Power System Analysis
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Lecture - 06
Resistance & Inductance (Contd.)

So, inductance of a single phase two wire line.

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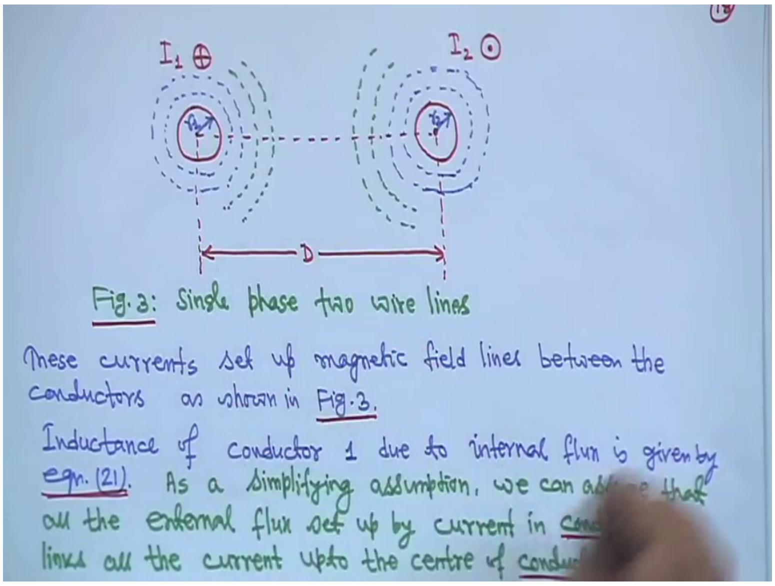

Just now I showed you the diagram right. So, it is that is your, so this is the 2 conductors carrying current I 1 and I 2 equal, but in opposite direction because 1 entering into phase this plus symbol and dot means it is leaving the phase right. So, these currents setup magnetic field lines these are shows magnetic field lines between the conductors as shown in this figure, figure 3.

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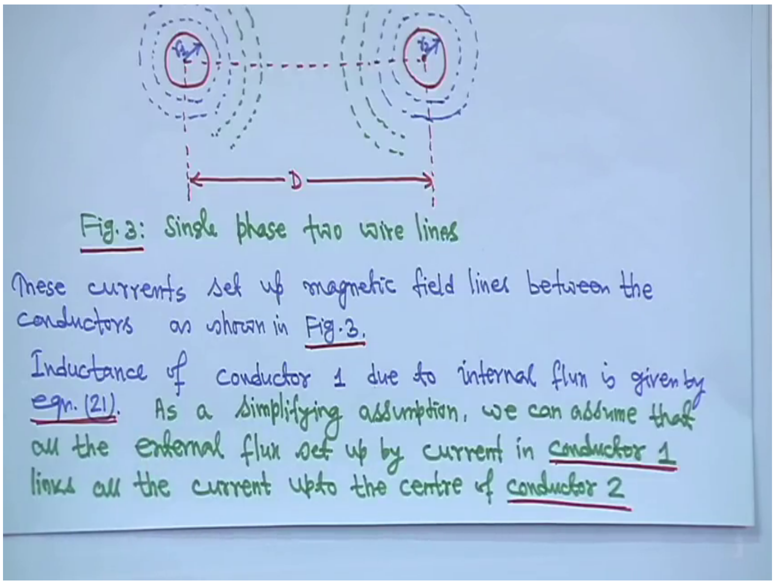

So, inductance of conductor one; that means, this conductor right due to internal flux is given by equation 21 that we have seen half into your 10 to the power minus 7 Henry per meter right. So, as you simplify assumption we can assume that all the external flux setup by current in conductor one, links all the current up to the centre of conductor 2; that means, we are making an assumption that we can assume that all the external flux setup by current, what, when this external flux setup by this current, it your what you call by current in conductor 1, this 1 we call conductor one, links all the current up to the centre of the conductor 2. That means, if we make the flux line up to the centre of that this conductor 2 not beyond that only up to the centre of this conductor not beyond that and this is quite your pure assumption.

So, similarly for this similarly under the total under the flux beyond the centre of conductor 2 does not link any current.

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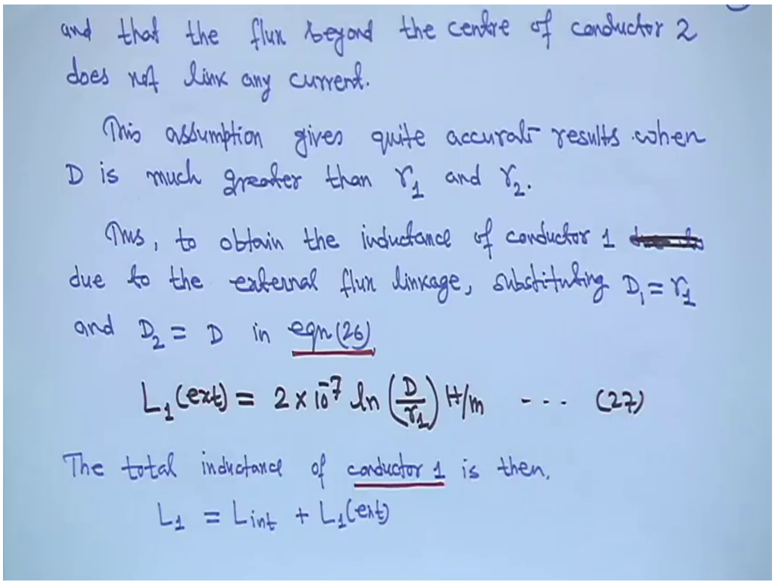

This is our assumption; that means, whatever extra this conductor 1 carrying current I saw external flux only we assume it links up to the currents up to the centre of the second conductor, conductor 2, but not beyond that. This we are making an assumption right and it does not link any link any current this assumption gives quite accurate result when D is much greater than r 1 and r 2. That means, the distance this distance D is not much greater than r 1 is radius of this conductor is r 1 and this radius is r 2.

