Power System Analysis
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Lecture - 48
Three phase fault studies (Contd.)

So, next is the type 3 modification; in this case an old bus j is connected to the reference bus, this is an old bus I told you I and j old bus.

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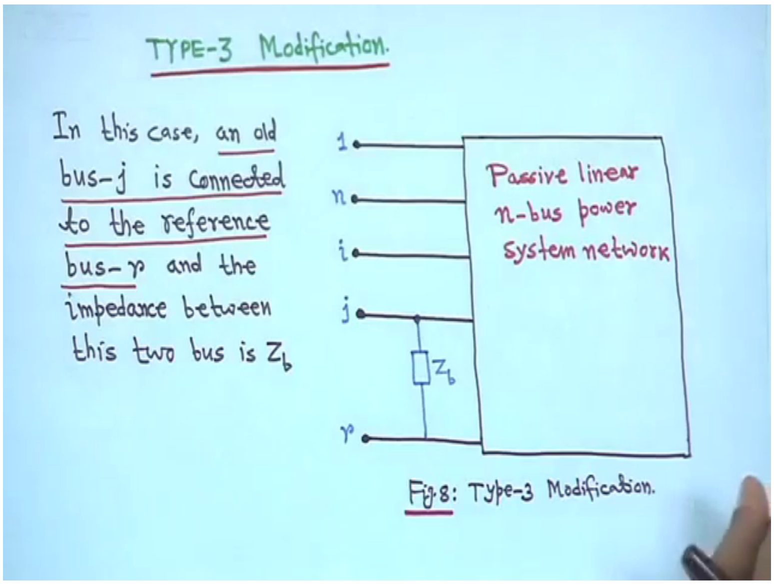

So, connected to your what you call to the reference bus r through an impedance Z b right. So, if you if we go back to this your I mean if we go back to that your figure 7 right.

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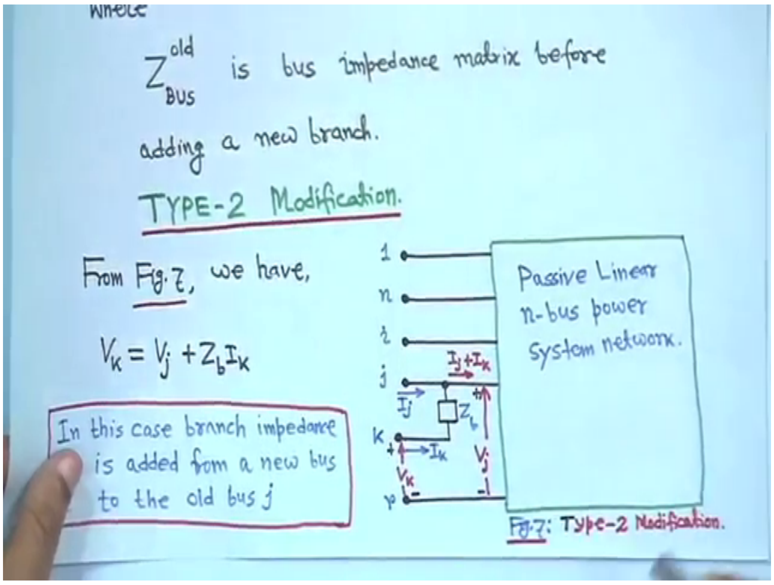

I mean this is your figure 7, it was it was shown that a new bus is connected to an old bus, but if you bring this per node connect this to this point right. So, in this case only Z b impedance will be there and in that case I k will be 0 right.

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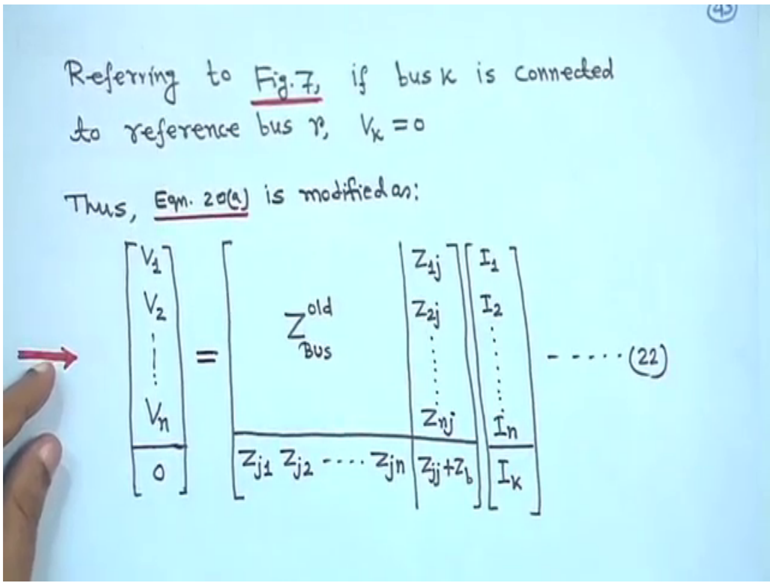

So; that means, that means that old bus connected to the reference bus. So, in that that is why I am showing to that figure your what you call 7 right if bus k is connected to reference bus r then V k will become 0 right sorry that in this case that this bus if you bring it then V k will be 0 right, so not the current. So, in this case your that seen equation 20 a that 20 a we rewrite everything only put I k is equal to 0 right because in this case that you are as soon as you bring this node 2 here right. So, no that is old bus connected to reference bus that means, there is no question of increasing the dimension of the Z bus matrix. So, in this case that your it is actually 0 right I k is 0 sorry V k is 0 and rest of the equation same from this equation what you have to do is, you have to find first that expression of I k. So, in this case this last row just hold on.

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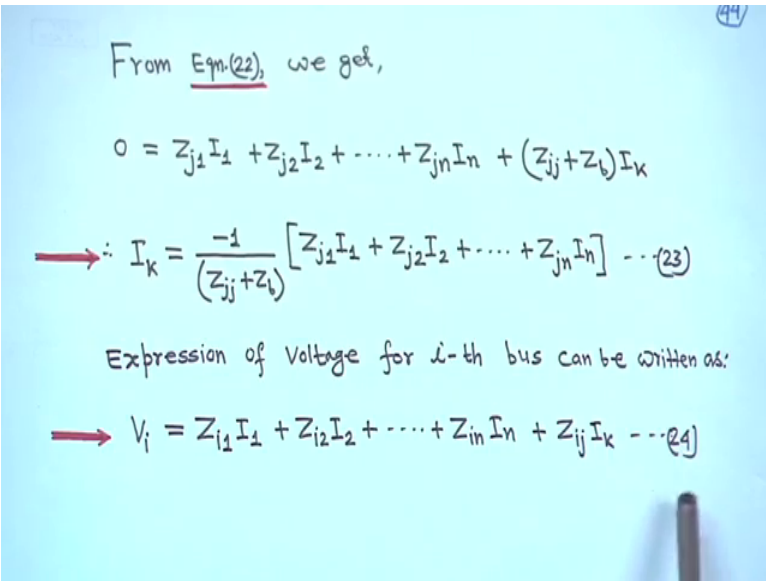

This last row can be written as that this one this last one can be written as that is as V k is 0 Z j 1 I 1 plus Z j 2 I 2 up to Z j n I n plus your Z j j plus Z b in to I k right. So, everything will remain same only V k will be equal to 0. From here you will get I k is equal to minus 1 upon Z j j plus Z b in to all this term right. So, expression of voltage for I th bus can be written as. So, suppose ith bus we write V i is equal to Z i 1 Z i 1 I 1 plus Z I 2, I 2 up to your Z i j I k right. In this expression here you substitute I k is equal to this expression you substitute right. If you substitute that is this is from equation 24 and 23.

