Power System Analysis
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Lecture - 58
Power System Stability (Contd.)

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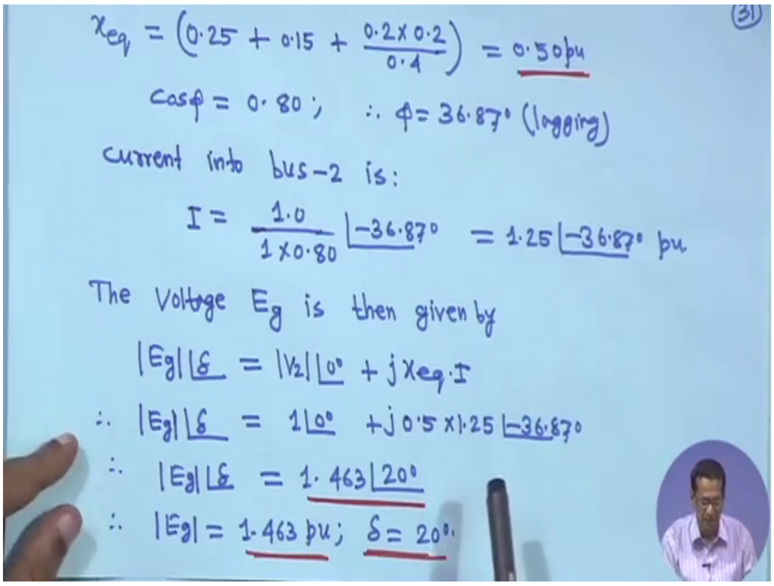

Now, voltage Eg is then given by magnate Eg angle delta is equal to magnitude your voltage V 2 angle 0 plus j Xeq I had very the circuit diagram I mean if you make it like a circuit.

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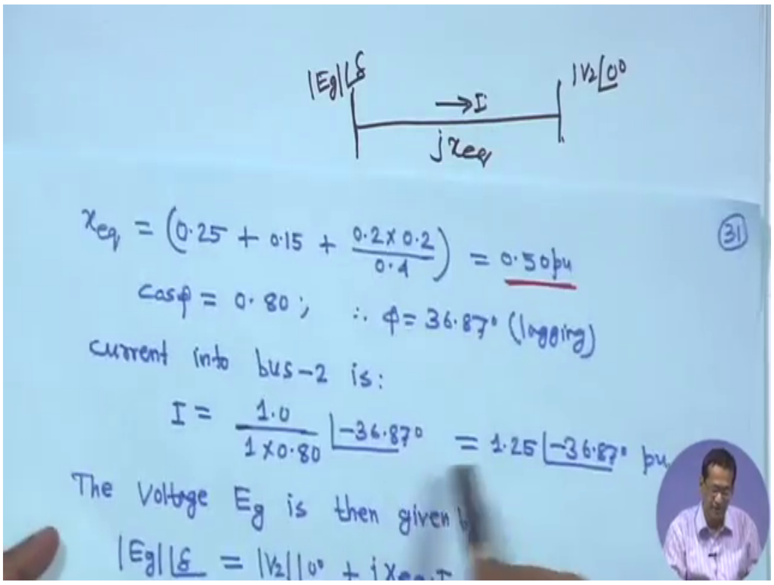

So, this is your one bus bar, this is your infinite bus equivalent size about this thing. This is your angle Eg delta and current flowing through this is I, it is j xeq and this is V 2 angle 0. So, that is the equivalent circuit if you do like this because x q any Xeq we have got 0.5 here. So, this way the current I also computed. This way if you make it then your magnitude Eg angle delta we can write magnitude V 2 angle 0 degree plus j Xeq into I, but you know this magnitude V 2 is given 1 and Xeq we got 0.5 and I also you have got this value if you put it you will get magnitude Eg angle delta is equal to 1.463 angle your 20 degree. Therefore, voltage magnitude 1.463 per unit Eg and delta is equal to 20 degree.

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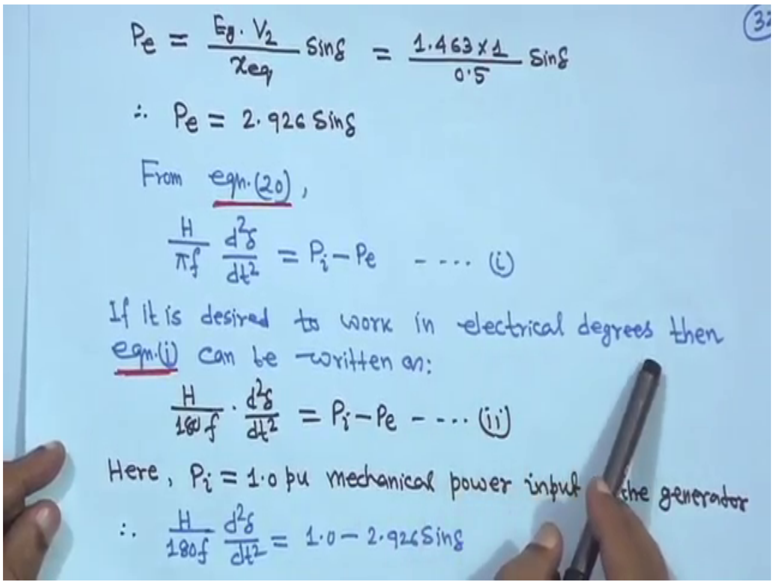

And next is that Pe is equal to Eg into V 2 up on x cube Xeq sin delta, therefore P is equal to Eg is 1.463 into 1 Xeq is 0.5 sin delta. Then with P is equal to 2.926 sin delta. Now from equation 20: this is the equation H up on phi d square delta dt square is equal to Pi minus Pe. This is from equation 20. Now if it is desired to work in electrical degrees then this equation 1, I mean we are repeating equation 20, right. That is H up on 180 f into d square delta up on dt square is equal to Pi minus Pe. This is equation 2 say.

