Power System Analysis
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 42
Optimal system operation (Contd.)

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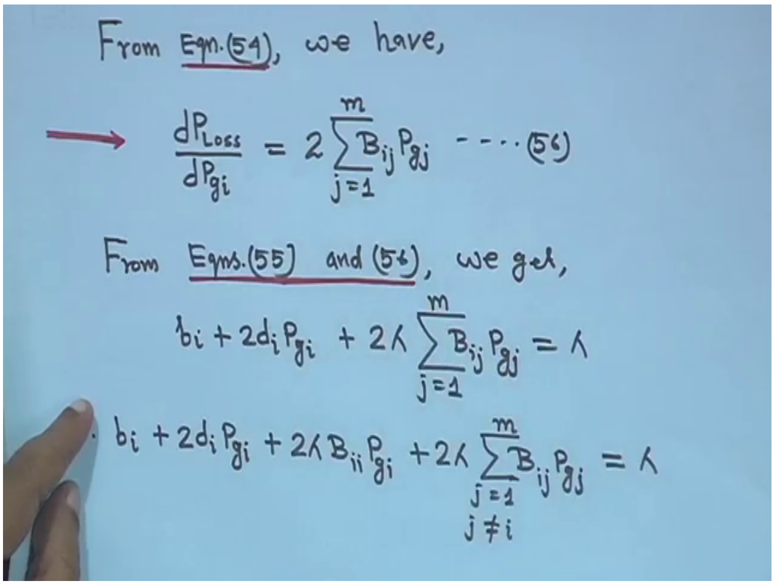

Then this one if you take that is a 2 (Refer Time: 00:25) that 2 is coming it is a quadratic actually.

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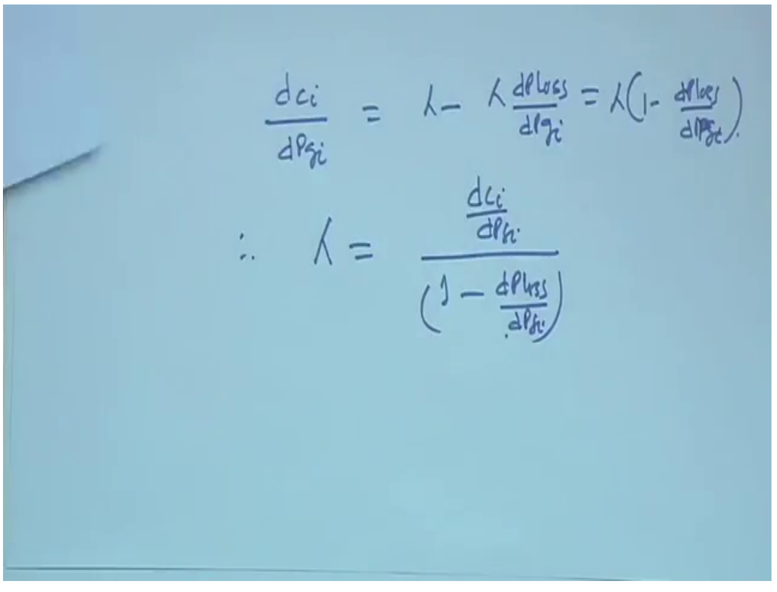

I will give you a small example for this equation take for example.

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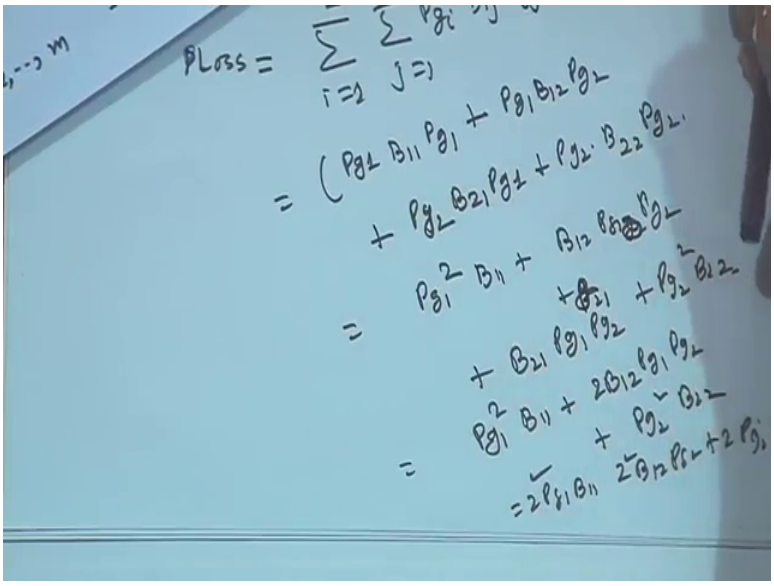

M is equal to 2; that means, my P loss just hold on my P loss is equal to take m is equal to 2, i is equal to 1 to 2, sigma j is equal to 1 to 2 then Pgi then Bij then Pgj right.

So, when i is equal to 1 it is a 2 form actually, 2 sigma means 2 form when i is equal to one if j vary from 1 to 2; that means, what will happen this term right when i is equal to 1 that is your Pg 1, i is equal to 1 right then j is vary 1 to 2, that is B 11 then Pg 1 right plus when your now your i is equal to 1 then your Pg 1, then your B 1 2, then your Pg 2 right plus when i is equal to 2, j vary 1 to 2; that means, Pg 2 right then B i is equal to 2 means B 2 1, j is 1 then your Pg 1 then when i is equal to 2 Pg 2, then your B 2 2 i is equal to 2 j is equal to 2 then jpg two; that means, this is Pg 1, Pg 1 it will be Pg 1 square right B 11 plus B 1 2 Pg 1 your Pg 2, Pg 1 Pg 2 right here also your B 2 1 sorry B 2 1 Pg 1 Pg 2 plus Pg 2 square B 2 2 right; actually B 1 2 actually is equal to B 2 1 right in general right

So, if you take derivative for example, this one I can write then Pg 1 square B 11 and B 1. Now if you take derivative with respect to Pg 1 say, then it will 2 term will come it will be 2 Pg 1 it will be 2 Pg 1 B 11 here also it will be 2 B 1 2, Pg 2 here also it will be 2 Pg 2, B 2 2. So, 2 2 2 2 term will be there it is a quad actually it is a quadratic one right if you expand this and if you take the derivative then you will get this one right. That is why this when you take that derivative of this term in general this is a general expression, at that time there is no 2 sigma because of that derivative only one sigma right

So, it will be 2 will be common and j is equal to 1 to m, it will be Bij Pgj right. So, that will be a this is this equation will be say this one sorry this one derivative one if you take with sorry sorry if you take derivative with respect to Pg 1, this one should not be there because this will be 0 this should not be there, it should be its it is it should be 0 sorry I thought it is Pg 2.

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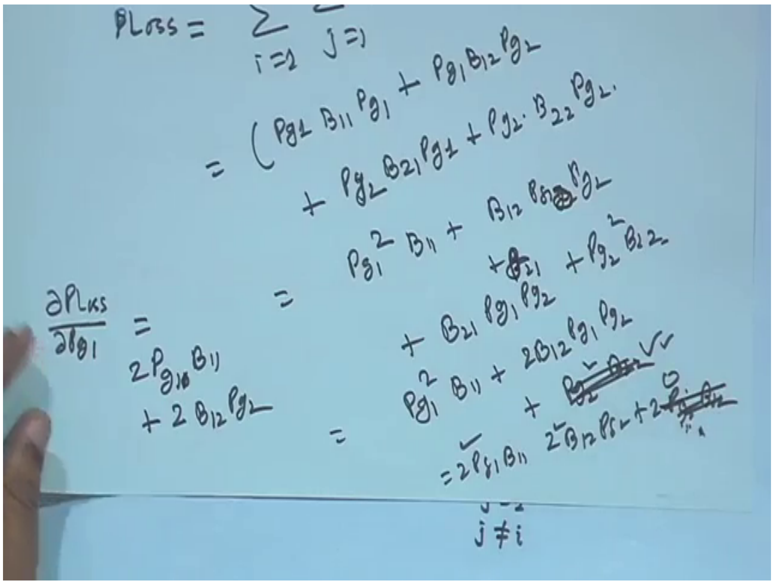

So, it will be 2 Pg 1 B 11 I am just writing, say it in this case a del P loss by del Pg 1 is equal to 2 Pg 1 B 11 plus 2 B 1 2 Pg 2 right.

So, actually it is Pg 2 right. So, its derivative will be 0 with respect to Pg 1. So, it should not be there right. So, that is why that is why this equation has become your a j is equal to 1 to m Bij Pgj. If you want to check you make m is equal to 2, then it will be it will be your j 2 then j is equal to that for a if you say for i is equal to this thing for i th one, if say I is one for example, B 1 2 your Pg 1, B 11, Pg 1 that is whatever you are coming to B 11 Pg 1 is coming and then 2 is there 2 B 1 2 Pg 2 right.

So, this is actually your what you call this 2 term comes right. So, equation actually whenever we are making all sort of things right if you are in front of me then little bit of minor writing error you can correct it to me right on the black board, but here nobody is hear right.

