Power System Analysis

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Lecture – 49
Symmetrical components

So next in that previous topic we have seen that your that 3 phase fault, but in that case our objective was mainly for z bus building algorithm and few numericals we have solved right, but question is that in the z bus that we have to have z bus matrix, so in anyway. So, today we will start the other thing that is your symmetrical component because symmetrical component is more important for your our fault studies, because most of the fault actually unsymmetrical or all unbalanced type of faults right therefore, symmetrical your what you call that symmetric unsymmetrical a symmetrical component will be that your is easier, I mean will be applicable for such kind of fault analysis.

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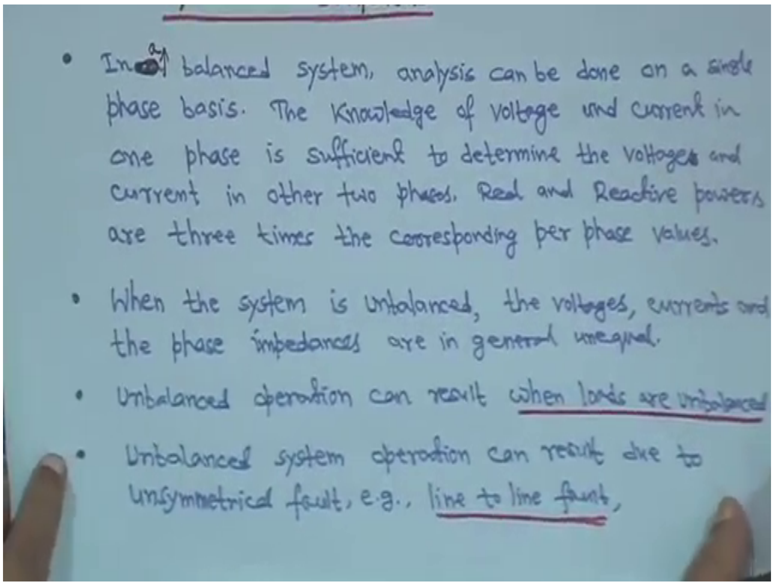

So, let us come to that your symmetrical component. So, generally in a balanced system analysis can be done on a single phase basis that we have seen also for three phase fault. The knowledge of voltage and current in one phase is sufficient to determine the voltage and current in other 2 phases because it is a balanced system. Real and reactive powers are three times the corresponding per phase values. So, for that is for balanced system, but when the system is unbalanced, the voltages, currents and the phase impedances are in general unequal. So, unbalanced operation can result when loads are unbalanced, this is one thing. So, loads are unbalanced means that I mean all the three phase loads may be different, and this is more your what you call more applicable to the your distribution low voltage distribution system for example, in our country say 11 k v distribution system were or are in other parts of this world.

So, sometimes what happened that in three phase distribution loads are not balanced, this is one thing second thing in many places that from the same substation same feeder right single phase 2 phase or 3 phase loads are supplied. So, in that case your that things are become system become unbalanced and another aspect is that unbalanced system operation can result that is due to unsymmetrical fault.

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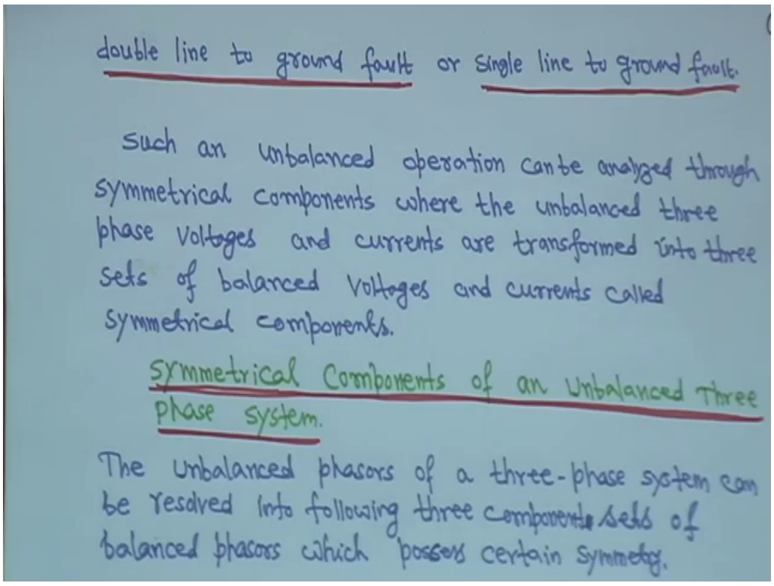

For example, that your line to line fault that is called l l g right then double line to ground fault or single line to ground fault right. Such an unbalanced operation can be analyzed through symmetrical components, where the unbalanced three phase voltages and currents are transformed in to 3 sets of balanced voltages and currents called symmetrical components. So, another type of fault happens, but very rare that is your conductor opening suppose one phase conductor your opening means cut right. So, these are very rare, but question is that is also called open conductor your fault, but that we will see when we will go for unbalanced or unsymmetrical fault analysis. So, symmetrical components of an your unbalanced three phase system right this one. So, the unbalanced phasors of a three phase system can be resolved in to the following three component sets of balanced phases, which possess certain symmetry; that means, for unbalanced phasor voltage or current right can we can dissolve in to following three component sets of balanced phasor and it has some symmetry that means.