Thus, to obtain the inductance of conductor 1 due to the external flux linkage substituting D 1 is equal to r 1 and D 2 is equal to D in equation 26; that means, this equation; that means in this equation. I told you know from this we can make out. So, in this equation, in this equation 26, in equation 26 what we are doing is that we are substituting D 1 is equal to r, D 1 is equal to r and D 2 is equal to D in equation 26.

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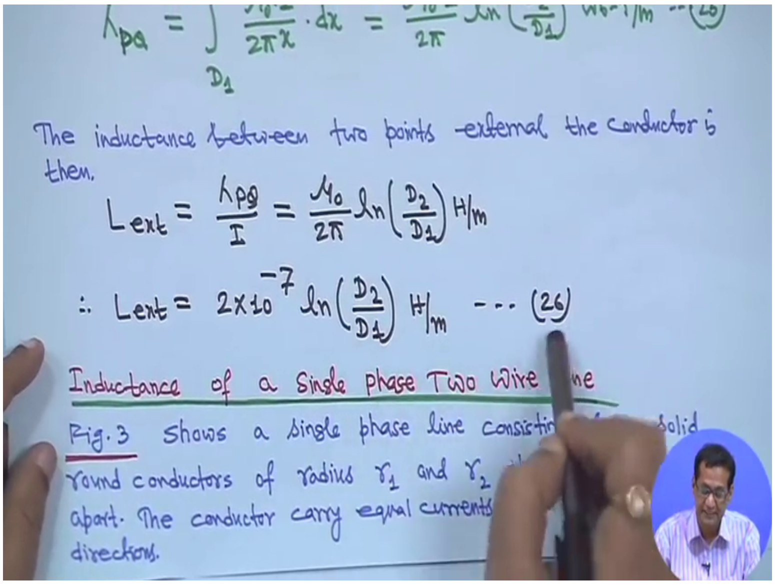

So, directly we are substituting in this equation D 1 r and D 2 is equal to your D. So, that is why you will get L 1 external you will get that 2 into 10 to the power minus 7 ln D upon r 1 Henry per meter. This is your equation 27. Therefore, the total inductance of conductor 1 is then given by L 1 is equal to L internal, internal inductance we have got it half into 10 to the power minus 7 plus L 1 external means this 1. So, this is the total inductance of the conductor 1.

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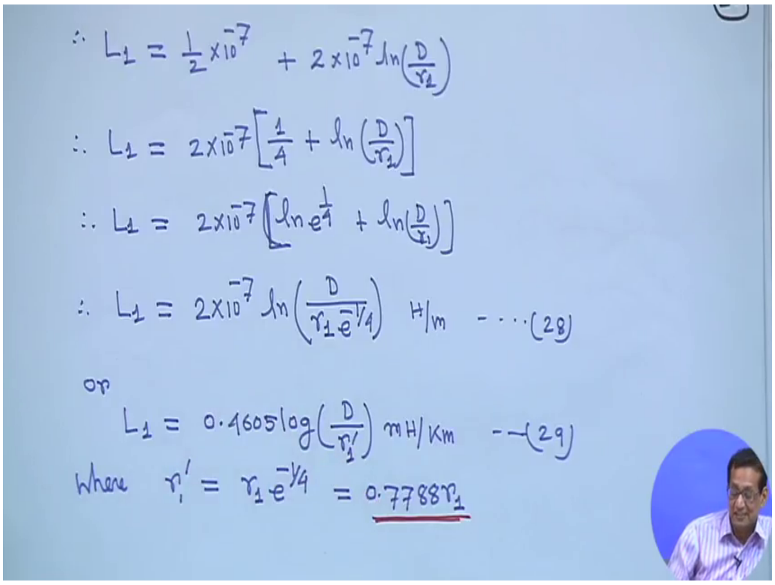

That means you substitute L 1 is equal to half 10 to the power minus 7 plus 2 into 10 to the power minus 7 ln D upon r 1 right you take from this 2, you take 2 into 10 to the power minus 7 common if you do so, this term will become 1 by 4 plus ln D upon r 1 this 2 common. So, it will be 1 by 4 ln D upon r 1 right.

Now, is equal to 2 into 10 to the power minus 7 you write ln. So, this 1 by 4 you can write e to the power 1 by 4 right. So, this you can write 1 upon 4 because ultimately it will be 1 upon 4 ln is actually because log of e base e right, so it is 1. So, that is why we can write ln e to the power 1 by 4. So, is equal to we can write ln is equal to 2 into 10 to the power minus 7 ln D upon r 1 e to the power minus 1 upon 4 Henry per meter, this is something like this.

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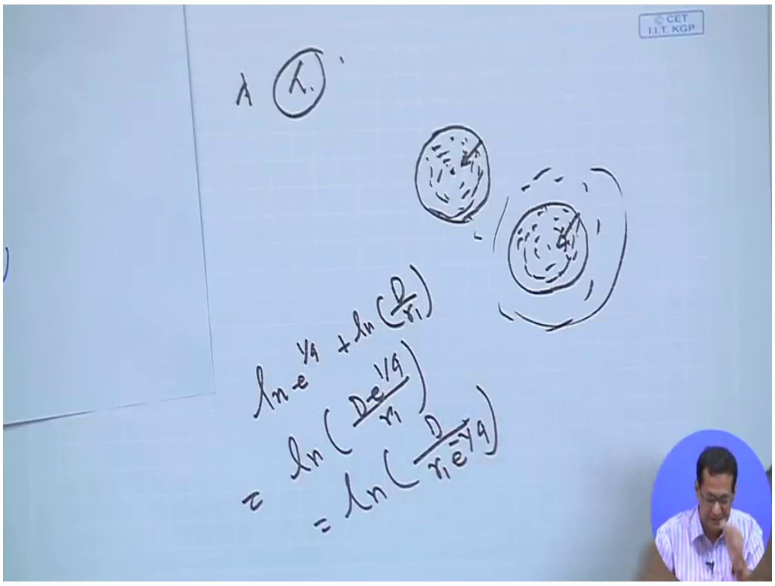

It is something like this, it is in the bracket term it is ln e to the power 1 by 4 plus ln D upon r 1 is equal to ln D e to the power 1 by 4 by r 1 is equal to ln D this e to the power for you being in denominator it will become it will be it will be r e to the power minus 1 by 4.

