Power System Analysis
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Lecture – 43
Optimal system operation (Contd.)

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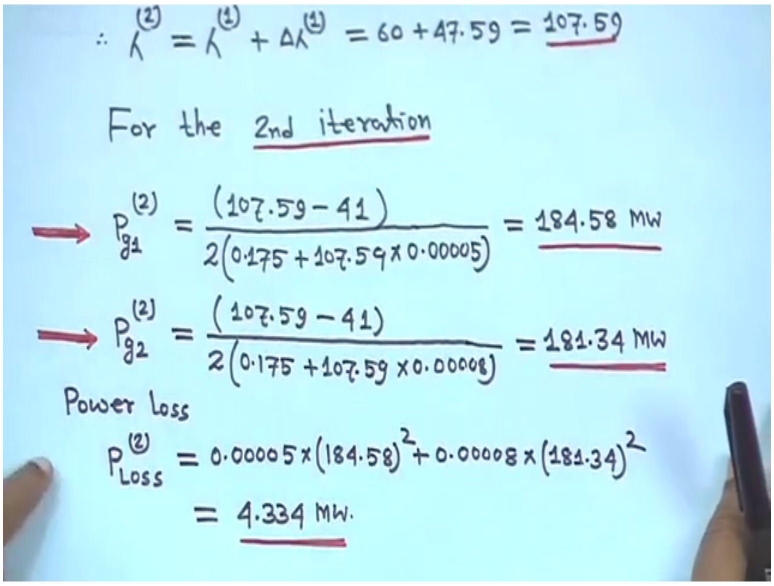

So, we will continue that problem. So, for the second iteration, we have seen that your what you call power loss is 4.334 megawatt and P g 1 and P g 2 after second iteration 184.58 and 181.31 megawatt. So, now, you have to check the mismatch that that is the your generation minus your load or load minus generation.

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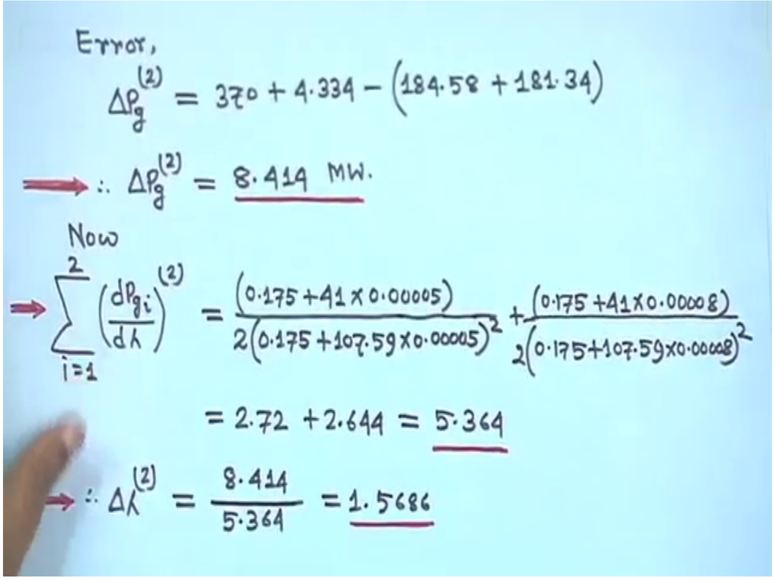

So, in that case delta P g 2 is equal to this is the total load and this is the loss we have obtained minus the sum of the generation two generator. So, we are getting delta P g 2 is equal to 8.414 megawatt; that means, still not converged. Next what we will do using those data of the generation and all these things whatever you have that you calculate using this i is equal to one two that your d delta P g i delta lambda iteration two you compute using these value of lambda whatever you have got. So, in that case compute this term and this term, these two terms all have been explained previously in terms of mathematical derivation. So, this is coming this first part coming 2.72 and second part is 2.644. So, total is 5.364. So, delta lambda 2 will be delta P g 2 divided by this thing basically this is divided by d lambda is equal to delta P g by d lambda d P g i by d lambda. So, delta lambda 2 will be 8.414 divided by 5.364 is equal to 1.5686.

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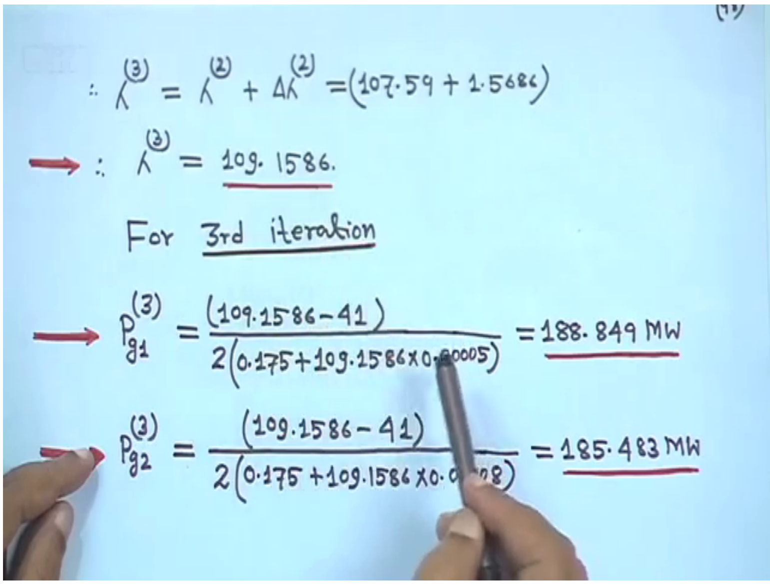