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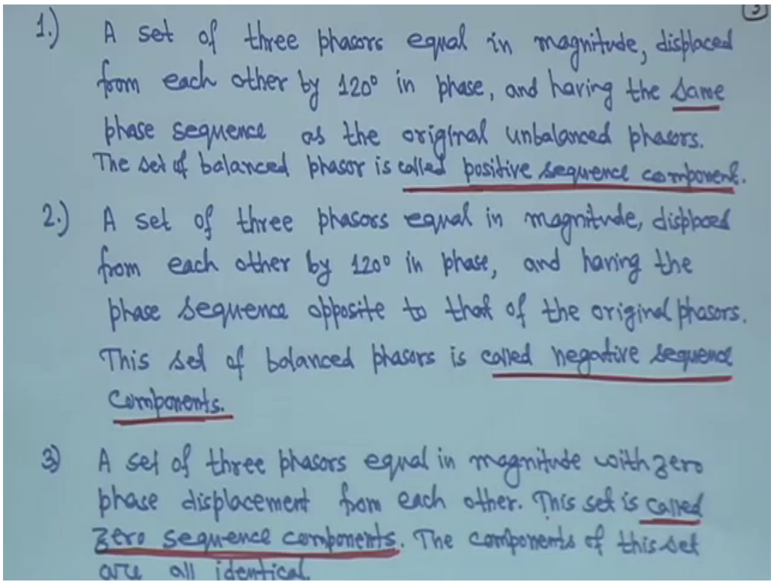

First thing is that is set of your three phasors equal in magnitude, displaced from each other by 120 degree in phase and having the same phase sequence as the original unbalanced phasors. And this set of balanced phase that is called positive sequence sequence component. So, a set of three phasors equal in magnitude, but displaced from each other by 120 degree. The way we have solved the balanced system right and your and the; or the original unbalanced phasor this set of balanced phasor is called positive sequence component. Similarly a set of three phasors again equal in magnitude displaced again from each other by 180 degree or sorry 120 degree in phase, and having the phase sequence opposite to that of the original phasor I mean if for this one for positive sequence if it is a b c say for this one opposite it will be a c b sequence right and this set of balanced phasor is called negative sequence component.

And last one that is a set of 3 phases they are equal in magnitude with 0 phase displacement. There is no phase displacement between them, but they have equal magnitude from each and this set is called 0 sequence component. So, the component of this set are all identical, I mean they all are identical because they have equal magnitude.

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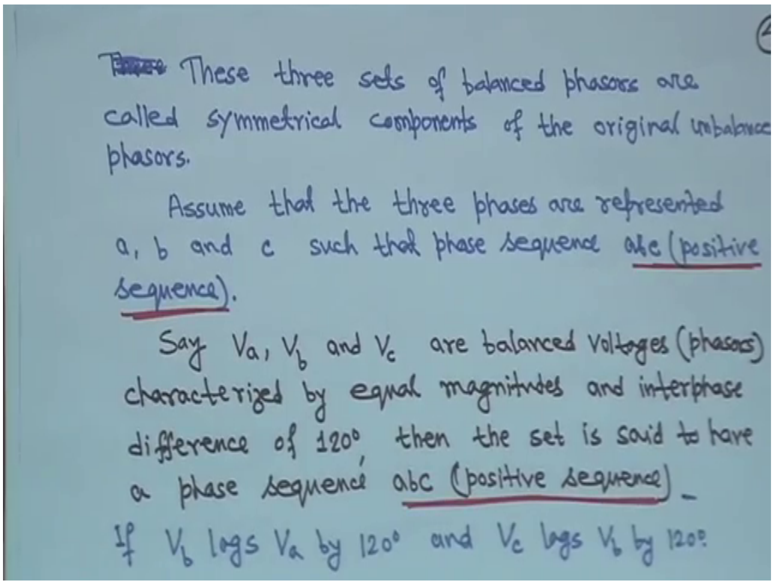

So, therefore, these three sets of balanced phasors are called symmetrical components of the original unbalanced phasor. So, assume that the three phasors are represented by a b c and such that phase sequence a b c is called positive sequence. So, this is this you know, this you know already we have done it right, this that three phase system. So, this is a b c. So, if a b c sequence this is actually positive sequence.

Now, say V a V b V c are balanced voltages that is phasors, characterized by equal magnitude and interface difference of 120 degree this also you know. Then the set is said to have phase sequence a b c or positive sequence right, that is whatever we have studied little bit for solving say 3 phase transmission line or these that almost all are a b c all the a b c your what you call phase sequence. So, if V b lags V a by 120 degree, this you know and V c lacks V b by 120 degree that also you know, then we can make this kind of your definition.

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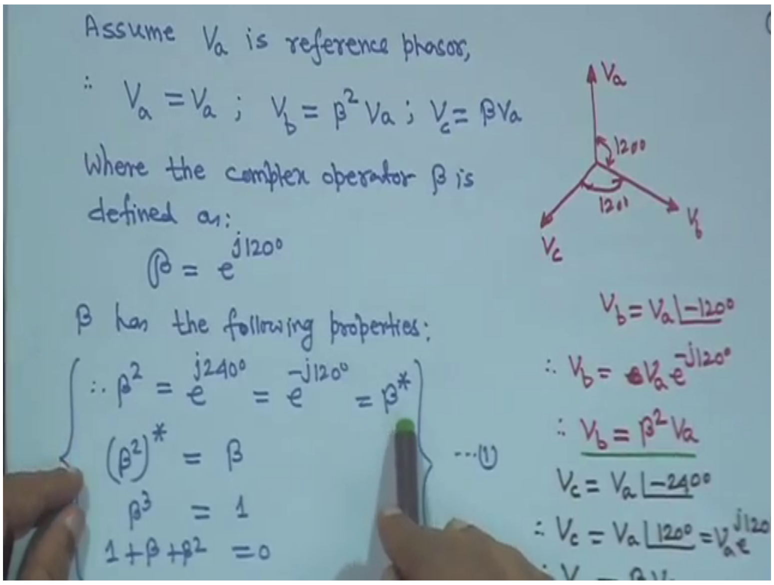

That is that assume your V a is a reference phasor. Suppose if we put like this say V a is reference phasor. So, V b lags from V a here by 120 degree and V c lags from V a by 240 degree or otherwise V c leads V a by 120 degree. So, if we if I define like this say V a is equal to V a, a reference phasor V b is equal to beta square V a I am coming to that and V c is equal to beta V a right. Where the complex power a complex operated beta is defined as we have taken beta is equal to actually angle 120 degree. So, e to the power j 120 degree; that means, this beta has the following properties. Beta square beta is e to the power j 120 degree, then beta square e to the power j 240 degree is equal to it is e to the power minus j 120 degree that is actually is equal to beta conjugate. Similarly beta square conjugate is equal to beta. If you take beta square this conjugate that will become beta and beta cube is one because beta q means e to the power j 360 degree that is cos 360 plus sin 360. So, it is one right and 1 plus and this ideal this also holds for my beta is equal to e to the power j 120 degree, then 1 plus beta plus beta square is equal to 0 this conditions will be applicable now right. Now in this case that V b is equal to V a angle minus 120 degree here this figure.