Now here Pi is equal to 1.0 per unit mechanical power input to the generator, therefore this can be written as H is by 180 f into d square delta dt square is equal to 1 minus 2.926 sin delta; so as a verification right. At steady state your it your at steady state you will find that 1 minus 2.926 sin delta is equal to 0 and if you compute delta delta will become 20 degree. And there also you have got where this here also we have got delta is equal to 20 degree.

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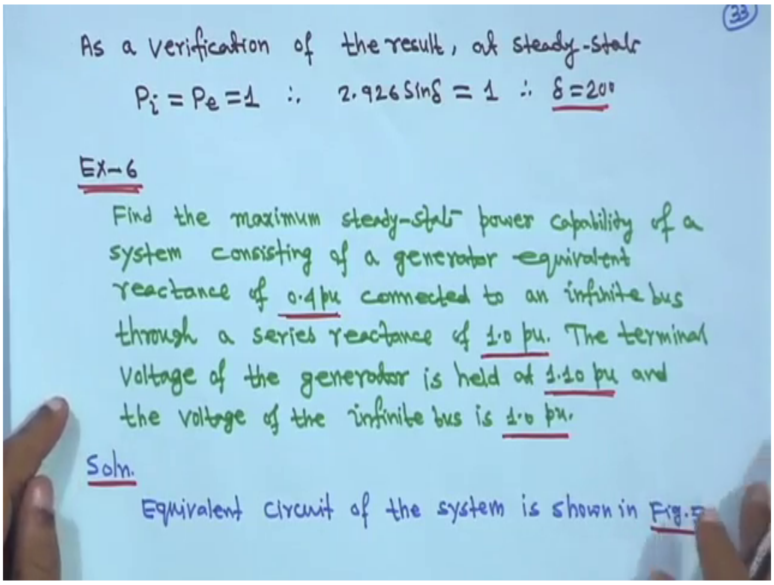

So, this is the swing equation. Here also you will get as a verification of the result a steady state Pi is equal to Pe is equal to 1 therefore 2.926 sin delta is equal to 1, therefore delta is equal to 20 degree right. Now example 6: it is given that find the maximum steady state power capability of a system consisting of a generator equivalent reactance of 0.4 per unit, connected to an infinite bus through a series reactance of 1 per unit. The terminal voltage of the generator is held at 1.10 per unit and the voltage of the infinite bus is 1 per unit. That means if you put that all this language in the diagram that how it is given that equivalent circuit of the system.

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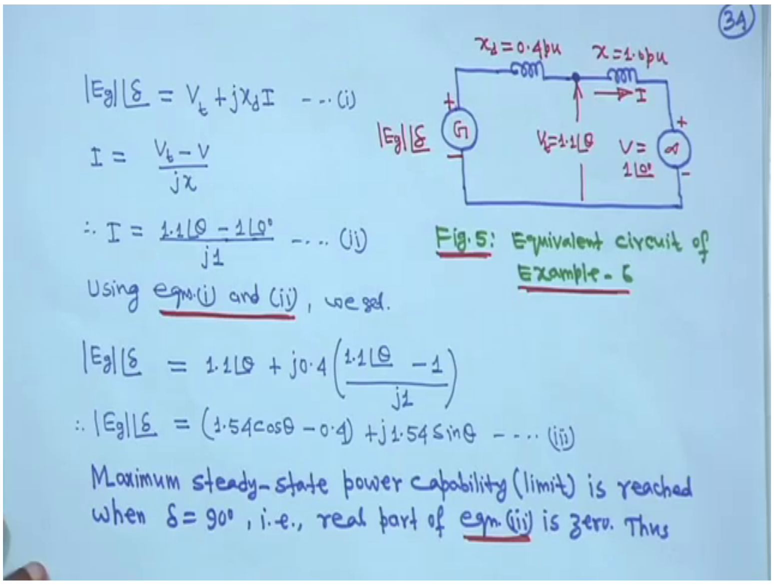

So, it is actually that generator voltage exited that is your Eg angle delta and this is given what you call generator equivalent reactance 0.4 per unit. So, this is x t we are writing say 0.4. And your through a series reactance of 1 per unit, so this series reactance here it is 1 per unit and terminal voltage it is given 1.10 your magnitude. So, we are writing V t is equal to 1.1 angle theta an infinite volt bus voltage is given it is 1 angle 0. So, you have to find out the maximum steady state power capability. So, this whatever has been x your mentioned here we are putting in this diagram.

Now what we will do that first you apply your what you call your KVL in this So, it is your magnitude Eg angle delta is equal to your j X d I j X d into current flowing through this is I plus V t. This is one equation. Similarly, here I is equal to you can write V t minus V divided by x; this is series this x I mean here. So, I is equal to we can write V t minus V up on x; that means, we can write I is equal to V t minus V up on your reactance it is, so j x right.

So, I is equal to 1.1 angle theta minus 1 angle 0 divided by j 1. So, now, this equation using equation 1 and 2; that means, this I you substitute here you substitute here. If you do so and then your magnitude Eg angle delta again and again not calling magnitude understandable easy angle delta is equal to 1.1 angle theta plus j 0 0.4 1.1 angle theta minus 8 upon j 1 if you simplify this if you simplify this you will get angle Eg your Eg angle delta is equal to 1.54 cos theta minus 0.4 plus j 1.54 sin theta. This is equation 3.

Now maximum steady state power capability limit is limit can be reached when delta is equal to 90 degree. That means real part must be is 0, if real part 0 then delta will become 90 degree. So, that means, if you put this one real part is equal to 0 then you will get the theta value. That means, 1.54 cos theta minus 0.4 is equal to 0, therefore you will get theta is equal to 74.9 degree. That means, if this part is 0 then magnitude Eg actually will become 1.54 sin theta.

So, that is why magnitude of Eg is equal to 1.54 sin theta is equal to 74.9 degree; that means, you will get magnet is equal to 1.486 per unit. That means V t is equal to actually 1.1 angle theta.