So, this is changing that means, in the your what you will call in the third iteration lambda 3 will take lambda 2 plus delta lambda 2, it was 107.59 plus 1.5686. So, lambda 3 is equal to 109.1586. So, for third iteration again you calculate using that same formula its P g 1 at the third iteration this is the lambda value minus 41 divided by 2 into whatever is there you will get 188.849 megawatt in the same formula just to substitute. So, P g 2 also at the third iteration 109.1586 minus 41 divided by or 2 into whatever parameters set here, you will getting 185.483, so that is your P g 1 and P g 2 after third iteration.

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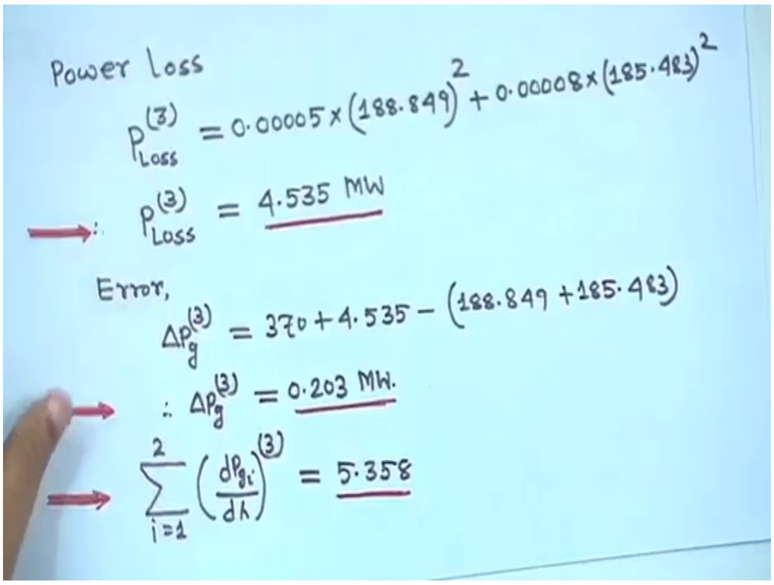

Now using these two generation values, using these two generation values you are what you call compute the power loss at third iteration. So, this formula was given. So, point it is four naught five into that P g 1 square this is your P g 1 plus point four naught eight into P g 2 square this is the value of P g 2. So, here we are getting loss third iteration 4.535 megawatt that means, error is that is delta P g 3, 370 plus this loss 4.535 minus the sum of the two generation which you have got add it. So, delta P g 3 is 0.203 megawatt. So, this is also still not sufficient to call I mean low for convergence.

So, what will do, will find out one more iteration. So, i is equal to 1 to 2 this d P g i by d lambda third iteration here directly I am writing the value 5.358. But you please compute using this your what you call the value of your that lambda 3 please you compute this one I am giving you the final answer this is 5.358. Therefore, therefore, your delta lambda 3 is the delta pg is 0.203 and your d P g d lambda summation is 5.358. So, this is coming 0.03788. Therefore, lambda 4 is equal to lambda 3 plus delta lambda 3. So, 109.1586 plus 0.03788, the lambda 4 will be 109.196. So, after these for fourth iteration, you calculate P g 1 at fourth iteration, P g 2 at fourth iteration. And substitute in the same expression this values, you will get 188.95 megawatt, and this P g 2 4 you will get 185.58 megawatt.

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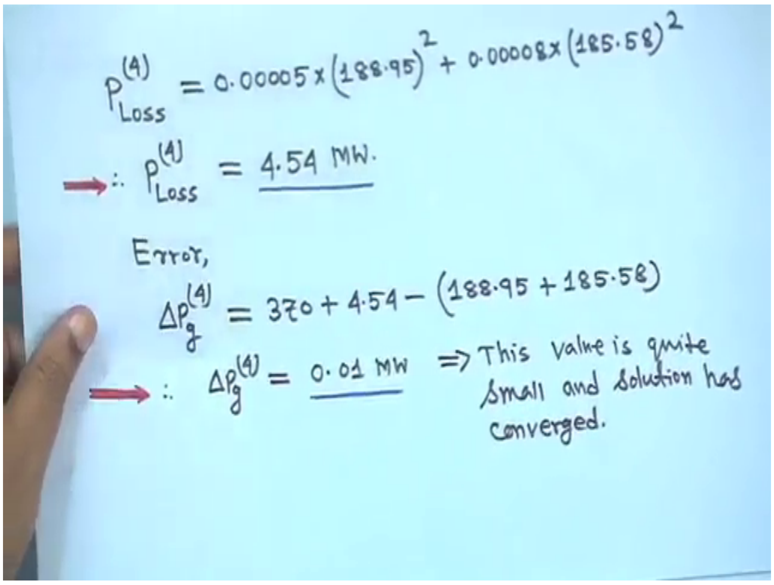