That means V b is equal to V a e to the power minus j 120 degree we can write like that therefore, V b is equal to we can write beta square V a because beta square is equal to e to the power j 240 degree is equal to e to the power minus j 120 degree. So, we can write V b is equal to beta square V a that is why we writing here V b is equal to beta square V a. Similarly V c is equal to V a angle minus 240 degree because these V c lags from V a that is 240 degree. So, V c is equal to V a angle minus 240 degree that is we have given that your this V c is equal to V a otherwise V c lags from here this thing or V c leading V a by 120 degree. So, same this can be written as V c is equal to V a angle 120 degree that is V a e to the power j 120 degree, that is V c is equal to beta V a; that means, we are writing here V c is equal to beta V a. So; that means, that means this V a V b V c this V b V c we are putting in terms of a complex operator beta, where beta is equal to e to the power j 120 degree.

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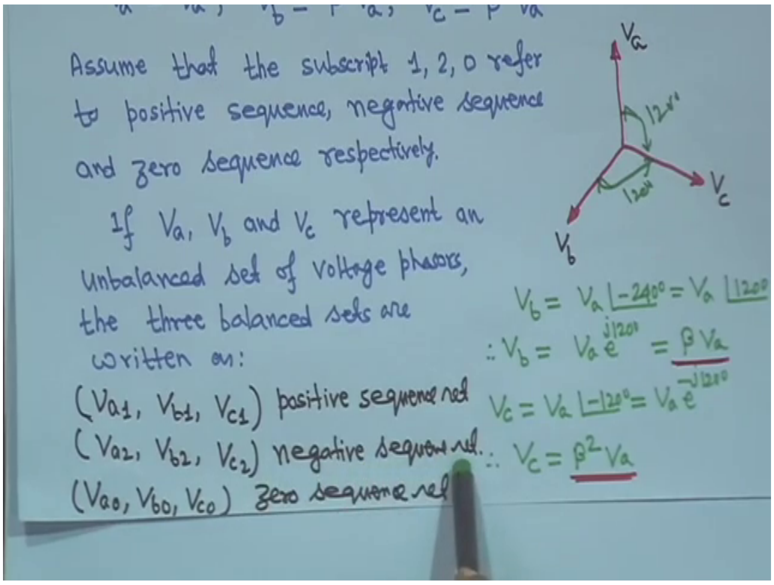

Next is if the sequence is a c b I mean it is if it is a negative sequence, right then V a is equal to V a now V b will become beta V a and V c will become beta square V a. Now sequence is a c b; that means, V c lags from V a by 120 degree this is your 120 degree and this one also your 120 degree. So, and V b lags from V a by 240 degree. Just whatever we have done previously it was beta square now it will beta it was beta now it will be beta square therefore, V b is equal to that is it is lags from V a by angle minus 240 degree, that is actually V a angle 120 degree. So, that is V b is equal to V a e to the power j 120 degree that is beta V a.

Similarly this V c V a angle minus 120 degree lags from V a is equal to V a e to the power minus j 120 degree, that is V c is equal to beta square V a. So, b V b is equal to beta V a here and V c is equal to beta square V a right. Assume now will assume now the subscript 1 2 and 0 refer to positive sequence, negative sequence and 0 sequence respectively. This is a this is actually convention everyone following this convention. So, if V a V c V a V b and V c represent an unbalance unbalanced set of voltage phasor, the three balance sets are written as. So that means, V a 1, V b 1, V c 1 this is positive sequence set V a 2, V b 2, V c 2 that is negative sequence set and V a 0, V b 0, V c 0 that is 0 sequence set the this way we made it; that means, will come later, but let means V a is equal to V a 1, V a 2 plus V a 0. V b is equal to V b 1, V b 2 plus V 0 and V c is equal to V c 1 plus V c 2 plus V c 0. So, these are that positive negative and 0 sequence set.

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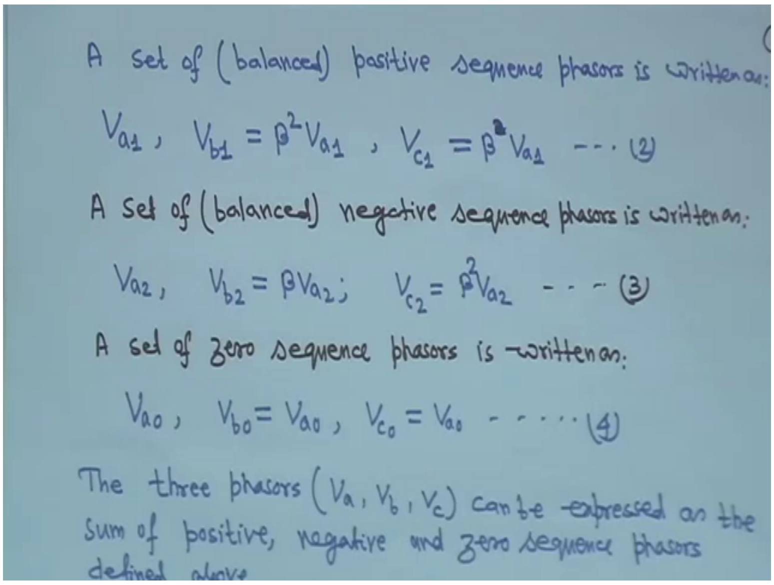