So, that is why this term we have written 2 into 10 to the power minus 7 ln D upon r 1 into e to the power minus 1 by 4 Henry per meter or this one you convert to a natural log to common log right and convert it to it is Henry per meter. So, convert it to milli Henry per kilometre this is a small exercise for you to do it right, when you will listening this lecture you please do it right. So, it just taken instead of natural log we have taken the log. So, if you convert it and remember Henry we have convert it to milli Henry and meter you have to convert it to kilometre.

So, it will become actually 0.4605 log D upon r 1 dash right r 1 dash is given here milli Henry per kilometre right where r 1 dash is equal to r 1 into e to the power minus 1 upon 4 that is equal to 0.7788 r 1 right. So, this r 1 dash actually called fictitious radius of the conductor right so; that means, this is a standard formula for this to say to find out the inductance right. So, 0.4605 log this is per single conductor.

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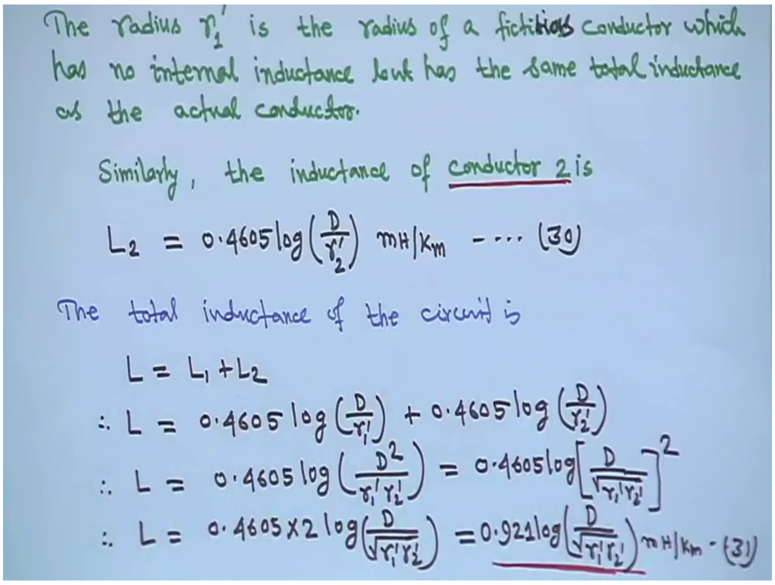

Now, this radius r 1 is the radius of the fictitious conductor actually which has we assume which has no internal inductance right this way you can imagine, but has the same total inductance of the actually conductor that is the meaning of that 0.7788 r the fictitious radius of the conductor.

Similarly the inductance of conductor 2 you can find out. So, this same thing L 2 is equal to 0.4605 log D will be there, but radius of the conductor where r 2. So, it is r 2 dash milli Henry per kilometre and r 2 dash also same thing is equal to 0.7788 into your r 2, if r 1 r 2 both radius are different right. So, r 2 dash is equal to 0.7788 r 2 it is milli Henry per kilometre; this is equation 30.

Therefore the total inductance of the circuits because one is current going another return path right, so total inductance is called loop inductance right. So, total inductance of the circuits is L is equal to L 1 plus L 2 right. So, 0.4605 log D upon r 1 dash plus 0.4605 log D upon r 2 dash. Therefore, L is equal to 0.4605 log then both you make it D square upon r 1 dash into r 2 dash, is equal to 0.4605 log D upon under root r 1 dash r 2 dash whole square this 2 will come here, so it is 0.4605 into 2 log D upon under root r 1 dash r 2 dash.

So, 2 into 0.4605 is equal to 0.921 log D upon under root r 1 dash into r 2 dash milli Henry per kilometre. So, this is that total this is that inductance of the your (Refer Time: 08:40) inductor this is called loop inductance and this is the formula of that you are what you call the total inductance of a circuit right. So, this is equation 31. This is milli Henry per kilometre.

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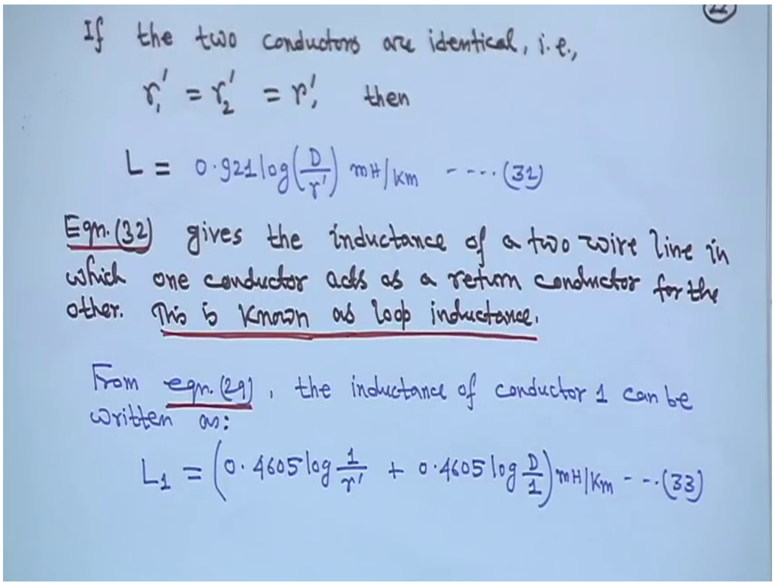

Therefore if the 2 conductors are identical; that means, we assume r 1 dash is equal to r 2 dash is equal to r dash that means you are what you call. That means, you are I mean r 1 is equal to r 2 is equal to r.