So, then with this you will get that p loss 4 again you compute using the same formula which p loss 4 is 4.54 megawatt. Now check the error. So, delta P g 4, 370 the load plus

4.54 minus sum of that is a generation. So, this is coming actually 0.01 megawatt this value is quite small and solution has converged that means, what that this takes when you introduce this loss formula this one takes more number of iteration. Here it is taking at least four iterations to converged, but gradient method has advantage that every iteration easily you can compute your delta lambda and accordingly you modify the your what will call lambda and the generation and losses. So, this is your final answer. So, this is your final answer. So, P g 1 is 188.95.

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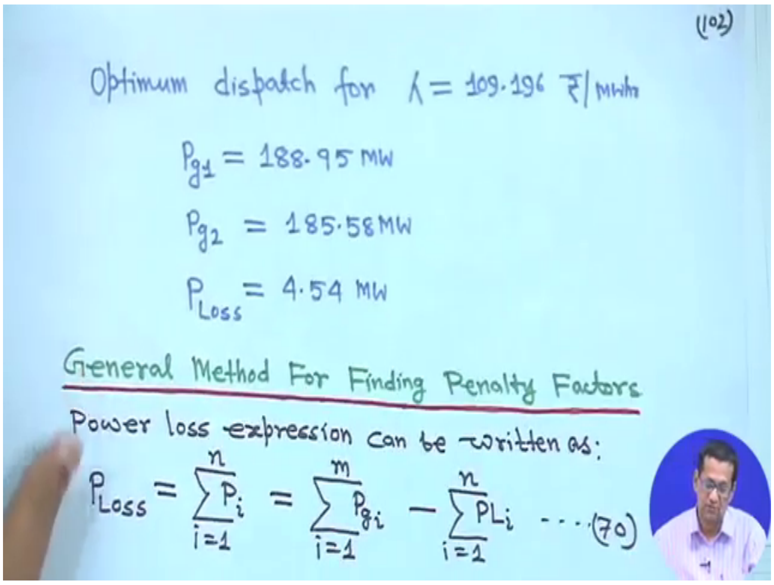

This lambda is equal to optimal dispatch for lambda is equal to 109.196 giving rupees per megawatt hour say. And this P g 1 is this much 188.95 megawatt P g 2 is 185.58 megawatt and p loss is 4.54 megawatt. So, this is the answer after fourth iteration. If you one more accuracy, you can go for further, but no need actually we assume the solution has converged. Now, general method for finding penalty factors. So, although expressions are seems to be big, but we will take small example to you know demonstrate all these application mathematical application. So, earlier I have told you that power loss is basically it is sum of the power injected at every bus bar. So, it is i is equal to 1 to n, it is sigma P i is equal to sigma i is equal to 1 to m P g i minus where i is equal to 1 to n P L i. So, this is actually power loss formula. So, this equation is say 70.

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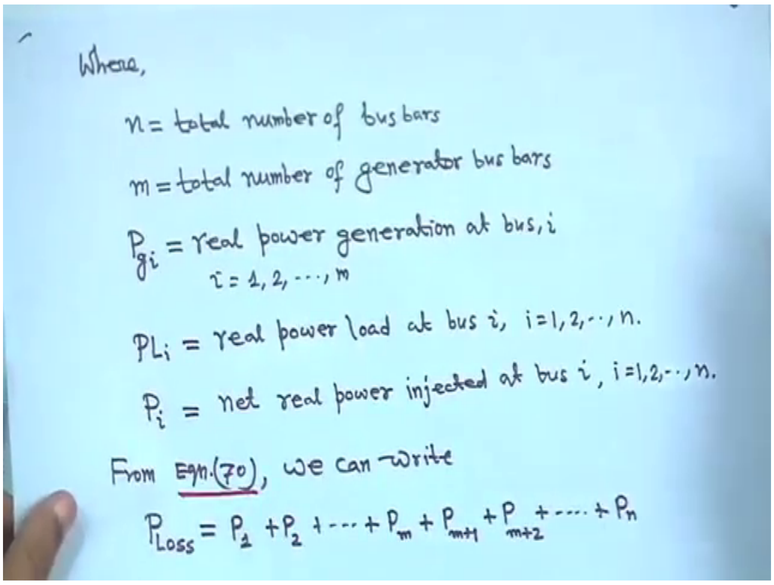

And in this equation the nomenclature is also given. So, in this equation n is the total number of bus bars, m is the total number of generator bus bars. Is mean buses where generators are connected. P g i is real power generation at bus, i this is known to you. And P L i the real power load at bus i for general bus i is equal to 1 to m; and for load bus i is equal to 1 to n and P i is the n is real power injected at bus i. So, if we expand this equation if you just hold on just hold on just one minute. So, if we if we expand if we expand this equation therefore, this one then we can write P loss is equal to this one we can write P loss is equal to P 1 plus P 2 up to P n this one we can write. That means your p loss is equal to P 1 plus P 2 plus a P m up to the that P m we are marking actually it is up to that we have m number of generator bus bar say up to P m plus next one will be P m plus 1 plus P m plus 2 plus up to P n this is that P loss.