A set of balanced positive sequence phase now same way we will write the way we have seen a b c sequence here also, this is also balanced positive sequence phasor and sequence is your a b c. So, the way we wrote same way that it is we are not writing again V a 1 is equal to V a 1, it is V a 1 understandable V b 1 will be beta square just one minute the same thing V b will be is equal to your we have made it know V b is equal to

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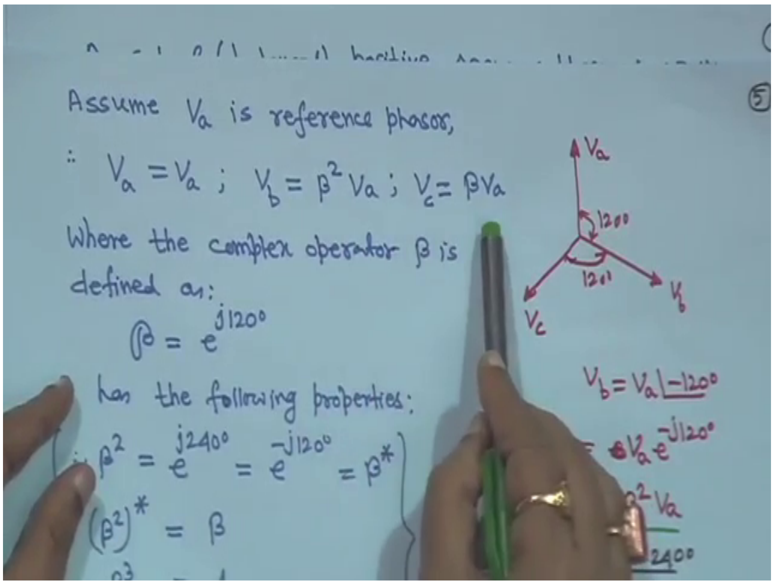

Beta square V a 1, and V c is equal to beta V a that is for a b c sequence this is also we are making it V a 1, V b 1, V c 1. So, here also V b 1 will be beta square V a 1, and V c 1 will be beta V a 1 this here it is a b c sequence here is their positive sequence negative sequence and 0 this thing your 0 sequence component same thing will come.

So, for positive sequence that V a 1 for phase a to this one, for phase v positive sequence V b 1 will be beta square V a 1, for phase c positive sequence V c 1 will be beta V a 1. Same your equations only you have to represent V a 1, V b 1, V c 1 same philosophy. Similarly the same thing same will follow an a c b sequence that is a set of balanced negative sequence phasor, it is V a 2 it will be V b 2 is equal to beta V a 2 and it will be V c 2 is equal to beta square V a 2 this is equation 3. Now a set of zeros sequence phasor they all are same. So, it will be V a 0, V b 0 will be V a 0, and V c 0 will be V a 0 this is equation 2 3 and 4. Now this set these three phasors V a, V b, V c can be expressed as the sum of positive negative and 0 sequence phasor defined above; that means, V a is equal to V a 1 plus V a 2 plus your V a 0.

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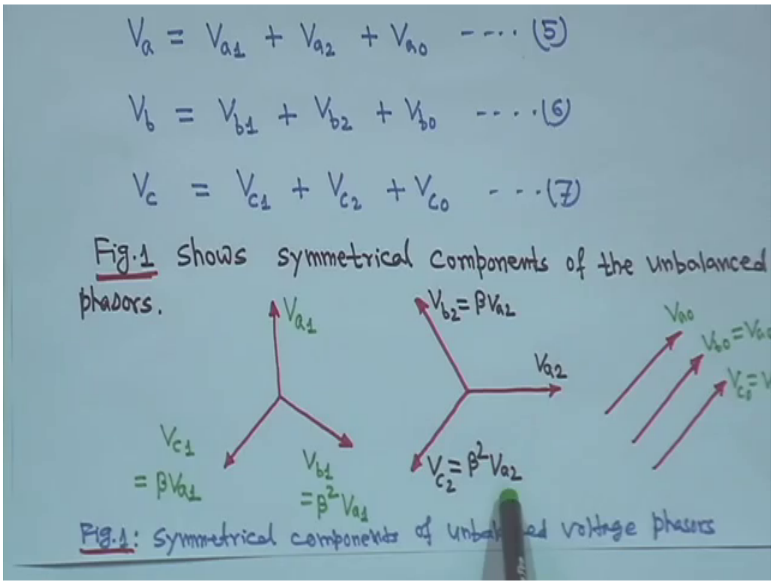

So, here we can write V a is equal to V a 1 plus V a 2 plus V a 0 this is equation 5 .V b is equal to V b 1 plus V b 2 plus V b 0 equation 6, and V c is equal to V c 1 plus V c 2 plus V c 0 that is equation 7. Now this is your positive negative and 0 sequence your component. Here I have made this is V a 1. So, V b 1 is equal to beta square V a 1 and V c 1 is equal to beta v 1 this is negative sequence. V a 2 at the reference one V c 2 is equal to beta square V a 2 and your V b 2 is equal to beta V a 2 and this is your 0 sequence this is positive negative this is 0 sequence they are all in same phase right I mean through it is V a 0, it is V b 0 is equal to V a 0 V c 0 is equal to V a 0 because they have equal magnitude. So, these symmetrical components of your unbalanced voltage phasor. So, easily you can make this one

Now, let us your express equation 5 6 and 7 in terms of the reference.

