Power System Analysis
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Lecture - 07
Resistance & Inductance (Contd.)

So, again you see that.

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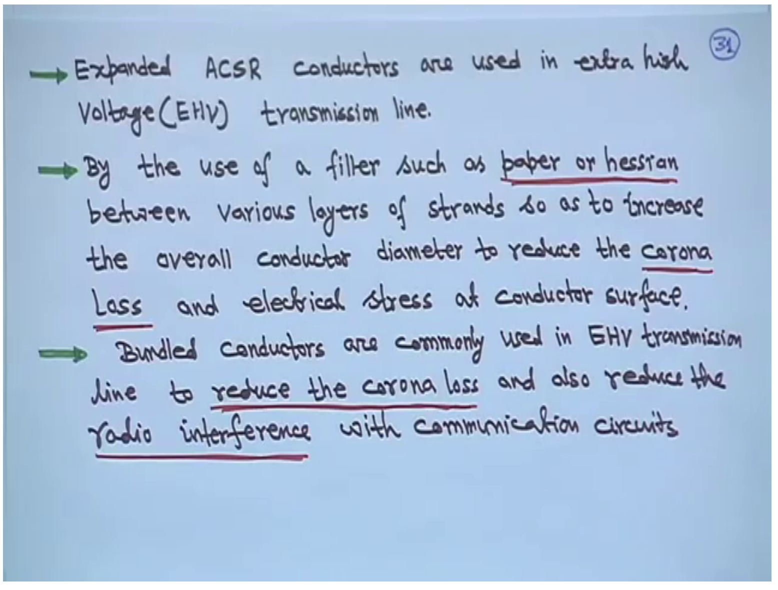

That expanded ACSR conductors are used in extra high voltage transmission line. So, by the use of filler or such as paper or hessian between various layers of strength, as to increase the overall conductor diameter to reduce the corona loss and electrical stress at conductor surface, and bundle conductors are commonly used in extra high voltage transmission line to reduce the corona loss and also reduce the radio interference with communication circuits.

So, this is that you what you call; expanded ACSR conductors are also used for your extra high voltage transmission line. So, next is the inductance of composite conductors.

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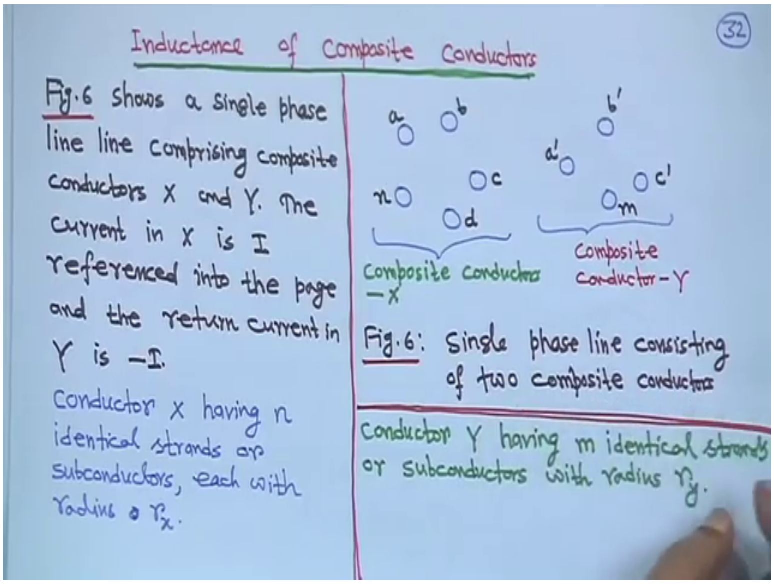

Suppose this is your figure 6, the single phase line say consisting of 2 composite conductor composite conductor X, this is actually I have written X composite conductor X suppose you have a, b, c, d, up to n number of conductor in this group composite conductor here also a dash b dash c dash up to m you have composite conductor another group you have n number of conductors. And we are calling this is composite conductor X and these are calling composite conductors Y right and what we are doing is that you are assuming that all this all this conductors they have their, your radius remains same.

