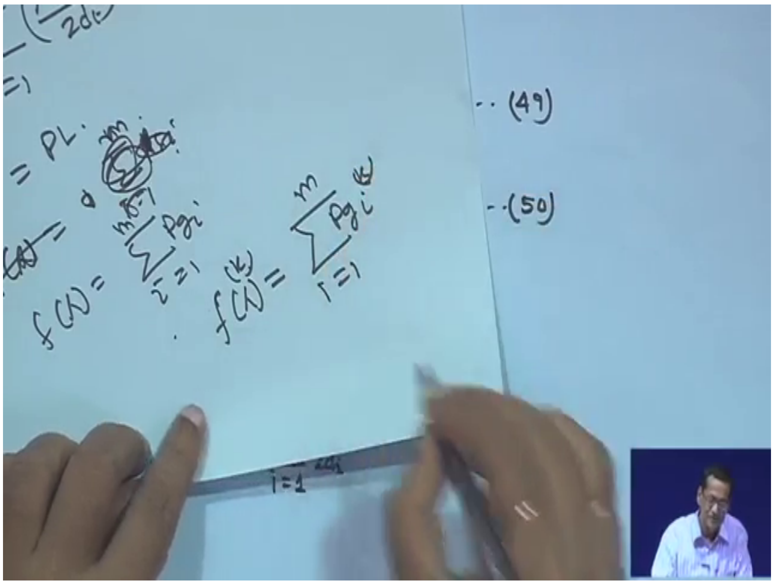

That is why that this one can be written as f lambda K is equal to this one also right.

So that means, your what you call that PL that is a that f lambda is equal to write PL as well as f lambda you write this one. If you do so, then this can be written as that is a mismatch, right. Between load and generate some of the generation actually. So, delta P g K will be PL minus i is equal to 1 to m P g K. So, this is actually equation 50, right. Now also you know f lambda is equal to i is equal to 1 to m lambda minus b i upon 2 d i you have to obtain because, you need the derivation of this one also with respect to lambda because here you need this derivative in equation 48 right.

That means, df lambda, d lambda, d lambda is equal to i is equal to 1 to m 1 upon 2 d i sigma i is equal to 1 to m if we take the derivative with that it will be sigma 1 upon 2 d i this is equation 51; that means, in this expression, in this expression this df lambda d lambda at K th iteration you substitute this is a constant actually for this case it is a constant because 1 upon 2 d i only, right. And this expression and this expression you have to call this delta P g K expression, delta lambda K it can be written as delta P g K because, PL because, PL minus f lambda K in equation 48 PL minus f lambda K is equal to actually delta P g K right.

So, in this equation you are putting in 48 delta P g K that is why it is written here that from equation 48 49 and 51 delta lambda K will be delta P g K divided by i is equal to 1 to m 1 upon to d i .

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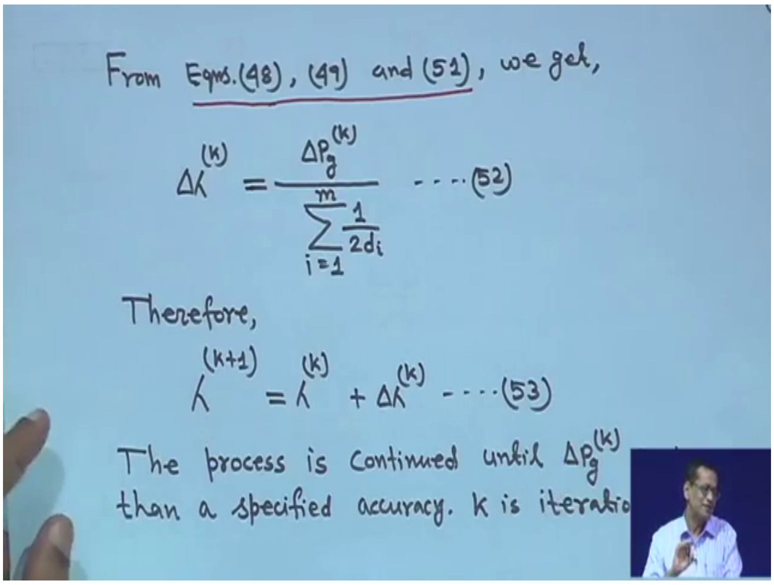

This is 52. So, at any iteration K plus 1 th iteration lambda K plus 1 will be lambda K plus delta lambda K right. So, this is equation 53. The process will be continued unless or until delta P g K is less than it is specified accuracy. K is the iteration count right. So, this f lambda is equal to PL and f lambda is also is equal to sigma of all that your generation P gi, right because both are function of lambda. So, there should not be any confusion for you, right. And this is your what you call numerical. So, we will take later right.

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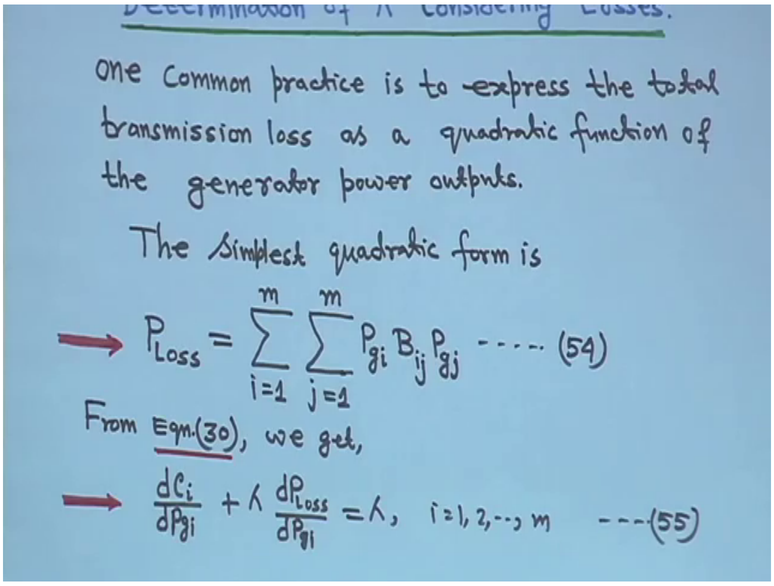