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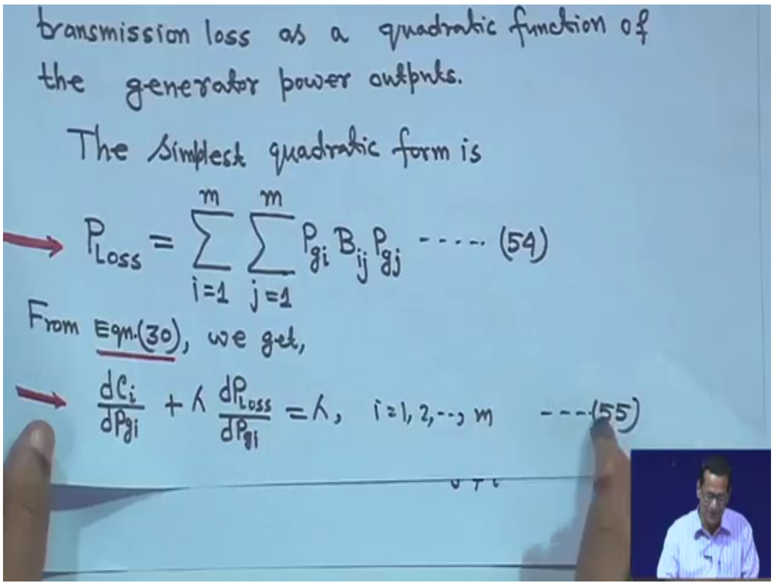

So, so this one; that means, this is your dP loss upon dPgi. Now equation 55 and 56 right. So, this is your equation 55, this is your equation 55 and 56.

So, what you do is this, you substitute this value for this one here you substitute dP loss upon dPgi you substitute right if you do so, this equation will become B i 2 di Pgi because dCi upon dPgi is B i 2 d plus 2 di Pgi this one right then this one this term is substituted here after that i th term is taken out. So, 2 lambda Bii Pgi from this summation i th term is taken out and that is why 2 it is written then 2 lambda j is equal to one time, j not is equal to I Bij Pgj that is equal to lambda that is from this equation from this summation i th term is taken out right.

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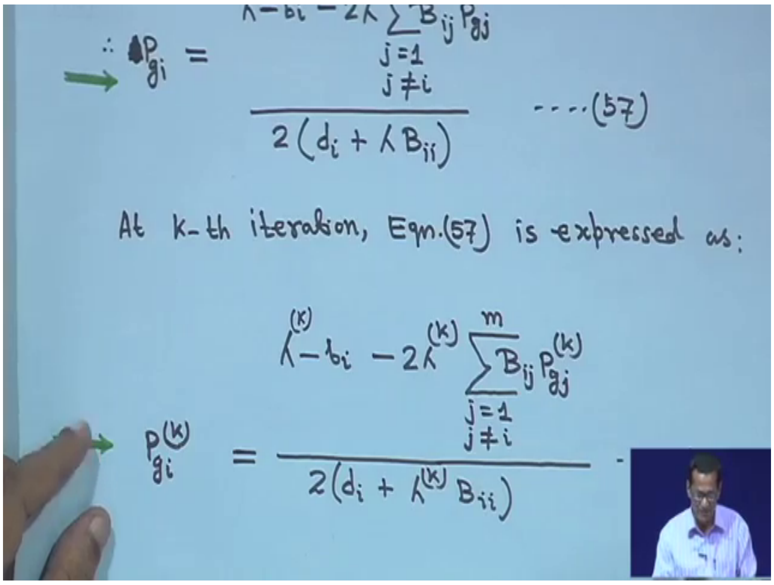

Then from this equation you can write lambda; from this equation you can write Pgi is equal to lambda minus B i minus 2 lambda this whole term divided by 2 into di plus lambda Bii. This is equation 57 right. now your k th iteration this equation 57 is expressed, as just put Pgik just put Pgik here this bd all are constant right that is equation 58 this is Pgik if you take the summation of this Pgik that will be your total load.

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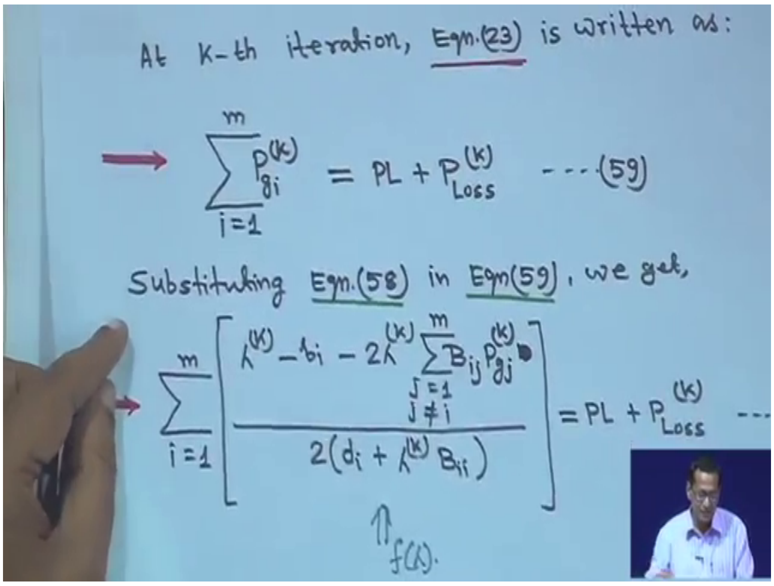

And that is function of also lambda; that means, this Pgik if you take a summation that is summation of whole thing right a summation of whole thing; that means, that total generation at k th iteration say, total generation is equal to i is equal to 1 to m, Pgik is equal to PL plus P loss k total generation include load plus loss right similar substituting equation 58 in equation 59.

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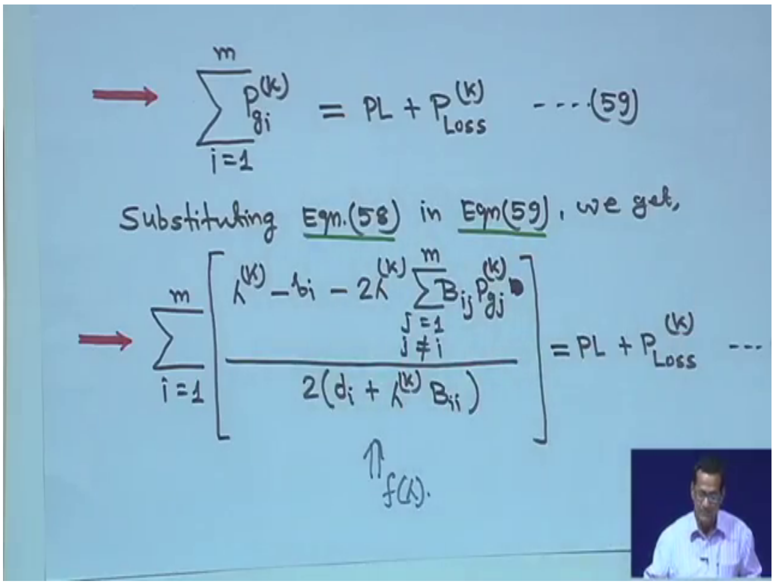

That means this equation this equation Pgik is this expression you substitute here in this sigma; that means, you will get sigma i is equal to 1 to m this whole thing is equal to PL plus P loss k. This whole thing actually is a function of f lambda; because if earlier we have seen f lambda is equal to this one is equal to f lambda is equal to P loss plus P loss k also right. So, this whole equation this thing as equally f lambda right; that means, you need df lambda upon d lambda; that means, whole thing you have to take the derivative right because you have you have to solve iteratively.

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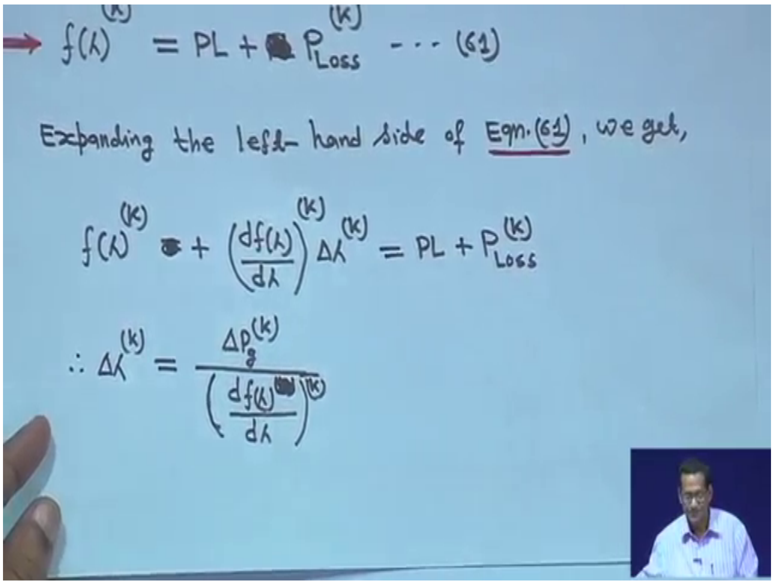

That means that and similarly if this is also f lambda summation of the generation is equal to PL plus P loss k this is also f lambda; that means, f lambda k is equal to PL plus P loss k the equation 61; that means, f lambda is equal to total generation because which is also function of lambda right. So, this is also function of lambda. So, f lambda k is equal to PL plus P loss k this is equation 61. I hope you have understood this right it is mathematics little bit you have to understood right.