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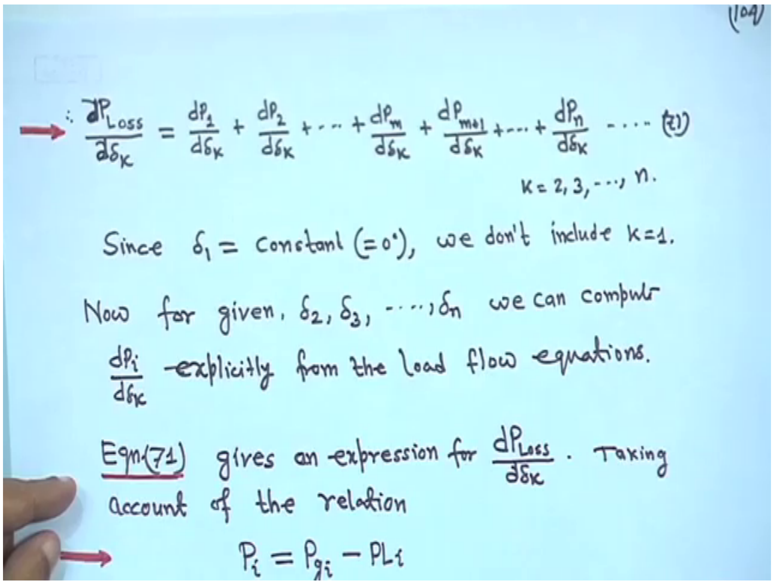

Now this equation this equation if you take derivative with respect to delta, this equation you take with this derivative with respect to delta, then what you called the dP loss by your d delta k is equal to it is dP 1 by d delta k plus dP 2 by d delta k plus dot dot dot plus dP m upon d delta k plus dP m plus 1 d delta k plus plus plus up to dP n by d delta k. It is equation 71. K is equal to 2, 3, n because delta 1 is constant is equal to 0 degree. So, we do not include k is equal to 1. So, this is k is equal to 2, 3 up to n.

So, now for a given delta 2, delta 3 and delta n you can we can compute delta pi upon delta explicitly from the load flow equation. I mean if you know delta two delta three delta n from the load flow studies everything is known then this one can be also computed. In general it can be told that this is actually your what you call delta P 1 upon delta two k plus delta P 2 upon delta k. So, in general it is actually delta p i upon your what you call that delta k that is.

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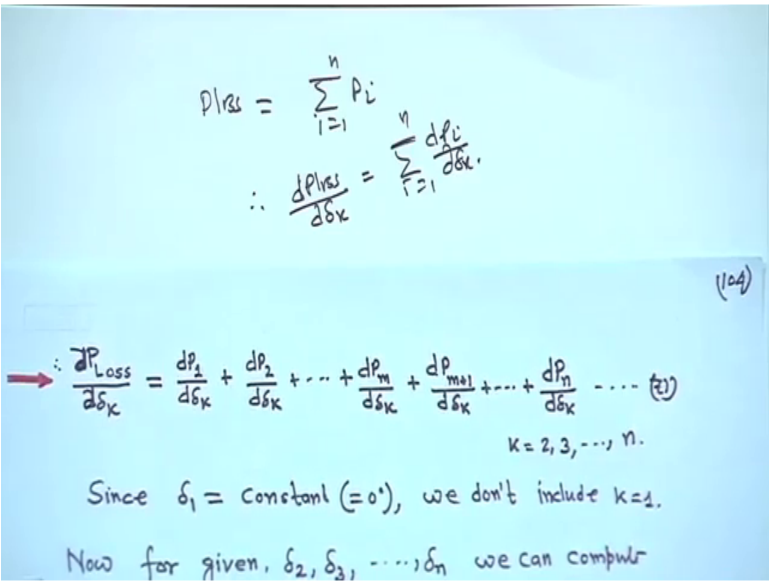

That means your P loss, P loss is equal to your i equal to 1 to n sigma P i that means, your this thing dP loss. Then if we put in general d delta k then this is actually your this equation can be written as what you to call that i equal to 1 to n dP i delta k, so that is that is your what you call in for d per k is equal to 2, 3, n this will come written. So, that is why we are telling that dpi by delta k explicitly from the load flow equations only, because you have to know the load flow solution. Now, equation 71, this means it is give the expression for dP loss upon d delta k taking account for the relationship we know P g i is equal to net power injection P i is equal to P g i minus P L i, but P L i actually is a constant P L i is a constant. So, you take the derivative the take the derivative this one with respect to delta k.