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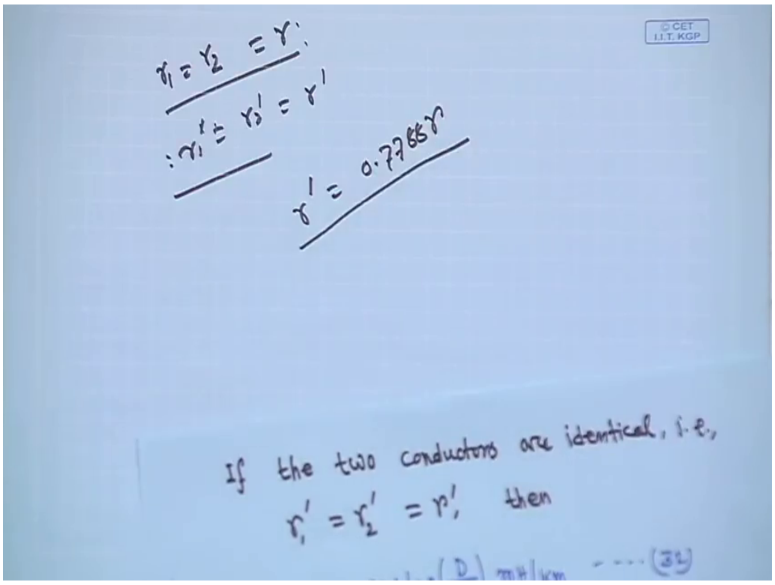

That means your r 1 dash is equal to r 2 dash is equal to r dash. So, we are assuming the radius is r r 1 is equal to r 2 is equal to r right therefore, r 1 dash is equal to r 2 dash is equal to r dash. So, r dash is equal to 0.7788 into r fictitious radius of the conductor.

Therefore, L will become point nine 2 1 log D upon r dash milli Henry per kilometre this is equation 32, right. So, now, equation 32 give the inductance of a 2 wire line in which 1 conductor acts as a return conductor for the other this is known as loop inductance right because you are considering the conflict circuit, one incoming another outgoing. So, totally this is called loop inductance.

So, this now from equation 29; that means, this equation now from equation 29; that means, this equation right this equation 29 the inductance of the conductor one can be written as it is given 0.4605 log D upon r 1 dash milli Henry per kilometre. What you do this equation you write 0.4605 log 1 upon we are assuming here that r 1 is equal r 2 is equal to r dash. So, instead of r 1 we are writing directly r dash understandable, there should not any confusion.

So, 0.4605 log 1 upon r dash plus second term 0.4605 log D upon 1, 1 upon r dash, but we are putting 1 because of some reason. So, here it 1 upon r dash is plus 0.4605 log D upon 1 milli Henry per kilometre this is equation 33. So, these equations you put in this form. Because our objective is find out some generalized formula.

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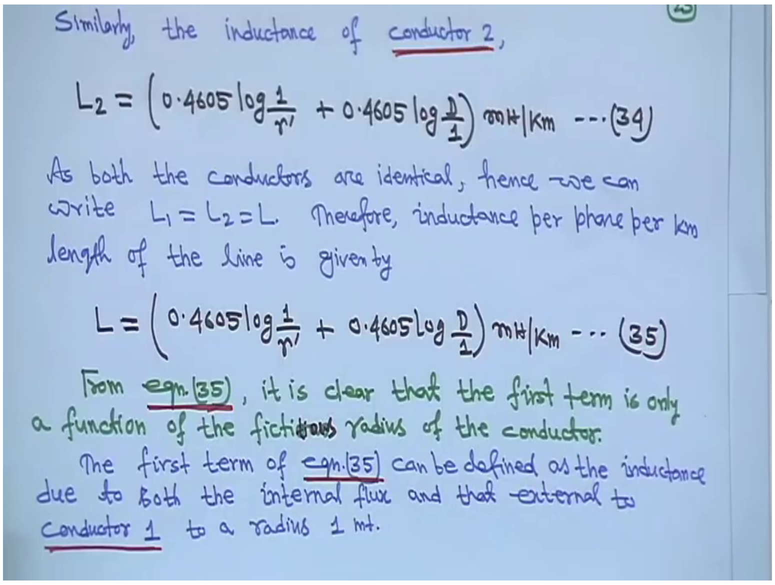

Therefore, similarly for the conductor 2, this is equation 34 similarly the inductance of conductor 2 right same way we can write L 2 is equal to 0.4605 log 1 upon r dash plus 0.4605 log D upon 1 milli Henry per kilometre this is equation 34.

As both the conductors identical hence we can write L 1 is equal to L 2 is equal to L they are same therefore, inductance per phase per kilometre length of the line is given by L is equal to 0.4605 log. Now we are writing in general therefore, the inductance per phase per kilometre length of the line is given by we are writing that L is equal to 0.4605 log 1 upon r dash plus 0.4605 log D upon 1 milli Henry per kilometre this is equation 35, because we are writing L 1 is equal to L 2 l.

Now, from this equation 35 it is clear that first term is only a function of the fictitious radius of the conductor this term is only function of r dash and second term is only function of D. So, first term here it can be that equation 35. That means, this term can be defined as the inductance due to both internal flux and that external to external to conductor 1 to a radius your 1 meter. That means, this term first term can be defined as the inductance due to both the internal flux and that external to the conductor to a radius up to 1 meter only that is why 1 upon r dash.