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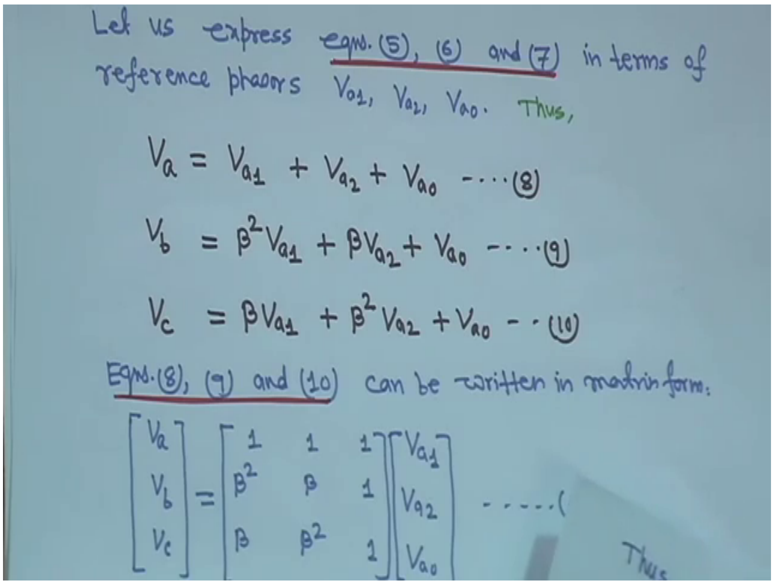

Phasor V a 1 your V a 2 and V a 0 therefore, current will come that current also same philosophy that V a is equal to V a 1 plus V a 2 plus V a 0. So, V b is equal to your beta square V a 1 plus beta V a 2 plus your V a 0 actually V b is equal to just hold on let me show you that thing. So, this equation it is V b 1 V b 1 plus V b 2 plus V b 0. So, V b 1 is equal to your beta square V a 1 and V c 1 is equal to beta V a 1.

So, in this equation it is V b is equal to your in this equation V b 1 you substitute beta square V a 1, V b 2 you substitute beta V a 2 and V b 0 is equal V a 0 V a your V b 0 is equal to V c 0 is equal to V a 0. So, V b 0 is equal to V a 0. So, this is V b similarly your V c is equal to V c 1 plus V c 2 plus V c 0 and your same way right you can make your this v or your V c 1 is equal to beta V a 1 and V c 2 is equal to beta square V a 2, same way you put here that the way we have made it that your V b 1 beta square V a 1, your V b 2 is equal to beta V a 2 same way you put it here V c 1, V c 2 and V c 0 is equal to V a 0; that means, this will come beta v beta V a 1 plus beta square V a 2 plus V a 0; that means, this V a, V b, V c all actually represented by V a 1, V a 2, V a 0 and the complex operator that is beta. So, this is equation 8 9 and 10, now this equation you put them in matrix form. So, it will be V a V b V c and it will be 1 1 1 beta square beta one beta square 1 and V a 1 V a 2 V a 0. So, these three equations you put it in matrix form.

Next is this equation this equation only we can write.

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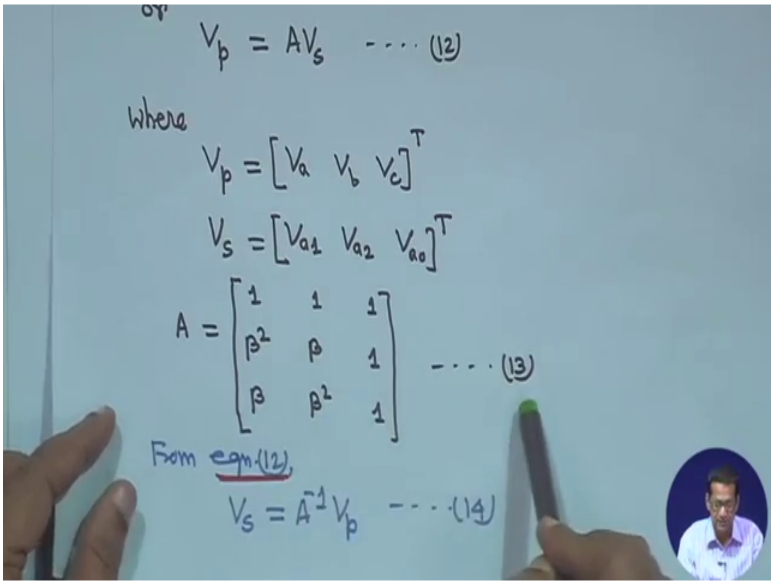

That Vp is equal to AVs this way we write where Vp is equal to V a V b V c transpose right and V s is equal to V a 1, V a 2, V a 0 transpose and A is equal to that matrix 1 1 1 beta square beta 1 and beta beta square 1 this is equation 13. So, this equation 12 this equation this equation can be written as V s is equal to A inverse Vp this is equation 14. So, the A inverse if you take the inverse of this matrix if you take the inverse of this matrix it will be.

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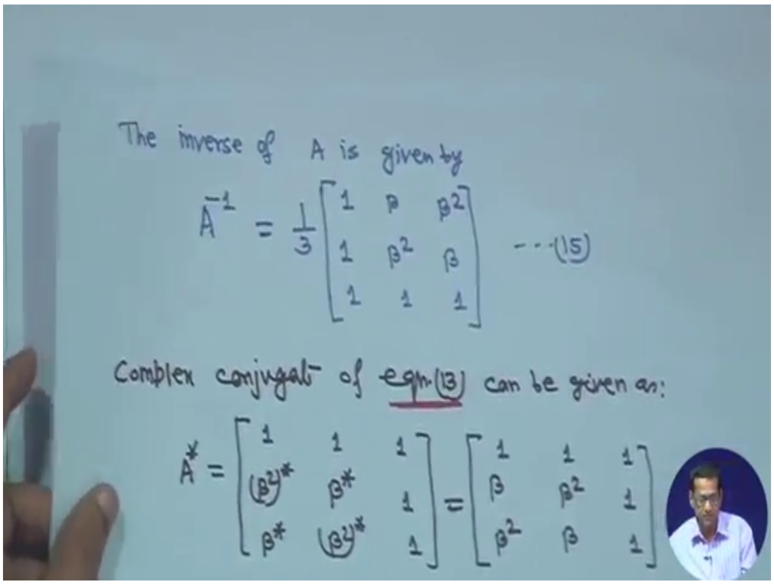

A inverse will become one third one beta, beta square one beta square beta 1 1 1, but you need not take directly that inverse I will show you one methodology that how one can remember at write basically A inverse is equal to one third A conjugate transpose I will come to that. So, if you take the complex conjugate of this matrix right that is your a matrix you take.