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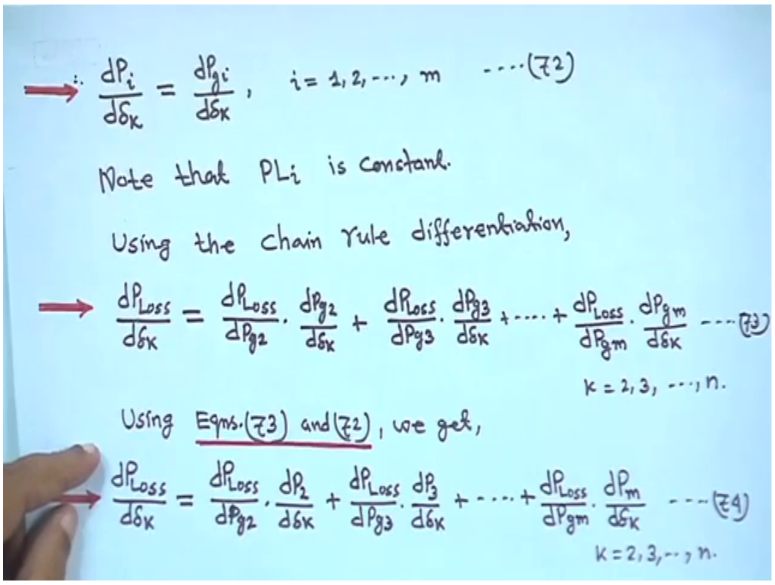

If you take the derivative of this one with respect to your what will call delta k then dpi upon delta k will be dP g i upon dpi d delta k is equal to dPi d delta k. That means, you take the derivative of this equation it will be dpi d delta k is equal to dP g i d delta k because P L i is constant. So, note that that is all written here also note that P L i is a constant. Now, using the chain rule of differentiation now use the chain rule of differentiation that means, this equation. So, if you make it like this that your what you call that dP loss upon d delta k that means, this equation we are writing that is in general this one this equation dP i by d delta k is equal to dP g i by d delta k at the same time this dP loss by d delta k is equal to dP 1 by d delta k plus dP 2 by d delta k and so on.

So, dP loss by d delta k that equation can be written as dP loss upon dP g 2 into it is a chain rule into dP g upon d delta k plus dP loss upon dP g 3 into dP g 3 upon d delta k plus up to dP loss upon dP g m into dP g m upon d delta k. So, it is chain rule of the differentiation. So, this way you can write for k is equal to 2, 3, n. So, this is chain rule. Now, using equation 73 and 72 you will get. So, in this case here it is a delta your delta P g 2 upon delta delta k delta P g 3 upon delta delta k in general upon your delta your dP gm upon d delta k.

So, because loss is a function of all your what you call all the generations except that slack bus one that is why that is why this is written up to the mth term because you have m generator beyond that nothing. So, that is why it is starting from P g 2, P g 3 and up to

P g m. So, taking the derivative of that. So, in that case your equation 73 and if you look at this equation general it is basically you can make it dP g 2 upon d delta k is equal to dP 2 upon d delta k dP g 3 upon d delta k is equal to dP 3 upon d delta k. So, this one this one actually you dP g 2 upon d delta k you can replace it by dP 2 upon d delta k from this equation only from your 72 only not again writing every term, but understandable in general the dP g i upon d delta k is equal to dpi upon d delta k. So, here we are replacing this one d P g 2 upon d delta k by d P 2 upon d delta k dP 3 upon d delta k from here only it is coming replacing this one by this one.

So, similarly your dP g m upon d delta k is equal to actually dP m upon d delta k again from this equation only, so that is we are it is equation 74 for k is equal to 2, 3, n. So, this is your what you call the chain using chain rule of differentiation. So, next stage since the function P loss does not include P g 1 because p loss is a function of P g 2, P g 3 up to the up to P g m, your P g 1 among its arguments here for delta p loss upon delta P g 1 is equal to 0, and is not included in equation 73.

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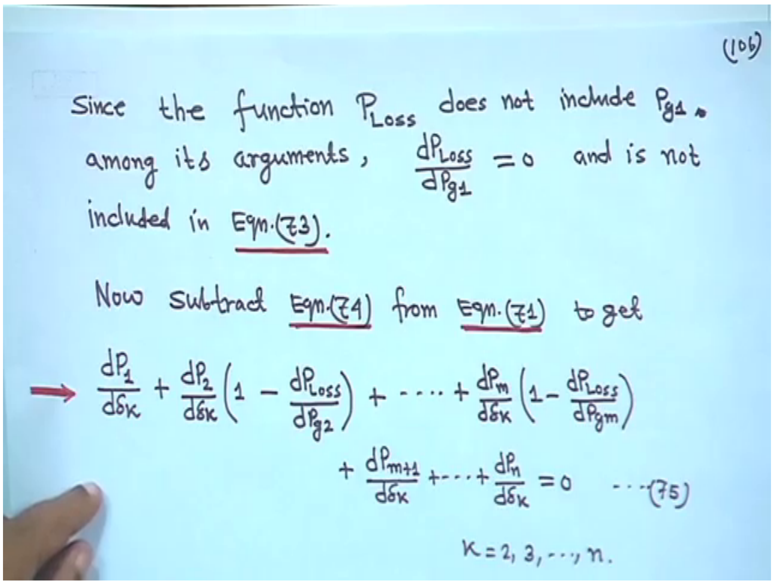

That is why in this equation it is not included, because P loss is not function of your P g 1 that is why it is P g 2, P g 3 up to P g m we have taken the derivative. That means, there now you subtract equation 74 from equation 71. Now, let me find out 74, you have to subtract equation 74 from equation 71. So, for your understanding only I am trying to write this equation I am subtracting it first what you called the subtract it from equation 71.