Ah. So, when I will take the numerical, see I have see you will understand more, but looked up look at. Now question is at any operating point lambda k same as before you expand it in Taylor series right; that means, f lambda k plus df lambda d lambda then delta lambda k, I am not telling again and again it has not count is equal to PL plus P loss k earlier loss was not there now losing included.

And we have seen the delta lambda k is equal to Pgk, delta Pgk upon df lambda upon d lambda k that also we have seen right.

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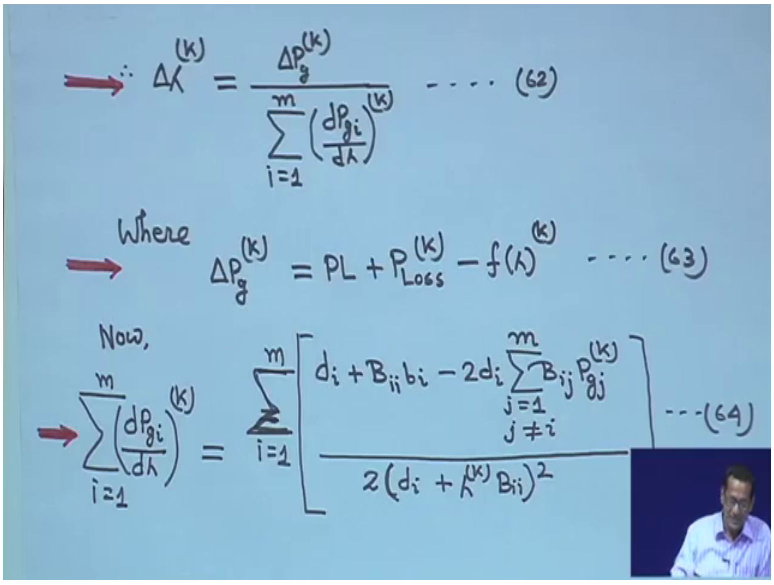

And delta Pgk right is equal to that means, this delta lambda k, this is also we have seen earlier therefore, delta lambda k we can right this one, this is equal to i is equal to one to m, dPgi upon d lambda k I mean this is the this is your this whole equation actually this whole this whole thing actually your f lambda right; that means this your what you call just hold on that your this is your delta lambda k is equal to delta Pgk upon df lambda by delta lambda k; that means, delta lambda k you can right is equal to delta Pgk divided by i is equal to 1 to m, dPgi upon d lambda because this equation if you look right this equation if you look that it is basically summation of i is equal to 1 to m actually right and whole Pgik expression we have substituted here.

So that means, this df lambda upon d lambda is nothing, but i is equal to 1 to m dP gi by d lambda k right; that means, this delta Pgk is equal to load plus loss minus f lambda k; because f lambda k is a what you call that your function of all the generation every iteration generation is changing load is fixed, but loss will vary right because loss is a function of generation.

So that means, load plus loss minus f lambda k, f lambda k is nothing but summation of all the generation at k th iteration right loss is also changing at k th iteration, but total load is fix right therefore, this sigma I 1 to m dPgi by d lambda; that means, if you take the derivative of this equation with respect to lambda, at least you take this derivative we get. If I write the derivative it will take another few pages for through another 2 pages.

So, I do not want that this derivative you please do it right. So, what I have done is this whole derivative actually I have written here it will be di plus capital Bii small b i minus 2 di j is equal to 1 to m j not is equal to I capital Bij into Pgj at k th iteration divided by 2 into di plus lambda k B ii whole square this is equation 64. This is whole square actually this is equation 64 right; that means, this is known this is also known from this equation 63, delta Pgk known and from this 64 right this is also known this is from 63 this is known right and from 64 this is also known; that means, you can easily compute delta lambda k in every iteration.

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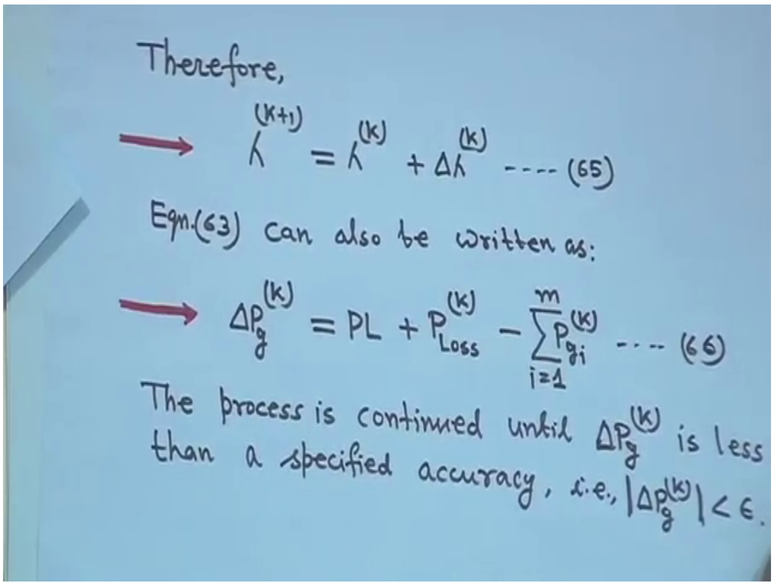

So, once this is done then delta also lambda k plus 1 is equal to lambda k plus delta lambda k this is equation 65. So that means, equation 63 actually; that means, I told you this equation 63 it can be written as instead of f lambda k you write PL plus P loss k minus sigma i is equal to 1 to k 1 to m, Pgi k that is 66. This process will continue till we get summation of this sorry absolute difference this thing of this one is less than epsilon then solution in is solution is converge right.

So that means, once whatever we have done that we have to determine lambda iteratively right and with loss and without loss with loss things are seems to be very simple sorry without loss things are seems to be very simple and with loss some more terms are added that is a loss term right. So, this is the thing. So, only there should not be any confusion, I repeat that f lambda that lambda that is function of lambda actually is equal to PL at the same time it is when which is loss is not there right at the time it is from the functional load as equal to generation right both the cases f lambda is function of the sum of the total generation right.

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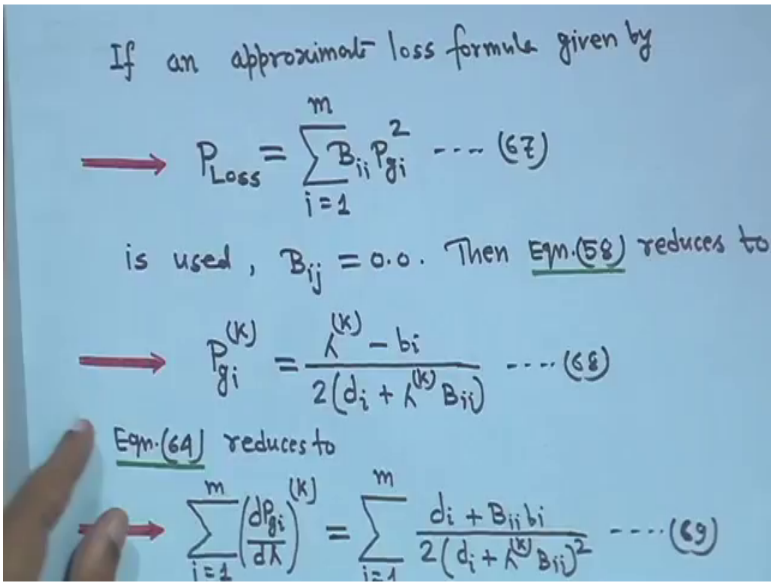

Now, in an approximate loss formula suppose it is given like this suppose it is given like this approximate loss formula is given like this. So, P loss is equal to i is equal to 1 to m Bii Pgi square, that is equation 67; that means, is used as Bij is equal to 0.0. So, no Bij only Bii is there then equation 58 reduces to; that means, in that equation 58 you please substitute Bij is equal to 0, but B ii all will be there; that means, Pgik will become lambda k minus B i divided by 2 into di plus lambda k Bii lambda k means lambdas value at k th iteration right this is equation 68.

Then in the equation 64; that means, this equation; your Bij term is not there right; that means, this equation will be simple di small di plus capital Bii into small B i right the Bij term is not there; that means, your this equation will simply become di plus Bii into small B i divided by 2 into di plus lambda th k th iteration into Bii whole square this is equation 69. This is a simple example I gave I mean if bijs are not there right. So, this is your what you call equation 69.

So, this is how one can find the what you call that your lambda values iteratively right. So, let us take one example on this. So, fuel cost function for three thermal plants is given in rupees per hours C 1 is given this one.