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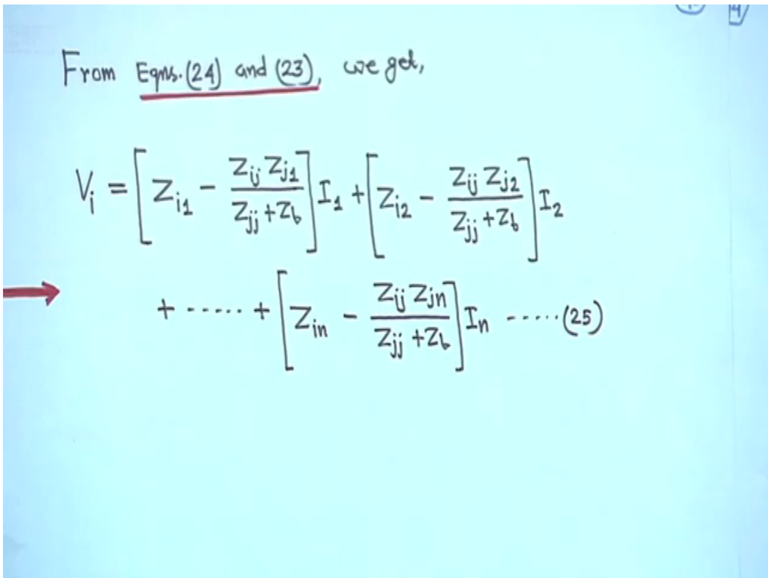

So, if you substitute then you will get your V i is equal to Z i 1 minus Z i j Z j 1 upon Z j j plus Z b in to i 1 plus Z I 2 minus Z i j in to Z j 2 divided by Z j j plus Z b in to I 2 up to n th term, Z I n minus Z i j in to Z j n upon Z j j plus Z b in to I n this is equation twenty 5 right. Now by inspection using this whole thing that this Z i 1 Z i 2 basically elements of the old j bus and these are the changes. So, if you put them in a you know from your intuition if you try to put them in mathematic matrix form, then Z bus new will become Z bus old minus.

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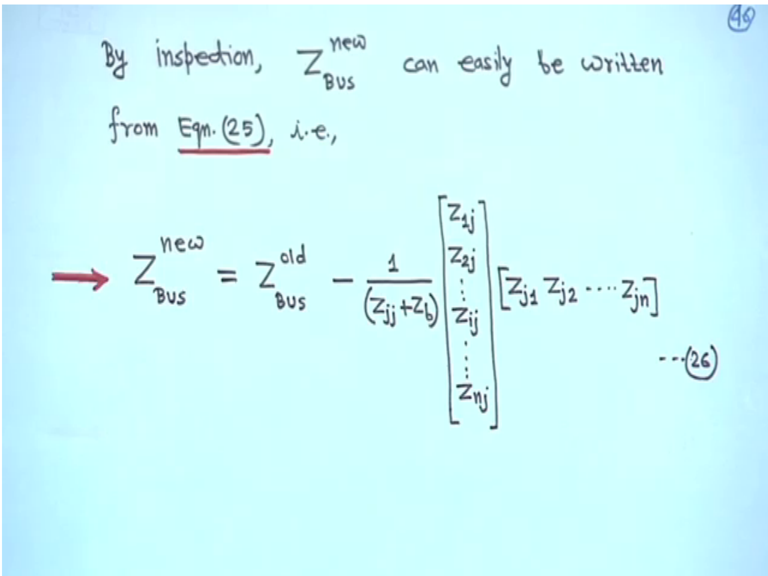

You have the term here Z j j plus Z b, Z j j plus Z b everywhere minus 1 upon Z j j plus Z b then you have the in general you have the product Z 1 j, Z 2 j up to Z i j then Z n j then Z j 1, Z j 2 Z j I up to Z j n right because this 2 product is Z i j, Z j 1, Z i j Z j 2, Z i j Z j n. So, this is call this is your column and this is your Z 1 j, Z 2 j, Z i j, Z n j and this is Z j 1, Z j 2 up to Z n

And this Z i 1 Z i 2 these are the element of the old Z bus matrix and this term this term this term have been written in this form such that computationally you can compute this way right and Z 1 j is equal to Z j 1 it is same symmetric matrix this is equation 26, this is actually type 3 modification right. So, little bit just little bit of practice right and in that case of type 4 modification here I will only give you the final expression and I will request you to derive this part right.

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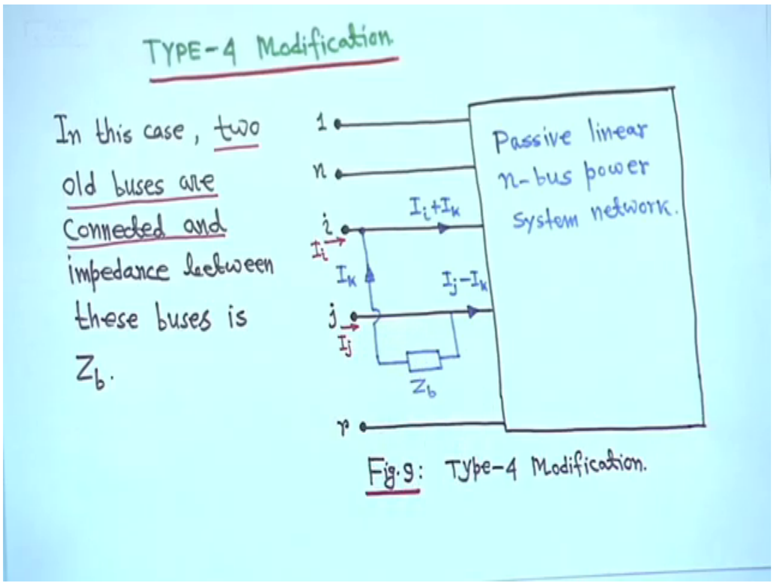

So, type 4 modification in this case old bus are connected your 2 old buses are connected and impedance between these bus is Z b. I bus and j bus these are 2 old bus and this impedance is Z b right. So, here direction of the current is taken here I k. So, here it will be Ii plus I k and here it will be I j minus I k and this is the passive linear n bus power system network. So, 2 old buses are connected through bus impedance Z b so; that means, in this case also that order of the Z matrix will remain same only it will be modified.