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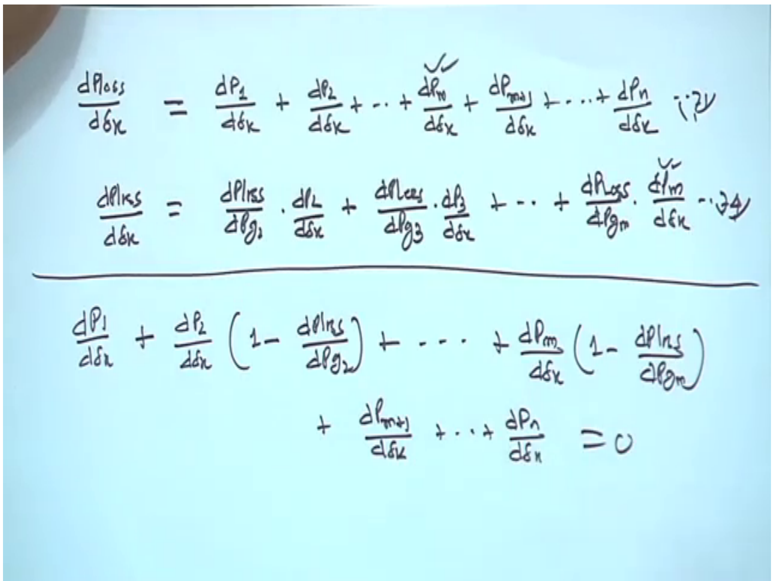

So, I am writing equation 71. So, hope everything will be readable to you. Just have a look delta P loss then delta delta k is equal to dP 1 d delta k, I am rewriting that dP 2 d delta k plus up to dP m d delta k, next term will be d P m plus 1 d delta k then up to nth term plus dP n d delta k. This is actually your equation 71. And then equation 74 is this one I am writing again that dP loss d delta k is equal to your dP loss dP g 2 into dP 2 d delta k plus dP loss dP g 3 into dP 3 d delta k plus mth term dP loss dP g m into dP m upon d delta k. This is what you call equation 74.

So, just for your understanding actually I have written this now you have to do is that will subtract your what you call equation your 74 from 71. So, what you will get if you subtract from this one, I mean these if you subtract from this one, so this one if you subtract 74 this time with this term will be zero left hand side will be zero. So, left hand side putting it on the hand side, 0. So, if you subtract that faster will remain as it is, first term will remain as it is, so that is here dP 1 d delta k plus.

So, you are subtracting 74 from 71, so if you do so then this term will come that dP 2 by d delta k you take common dP 2 by then it will be your dP 2 d delta k in bracket it will be your dP loss by dP g 2. Next term, if you take similarly for the second term also this way it will come up to your what you call up to the because it is up to m. So, this is up to dP m this is up to dP m. So, it will be your dP m by d delta k in bracket it will be 1 minus dP loss by dP g m. So, it will up to your mth term up to mth term. After that no other extra term is here, after that it will remain as it is. After that it will be dP m plus 1 by d delta k and so on I mean it will continue up to dP n upon d delta is equal to 0 that means, I mean that means, we are subtracting actually equation 74 from 71.

If you do so actually we get I have shown you how we are getting it. So, this way you are what you call we are getting this equation dP 1 upon d delta k plus d P 2 d delta k 1 minus dP loss dP g 2 plus up to dP m d delta k 1 minus dP loss by dP g m. Then up to the mth term is ok from here to here i mean from generator two to your what you call m mth one you will get like this plus in the equation 74, no other terms like that. So, it will be dP m plus one upon d delta k plus up to dP n upon d delta k that is equal to 0. So, this equation is 75 is per k is equal to 2, 3, n. Now, this is actually for your delta k then k is equal to 2, 3, n. So, when you put k is equal to 2, we will get one equation; when you put k is equal to 3, you will get another equation. This way you have your what you call that your n number of equation because it is going up to nth term. So, then what will happen that if you put them in or what you call in that this term if you bring it to that your right hand side that means, let me put it like this otherwise again you may have some problem.