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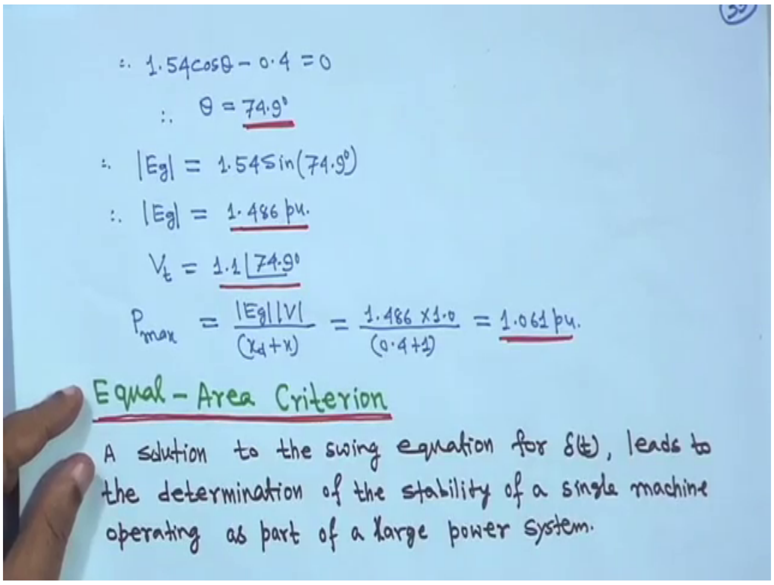

So, theta we have got. So, V t is equal to 1.1 angle 74.9 degree. Then with P max is equal to Eg into v divided by x d plus x; that is your here magnitude Eg into v divided by x d plus x the whole reactance of the line. So, it is actually 1.486 into 1 divided by 0.4 plus 1 that is equal to 1.061 per unit. So, with that we have taken few examples from the point of view of swing equation as well as steady state stability.

Next will come to the equal area criterion; now a solution to the swing equation for delta t delta is a function of t leads to the determination of the stability of a single machine operating as a part of large power system; that means whenever you try to it is this thing your solution for swing equation you have to go for some iterative technique. It leads to a determination of the stability of a single machine operating as a part of large power system. But we will see some, so that what you call that equal area criterion.

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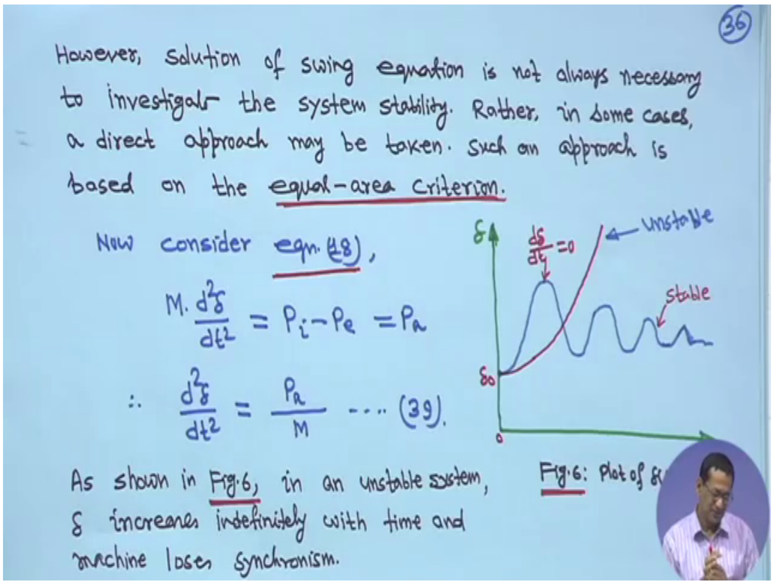

However, solution of swing equation is not always necessary to investigate the system stability. All the time we do not. So, rather in some cases a direct approach may be taken such an approach is based on equal area criterion. For example, just come to that just consider equation 18 that M d square delta dt square is equal to Pi minus Pe is the accelerating power. That means, d square delta dt square is equal to P a up on M. This is equation 39.

Now if this is delta and this is t if that graph delta versus time we have moves like this then system is unstable. But if it oscillates and die out that at particular at any point that d delta by dt will be 0. So, as shown in figure 6 in an unstable system delta increases in the in your indefinitely it is continuously increasing with time and machine loose synchronism. So, it is unstable system, because delta is continuously increasing. But if the stable system delta undergoes oscillation; that means, in a stable systems which undergoes oscillation which eventually die out due to damping. So, it will go like this and finally it will die or what you call that oscillation will die out due to damping.

So, from this it is clear that it must be what you call for a stable system then at some point d delta by dt has to be 0. This criterion can simply be obtained from equation 39; from this one that if d delta by dt is 0 like this then system will become stable it will oscillate; and finally oscillation will die out.

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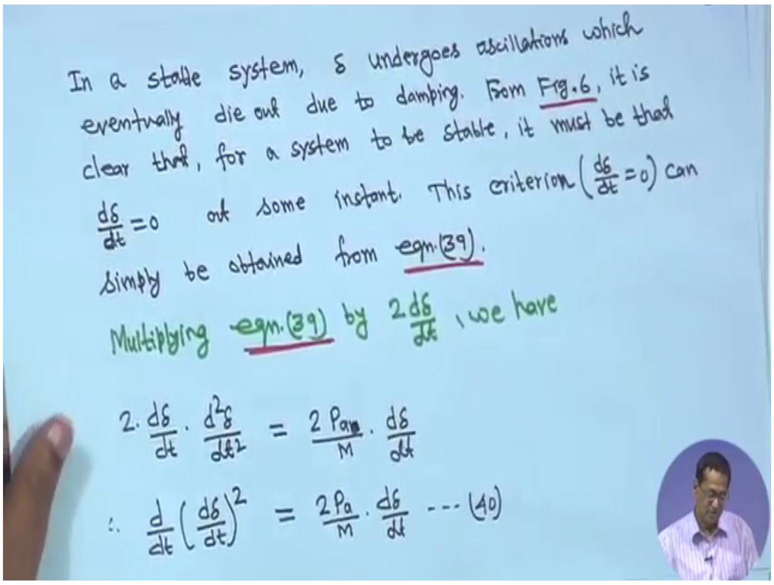

So, multiply equation 39; that means this equation both side you multiply by 2 into d delta by dt. So, if we multiply this two delta by dt on both side then 2 d delta dt into d square delta dt square is equal to 2 P M into d delta by dt. So, left hand side can be written as d dt or d dt of d delta by dt whole square if you take its derivative it will come like this. So, we are writing d dt of d delta by dt whole square is equal to 2 P a up on M into d delta by dt. This is equation 40.