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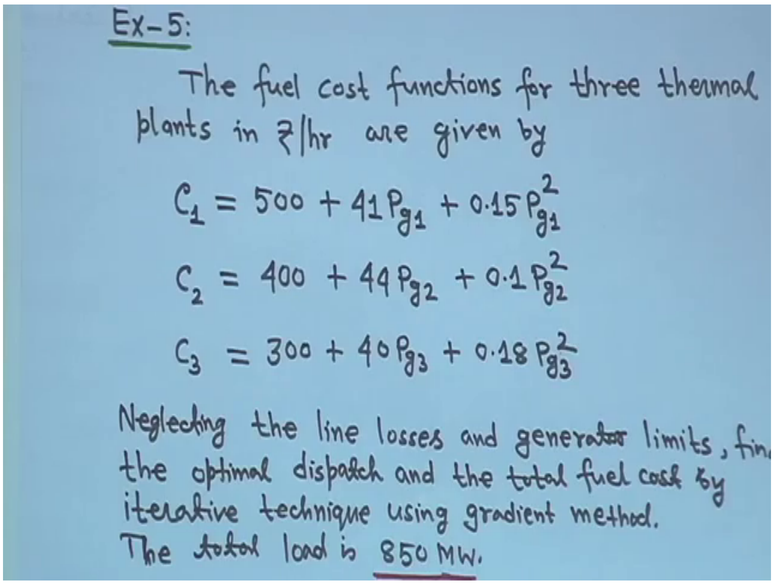

500 plus 41 Pg 1 plus 0.15 Pg 1 square C 2 is given 400 plus 44 Pg 2 plus 0.1 Pg 2 square. C 3 is equal to 300 plus 40 Pg 3 plus 0.18 Pg 3 square right. You neglect the line losses first you will neglect the loss right and generator limits, generator limit is also not there we will consider in the next problem right you have to find out the optimal dispatch and the total fuel cost by iterative technique using gradient method, total load is 850 megawatt right that is the thing.

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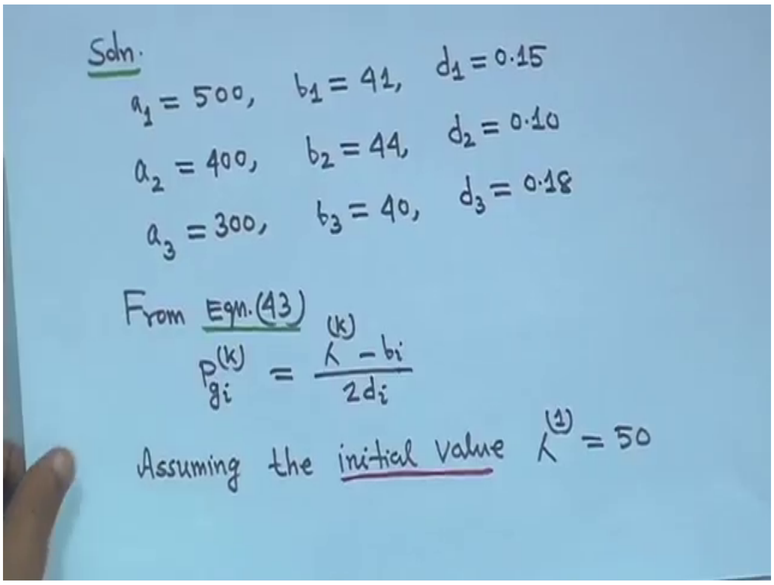

So, using this data right a 1 is equal to 500, B 1 41, d 1 0.15; a 2 400, B 2 44, d 2 your 0.1 and a 3 300, B 40 and d 3 is equal to 0.18. Using equation 43, it is given Pgik is equal to lambda k minus B i 2 k th iteration. So, assume the initial value. So, lambda 1 when k is equal to 1 lambda 1, you assume the initial value is equal to 50 lambda 1, B i and di known to you right for all that generating a characteristics that c 1 c 2 c 3 is given right.

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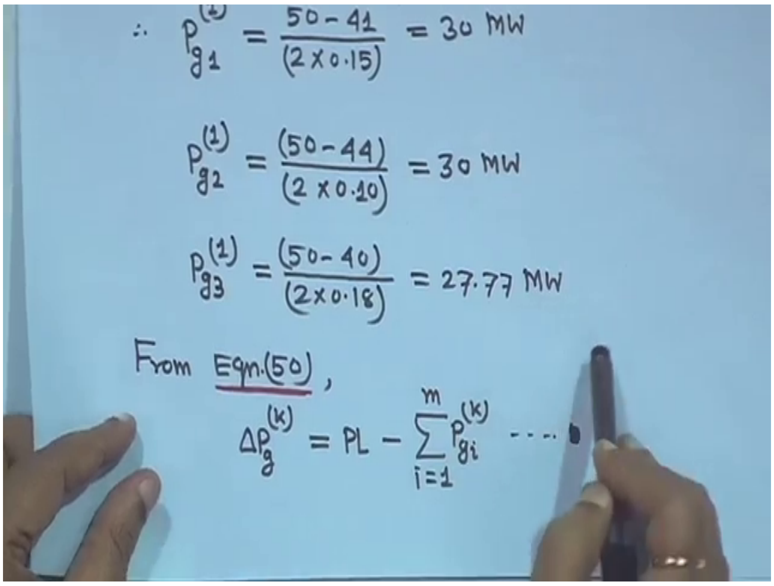

Therefore, using the initial values you calculate using this one using lambda 1 is equal to 50, calculate Pg 1, Pg 2, Pg 3 at the first iteration, right.

So, in that case your Pg 1 will become 50 minus 41 and upon 2 into 0.5 that is 30 megawatt. Pg 2 at first iteration 50 minus 44 divided by 2 into 0.1 that is 30 megawatt and Pg 3 at first iteration 50 minus 40 divided by 2 into 0.8 that is 27.77 megawatt.

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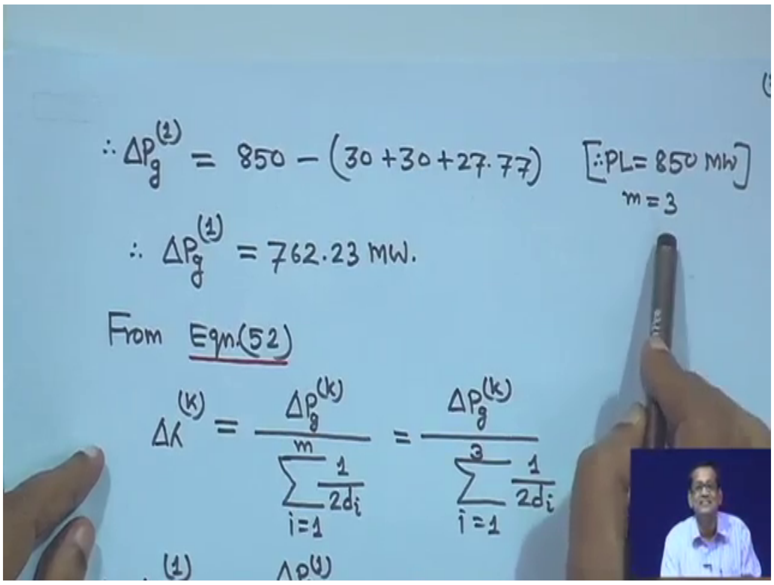

Now you have to mismatch you have to make it delta Pgk is total load minus total generation this is the formula. So, at first iteration you find out what is the. So, in the first iteration when k is equal to 1 delta Pg 1 is equal to 850 minus 30 plus 30 plus 27.77 right total load is 850 megawatt written here right and m is equal to 3 generators are there; that means, delta Pg 1 is equal to 762.23 large difference. So, you have to go for the next iteration.

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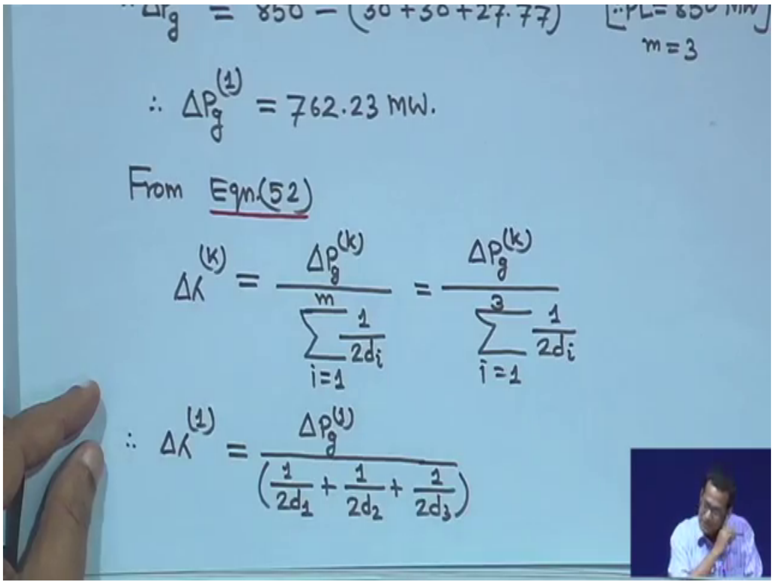

So, then equation 52, you calculate use equation 52 delta lambda is equal to delta Pgk divided by sum of i is equal to 1 to m 1 upon 2 di right. So, that is; that means, delta Pgk it is i m is equal to 3. So, i is equal to 1 to 3, 1 upon 2 di that is delta k is equal to 1 then delta lambda 1 delta Pg 1 divided by 1 upon 2 d 1, plus 1 upon 2 d 2 plus 1 upon 2 d 3. All these d 1 d 2 d 3 values known, and from here your delta Pg 1 values are known. So, calculate what is your delta lambda right.