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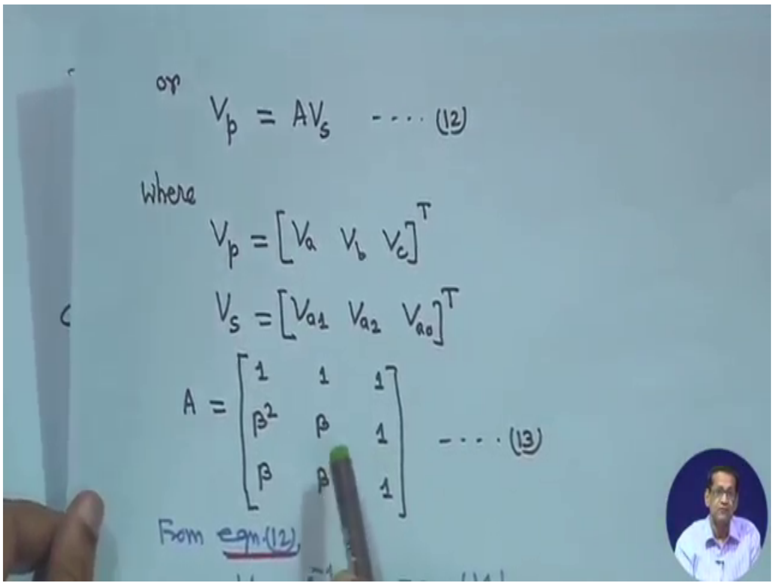

The complex conjugate of this matrix that is equation 13, then A conjugate will be this is 1 1 1 it will beta square conjugate it is beta conjugate it is one, beta conjugate beta square conjugate one. Earlier in the beginning I have given this is 1 1 1 that beta square conjugate is equal to beta at the beginning this has been given beta conjugate is equal to beta square that also given and one same is here. Beta conjugate is equal to beta square and beta square conjugate is equal to beta and this is one. So, first you take the A conjugate, after this this is actually A inverse if you take to that after this.

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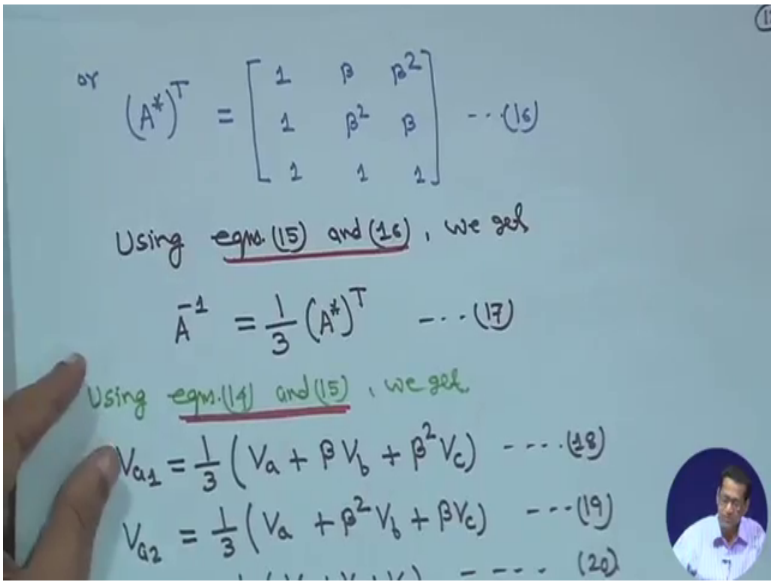

You take that transpose of this matrix that is A conjugate transpose, you take the transpose of this matrix if you take the transpose of this one. So, A conjugate transpose is equal to 1 beta beta square, 1e beta square beta and 1 1 1. So, using equation 15 and 16 this is your A inverse if you try to find out A inverse of your own it will become like that this is actually A inverse so; that means, A inverse actually one third A conjugate transpose; that means, if you know the a matrix a you take the A conjugate, then you take the transpose and divide it by 3, that is A inverse one third your A conjugate transpose this is equation 17. So, this is this is easy to remember if you know that you know the your what you call that A matrix.

So, and A inverse will be one third, A conjugate transpose it is easy to remember. So, equation using equation 14 and 15 right we get, so equation 14. So, let me go back to equation 14. So, this is equation 14 that V s is equal to A inverse Vp and that means, if you that means, Vp is equal to your your V a, V b, V c and V s is equal to V a 1, V a 2, V a 0; that means, positive sequence component that V a 1 you can write one third because your one third is here right it is in that right because it is a inverse. So, one third is there. So, V a 1 is equal to one third V a plus beta V b plus beta square V c this is equation 18, and V a 2 is equal to one third V a plus beta square V b plus beta V c this is equation 19, and V a 0 is equal to one third in bracket it is V a plus V b plus V c this is 20. So, similar philosophy is applied to current same thing same positive negative 0 sequence component right.