But this is your this concepts should be cleared right. So, this is for without your what you call losses. If we consider the loss right. So, determinants of lambda considering losses. So, what happened actually that this loss formula before say anything whatever you see it here i is equal to 1 to m j is equal to 1 to m P gi B ij P gj B j B ggi sorry, P gj right. This will be derived at the end. Right now I do not want then this whole continuality will be lost actually. And you will be also thinking that after that suddenly this is coming. So, this formula for the timing will assume that this is a your what you call this is the given, but I will derive it for you it will take some time, but I will derive at the end of after making all these thing that loss formula, right. And how then all the B coefficient also how one we will get it? Now one common fact is actually to express the total transmission loss as a quadratic function of the generator power outputs.

The simplest quadratic form loss is P loss is equal to i is equal to double sigma i is equal to 1 to m then sigma another j is equal to 1 to m P gi B ij P gj this is equation actually 54, right. From equation 30 actually we get C i upon dP gi plus lambda d P loss dP gi is equal to lambda that is i is equal to 1 to m. This is equal this is actually viewed equation viewed 55. We go back to equation 30 this equation actually written there, right. It was written actually

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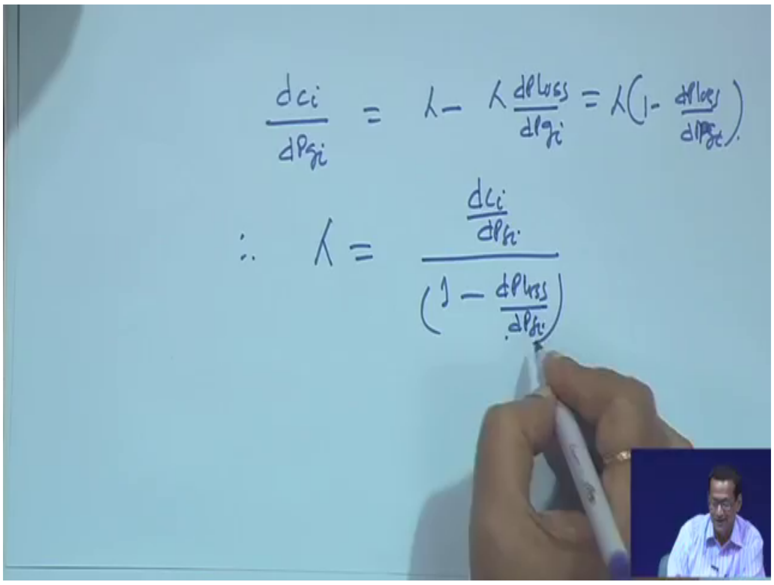

That your lambda if you take the your what you call, it is a lambda into your this way it is written know that your basically you know this one it is written know, d C i I am writing I am not going to equation 30 I have just writing here. This one is equal to lambda minus lambda.

Student: (Refer Time: 30:12).

D P loss then dP gi that is lambda common 1 minus d P loss then d P gi right; that means, lambda is equal to d C i dP gi divided by you 1 minus d P loss then dP gi this one you have made know and this is your we have made a lie.

Student: (Refer Time: 30:25).

Right so, from that equation 30 only we are getting this formula d C i dP gi plus lambda is equal to this one.

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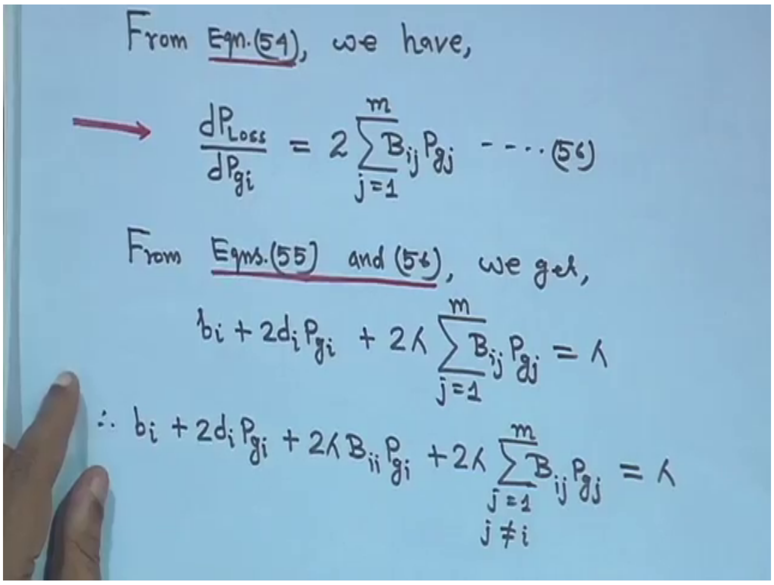

This is equation your 55 right. So, and this equation if you take the derivative of this equation with respect to P gi it will be, it will be your what you call 2 into your j is equal to 1 to m B ij P gj this is equation 56.

Thank you, I am coming again.