So, for this group as composite conductor X we are typing for this group radius of this conductor all this conductors we are assuming r x small r suffix X here it is a r x right and here also that it is your a dash b dash, it is a group Y. Similarly this Y having m identical conductors right their radius is r y here, it is r y here, it is r y right and another thing is that and suppose these group this total current right they are this things the current in this group is I right referred into the page and that means the return currents is minus I.

That means, if one is entering into the page another is leaving to the page. So, this current total current actually I and here it is written I means it is something like this right there should not be in suppose.

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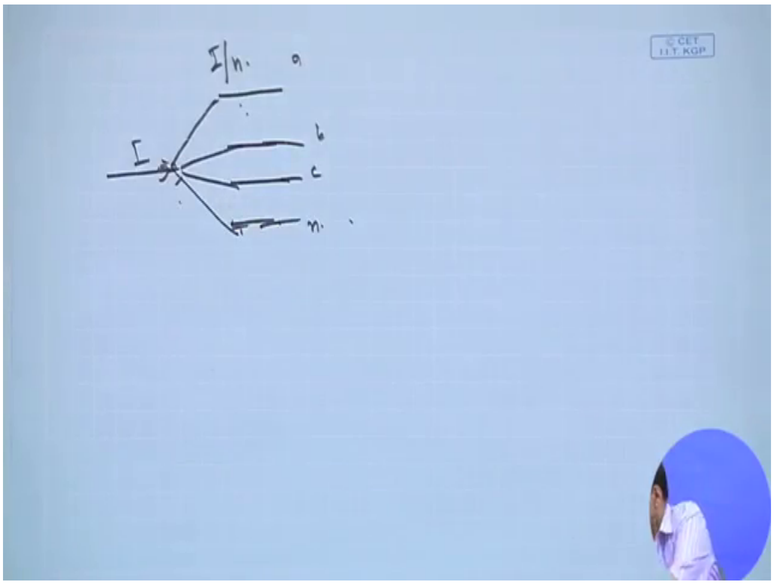

Suppose you have and just for the sake of understanding this thing suppose this is suppose if you have suppose a b c n this kind of four conductor sides suppose you have this much of conductors and this is the current I all the radius of this conductor is you are what you call we assume that it is your r x, r x, r x. So, total current is I. So, each conductor will carry currents I by n because they all are in parallel and we assume they are all similar conductor. So, each conductor will carry current I upon n.

So, here in that group you have n number of conductors. That means, in this group total current is I total, but after that current is getting divided in equally in each conductor therefore, each conductor actually is carrying I by n current. So, later we will see next page and here also if the current is high, but its return path. So, if this current positive is I by I this current will be minus I and each current returning at which will be minus I by m because here you have m number of conductors. That means, that means. So, we have assumed that current is equally divided among the sub conductors.

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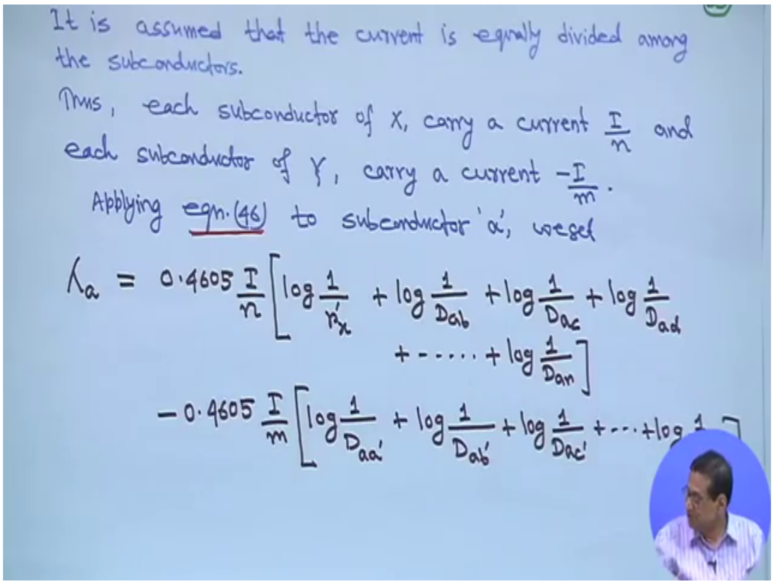

So, that c sub conductor of X carrying a current I by m I just told you and each sub conductor of Y carrying a currents minus I by m minus sign it is a return path that is why minus I by m. Now we will use this equation 46 we will use this equation, we will use again and again.