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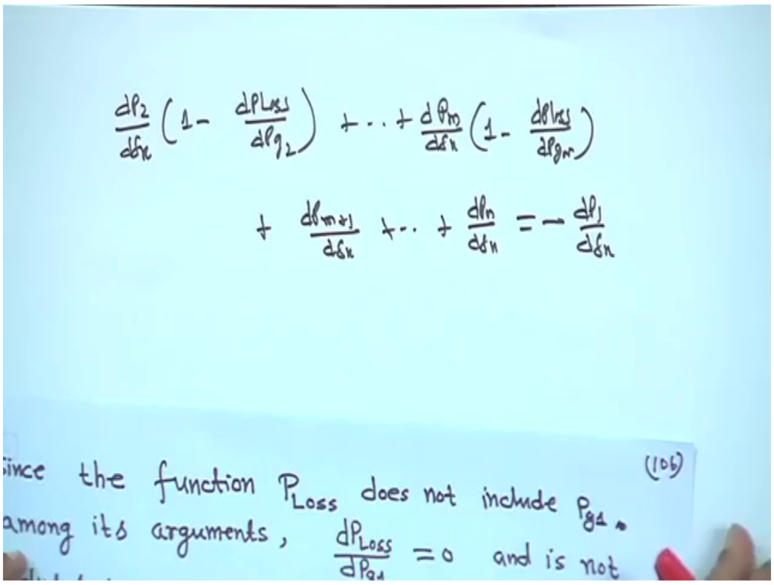

Suppose this equation is 75, this equation is 75. Now, this equation we can this term this whole this term is equal to we will write minus dP 1 upon d delta k. That means, what we will do we will write dP 2 d delta k that is 1 minus dP loss dP g 2 plus up to this one dP m d delta k in bracket 1 minus dP loss dP gm plus d P m plus 1 d delta k up to dP n d delta k is equal to minus dP 1 upon d delta k. I mean with all these terms all this term keeping to the left hand side is equal to a writing that is equal to minus dP 1 upon d delta l. And for k is equal to your what you call for k this is for k is equal to 2, 3, it will continue.

So, in that case what will happen k is equal to 2, 3 n, it will be continuing in that case you will get k put k is equal to 2, k is equal to 3, k is equal to n this way this way if you put, so will get sets up your non-linear equations. So, that means, in; that means, this equation if we want to put in matrix form this equation if we want to put in the matrix form that means, right hand side that column vector will be dP 1 upon d delta 2 dP 1 upon d delta 3 this way it will continue. So, and this one this term then dP 2 above d delta k up to dP m d delta k sorry up to dP n d delta k how we can relate to matrix form.

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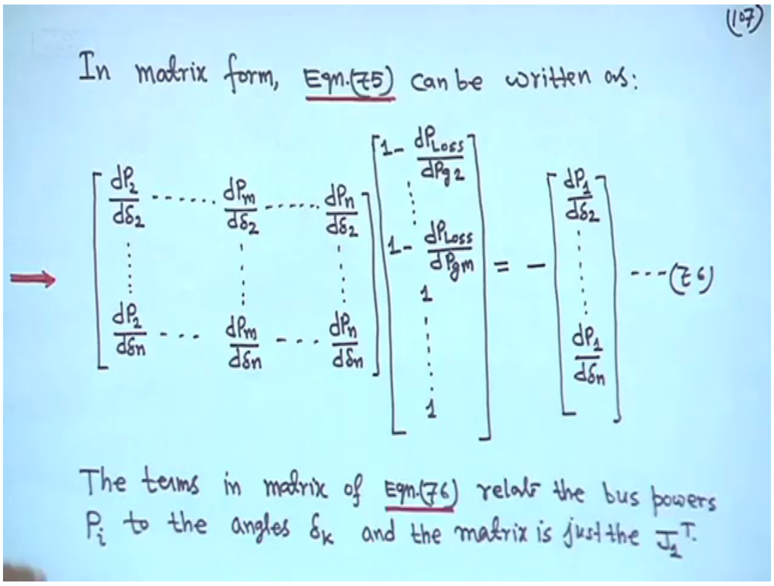

So that means, your this equation that means, this equation we can write that equation 75 this one can write that dP 2 d delta 2 up to dP m d delta 2 then dPn d delta 2, so that dP 2 d delta 2 its coefficient in 1 minus dP loss dP g 2 that is here 1 minus dP loss dP g 2. Similarly, it is next term if you come mth term if you come, this is a mth term because to you know you are to accommodate this say this I did not write all the terms, next one will be dP 2 upon d delta three and so on. Then it will come your dP m upon your d delta k if this thing.

So, when it is coming dP m upon d delta k terms, it is 1 minus delta P loss or dP g m. So, your 1 minus dP loss by dP g 2 then dot dot dot is come 1 minus dP loss up by dP g m. That means its next term will be 1 minus dP loss by dP g 3, then next will be 1 minus dP loss by dP g 4 and so on up to 1 minus dP gm term. After that there is all the coefficient attached with this dP by d delta k in general one only that is why this is 1, 1 1.

For example, that this one what you call this dP 1 upon d delta k taken to the right hand side, so that is so is equal to minus it is dP 1 upon d delta 2 dP 1 upon d delta 3 and so on then dP 1 d delta n is equal to minus of this one. I mean this term has been taken to the right hand side I showed you that means, this is actually it is where that Jacobian load flow studies we have done now. So, they are j 1, j 2 j 3, j 4 matrix is there actually this one basically it is your j 1 transpose.