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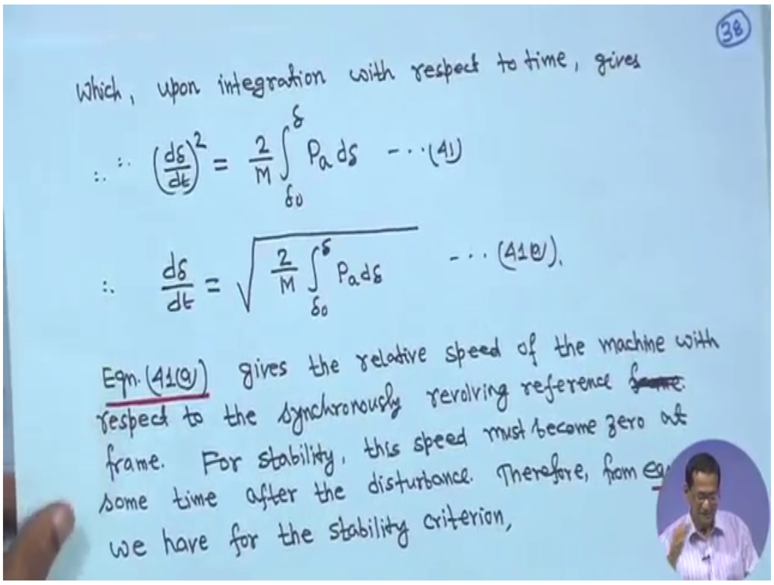

So, that means, which up on integration if you integrate this equation 40 then it will be a d delta by dt whole square is equal to 2 by M your delta 0 to some angle say delta into P a into d delta. This is equation 41. So, or d delta by dt is equal to your root over 2 by M delta 0 to delta P a delta. This is equation 41 a. Now this is this is 41, this is 41 a.

Now this equation 41 a actually gives the relative speed of the machine with respect to the synchronously revolving reference frame here.

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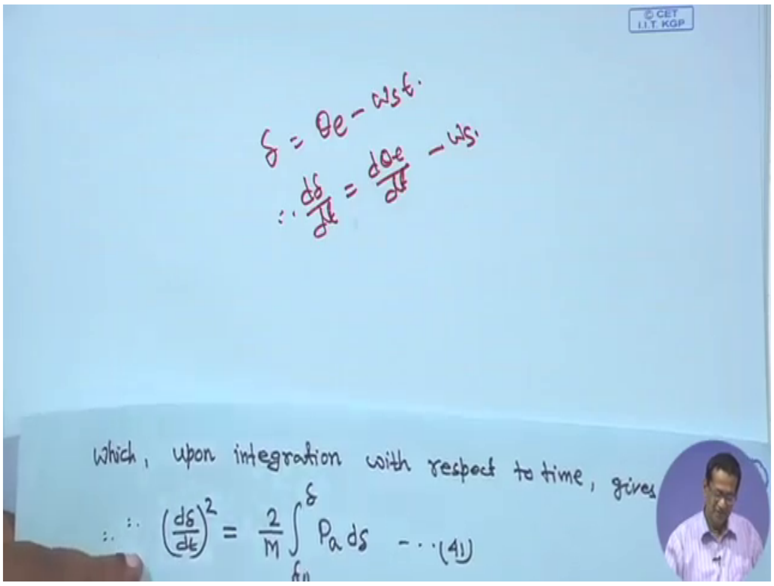

We assume know somewhere that delta is equal to theta e minus omega s t; that means, d delta by dt is equal to your d theta e dt minus omega s; That means, the same thing it gives the relative speed of the machine with respect to the synchronously revolving reference frame. So, for stability the speed must become 0; that means d delta by dt it must be your equal to 0.

So, that means, at some time after the disturbance. That means, your what you call therefore, from equation 41 a we have the we have further stability here. So, for stability the speeds must become 0 at some time after the disturbance. That means, I showed you know this graph that for stability somewhere d delta by dt has to be 0. So, in that case from equation 40 if d delta by dt 0 then integral of delta 0 to delta P a d d delta it can be written as 0.

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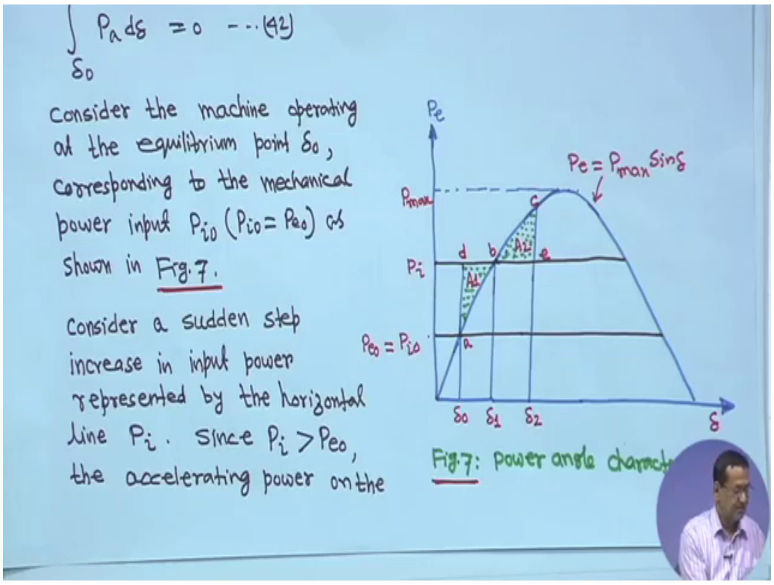

Therefore, we can write this that delta 0 to delta P a delta is equal to 0. Say this is equation 42. Now you consider the machine that now will come to that what is equal area criterion. So, you consider the machine operating at an equilibrium point delta 0. This is the equilibrium point that is your delta 0. And corresponding to the mechanical power input Pi 0; this is the mechanical power, but this horizontal line this horizontal line mechanical power Pi 0, but we are writing Pe0 is equal to Pi 0; as shown in figure 7.