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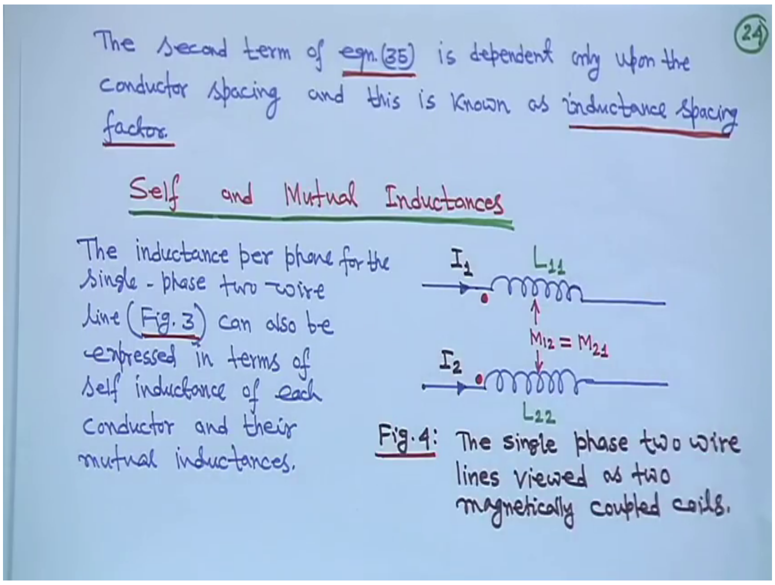

Similarly, the second term similarly the second term of equation 35 right; that means, this term second term of equation 35, second term of equation 35 right is depended only upon the conductor spacing because between that conductor distance is D then the conductor spacing and this is known as inductance spacing factor. So, this is the 2 terminology. So, this term is called inductance spacing factor. Later from this one we will see some generalize thing.

Now next thing will come that self and mutual inductance right. Suppose you have single phase 2 wire line viewed as 2 magnetically coupled coils you assume they are magnetically coupled right. So, current is one case this is dot convince and is taken right as currently entering the dot. So, you tacking that is a plus if leaving the dot it is minus, but in this case both dot are taken here right. So, one is incoming another is outgoing path. So, I 1 actually I 1 is equal to minus I 2, but direction is taken like this and they are entering into the dot and self inductance of this per conductor 1 it is L 11 self inductance of conductor 2 is L 22 and mutual inductance between them is M 12 is equal to M 21. So, this is the single phase 2 wire line viewed as 2 magnetically coupled coils.

Therefore the inductance per inductance per phase for the single phase 2 wire line that is figure 3, just hold on figure 3, I will show you again just hold on - this is your figure 3, this is your figure 3. So, these circuits also we are taking as a magnetically coupled circuit right representation is like this, this is actually your figure 3. So, can also be expressed in terms of self inductance of each conductor and they are mutual inductance the way it has been represented here; later figure 4 is marked later.

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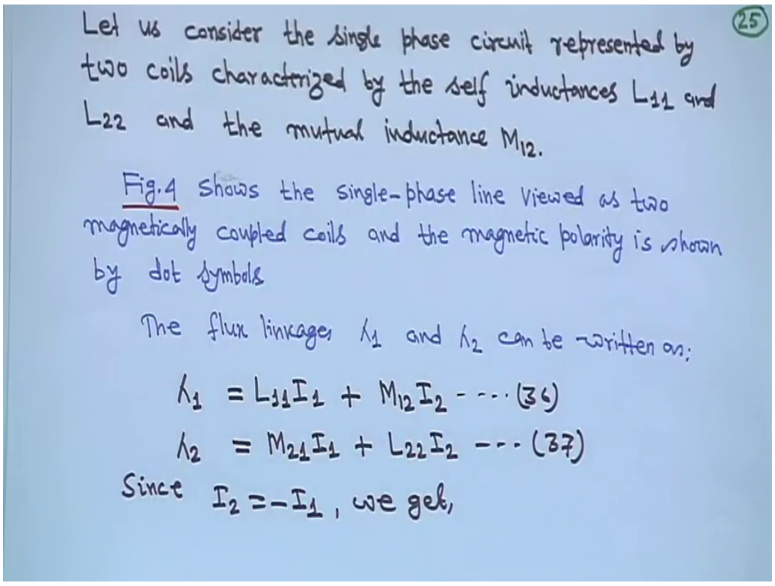

So, let us consider the single phase circuit represented by 2 coils characterize by the self inductances L 11 and L 22 and the mutual inductance M 2 I mean this one. So, that is why figure 4, this is the figure 4 right; shows the single phase line views us to magnetically coupled coils and the magnetic polarities shown by dot symbol now flux linkage lambda 1 and lambda 2 can be written as that dot current entering dot. So, we will take the positive 1.

So, flux linkage 11 lambda 1 lambda 2 you can write lambda 1 is equal to L 11 I 1 plus M 12 I 2 right M 12 and this is due to the current I 2. So, M 12 I 2, this is equation 36. Similarly, lambda 2 for this 1 is equal to M 21 I 1; M 12 is equal to M 21, but we have 2 write from the mathematical point of view M 21 I 1 plus L 22 I 2 right this is equation 37. This plus plus here also plus sign here also plus sign because current both the cases we assuming entering into the dot.

Now, but current 1 is incoming another is outgoing, but I 2 is equal to minus I 1. So, if you put I 2 is equal to minus I 1 we will get both the equation you put I 2 is equal to 1 equation you put I 2 is equal to minus I 1 another equation we will put I 1 is equal to minus I 2.