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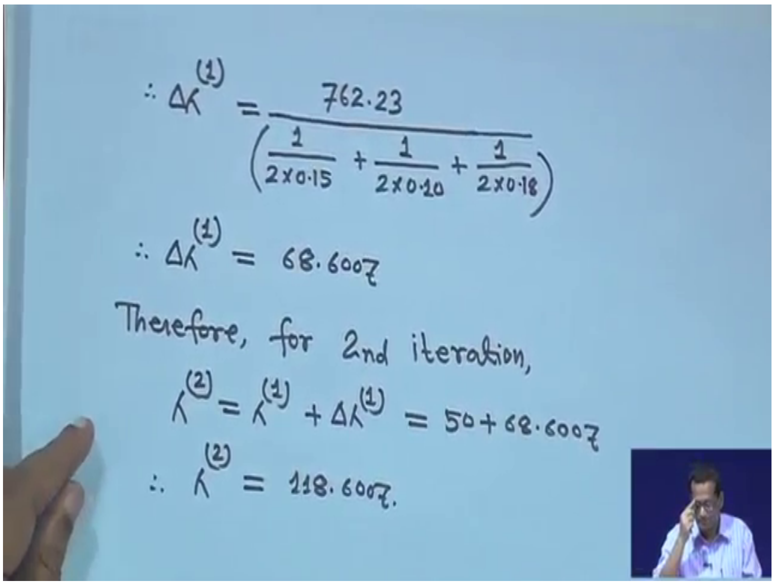

So, in this case delta, delta lambda 1 is equal to 762.23 divided by 1 up to or 1 upon 2 into 0.15 plus 1 upon 2 into 0.1 plus 1 upon 2 into 0.1. So, it is coming delta lambda 1 is equal to 68.6007. Therefore, for second iteration lambda 2 is equal to lambda 1 plus delta delta lambda 1. So, 50 plus 68.6007 that is lambda 2 is equal to at second iteration 118.6007 with that you calculate again Pg 1, Pg 2 and Pg 3 right.

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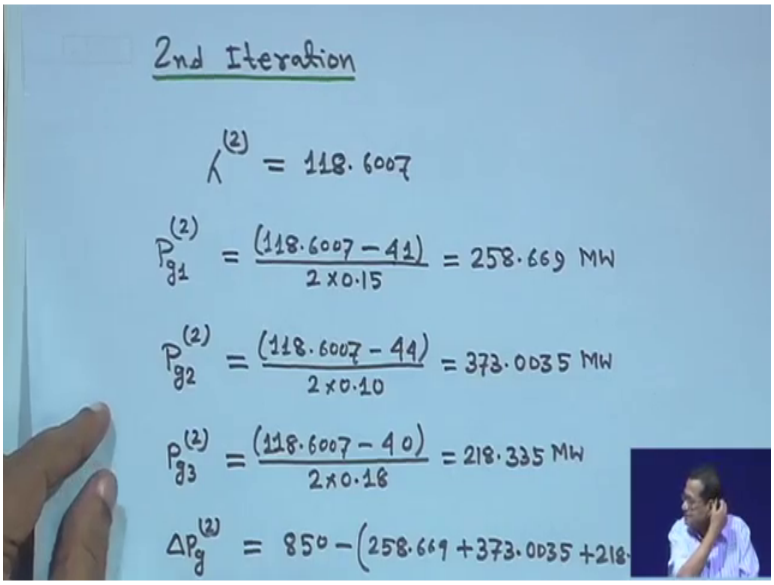

So, second iteration lambda 2 is 118.6007 right. So, in this case it will be a this one will be Pg 1 2 you calculate substitute lambda again you get 258.669 megawatt. Similarly Pg 2 at second iteration you will get substitute lambda here right 373.0035 megawatt. So, Pg 3 here you substitute lambda is equal to this one, you will get 218.335 megawatt. So, delta Pg 2 will be 850 the total load minus sum of all the generation.

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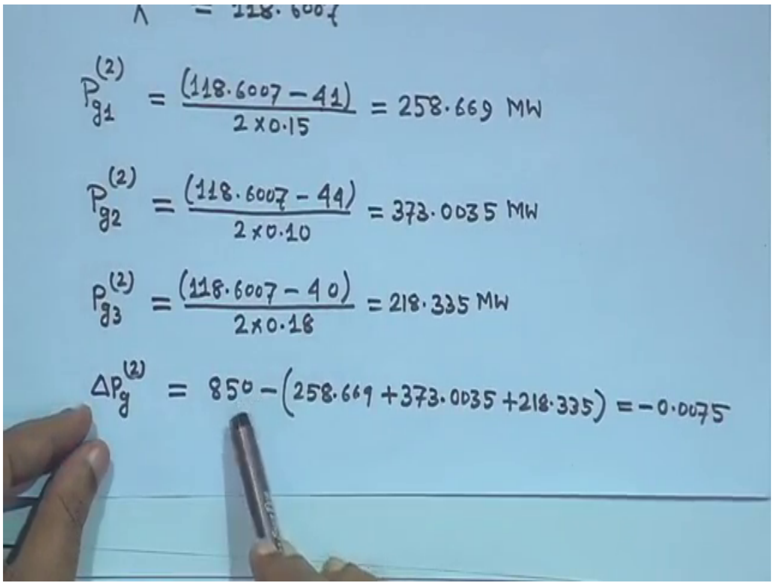

We are getting minus 0.0075 right; that means, if you take the absolute of it that your 0.0075 absolute of it or it is very very small.

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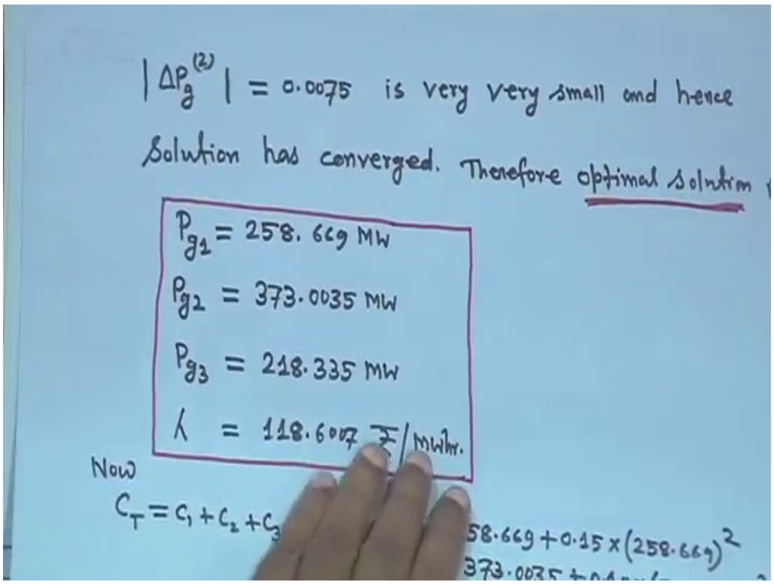

And hence the solution has converge after the second iteration right and therefore, the optimal solution will be Pg 1 will be this much Pg 2 will be this much Pg 3 will this much and this will be lambda values rupees per megawatt hour right.

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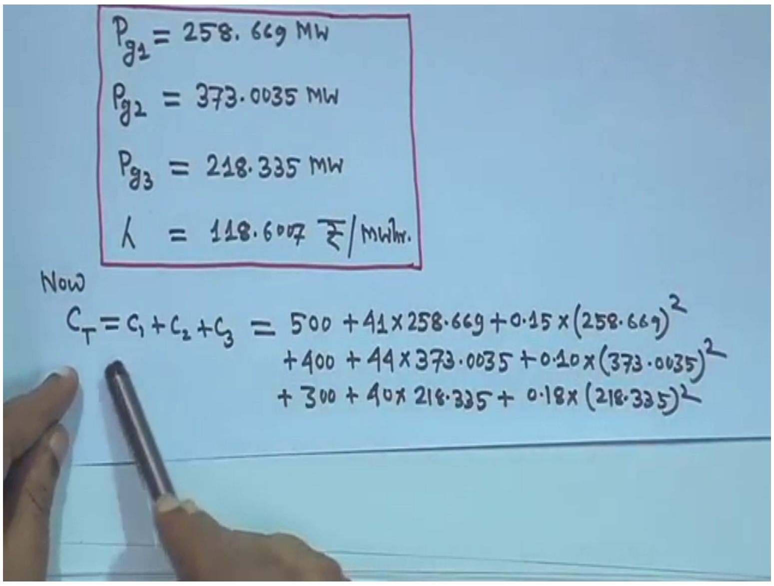

Therefore that total CT will be C 1 plus C 2 plus C 3 right. So, how much cost will be? Those Pg 1 Pg 2 Pg 3 substitute in their expression and simplify right then you will get this value right just to substitute all these values right and you will get ct will be 69481 rupees per hour right.