So, consider a sudden step increase in input power represented by the your horizontal line Pi. That means it was operating at steady state condition, but suddenly that your input power that increased for up to your Pi. So that means, say Pi greater than Pe0 or Pi 0, I say Pi greater than Pe0. Suddenly your what you call input power is increased. So, in that case and Pi greater than your this Pi then greater than your Pe0 then; that means, the accelerating power on the rotor is positive. And the power angle delta increases.

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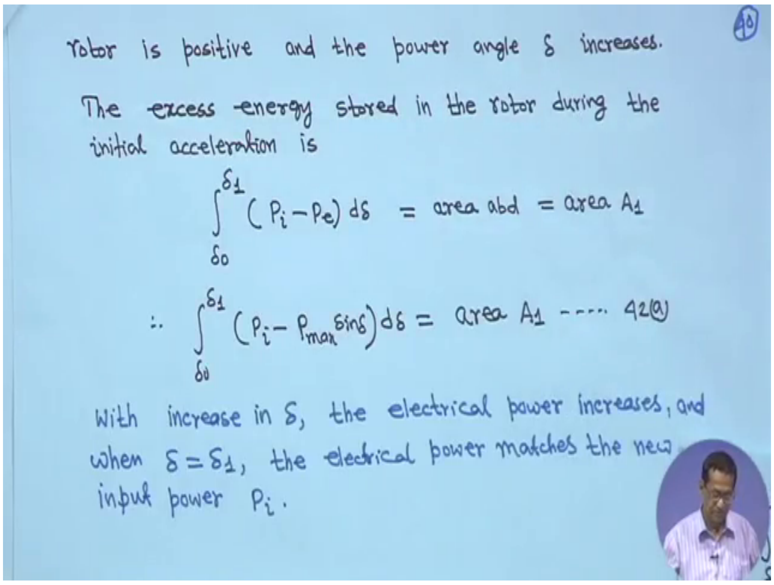

So, as soon as your because this Pi actually your greater than your Pe0 the accelerating power on the rotor is positive and the power angle delta increases, because delta now will increase delta will increase. So, what will happen that excess energy stored in the rotor during the initial acceleration is it will be delta 0 to delta 1 that is your delta this diagram that delta 0 to delta 1, because it has increased from your suddenly to Pi. So, this is the horizontal line it is cutting here this sin curve that is your power your power curve at this point that is b right.

So, whenever that means, delta 0 to delta 1 that is that excess energy stored in the rotor due to initial acceleration that Pi minus Pe delta that is area a b d; that is this area a 1 area a b d right. And that can be equal to I am writing area a 1 or delta 0 to delta 1 is Pi and Pe is equal to P max sin delta. So, we are writing Pi minus Pe max sin delta d delta this is area a 1. So, this is I am marking as equation 42 a. This is that excess energy stored in the rotor during the initial acceleration, because as soon as this Pi suddenly load has increased input power increase to Pi then rotor will start accelerating. So that means, what you call this excess energy stored will be only this area you have to find out a 1.

Similarly with the increase in delta the electrical power increases, because as delta is increasing then electrical power P also will increase. So, electrical power increases. And when delta is equal to delta 1 right I mean this value when delta is equal to delta 1; the electrical power matches the new input power Pi. That means, when delta is equal to delta 1 you will get P is equal to P max sin delta 1, but that means, at that time your Pi will become P max sin delta 1. It matches your new input power Pi.

So, if it is so even though; that means, at this point your accelerating power is 0. At this point because Pi when it is coming to this point Pi is becoming your Pe is becoming P max sin delta 1 when delta is equal to delta r. So, at that time Pi will be I can say this is my P 1 P max sin delta I have written when delta is equal to delta 1 P 1 is equal to P max sin delta 1 at this point Pi is equal to P 1. So, it is to that means P accelerating power is 0 at this point if it matches.

So, that is why that is the electrical power matches the new input power Pi. So, in this case even though the accelerating power is 0 at this point the rotor is running actually above the synchronous speed, because it was accelerating. Although at this point that you are electrical this power is equal to that electrical power matching with this input power, but rotor is accelerating, so it is running above the synchronous speed.

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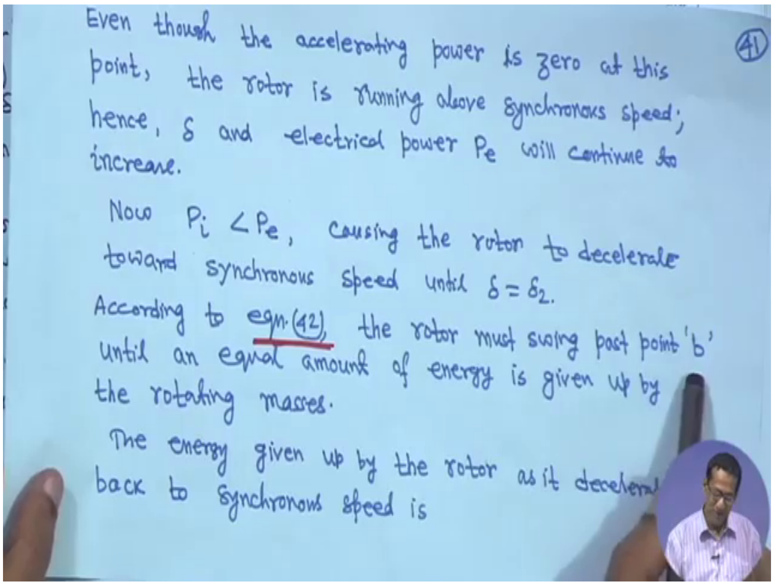

That means, what will happen the; hence delta and electrical power P will continue to increase. So, your delta will continually increase, so P also will increase the electrical power continuously will increase, but how long how far?