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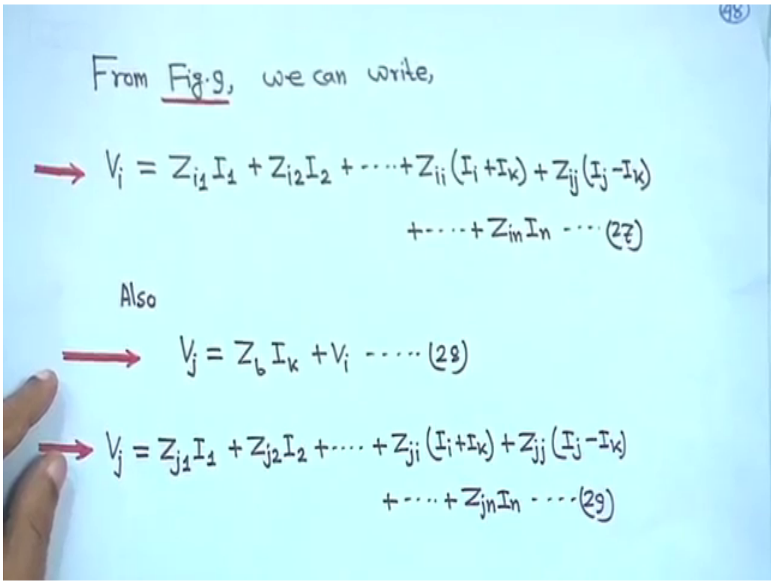

So, if you write the equation of V i using look at these diagram if you write the equation of V I, you can write V i or V i Z i 1 I 1 plus Z i 2 I 2 plus up to Z i i in to I i plus I k because current injection here I i plus I k right plus Z i j that is I 2 j here current injection is i j minus I k, that is plus Z i j i j minus I k up to Z I n I n and also that V j is equal to Z b I k plus your V i. So, in this case you just what you do you just apply your this thing k v l in this you draw this diagram like this right

I have to show that just hold on just hold on just hold on right this is your V i V j and this connected. So, this is your this is your bus I and this is your bus j right and this is a this is your reference bus voltage V i and this is your bus j, this is your reference voltage V j right.

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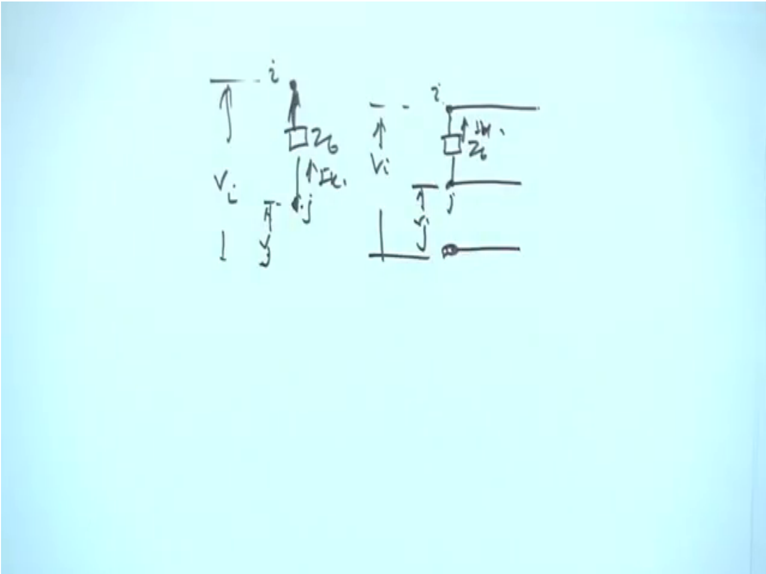

And this is actually connected your what you call by impedance this is connected by impedance Z b right, and your this direction of the current is taken as I k right. So, this is the simplification that how you can make it. So, if you connect here I mean if you draw this and put the Z b, then apply k v l then your v your this thing V j will become Z b I k plus V i right. So, that is why where it has gone just on just hold on just hold on 2 pages. So, this equation that equation 28 right that is your V j will be is equal to your what you call that your Z b I k actually it is simple thing this Z b this is your if you take this line like this if you write like this if you like this, this is your I this voltage actually it is V i right this is your reference say and this is your j this voltage is V j, and then you have i j you have the impedance Z b and this is your current direction is this way I k right I k. So, it will be actually Z b I k plus V i minus V j is equal to 0; that means, your V j is equal to Z b I k plus V i right. So, though this V j is equal to your Z j 1 I 1 plus Z j 2 I 2 plus Z j i I i plus I k right. So, Z j I that is your I i plus I k right plus Z j j i j minus I k plus your Z j n I n, because we have written V i is equal to all this things and V j is equal to your this expression.

Z j 1 I 1 plus Z j 2 I 2 plus Z j I i I plus I k, plus Z j j I j minus I k plus Z j n I n right. So, v; that means, this expression is equal to this one plus this expression. So, if you that means, this is V i is there, V j is there and this expression is this expression is there. So, from equation 28, 27 and 29 right.

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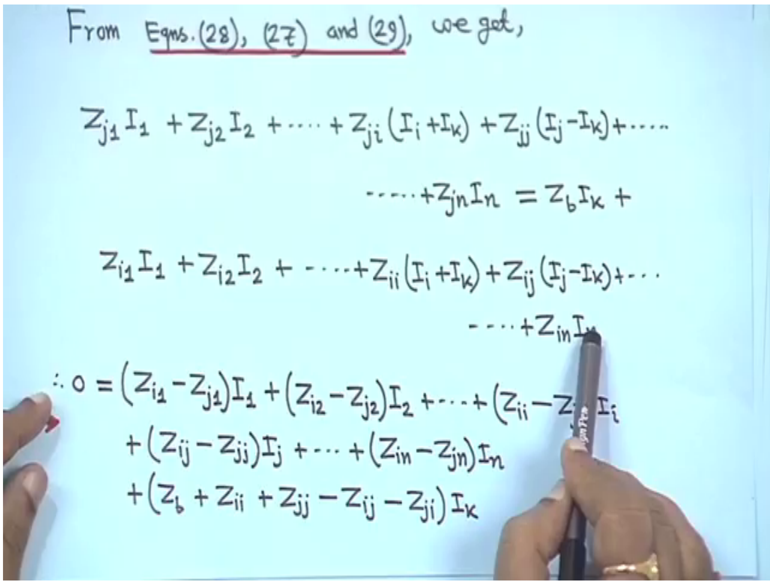

So, this is actually your that; that means, this one actually your this is your what you call that V j is equal to this is Z b I k plus V i, the whole expression then you 0 is equal to bring everything together left hand side you 0 is equal to bring everything in one side.

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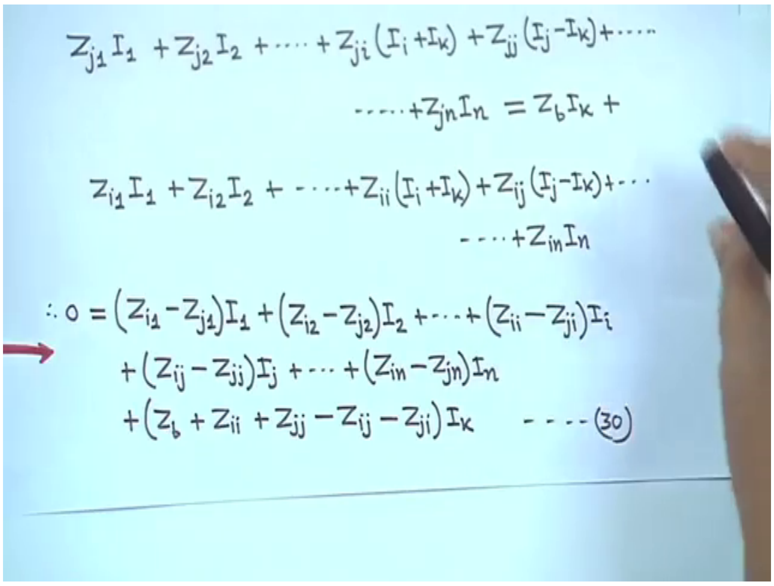

Then you simplify this equation will be coming like that, from here you get I k expression of I k this is a big expression, but you will get expression of I k in terms of I 1 I 2 I i, i j everything then in the expression of V i you substitute I k, and after that you will get that expression.