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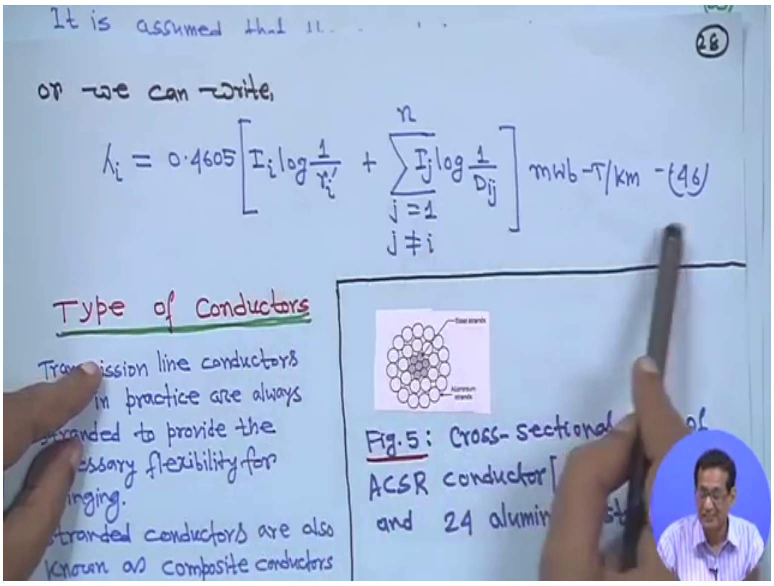

Now, this 46 equation number this equation this is generalized fact linkages these equation we will use right, so, applying equation 46 to sub conductor a; that means, for this conductor say. So, many conductors are here we are applying for a; for say this conductor a conductor a right.

Therefore this formula if we use then you will see lambda a we are now replacing for conductor a lambda a is equal to 0.4605 current is your instead I it will be I by n each conductor is carrying I by n right. So, I by n then your log 1 upon r dash r x dash because r x is the radius of the sub conductor in group X right. So, r dash x plus all the all the all the distances log; log 1 upon D a b here, here it was given in terms of general 1 j is equal to 1 to n and j naught is equal to I i j log 1 upon D i j the general thing, but i j all can be replaced to a b c m and this right.

So, this one we can write log 1 upon D a b; that means, this distance within this within this group D a b then log 1 upon D a c plus log 1 upon your a b a c then log 1 upon D a d right up to plus log 1 upon D a n the first step first this thing is all the all the distances from a; from sub conductor a to all other sub conductors in this group that is why log 1 upon D a b plus log 1 upon D a c plus log 1 upon D a c plus dot, dot, dot, plus log 1 upon D n.

So, this is the first thing or you within that a to b, a to c, a to d, dot, dot, dot, a to n all that distances right using this formula only; using this formula only. Now next one is this return path is also there this current is minus I by m.

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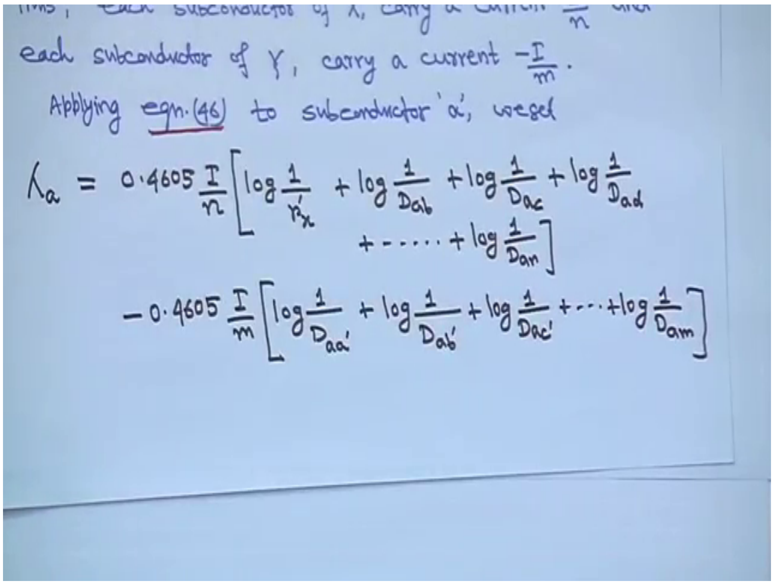

So, that is why next one is your minus 0.4605 minus for I by m then log 1 upon D a right that is your D a a dash; that means, mutual 1 mutual distances a a dash then you look a look here a a dash a a dash then you look a b dash a b dash plus look a c dash a c dash up to plus log 1 upon D a n a m mutual distances.