So, it will happen; now what will happen as rotor is above the synchronous speed and as it is your what you call it is continuously the delta will increase; that means, that P at that time if it is delta 1 is increasing; that means, P will be greater than Pi. At that time Pe will be greater than Pi, because at this any point here Pe will be greater than Pi then what will happen rotor will start decelerating till it achieves its synchronous speed say at point c. That means, the rotor is running above synchronous speed hence delta and electrical power P will continue to increase.

That is why I am telling, now when Pi less than Pe negative. So, even Pi less than negative my rotor will decelerate. They consider the rotor to decelerate towards synchronous speed until delta is equal to delta 2. When it achieves here delta is equal to delta 2 that it reaches to synchronous speed that is your say at point c. So, until delta is equal to delta 2.

So, according to equation 42 that means your this equation according to equation 42 this equation the rotor must swing past point b until an equal terminal amount of energy is given up by the rotating mask. Up to this it was energy was stored right, and it will reach to a point; it will state our oscillate until past point b until a point. That is when delta is equal to delta 2 it will what you call it will that your until and it equal amount of energy is given up by the rotating masses. Then whatever energy stored same amount of energy will be given up till it reaches at point delta is equal to delta 2.

So, the energy given up by the rotor as it decelerates back to synchronous speed is this one.

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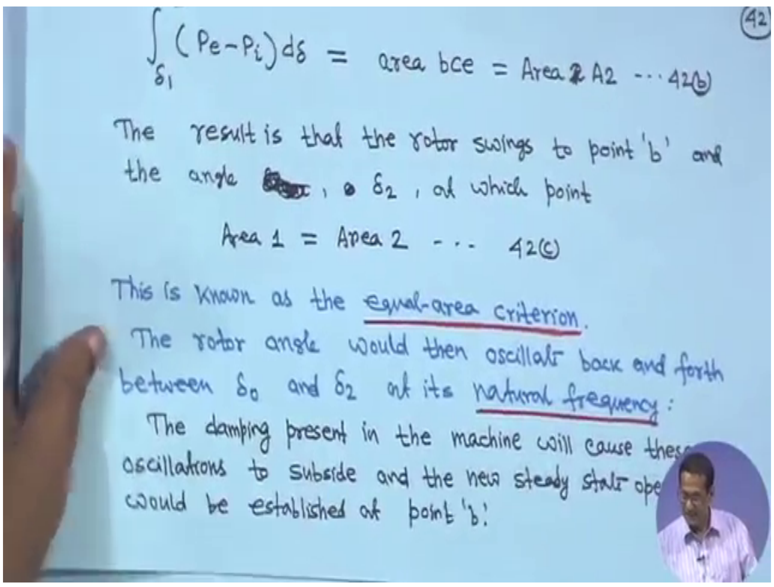

So, the energy given up by the rotor as it decelerates back to synchronous speed it can be given as your you do your Pe, because at that time your Pe is greater than Pi. So, it is del integration will be delta 1 to delta 2 and it will be the integration of the Pe minus Pi d delta is equal to area bce area is equal to area this area a 2 bce; is equal to area a 2.

Actually, somewhere you can also write some book also you can find that decelerating energy; you will find some somewhere there I also writing delta 1 to delta 2 they are writing Pi minus Pe d delta then we area will be this inter area will become negative after that they are telling taking that absolute of area 1 is equal to absolute a period; that is also true. But directly we are writing in that case if you take that Pi minus Pe e decelerating means it will become negative. And here it is storing, so it will be positive and when it is your what you call a decelerating side it will become negative. But here we have written delta 1 to delta Pe minus Pi d delta; that is area bce is equal to area a 2. This is equation say 42 b.

Now result is that the rotor swings to point b and the angle delta 2. That mean rotor will swing till delta 2 and this area delta 2 that area 1 must be equal to area 2. So, this area 1 must be equal to area 2, because I said the rotor must swing past point b until an equal amount of energy is given up by the rotating masses. So, in that case it will be area 1 is equal to area 2; this equation I am giving 42 c.

Now what is the basic philosophy? That suddenly power has increased to Pi. So, in that case what will happen that Pi will be greater than Pe0. So, rotor will start accelerating, and when it is accelerating it will come to a point when delta is equal to delta 1. At the time Pi is equal to your this P is equal to P i; that means, accelerating power is 0. And if this term the rotor is storing energy. Now at this point the rotor is running above synchronous speed; that means, rotor will continue to; that delta will continue to increase because it is running your above synchronous speed till the point say c that is delta 2 and until it give up the equivalent amount of energy as it is a 1. So, it is going up to c.

So, this way this is accelerating energy this is decelerating energy both have to be equal; though area a 1 must be equal to area 2. But question is that is up to the synchronous speed till it is a synchronous speed. So, ultimately what will happen that area 1 must be, this is known actually equal area criterion. So, this is area 1 and this is area 2.

Actually, it will swing back and forth back and forth and because of the machine damping finally after some time it will settle to point b. So, these is the rotor angle would then oscillate back and forth between delta 0 and delta 2. I told you between delta 0 and delta 2 it will be swing back and forth and finally at its natural frequency. So, as damping present in the machine right will cause these oscillations to subside and new what you call new steady state operation would be established at point b, after some time this will oscillate back and forth. Finally, it will be stable at this point b. So, this is actually is called equal area criterion.

So, I hope this part is understandable. So, idea is that when it is going pass to up to this that power has increased to Pi and then it is accelerating, but it is above synchronous speed so it is further it will your actually your accelerate, but still it achieves to synchronous speed at the time P greater than your what you call Pi so it will decelerate. And finally, it will achieve a point your delta 2. So, this is actually power angle characteristic of your machine.