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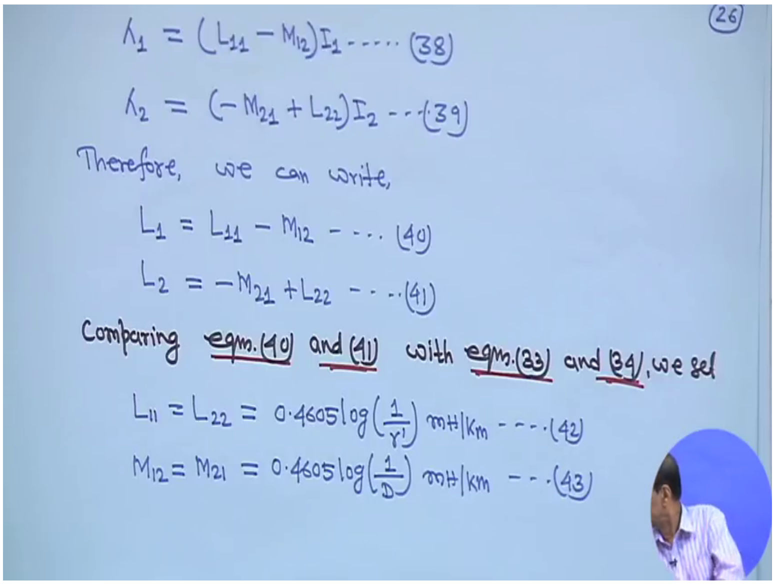

If you put you will get lambda 1 is equal to L 11 minus M 12 bracket close into I 1 this is equation 38. Similarly, lambda 2 is equal to minus M 21 plus L 22 bracket close into I 2 this is 39. Therefore, we can write L 1 is equal to L 11 minus M 12 I mean this is your L 1 and L 2 is equal to minus M 21 plus L 22 this is your L 2 this is equation 40, this is equation 41. Therefore, you compare equation 40 and 41 with equation 33 and 34 just hold on I will bring equation 33 and 34 just hold on.

Yes, so this equation 40, equation 40 you compare with equation 33 right. So, if you compare with equation your equation 40 with 33. So, this is equation your 33 right. So, what you can, what one can do is I am putting it here look, just hold on.

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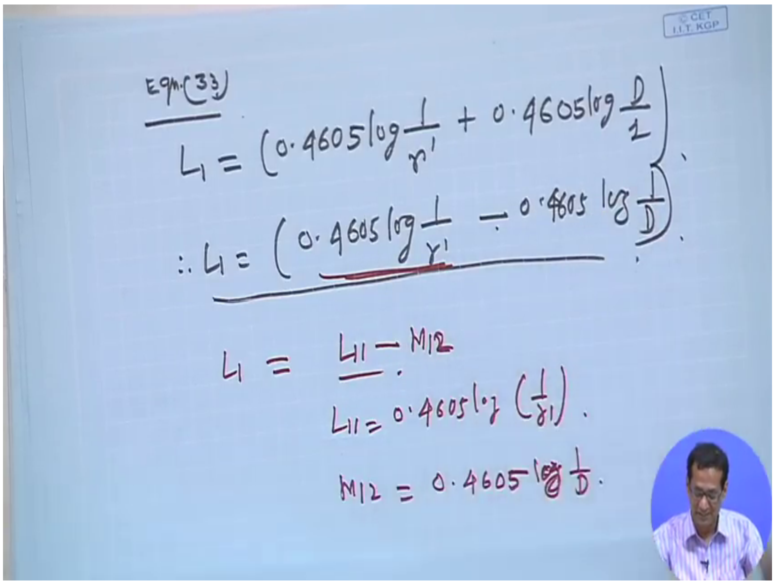

It is written like this I am rewriting equation 33 I am rewriting equation 33 right. So, in this case your L 1 is equal to 0.4605 log 1 upon r dash plus 0.4605 log D upon 1 right this one you can write L 1 is equal to 0.4605 log 1 upon r dash. So, minus 0.4605 log I am writing instead of D by 1 it is minus is taken here, so 1 upon D, this is actually your equation 33.

Now, we are comparing this equation 33 with your equation 40. This is now equation 33 and next if I put equation 40 that is L 1 is equal to my L 11 minus M 12 right. So, when you compare in this 2 equation 40 and 41 just hold on while it as gone this thing. So, when you are comparing actually L 11 L 11 is equal to this 1 that your L 11 is equal to your 40 and 33 we are comparing right. So, L 11 is equal to just I am writing 0.4605 log 1 upon r dash right and M 12 this is here it is L 11 minus M 12 already minus sign is here. So, M 12 is equal to 0.4605 your log 1 upon D.

So, if this is just we are comparing this just we are comparing this 2 that is why when you similarly for L 22 also both are same when you compare 41 and 34 it will remain same therefore, L 1 is L 11 is equal to L 22 actually this is 0.4605 log 1 upon r dash milli Henry per kilometre this is equation 40 2 and M 12 and M 21 same 0.4605 log 1 upon D milli Henry per kilometre

So, this is what D by 1 taking minus here making log 1 upon d. So, this is actually L 11 and this is actually M 12. So, this is that this is just the same thing right how we how we can go for this comparison right. So, hold on. So, now, this is whatever you have seen we have seen for 2 conductors right one outgoing and other is incoming now if you have say n number of conductors, if you have n number of conductor. And that is this above described approach of self and mutual inductances can be extended to a group of conductors.

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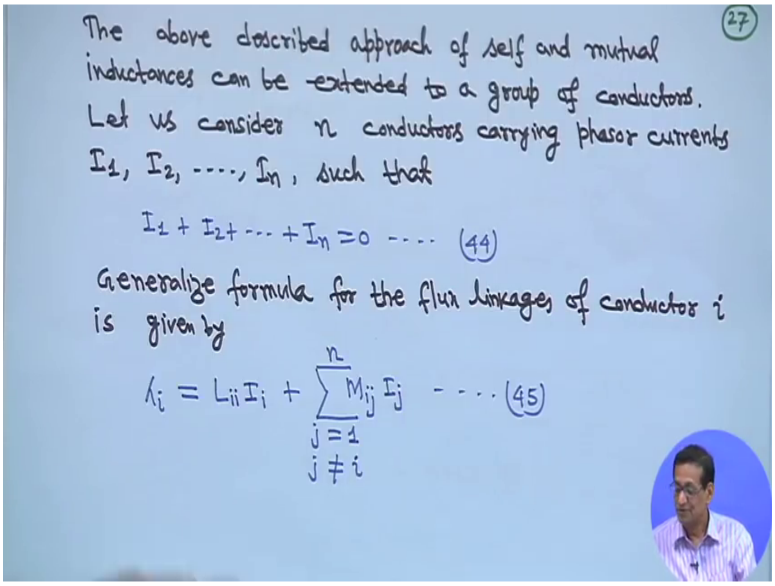