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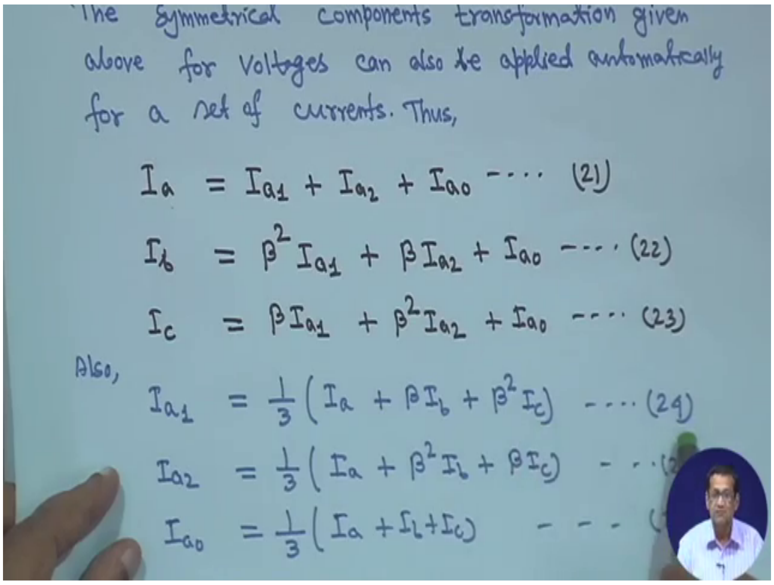

So, in this case you are the symmetrical component transformation given above for voltages can also be applied your automatically for a set of current same philosophy.

Therefore I a is equal to I a 1 plus I a 2 plus I a 0 this is equation 21. I b is equal to same as same as voltages beta square I a 1, plus beta I a 2, plus I a 0 and I c is equal to beta I a 1 plus beta square I a 2 plus I a 0. So, this is equation 23 same as voltage you will get I a 1 that positive negative and 0 sequence component of say phase a, I a 1 is equal to one third I a beta plus beta I b plus beta square I c this is equation 24. I a 2 is one third in bracket I a plus beta square I b plus beta I c say it is 25, and I a 0 is one third I a plus I b plus I c this is equation 26. So, voltage and current same philosophy.

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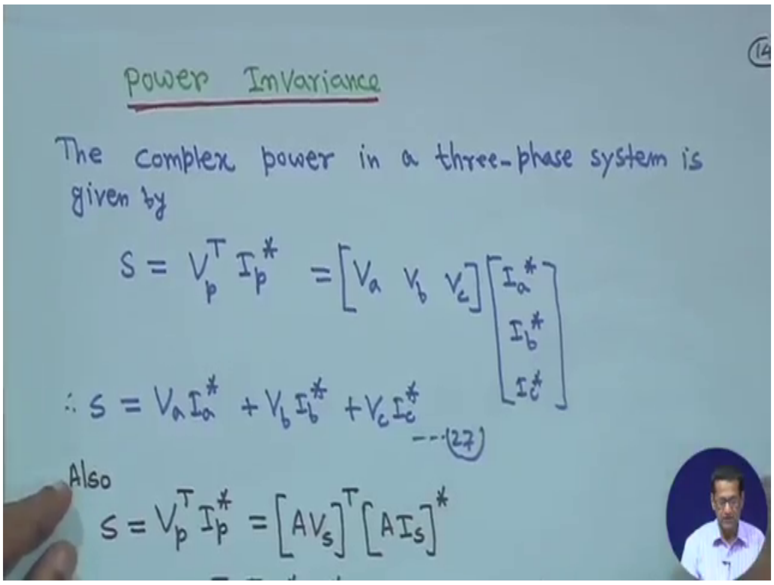

Next is your power invariance. So, we have to show that that is the power equation also original thing and transformation both are matching. So, the complex power in a three phase system is given by we have already seen it before that s in general it is be your your what you call V transpose your I i conjugate. So, we can write s is equal to Vp transpose I p conjugate that is Vp transpose is V a V b V c that we have given and I p transpose actually I A conjugate, I p actually I a I b I c. So, it is conjugate. So, I A conjugate I v conjugate I c conjugate multiply you will get s is equal to V a I a conjugate plus V b I b conjugate plus V c I c conjugate this is equation 27. Also s is equal to your Vp transpose I p conjugate this one, we have seen before that Vp is equal to AVs that we have defined let me show you once again that Vp is equal to AVs that is equation 12. So, put here Vp is equal to AVs similarly your same philosophy I p also will become AIs same philosophy I this is directly I have written, but philosophy remains same for voltage and current.

So, it will be AVs transpose there AIs conjugate. So, AVs transpose means this one, V s transpose then A transpose then AIs conjugate; that means, A conjugate I s conjugate this is actually equation 28, but AIs I have not shown that, but understandable both are same voltage and current philosophy remain same. Now if you make it a matrix is known to you right a that means.

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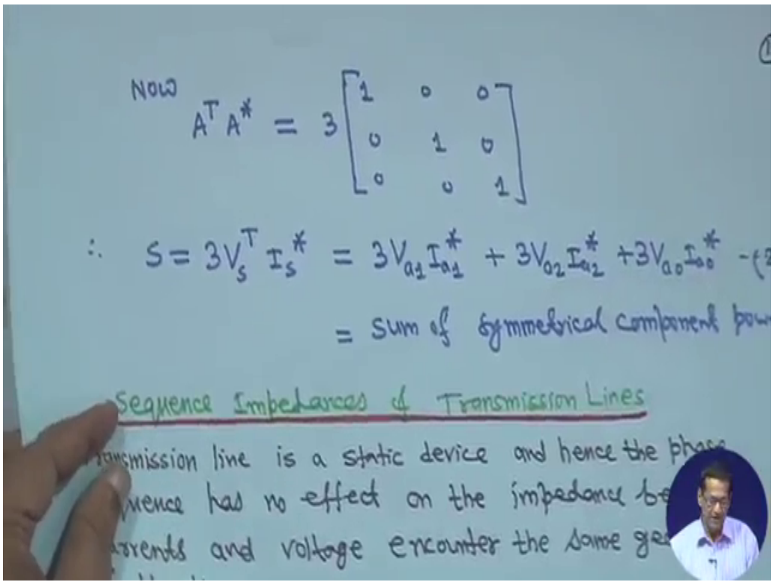

This is your A transpose A conjugate. If you multiply A transpose A conjugate it will be three in to an identity matrix that is 1 0 0 0 1 0 0 0 1. So, A transpose if you do so, it will be three in to an identity matrix; that means, s is equal to actually 3 V s transpose I s transpose; that means, it turns a A transpose A conjugate is a A transpose A conjugate is an identity matrix right multiplied by 3 of course, that means, it will become 3 V s transpose I s conjugate that is why we are writing that it is 3 V s transpose I s conjugate.