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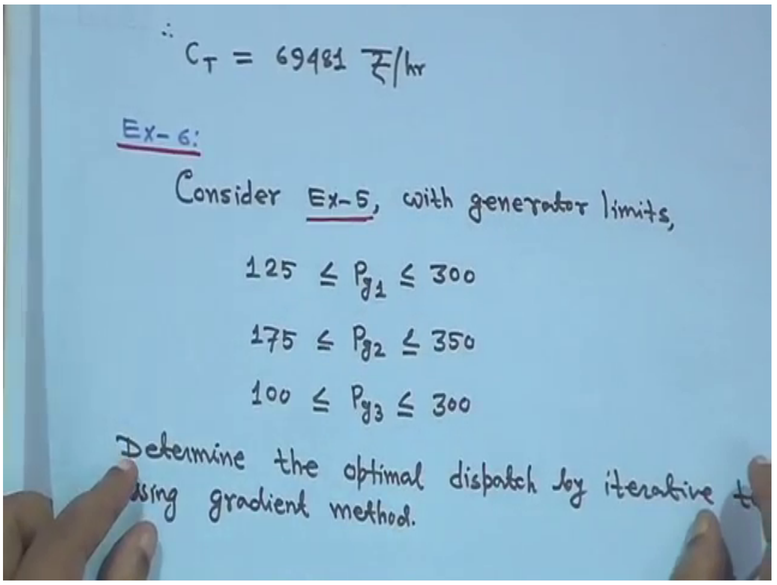

So, now we will consider the limit we will consider the generator limit same example, but we consider the generator limit. So, Pg 1 the minimum is 125 megawatt, maximum is 300 megawatt right Pg 2 a minimum 175, maximum 350 and Pg 3 minimum 100, maximum 300 megawatt.

So, you have to determine the optimal dispatch by iterative technique using gradient method right. Now same data everything is same example same data, but we will take this limit and you will see what is the change in the result right.

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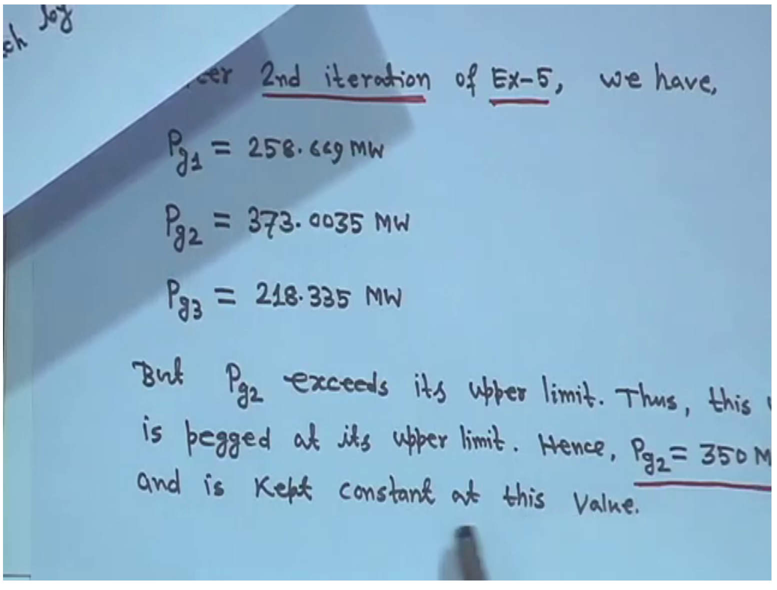

Now question is that after second iteration of the previous example that is a example 5, after second iteration we got Pg 1 250 megawatt, 258.669 megawatt, Pg 2 373.0035 megawatt and Pg 3 218.335 megawatt right.

So, if you see that Pg 2 is limit is lower limit is 175, upper limit is 250 right, but here Pg 2 is equal to 373.0033 megawatt 0 0 35 megawatt. So, we cannot take this value we have to take only this limiting value 350 megawatt right; that means, thus this unit is pegged at its upper limit that is Pg 2 must be taken as 350 megawatt and is kept constant at this value; that means, Pg 2 we have to fix at 350 megawatt you cannot take this one; that means, this generator 1 and generator 2 this will be in operating right because their limits are not violated.

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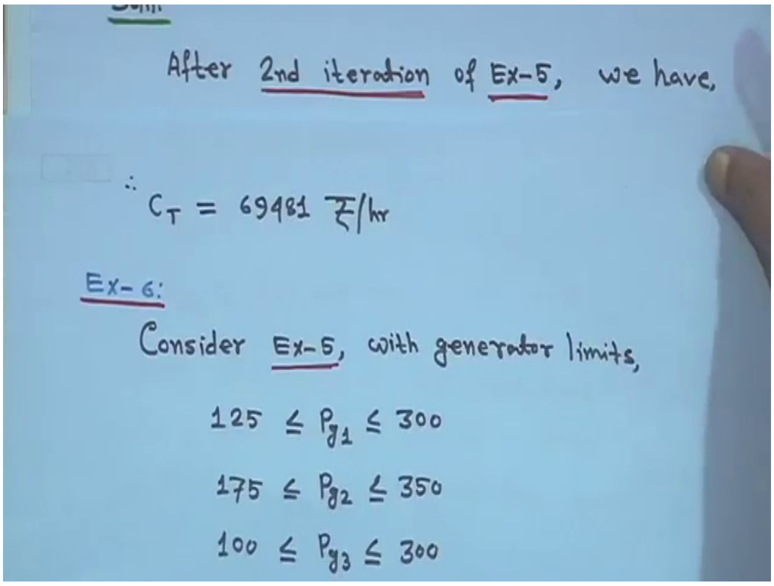

If you do so, then what will happen after this second iteration that just hold on, a second iteration this the new that delta Pg 2 the new imbalance 850, total load minus 250.669 is there.

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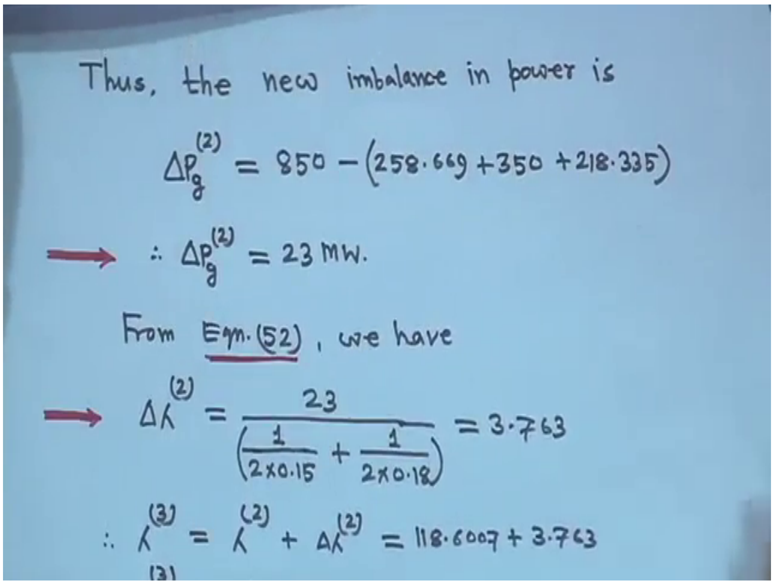

Now, we will not take this one 373 right because it is take that 350, it is violating the limit we will take this 350 only plus 218.335 these 2 are not violating the limit.

So, in that case delta Pg 2 is 23 megawatt, now equation 52 what we will do this limit actually Pg 2 has your fixed at 350. So, in that expression that 1 upon sigma di we will take one upon 2 d 1 plus 1 upon 2 d 3, but we will not take one upon 2 d 2, because this has fixed at this limit earlier we are taking all the three terms we are not considering the limits.

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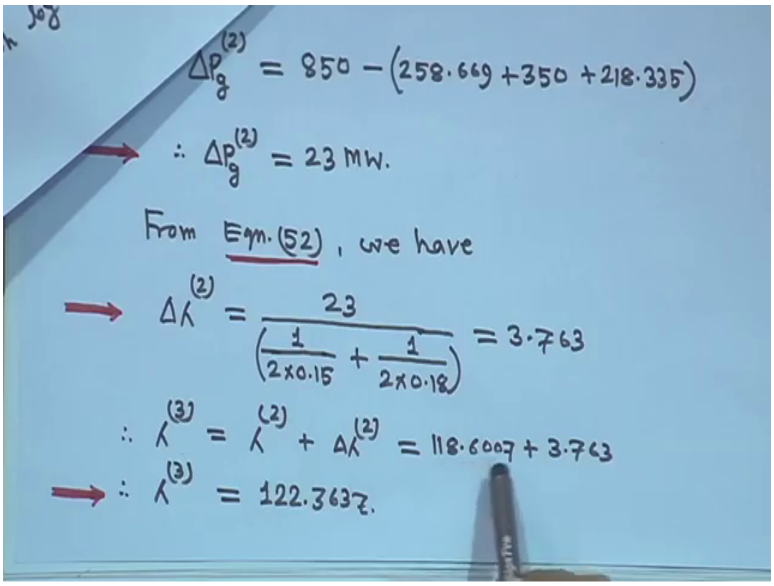

Now, this generator 2 has your touched the limit then naturally one upon 2 d 1 plus 1 upon 2 d 3, 1 upon 2 d 2 will not be there because it has touched the limit I hope you have understood this right; that means, delta lambda 2 will be now it is coming 3.763 right therefore, lambda 3 lambda 2, 2 plus delta lambda 2. So, it will 118 earlier it was converge at these value, now it will be 118.6007 plus 3.763. So, lambda three will be 122.3637 right.