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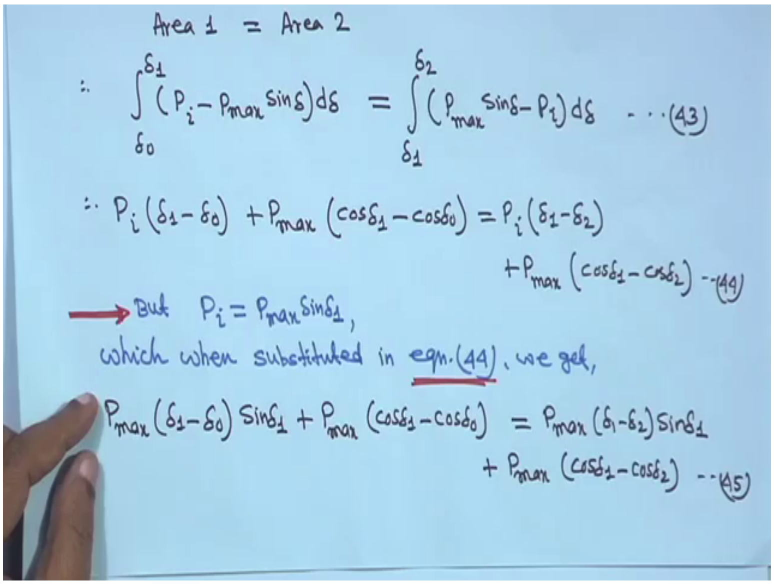

Therefore, if area 1 is equal to area 2 then this we can write all this equations shown that delta 0 delta 1 is equal to Pi minus Pe max sin delta d delta is equal to integral delta 1 to delta 2 P max sin delta minus Pi d delta. This is equation 43.

Now if you integrate and if you simplify it will come like this: Pi into delta 1 minus delta 0 plus P max cos delta 1 minus cos delta 0 is equal to Pi into delta 1 minus delta 2 plus P max into cos delta 1 minus cos delta 2. This is equation 44. But Pi is equal to P max sin delta 1 from this equation that Pi here is equal to when delta is equal to delta 1 Pi will be P max sin delta 1. So, if you substitute here Pi is equal to P max sin delta 1 and if you put it here and simplify it will become P max delta 1 minus delta 0 into sin delta 1 plus P max cosine delta 1 minus cosine delta 0 is equal to P max delta 1 minus delta 2 sin delta 1 plus P max cos cosine delta 1 minus cosine delta 2. This is equation 45.

If you simplify this something will be canceled; I am giving you the final form.

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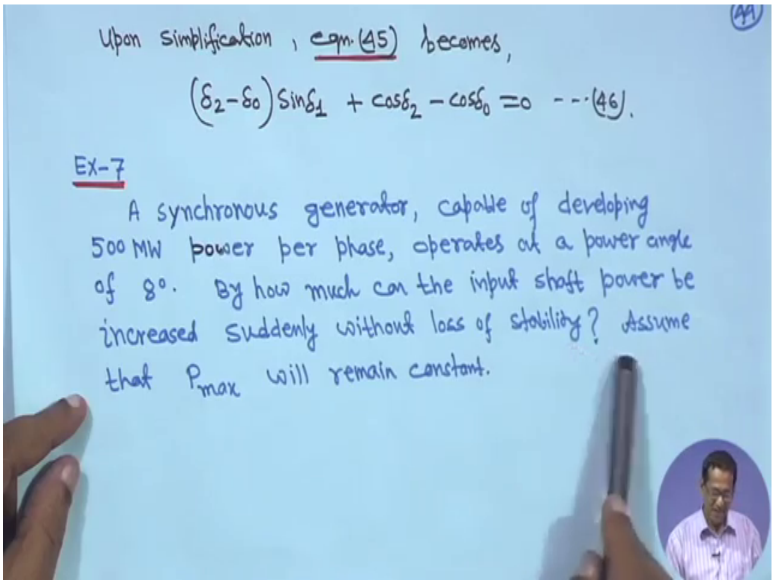

If you simplify this then it will become that delta 2 minus delta 0 sin delta 1 plus cos delta 2 minus cos delta 0 is equal to 0. This is equation 46. So, this one actually that equal area criterion whatever you have got different conditions are there; different conditions are there. So, for this thing although will solve in the next class, but I am giving you.

A synchronous generator capable of developing 500 megawatt power per phase, operates at a power angle of say 8 degree. By how much can the input shaft power be increased suddenly without loss of stability? Assume that P max will remain constant. Somewhere we will find this kind of problem has been taken as an mathematical development.

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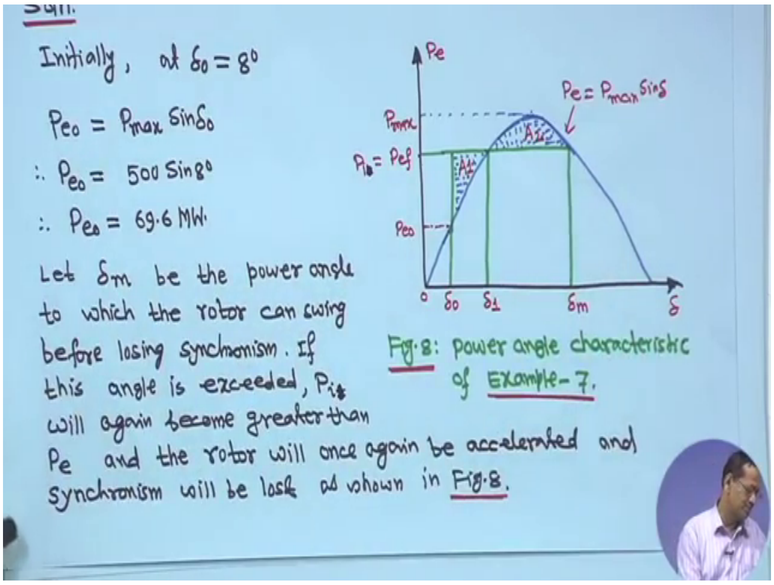

But what we will do it; that we have taken this one. So, maximum will be your what you call that this is P 0 suddenly it has increased to Pi and this is that maximum angle it can go up to delta f. It it goes somewhere I mean more than delta M then what will happen P will be less than your Pi system will become unstable. So, I will come to that.