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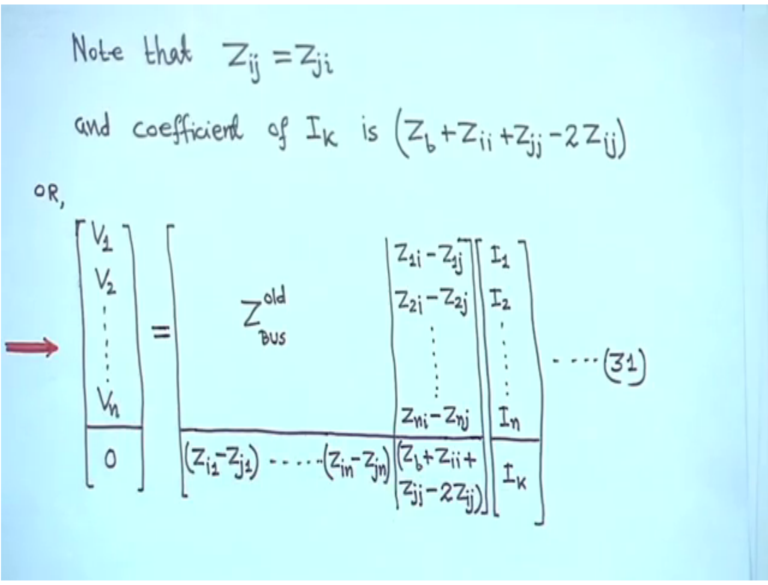

That your what you call that expression of your Z new Z bus. So, Z i j is equal to Z j i and coefficient of I k is Z b plus Z i i plus Z j j minus 2 your what you call Z i j, because in this expression Z i j is equal to Z j i. So, it will be minus 2 Z i j in this expression it is that is equation 30. So, and coefficient of I k is this one; that means, we can write same thing V 1 V 2 V n that as if V k is equal to 0 and this will be this 0 will be is equal to whatever will come this I k you have to find it out. So, this way it is written that 0 is equal to this whole thing right whole thing.

So, that is actually written here, after that you will from this equation only or from here you find out I k is equal to in terms of I 1 I 2 divided by this one right whatever way it comes and then if you do.

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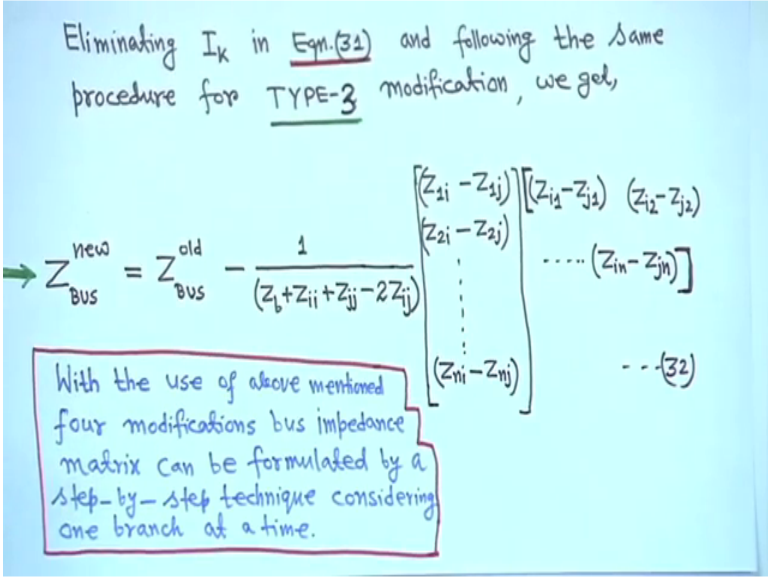

So, you will get some expression and from there this part you will derive right because this Z bus old will be there, but this modification will be there, it is it will become Z b plus Z i i plus Z j j minus 2 Z i that is the coefficient of I k I told you that you, I k you find out and put in the expression of V i then you generalize right. It will become Z 1 i minus Z 1 j Z 2 i minus Z 2 j up to Z n i minus Z n j right and in to it will be Z i 1, it is one I Z i 1 here it is Z i j minus Z j 1 same Z i 2 minus Z j 2 up to Z I n minus Z j n. So, this is actually type 4 modification.

So, there are 4 types of modification that for your question is first is your we have to say some old busses, new busses and the reference bus and from there we have to start the construct your forming that Z bus matrix. So, this is actually we have assume some old bus, new bus and reference bus, if some problem is given suppose some problem is given you have to start from the scratch then how one can start. Reference bus is given and any bus you start from the scratch means that bus will be new bus, once a bus is already consider means that bus bar will be an old bus right. So, this expression of this type 4, type 3 looks like little bit of you know huge mathematics, but you have to do it you know systematic way. In any way will take for this one simple example.

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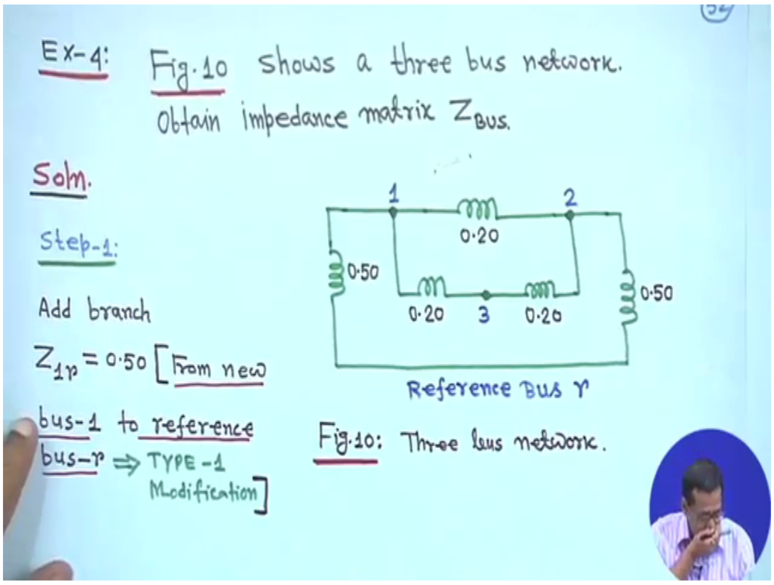

For example this figure ten that is shows a 3 bus power network where reference bus is there bus 1 to 2 it is given 0.2, j and other thing not putting an again and again, but ultimately j is there, but understandable, which is also bus 3 is also here this is bus 3 this is 0.2, 0.2, 0.5 0.5 right sorry. So, now, we have to start from step one because here no Z bus old is there we have to start from the scratch look how simple it is now we have to do add branch Z 1 r that is 0.5 per unit say; that means, from new bus one to reference bus one because anything nothing is there. So, any bus you are considering it is a new bus first right. So, one 2 reference bus only single element. So, it is a type one modification; that means, your initially and this once this bus one is considered as in considered already right. So, next step this bus one you will become an old bus because already you have considered that one, so only one element.