So that means, with this lambda you calculate your what you call third iteration the Pg 1 Pg 2, Pg 3.

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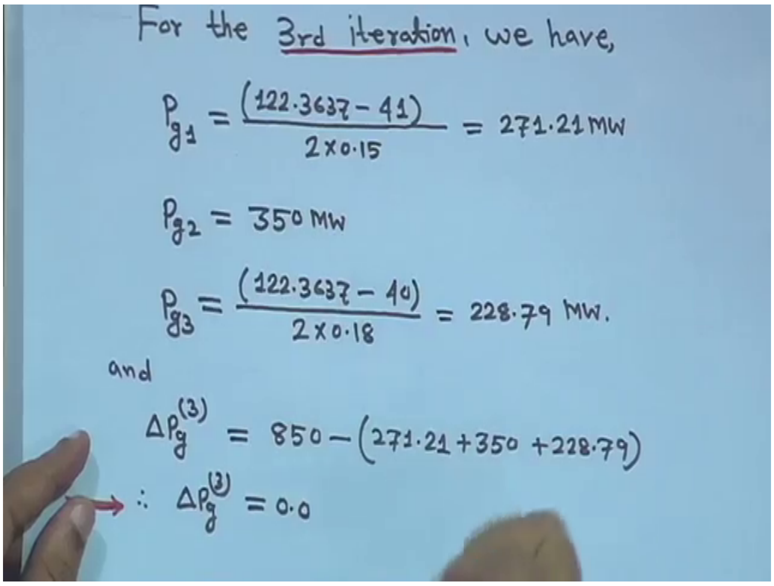

So, Pg 2 is fixed at 350 megawatt P 2 will not change it is touched its limits, Pg 1 is 271.21 megawatt right its maximum limit 300 So, not touched. Pg 2 350 fixed and Pg three is 228.79 his maximum is 300. So, it is this 2 not touch the limit, but this is there. Now you calculate delta Pg 3; as soon as you calculate delta Pg 3 it is 850 minus 2 271.21 plus 350 plus 228.79.

So, delta Pg 3 at third iteration is becoming 0; that means, solution has perfectly converge right. So, so that means, after third iteration that your delta Pg 3 is equal to 0.

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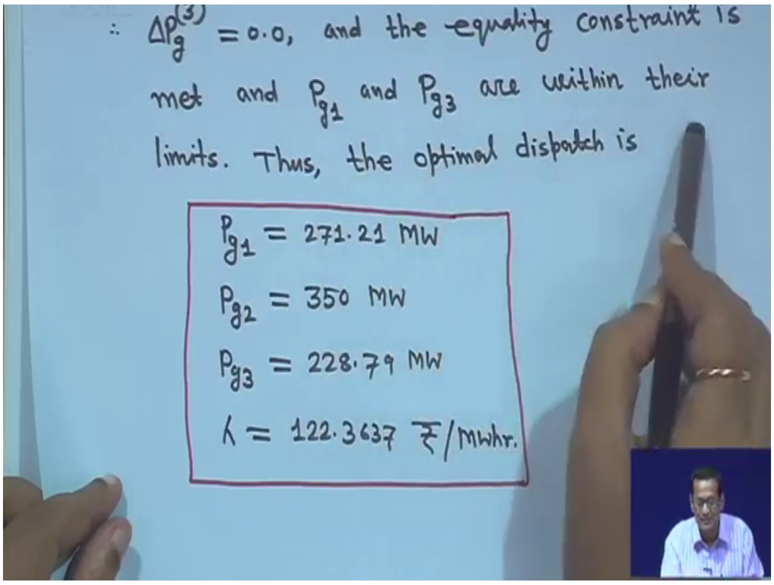

And the equality constant is met and Pg 1 Pg 3 are within their limit therefore, this is the optimal solution when constants are introduced for the generator right therefore, Pg 1 is equal to this one, Pg 2 is equal to this one, and Pg 3 is this one and this is the value of the lambda right.

So, only my request to you that look so many calculations whatever you see here all calculations actually have been done by me right. So, if you find any error any mistake right and anything a calculation error or anyway any writing error right please let me know this such that I can rectify this things, I repeat many times right because all these calculations I have to do of my own right and that is why I am telling you that you I mean your cooperation is required when you listen this right.

So, next one we will take another example we will start right. So, consider now example 4, we will take the example 4 right with power loss expression.

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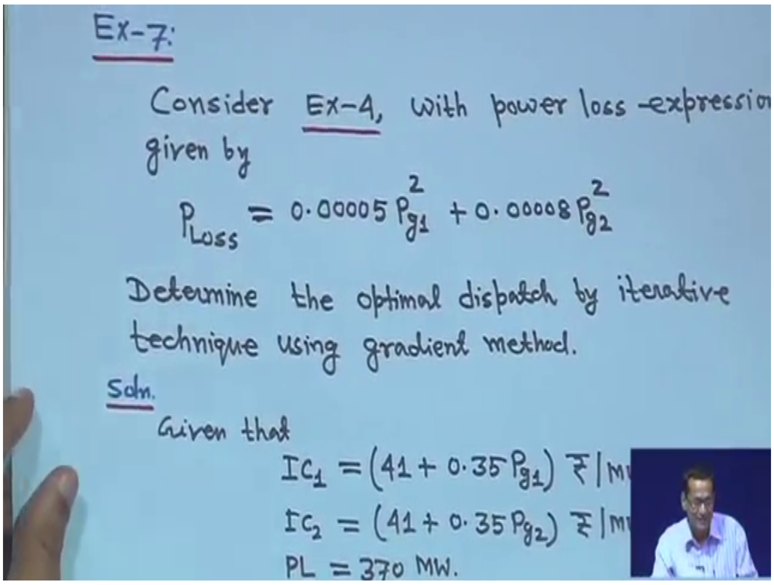

This loss expression is given example four same data, but this loss expression we will take it is P loss is equal to 0.00005405 Pg 1 square plus again 0.408 Pg 2 square. This loss expression is given that is a B 11 and B 2 2 this B coefficient other things all I will tell you at the end of the I mean when this economic load dispatch will lower after that I will derive for you right.

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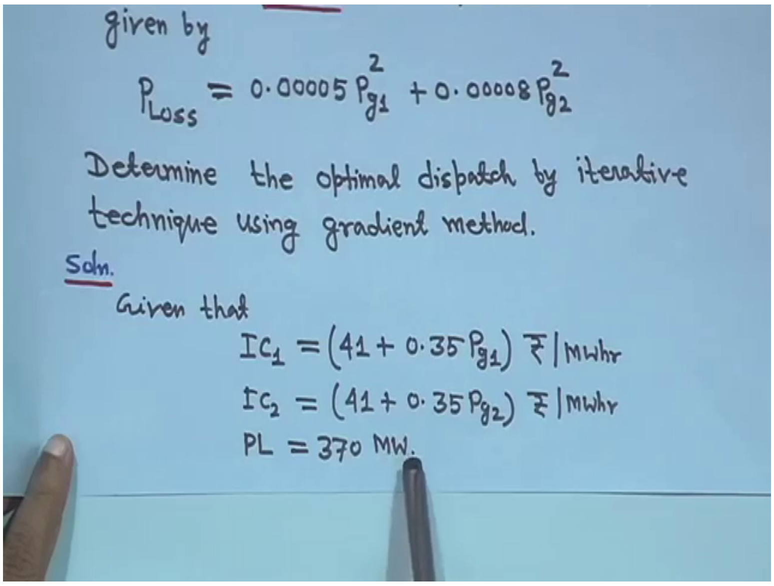

So, solution this IC 1 IC2 both are same earlier we have seen that both are same, same data you are taking and P L is equal to 370 megawatt the same load same data right so; that means, b 1 is equal to 41 and d 1 is equal to 0.35 by 2 because

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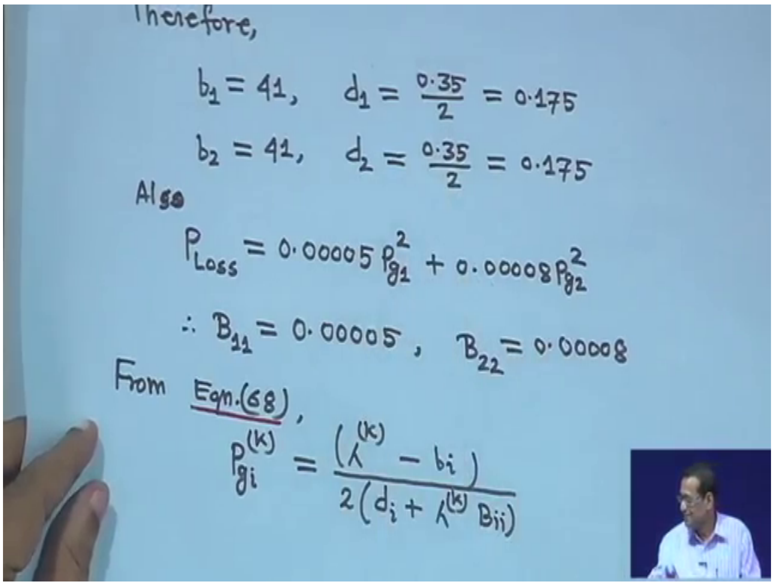

when you take the derivative this one actually it is b 1 plus 2 d 1 which is b 1 plus your B 2 plus 2 d 2. So, 2 d 1 is equal to 0.35 here also 2 d 2 is equal to 0.35 that is why d 1 is equal to 0.35 by 2 right.