So, this within the group this is between the group right. So, first term is all like this second term is only the D a a dash a b dash a c dash up to a a right and minus sign because this is a return path. So, this is only the expansion of this of this particular equation right. So, I hope you have understood this right very simple actually this is one is incoming another is outgoing return path only you have n number of conductors there, but if n is equal to m then different issue, but this term will remain as it is. This is within the group this is between the group the distance between the groups.

So, now we will simplify this equation therefore, this term lambda equal to this term I n all should be log 1 upon r x plus log all this things can be written as log 0.4605 I by n log 1 upon r x dash into D a b into D a c into D a d up to dot, dot, dot, D a n.

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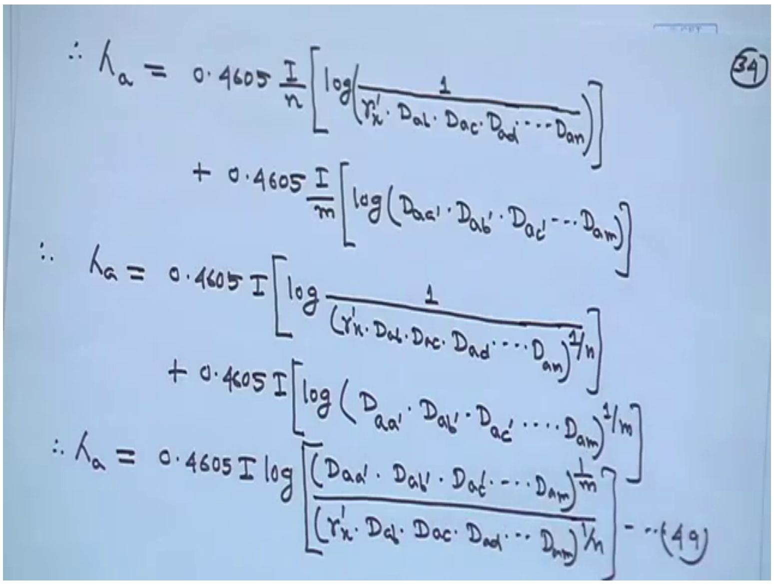

A next; now this is actually 1 upon D a dash log of 1 upon D a dash D a dash plus log upon 1 upon D a b dash like this. So, rather than you take this one you make plus right and this 1 upon d a actually you can write D a a to the power minus 1; 1 upon D a b to the power minus 1. So, all minus 1 minus will come ultimately it will be plus. So, 0.4605 I by m called log then D a d a a dash into D a b dash into D a c dash up to D a m right.

So, this plus because this 1 upon D a we have made d a 1 upon D a b; D a b; right, so, just you make this; this like this why right; next this 2; this 2 you can write that your 0.4605 I, this 1 upon n is here; here 1 upon m is here. So, log of all this thing and this power to the power 1 upon n. And similarly this one also 1 upon m is here I by m. So, this power to the power 1 upon m right therefore, it can be written as lambda a is equal to 0.4605 I log D a a dash into D a b dash into D a c dot, dot, dot, D a m to the power 1 upon m r x dash into D a b into D a c into D a d up to D a m to the power 1 upon n, this is equation 49; I think it is a understandable to you right.