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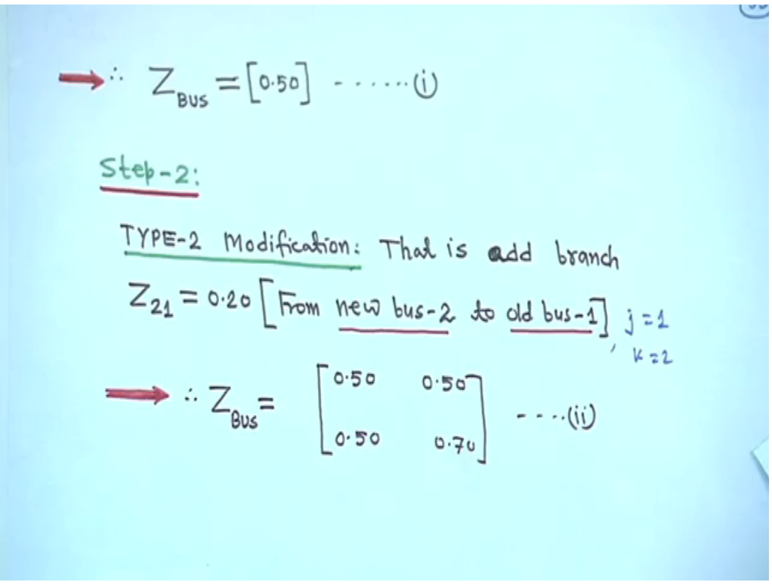

So, what will do Z bus first you start 0.5 right because 1 to bus 1 2 are the reference bus it is 0.5 right next step to type 2 modification, that is add branch Z 2 1, that is from new bus to old bus one because already this bus one we have considered previously. So, this bus will become old bus, now this bus is a new bus. So, and you are from an old bus to a new bus. So, if you add this old bus to a new bus. So, it will become Z bus will become 0.5, 0.5, 0.5, 0.7 that how it is coming. So, if we if we just hold on let me bring that that this thing that that expression just hold on right just let me find out just 1 minute every everything is mixed up just hold on.

If I tell from my mouth you may not understand right. So, just hold on if I find it here it is fine just hold on just hold on where it has gone just hold on. I will show you that formula from that you can do it just hold on right where it has gone strive to this is actually strive to modification, 40 just come back where it has gone q r this is 49 47 anyway I am not where it has gone I am here, here I have I have got it here right.

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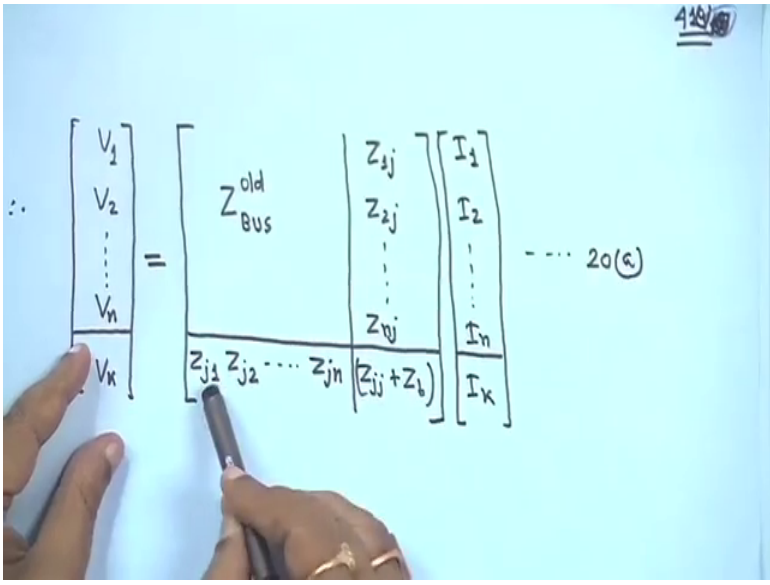

For example this one right this is actually type your this thing 2 modification. So, when your this if this old one to new one you are adding. So, this will be actually this is a 2 in to 2 matrix. So, Z when j is equal to 1, this is j is equal to 1 and your k is equal to 2 right. So, in this case when Z is j is equal to 1 it will be j 1 1 Z 1 1 rather right and this is actually here also it will be Z 1 1. So, Z 1 1 the previous one it is actually 0.5 right. So, that one your Z bus first one we got this is 0.5 this is actually Z 1 1. So, it will be also Z 1 1 it will be also Z 1 1. So, point because it is only 3 bus problem. So, j is equal to one and old bus it is also Z is equal to j is equal to one old bus and then this is Z j j plus Z b. So, Z 1 1 because j is equal to 1, and k is equal to 2 and this is the branch impedance we are adding 221.2; that means, it will be 0.5 plus 0.2. So, 0.7 this is type 2 modification. So, this is actually Z bus when we are adding new bus to old bus right.

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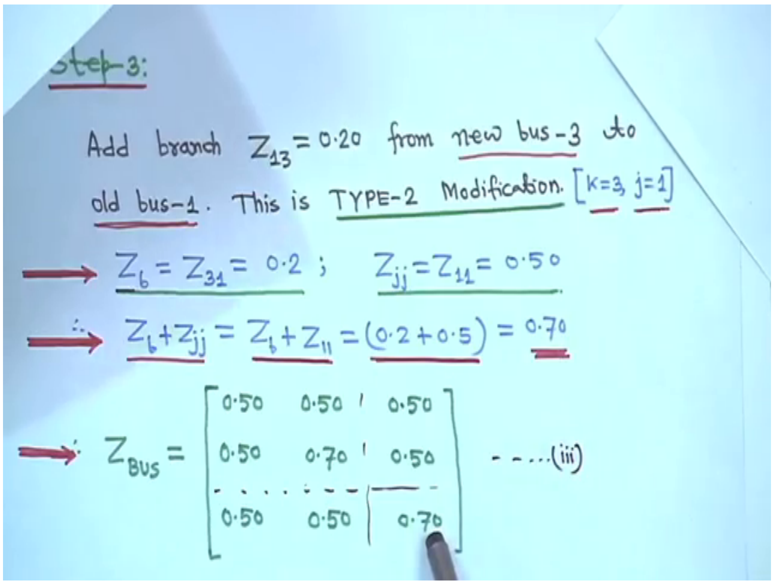

Next is your this step 3, in this case add branch Z 1 3 your from new bus 3 to old bus one this is also type 2 modification; that means, you add bus 3 to a new bus one new or this new bus 3 to old bus one in this case what will happen that new bus K is equal to 3 and j is equal to 1 this way you will do it. And Z b is equal to Z 3 1 is 0.2 because 3 to 1 0.2 and Z j j that is 11 because j is equal to 1 is 0.5 so Z b plus Z j j that is Z b plus Z 11 0.2 plus 0.5 that is 0.7.

Using this same expression that when j is equal to 1; that means, Z 1 2 when j is equal to one Z 2 1. So, 0.5, 0.5, 0.5, 0.7 it was there that this matrix is there in addition to that this is also coming 0.5, this is also coming 0.5, and Z b plus Z j there is 0.2 plus 0.5 this is 0.7 this is also type 2 modification I mean next step because you have to consider in this network you have to consider all the your nodes right. Under reference bus reactance impedance also so that means, here when j is equal to 1 because dimension is increasing. So, k is 3 j is one. So, this already we have got when j is one Z 1 2 and j 1 your Z is 1 Z 2 1. So, this equation Z 1 2 and Z 2 1 0.5, 0.5 so, here also it is 0.5, 0.5, 0.5 0.5 earlier it was same thing 0.5, 0.5.

And this one Z your Z j 1 one Z 1 1 plus Z b that is your this is your what you call coming at 0.7. So, here it is 0.7. So, here it is also 0.7 here it is also 0.7 right and similarly if you put Z j is equal to one, you will get all this thing Z 1 1, Z 2 1, Z 1 1, Z 1

2 0.5, 0.5, 0.5, 0.5 this is type 2 modification still something is left next one is that is step 4 this is step 3.