Similarly, d 2 is equal to also 0.35 by 2, 0.175, 0.175 right. Now this loss expression is given B 1 is equal to this much and B 2 2 is equal to this much. Now equation 68 right equation 68 there we ignore that Bij right that Bij term we ignore. So, for equation 62 we get Pgik is equal to lambda at k th iteration minus B i divided by 2 into di plus your lambda at k th iteration into Bii right.

So, in this case what you have to do you have to start some initial values of lambda. So, what you can do is you take lambda is equal to is a initial value at it 60 that is lambda 1. So, with this initial value you get Pg 1 substitute in this expression equation 68.

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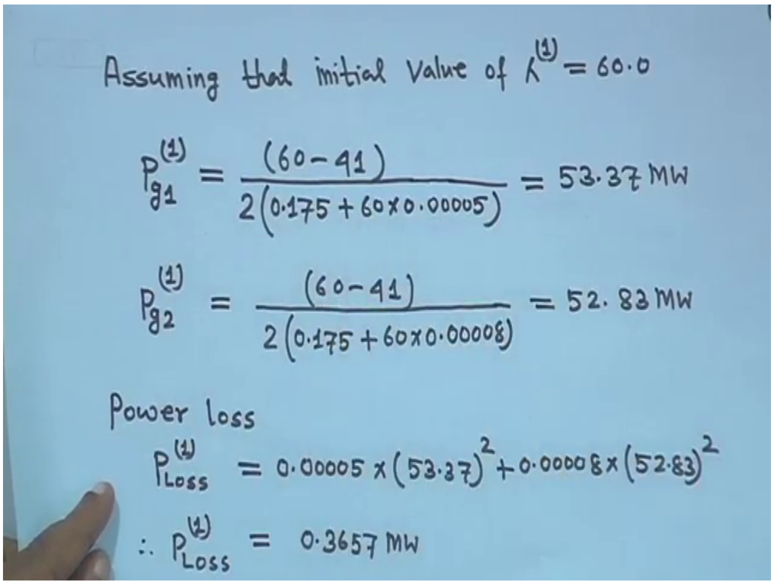

In this expression you substitute right then you will get Pg 1 is equal to 60 minus 41 divided by 2 into all sort of things right that is you will get 53.37 megawatt right.

Similarly, Pg 2 1 you will get 52.83 megawatt right then power loss expression is given you substitute Pg 1, Pg 2 you will get P loss at first iteration 0.3657 megawatt right. So, once you get it right then you check the mismatch.

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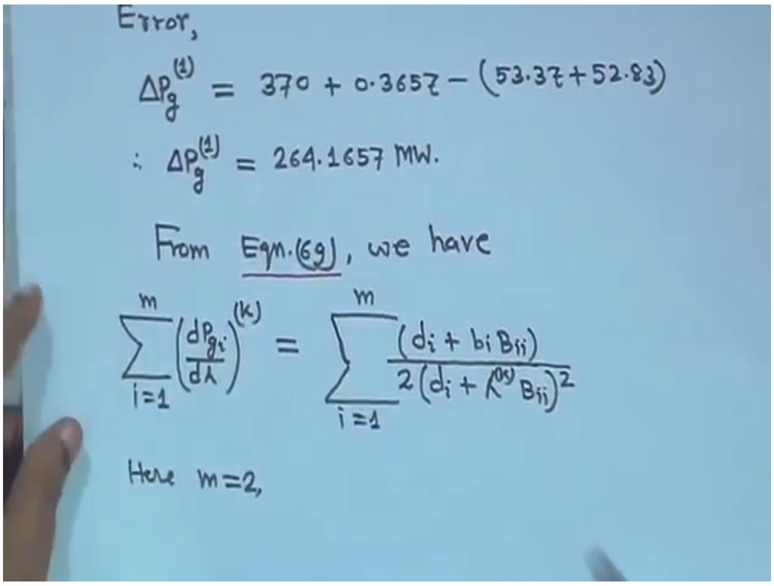

Then what you will get that delta Pg 1 is equal to 370 plus loss 0.3657 this is the loss minus the 2 generator generation, it is coming delta Pg 1 264.1657 megawatt this is actually too high we have to go for next iteration right and delta find the delta lambda.

So, from equation 69 we have i is equal to 1 to m, this value actually this is derived in equation 69 in this expression you put this lambda value Bis value, dis value everything for m is equal to 2 because 2 generator units you put m is equal to 2 right you expand this term for i is equal to 1 to 2 and just put those values.

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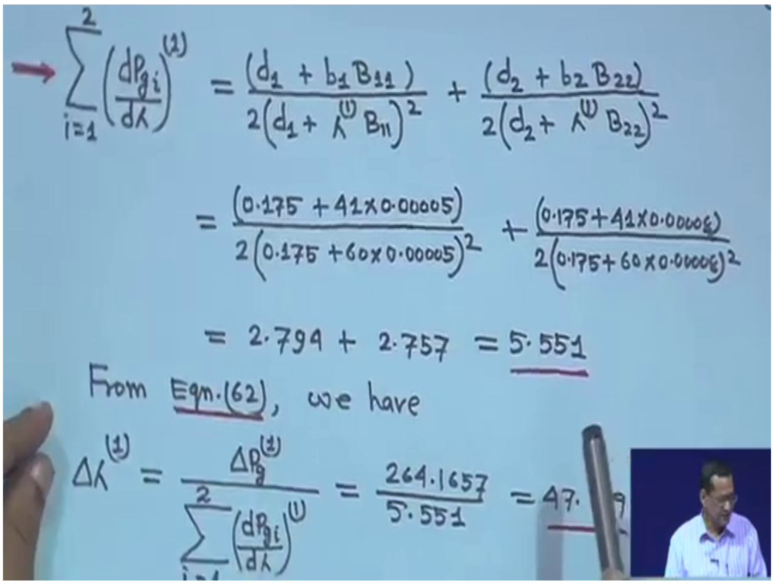

If you do so, then sigma i is equal to 1 to 2 this will be this term right this one plus these one this 2 term.

You substitute all the values are known to you substitute, you will get your what you call this to 5.5511 right.

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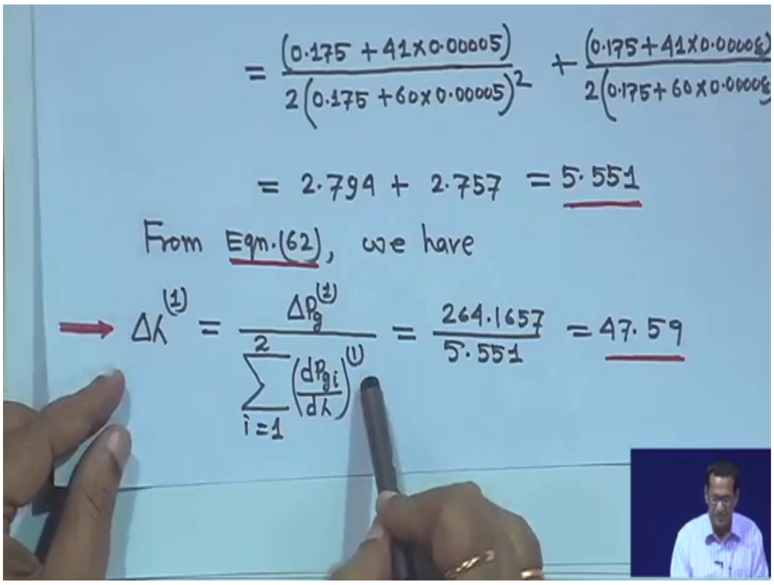

Therefore, this is your this one therefore, equation 62 delta lambda 1 delta Pg 1, i is equal to one to 2 dPgi by d lambda at iteration one. So, you have got this value 5.551 you substitute in the denominator and numerator we have got the mismatch 264.1657 that is actually is equal to delta lambda 47.59 right.

That means lambda 2, lambda 2 will become your lambda 1 plus delta lambda1 right this is 60 plus 47.59, so 107.59. So, for the second iteration this lambda value you substitute in Pg 1 in Pg 2 expression, you will get 184.58 megawatt. Pg 2 will get 181.34 megawatt and power loss you will get the same expression put Pg 1 and Pg 2 you will get 4.334 megawatt right, so with this right.

Thank you again we will come right.