So, this that means, last one is dP 2 d delta n up to dP m d delta n then dP n this over dP 2 d delta n dP m d delta n if you do like this up to a mth term it is fine after that is all coefficient. I mean dP m plus 1 d delta n dP m your m plus 2 d delta n up to dP n delta n these all are one. So, this coefficient attached here as long as up to a mth one and after that all these things are 1 1 1 1, I mean done and these terms are here only and and is equal to minus d P m d delta 2 like this.

So, that means, in equation they are what you call this equation 76 actually relates the bus powers P i two the angles delta k and the matrix is just the j 1 transpose the load flow studies we have seen j 1, j 2, j 3, j 4. So, this matrix actually it is j 1 transpose. So, that means, after the after load flow studies you know everything. So, easily one can compute this our object then this one actually this is actually reciprocal of the or what you call that penalty factor, penalty factor in generally we have taken L i is equal to 1 upon 1 minus dP loss dP g i. So, these are basically penalty factor so that means, we can if you know this this then easily you can have then take they are what you call you multiply by this j one transpose inverse on both side and then you what you what whatever the column vector will get you will pay a fine up to here to here term something will come. And after that your this one all will come to the hand side also 1 1 1 so that means that your right hand side of equation 76 can easily be obtained.

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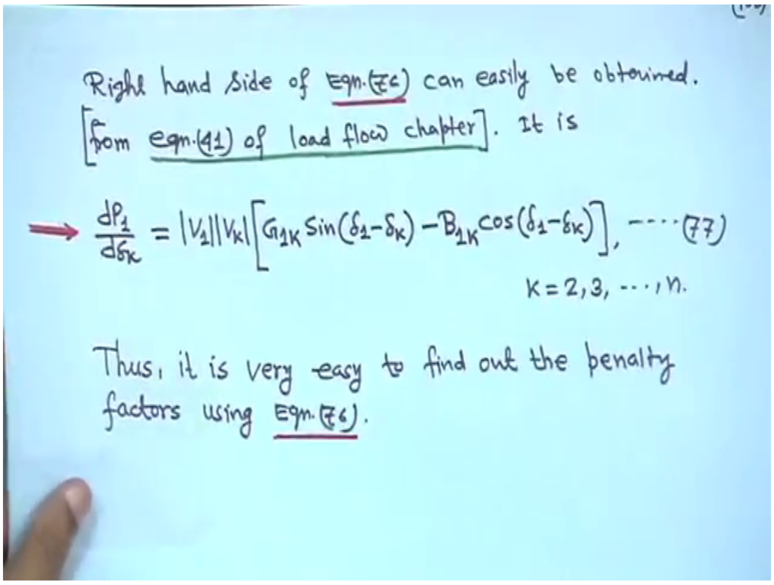

And from equation and this is actually your what you call equation 41 of the load flow topic that load flow we have studied their equation 41 is there. So, from that you will get this the equation 76 can easily be obtained the after the load flow studies. So, now and it is actually if you take the derivative of delta P 1 upon delta delta k that is in general this equation I mean this dP 1 this one dP 1 delta delta k which I have written here that in the right hand side. This is column vector minus dP 1 upon d delta 2 up to dP 1 d delta n.

So, this equation if you that if you equation 41 only the load chapter. So, is equal to mod v 1 v k the G 1 k sign your delta 1 minus delta k minus B 1 k cosine delta 1 minus delta k this is equation 77. And this is k is equal to 2, 3, n. Therefore, thus it is very easy to find out the penalty factor using your equation 76 that means this one. That means, from the load flow studies, after the load flow studies you know this because this is j 1 matrix and g 1 transpose. And right hand side also this dP 1 after the load flow studies all these parameters are known the v and delta all are known and G and B are line parameters that also you know.

Therefore, this is actually j 1 transpose and you multiply both sides by j 1 transpose inverse then what will happen this 1 minus dP loss upon dP g 2 then one minus dP loss upon dP g m up to this is equal to your minus j 1 your transpose inverse into this one. So, a column vector will come and from that you will find 1 minus dP loss dP g 2 1 minus whatever m number of generator this will find out and this 1 1 1 is there are know, in the right hand side also if there column we will see there will be 1 1 1.

So, already that means, this if you get then whatever value you will get its reciprocal will give you the penalty factor because penalty factor is equal to the reciprocal of this terms. So, this is actually that how to find out the penalty factor actually from your this thing using that chain rule of differentiation and you are from the load flow studies. That means, for this one evaluate this you have to have a load flow studies. If you know the load flow studies and if you know the loss formula P loss that loss formula, we will after finishing this will go to the loss formula how to get the P loss in terms of P g 2, P g 3 and so on so that will derive today.

So, these are the simple thing actually only thing is that little bit of practice. As far as classroom is concerned as well as classroom work is concerned only two bus example is possible or maybe three bus one is a slack bus. So, not more than two into two in the classroom exercise, very small example we can talk. If you take bigger example larger dimension size then without coding we cannot do that.

Thank you.