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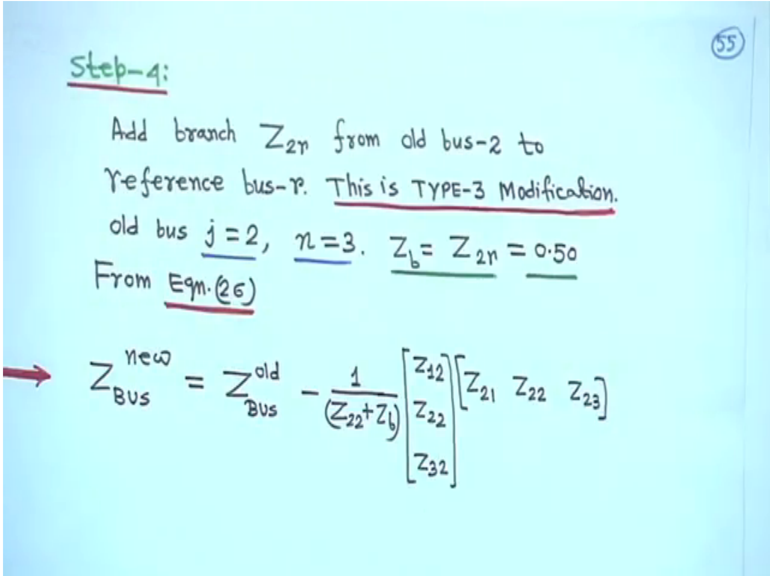

Now step 4; step 4 is that add branch Z 2 r from old bus 2 to reference bus r this is old bus 2 and this your reference bus r. So, Z 2 are actually 0.5. So, in that case this is type 3 modification right. So, bus in that case old bus j is equal to 2 because already bus is considered. So, that bus will never get new bus it is an old bus bar. So, j is equal to 2 and n is equal to 3, and Z b is equal to Z 2 r that is 0.5. So, Z bus new is equal to Z bus old Z bus old means this one is the step 3 calculation, this is your Z bus old right minus 1 upon Z 22 plus Z b as for your as per your equation 26 that is a huge equation, but it is only 3 bus problem it will become one upon Z 2 2 that is Z j j plus Z b j is equal to 2. So, Z 2 2 plus Z this is Z 1 2, Z 2 2, Z 3 2 and this is Z 2 1, Z 2 2, Z 2 3 only from this equation. So, due all this Z 1 2, Z 2 2, Z 3 2 all this things are known from this your step 3 calculation right all this things are known.

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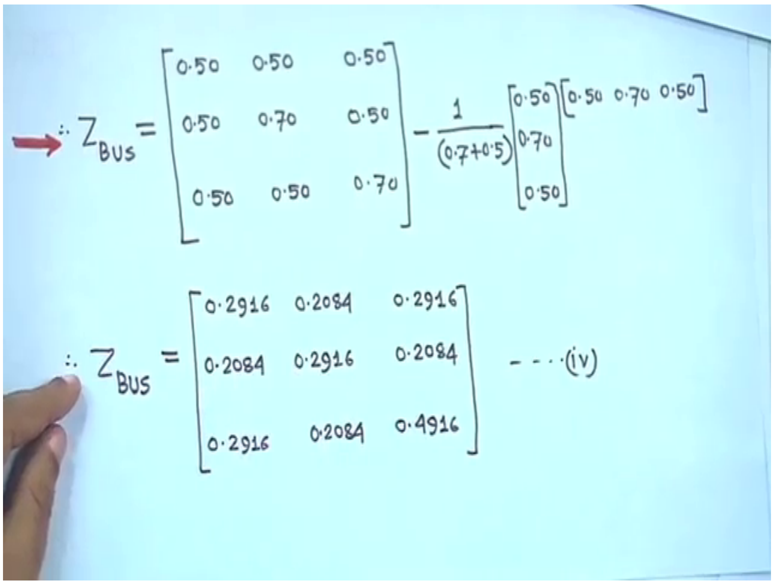

So, in this case if you compute Z bus new then it will become your just hold on. So, it will become your this one this one. If you do so, this Z bus all computation you have to make it will become Z bus your this thing, but still it is not completed one more branch is there that is step 5 in this case at branch your 2 to 3 because that is not added step 5.

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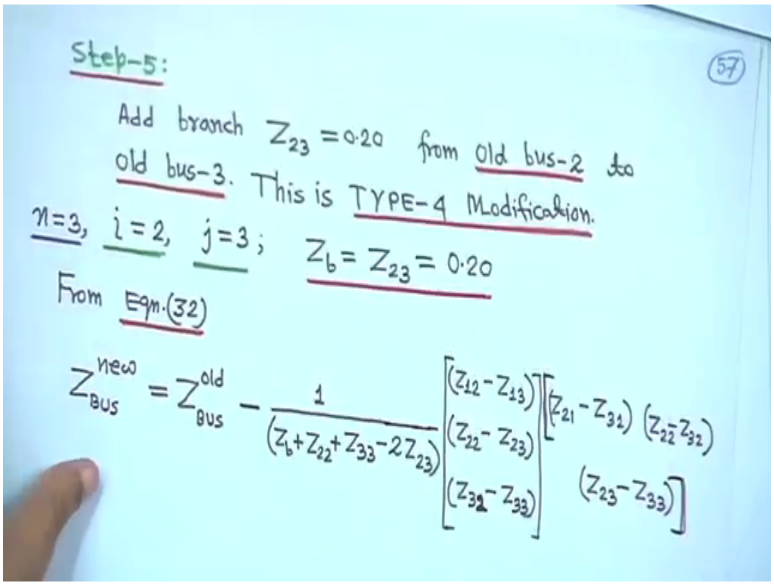

So, add branch 2 to 3 that is Z 2 3, that is from 2 to 3 you add right 2 to 3 you add. So, in that case your old bus 2 to old bus 3 because this bus 2 and 3 already you have considered. So, this are actually bus 2 and bus 3 are the old bus right to your old buses.

So, in that case it is a type 4 modification. So, i n is equal to 3, i is equal to 2, j is equal to 3 and Z b is equal to Z 2 3 0.20. So, equation 32 if we write like this, this Z bus old minus 1 upon Z b plus Z 2 2 plus Z 3 3 minus 2 in to Z 2 3 and Z 1 2 minus Z 1 3, Z 2 2 minus Z 2 3, Z 3 2 minus Z 3 3 in to Z 2 1 minus Z 3 1, Z 2 2 minus Z 3 2 and Z 2 3 minus Z 3 3 right. So, from equation 3 2 you can write this equation and from Z bus old means previously whatever we have obtained that will be always Z bus old it is something like iterative process right. So, once you substitute all this values and this all this Z elements will come from this previously computed one from here only.