Suppose let us consider n conductors carrying phasor currents I 1, I 2, I n and so on right. So, you have a group of conductors n number of conductors and we can make it I 1 plus I 2 plus dot dot dot up to plus I n is equal to 0 this is equation 44 right. So, generalized formula for the flux linkage of conductor I is given by lambda I is equal to L ii, capital L I i; I i plus j is equal to 1 to n j naught is equal to I M I j I j this is equation 40 five right loop here we have written only for 2 conductors L 11 is 1 L 1 is equal to L 11 minus M 12 this is for what if you are another one, one more term will be added and so on. So, that is why this instead of this is your that is why this your this lambda 1, lambda 1 is equal to basically L 11 I 1 minus M 12 I 1, but this sign minus sign will come because of this direction of the current i j.

So, in general this lambda 1 instead of lambda 1 we can write for ith conductor lambda I is equal to L ii capital L suffix I i into I i plus j is equal to 1 to n M i j into I j and j naught is equal to I this is the generalized formula for the flux linkage of conductor i. So, only from this simple thing, small thing we are giving this generalized formula this formula will view again and again for other things.

So, this mathematical expression for flux linkage for conductor I is very important right. So, this is actually equation 45. So, if it is so, hold on therefore, if you have this 1 therefore, from that we have seen lambda you are what you call that lambda 0.4605 this thing common all the time.

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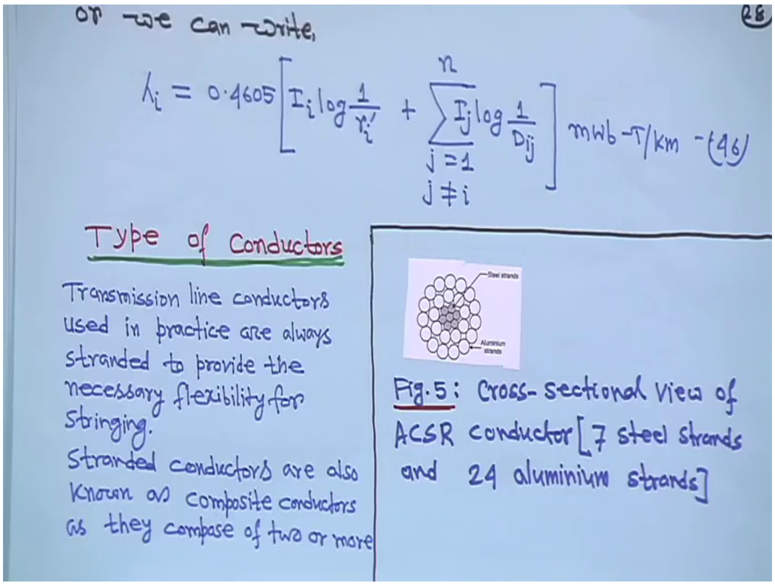

So, in this case what we have seen is that lambda it can be written as 0.4605 take common I i into log 1 upon r i dash same formula we are using plus i j log 1 upon D i j, j is equal to 1 to n j naught is equal to i milli we have got tons per kilometre.

So, there we have seen 2 terms, there we have seen 2 terms this is; this one there we have seen first term is r upon r dash second term is 0.4605 log 1 upon D and this is 0.4605 log this is the first term, this is the second term the mutual term right. So, in this case, in this case so this mutual term we are writing the same one we are writing it was given 1 upon D. So, general thing you make j is equal to 1 to n i j log 1 upon D i j, j naught is equal to I and first term 0.4605 will be common first term I i log 1 upon r i dash.

So, this flux linkage formula will be used again and again, your what you call for your calculating the flux linkages of other configuration. Now there is a figure here actually I check carefully this figure I have taken from a book right, but this figure is actually incorrect. So, we will correct it, we correct it, how many conductors will be their around this (Refer Time: 25:58) first second third layer this counting is not matching. So, this figure you should not do it I will tell you how to do it.

So, figure 5, so next is type of conductors. So, transmission and conductors used in practice are always standard. So, provide the necessary flexibility and standard conductors are also known as composite conductors right as they compose of 2 or more elements or stand electrically in parallel. So, this configuration I will give you the formula this how many conductors where. So, outsider one while they totally white one it is aluminium strand and inside that it is steel strand, this is steel stand the shaded one and other one this aluminium strand just to increase the mechanical strength we will give that.

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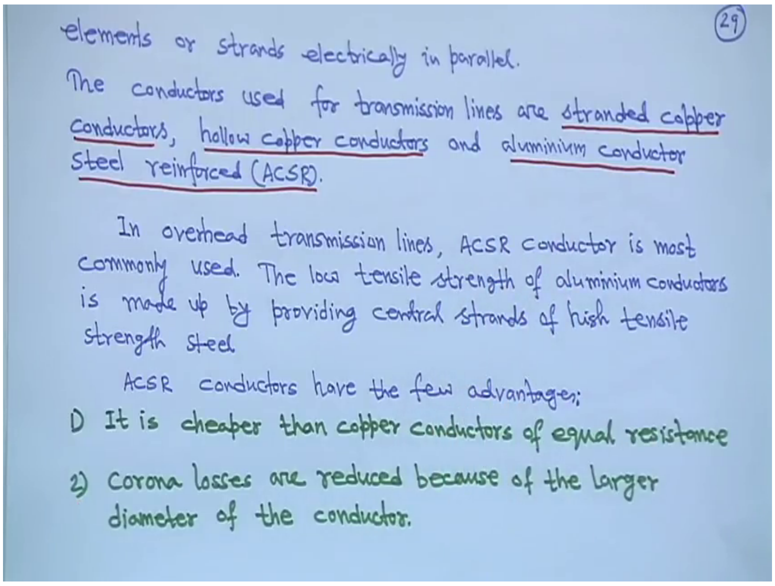

So, the conductor use for transmission lines are standard copper conductors, hollow copper conductors and aluminium conductors steel reinforced that is ACSR that is we call aluminium copper steel reinforced. In overhead transmission lines ACSR conductor is most commonly used right we will see that that ACSR conductor. The low tensile strength of aluminium conductor is made of by providing central strength of high tensile strength is steel.