So, directly we are multiplying not writing this that V s transpose is V a 1, V a 2, V a 0 and I s transpose is I a 1, I a 2, I s 0. So, if you multiply it will be 3 V a 1, I o 1 conjugate plus 3 V a 2, I o 2 conjugate plus 3 V a 0, I a 0 conjugate sum of symmetrical components power. So that means, s is equal to whatever you get here equation 27 that is actually equation 29. They are their same I mean they have to be same right. So, now, sequence impedance of transmission lines. So, that is now we have to go for sequence impedance of each component, particularly the transmission line transformers then synchronous machine. So, just see how will make it. So, sequence impedances of transmission lines.

So, transmission line actually is a static device and hence the phase sequence has no effect on the impedance, because currents and voltage encounter the same geometry of the line. So, for the transmission line that it is basically again the again as I am saying there is a static device and hence the phase sequence has no effect on the impedance because currents and voltage encounter the same geometry of the line therefore, positive and negative sequence impedance of transmission lines.

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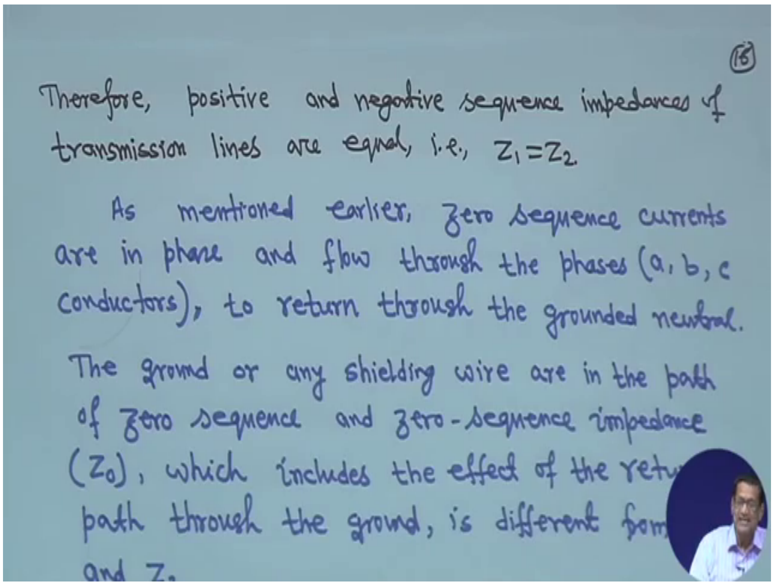

That is Z 1 is equal to Z 2 these 2 are equal for transmission line and, but as means we have discussed before that 0 sequence current are in phase with V a 0 your I s 0 same as voltage I s 0 I v 0 I c 0 they are in phase and flow through the phases that is a b c conductors and to return through the grounded neutral. So, the ground or any shielding wire are in the path of 0 sequence and 0 sequence impedance Z 0 right.

So, transmission that means for the transmission line ground on any shielding wire right are in the path of 0 sequence and 0 sequence impedance that is Z 0 which includes the effect of the return path through the ground that is different from Z 1 and Z 2. So, Z 0 sequence impedance for transmission line is different.

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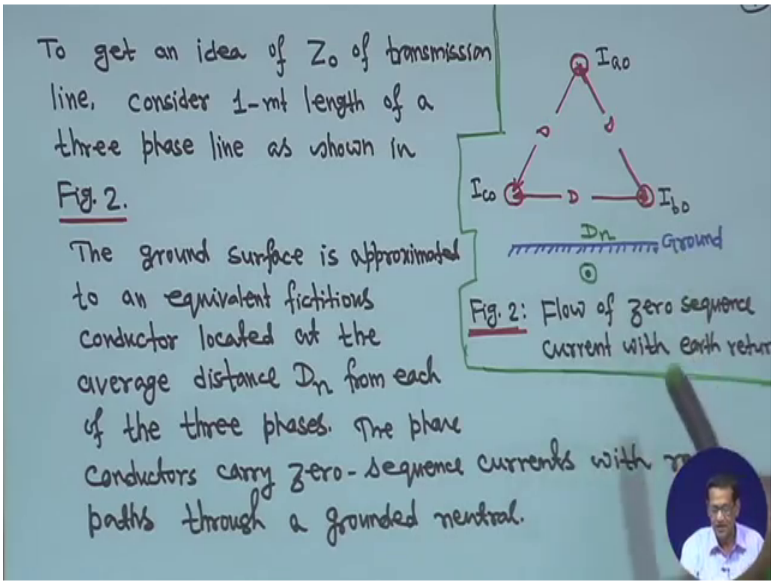

So, to get the get an idea, to get an idea of your Z 0 the 0 sequence impedance of the transmission line right you will consider only one meter length of a three phase life as shown in figure 2, I mean this figure right. So, this ground surface actually is approximated as an equivalent fictitious conductor located an average distance right from your what you call from each of the three phase. Actually this D n I have shown it very close I should have shown a little bit away from this, but anyway the ground surface is approximated to and this is the ground, this is the ground right to an equivalent fictitious conductor and located at the average distance D n from each of the three phases, so the this is an assumption.

So, the phase conductors carry 0 sequence current with written first through a ground neutral grounded neutral. So, for therefore, this is this is you have take a simple thing that is equilateral spacing distance is D D D, 0 sequence current here I a 0, I b 0, I c 0 right and this 0 sequence current your what you call the phase conductors this is phase conductors carrying the 0 sequence current. We return path through a grounded neutral and this approximate and D n actually that is distance as distance is taken from any phase to the neutral and average distance, this an approximation only to represent that 0 sequence impedance.

Thank you.