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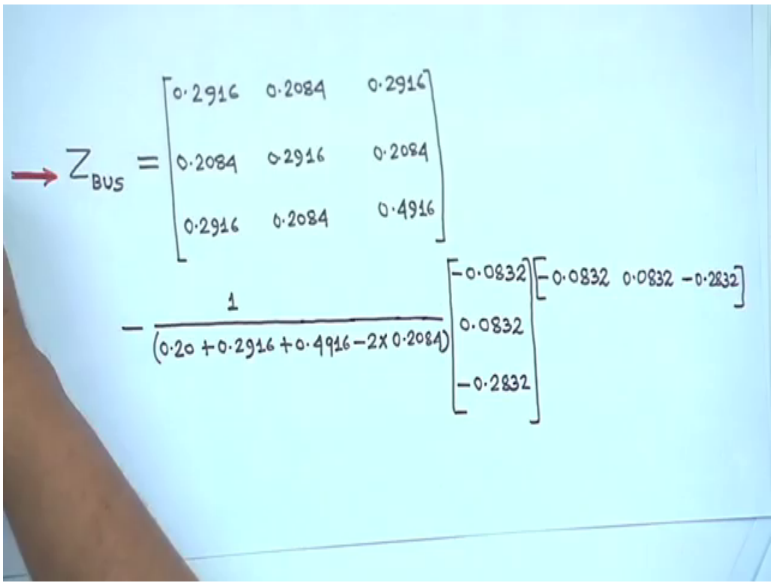

And if you substitute all then Z bus will become this one, minus whatever parameters are coming you put right then you see what you call you simplify. If you simplify then Z bus finally, I am putting a j here throughout I did not put again and again.

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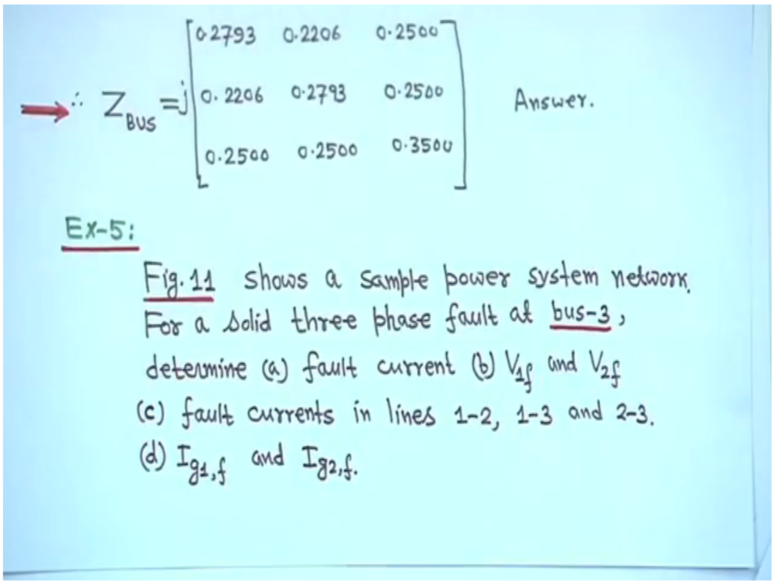

So, this is your 3 in to 3 Z bus matrix this is actually answer. So, computationally it is I mean it mean or your what you call that computer thing is required right and coding is require, but in a this problem also this problem for 3 bus problem it will take it also takes time right. So, because lot of your computations are involved so, but this problem can be simplified right because here one 2 reference bus this is a reference bus 2 r is given right this way.

So, if we give it that is suppose 2 bus, bus 1 and 2 and only one reference bus suppose this thing is not there suppose there is one 3 and 2 3 is not there for you only one 2 is there one 2 reference bus and 2 two reference bus as an exercise you can compute what will be the Z bus matrix, because as far as this is just to show you that how to do it right that lot of pages are there sometimes getting mixed up. So, this is your what you call that Z bus formulation algorithm right. So, and your, so another small example you will take that is your figure one that is shows a sample power system network for a solid 3 phase fault at bus 3. So, you have to determine in a fault current then V 1 f and V 2 f and c fault currents in lines 1 2, 1 3 and 2 3 and d I g 1 f and I g 2 f.

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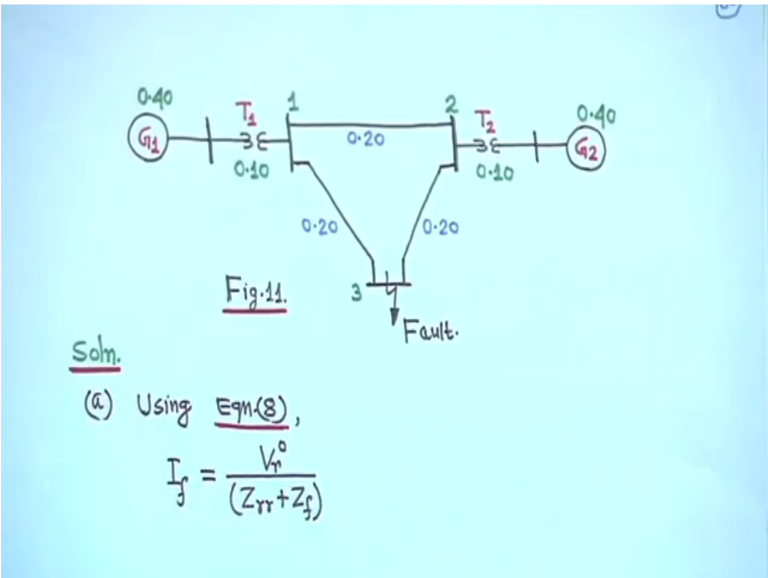

So, this is the diagram right and transformer reactance is given 0.1, generator 0.4, 0.4, but if you look at the 1 2 3 that 0.2, 0.2, 0.2 this is actually 0.2, 0.2, 0.2 right same data taken right and you have to find out fault currents and your and the other thing that line where fault line fault currents and I G1 f, I 2 f and your V 1 f and V 2 f. So, using equation 8 this one you can write I f is equal to V r 0 upon Z r r plus Z f fault has occurred at bus 3; that means, r is equal to 3 that is faulted bus. So, in this case this is 1 2 3 right. So, thevenin passive network for this system is shown in figure 10 that; that means, this figure; this figure10 because here it is generator is 0.4 and this is 0.1. So, 0.5 here you know 0.1 and 0.4 it is 0.5 right and when fault has occurred this generators that all that is ground is the reference, because yes fault has occurred. So, basically this is 2 if you add this two 0.5, if you add this two 0.5 and ultimately this is actually equivalent this network equivalent is this one and this Z bus is its Z elements will be used Z matrix will be used. So, this one this Z value will be used from this one only right.

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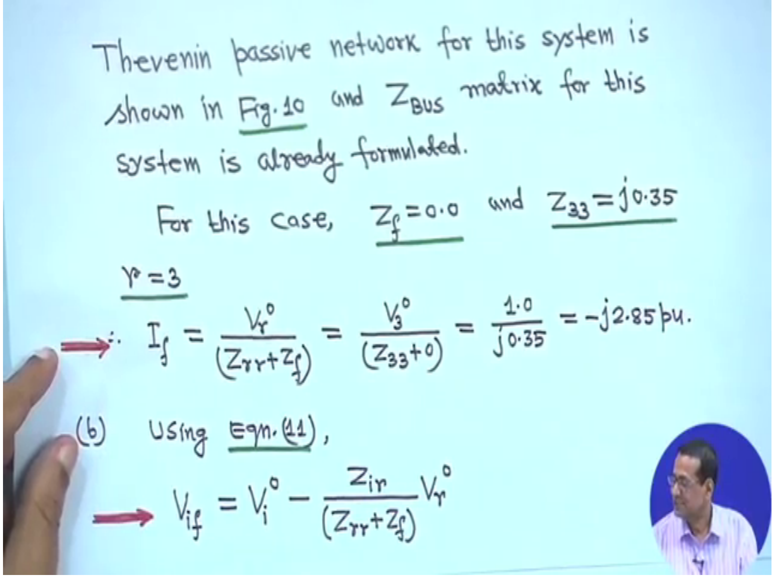

Whatever we have got this this Z because similar identical one this Z will be used. So, question is that for this kind of problem if it is given, I mean if you solve generally we supply that Z bus matrix.