So, Pe0 is equal to P delta 0 8 degree Pe0 is equal to P max sin delta 0. That is it is coming 69.9 megawatt. Now delta M be the power angle to which the rotor can swing before losing synchronous. This is the maximum one can go. If this angle is exceed Pi I mean if it comes somewhere here say then that again it because that P will be what you call; that Pi will be again become that Pi will be greater than your Pe, because if it comes somewhere here. If it goes below that then Pi will be greater Pi somewhere here; Pi will be greater than Pe. And rotor will again we accelerated and it will lose synchronism.

So, maximum it can go up to this delta M P is this is your delta is equal to delta M and you have to find out this delta M. So, P your Pi cannot be greater than Pe, if it goes more than delta M then it will start accelerating and system will your go out of your what you call step. So, this is power angle characteristic of this one.

Now from this figure only and this equation we have derived that; therefore the equal area criterion requires that equation 46. That means, in this equation: equation 46 what you will do is that from this graph delta M is equal to pi minus your delta this is a sin curve. So, from symmetry delta M is equal to pi minus your delta 1, because this is pi this is your what you call pi. So, it will be pi minus your delta 1. So, this delta M is equal to pi minus delta 1. That means, actually in this equation that delta 2 is equal to delta M is equal to your pi minus your delta 1. So basically, what you call here we have gone up to delta 2, but up to this it can come up to this delta m. So, delta 2 is equal to delta M actually what you call this.

So, instead of in this equation the delta 2 actually is equal to delta M is equal to pi minus your delta 1. So, that is for your understanding such that you should not make any this thing. This actually is equal to say delta 2. So, this delta 2 is equal to in this equation 46 you please put pi minus delta 1. Because in this equation is it up to delta 2 in this figure, but it can go up to your maximum delta M; delta 1 to delta M.

So, that is why for your understanding I am writing this is delta 2. So, put here pi minus delta M. So, here also delta 2 you put pi minus delta and minus cos delta 0.

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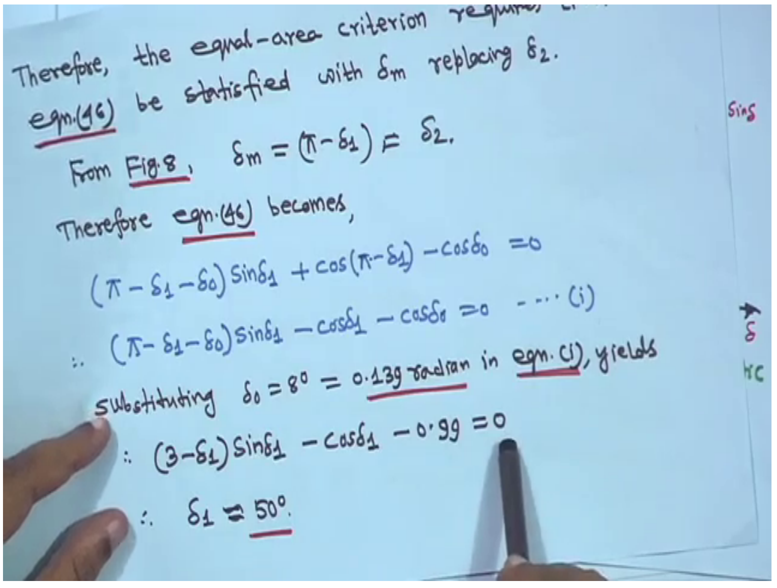

So, if you do. So, then as delta 0 is equal to 8 degree is equal to 0.139 radian. So, approximately 0.14 and equation 1 if you put that. So, if you put all these values then you will get 3 minus pi 3.14 and it is approximately 0.14. So, it is three approximately I am taking minus delta 1 into sin delta 1 minus cosine delta 1 minus 0.99 is equal to 0. Then with delta 1 is approximately 50 degree.

So, again repeat the delta M is nothing but delta 2; in the delta 2 is that previously previous figure equal area criteria

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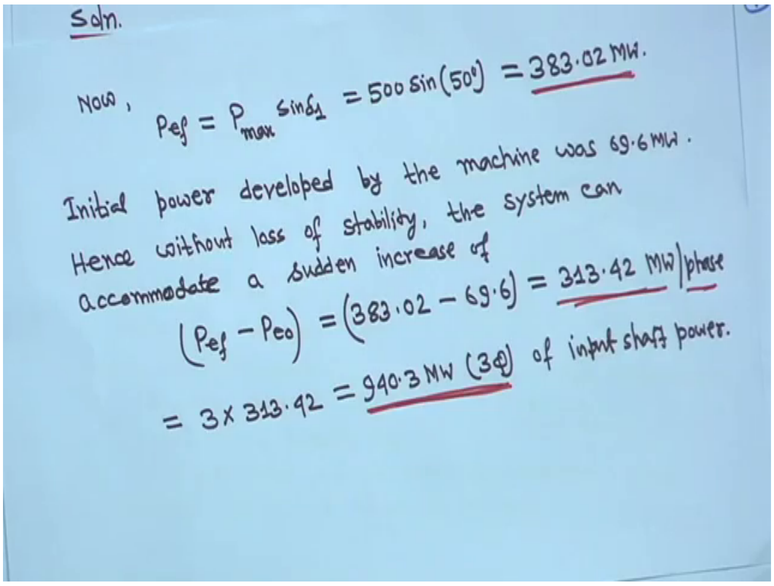

So, with that that P f this one Pe f right just where has diagram has gone that just; let me see. But anyway the diagram here; P f, P f is equal to P max sin delta 1. So, it is delta is equal to delta 1. So, P max sin delta 1. So, 500 sin 50; so 383.02 megawatt. So, initial power delivered by the machine was 69.6 megawatt that was at this point we computed 69.6 megawatt and now hence without loss of stability the system can accommodate a sudden increase of P f minus Pe0; that is 383.02 minus 69.6. So, this is your 383.02 minus 69. So, that is 313.42 megawatt per phase. So, if it is a three phase multiplied by 3 it will become 940.3 megawatt of the your input shaft power.

Only once again before closing this lecture; that it cannot be more than your delta M then Pi will become Pe because any point as you take Pi will become greater than Pe then machine will start accelerating and it will lose synchronism.

So with that, thank you.