So, the central strands these are actually steel, these are actually steel strands. So, ACSR conductors have the few advantages. I have written few such that I should not miss anything. It is cheaper than the copper conductors of equal resistance, corona losses are reduced because of the larger diameter of the conductor, so this is second point.

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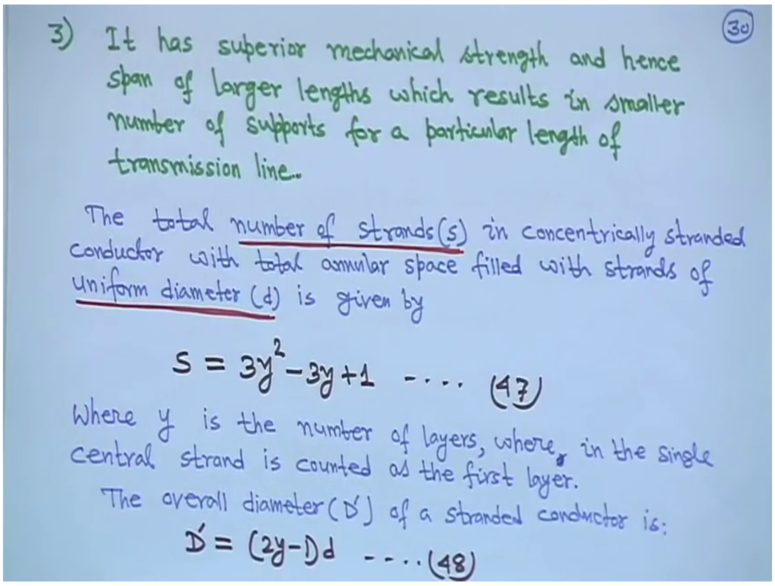

And third point is it has superior mechanical strength and hence span of larger lengths which result in smaller number of supports for a particular length of transmission line and is mechanical strength is a or this thing what you call your superior very strong then the span between the 2 towers can be increased such the number of towers will be less.

So, that is why it has superior mechanical strength because it is steel strand centrally right. So, the total number of strands in bracket is in a concentrically standard conductor with total annular space filled with strands of uniform diameter D is given by there is a formula that, why is the your what you call number of layers there is a formula S is equal to 3 y square minus 3 y plus 1. That means, this one this formula if you say this one.

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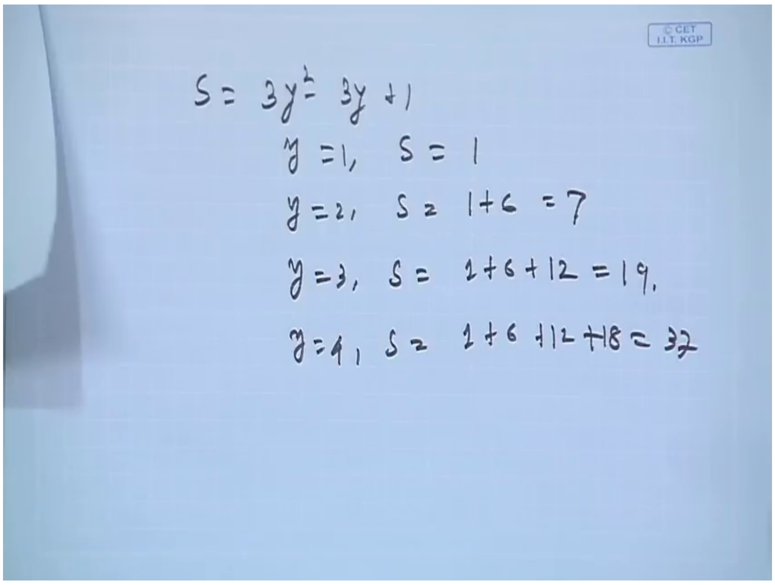

S is equal to 3 y square minus 3 y plus 1 for the first one when y is equal to 1 you substitute y 1 you will get S equal to 1. When y is equal to 2 you will get substitute 2 2 square 4 12 minus 6 6 plus 1 it is 7, 1 plus 6. Then when y is equal to 3 then S is equal to your 3 into 3 9 - 27 minus 8 18 plus 1 - 19. So, 1 plus 6 plus 12 is equal to 19. So, in y is equal to 4 then S is equal to I think it will be 1 plus 6 plus 12 plus 18 is equal to 37; that means, in this diagram; that means, in this diagram actually when y is equal to 1 first is central 1, when y is equal to 2 1 plus 6 - 7 so. Secondly, it will be 6 conductors.

When y is equal to 3 third layer there will be 12 conductor 1 plus 6 plus 12; 1 6 12 we are counting 12 not coming. So, this diagram is incorrect. So, around this there will be twelve conductor right and next one if you go for the forth one then there will be 18 conductors and the last one here 1 2 3 4 layer is there. So, total will be last layer will be 18 conductor, but total will be 37. So, you can make it of your own, so that is the thing.

So, this diagram this is our counting it was I think some less than 12 it is not correct one. So, some drawing error is there so that means, and the total diameter, total diameter it is given D dash is equal to 2 y minus 1 D, if y is equal to 1 then D dash is equal to D, if y is equal to 2 then D dash will be 3 D and so on. So, this is the overall diameter of the standard conductor D dash is equal to 2 y minus 1 into D this is equation 48.