Otherwise you know one can you know in the one cannot solve this one cannot it will take more time to get this your what you call Z bus matrix generally Z is supplied Z will be given. So, now, that I f is equal to V r 0 upon Z r r plus Z f. So, this Z 3 3 actually faulted bus is this 1 bus 3. So, it is your Z 3 3 and Z f fault impedance it is a solid fault. So, Z f is 0. So, I f is 1 upon j 0 0.35 that is Z your Z 3 3, that is minus j 2.85 per unit. This Z matrix we are using from that previous example. So, using equation eleven that is V i f is equal to this already you have derived is equal to V i 0 minus Z i r upon Z r r plus Z f Z f in to V r 0.

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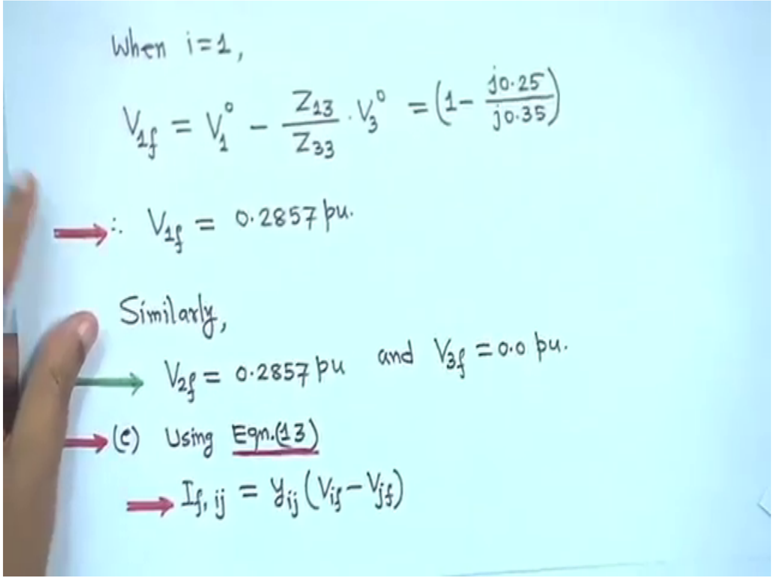

So, when i is equal to 1 you can calculate V 1 f is equal to this much 0.2857 similarly for I is equal to you calculate you will get 0.2857 per unit and fault has occurred at bus three. So, V 3 f is equal to 0. Now equation using equation 13, so I f i j is y i j V i f minus V j f this is general thing right now I is equal to one j is 2 all you substitute. So, in this case you all substitute all this values known to you right.

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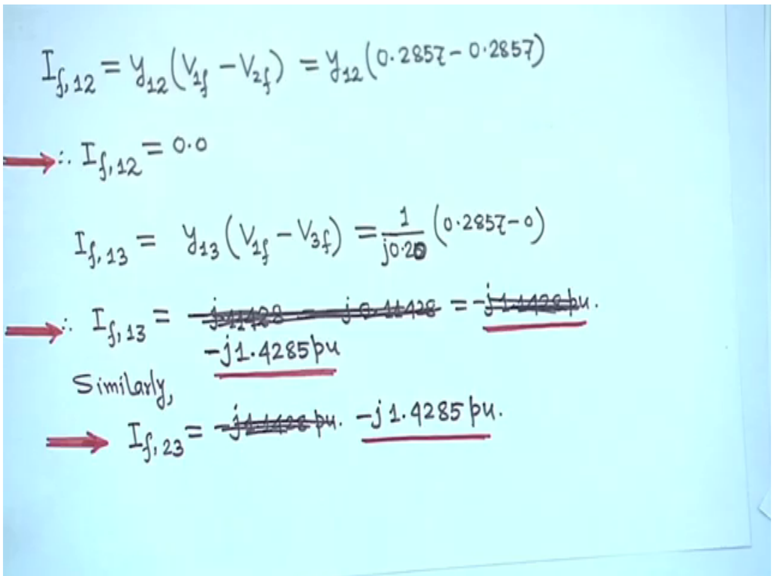

So, you can get I f 1 2 will get 0 all are calculations given I f 1 3 calculate minus j 1.4285 per unit, similarly I f 2 3 you calculate you will get minus j 1.4285 all you will put right at last one is that your using equation 14 this is the last one right.

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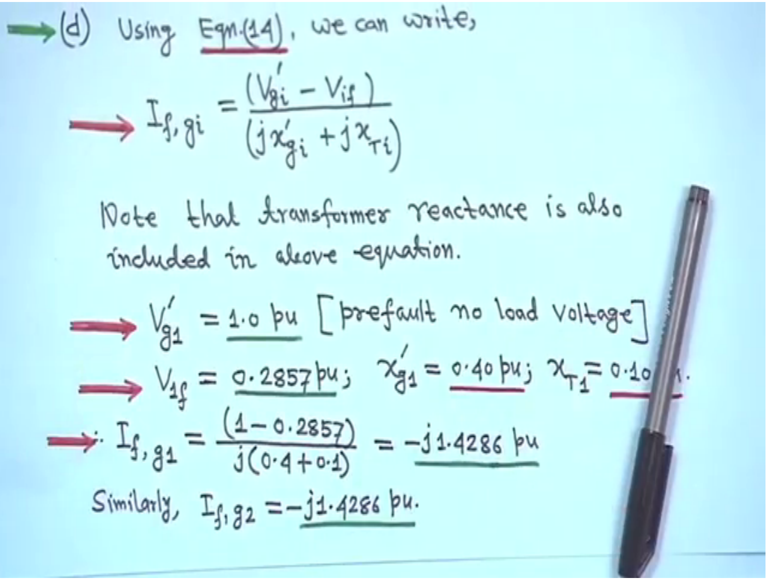

So, I f g I is equal to v g i dash minus V i f divided by j x g i dash plus j x T i. So, note that transformer reactance is also included in the above equation right. So, V g 1 dash is one per unit pre fault no load voltage V 1 f is calculated, x g 1 is given x g 1 dash 0.4 is given x t 1 is given. So, I f g 1 you can easily calculate substitute here all this values you will get minus j 1.48 sorry 4286 similarly I f g 2 also minus j 1.4826. So, in this case actually. So, 3 phase fault whatever I mean we have taken some critical thing and particularly Z bus building algorithm. So, step by step your what you call you have to formulate and couple of examples also that consider has been shown to you that how to compute the fault voltages and fault your line fault current. So, and we have shown that how to go for Z bus building algorithm.

So, in that case what you call that just step by step you have to move, but lot of computation is involved. But whenever problems are given numericals and other things generally Z bus will be provided otherwise very in the classroom it is not possible to compute Z bus without your computer, but one thing I can tell you that Z bus coding actually is not difficult it is quite easy. But my suggestion to all of you when you will practice this particularly the Z bus building algorithm just have patience and make calculations on just for a simple 3 bus problem with reference bus we have seen that how difficult it is even here also I have so, many papers it is getting mixed up right. So, thank you very much for listening this.

Thank you.