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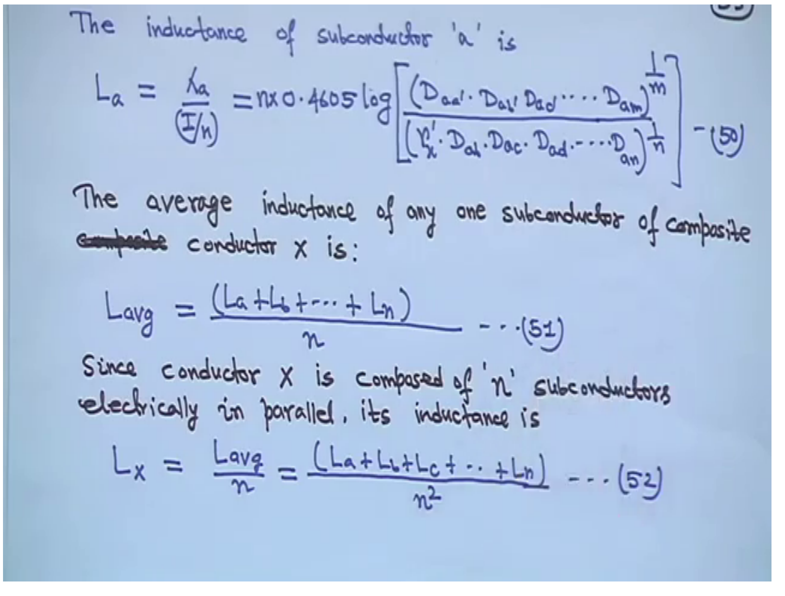

Now therefore, inductance of sub conductor a is that flux linkages L a is equal to lambda a and each that current is conductor is carrying current I upon n divided by I upon n do not make I because I is the total conductor each conductor is carrying currents I upon n. So, it should be lambda a upon I upon n is equal to, then lambda you know is equal to n into 0.4605 log the same expression D a a dash into D a b dash into D a c dash up to D a m to the power 1 upon m then r x dash into D a b into D a c into D a b up to D a n to the power 1 upon n this is equation 50.

Now, the average inductance of any one sub conductor of composite conductor a X can be given as L average can be given as L a plus L b plus up to dot L n divided by n this is equation 51. And since conductor h is composed of n sub conductors electrically in parallel it inductance is L average by n the idea is something like this know suppose you have that average inductance of any sub conductor this is the average inductance of any sub conductor.

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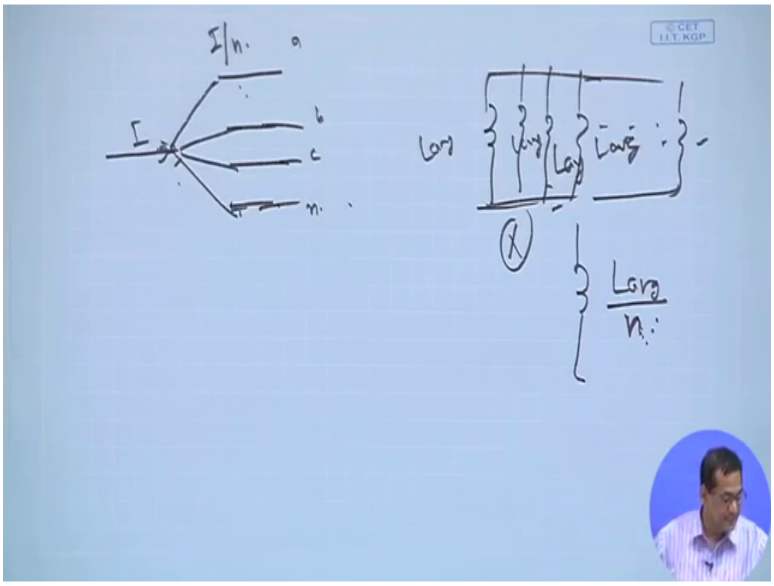

And if you have so many things are parallel; right. So, many things are parallel dot, dot, dot, up to n n n in all are L average L average then L average L average all are L average then they are equivalent is then L average by n right. So, all are electrically parallel.

So, that is why L average will be your total; that means, L average we got first right then total this is the composite X that conductor group x. So, that is L average n L average divided by n right because everyone has average inductance L average L average we are computing here L a plus L b up to this up to L n by n, because in group X n number of conductors are there right therefore, and all the conductors electrically are in parallel I showed you also this all conductors are in parallel right the total current I is conductor is assuming carrying current I by n.

Therefore, this L X is equal to that group your that sub since conductor X is composed of n sub conductors electrically in parallel its inductance is; that means, group you are what you call group X that inductance L X is equal to L average by n is equal to L a plus L b plus L c up to L n divided by n square because L average is L a plus L b up to L n upon n and this one is equal to here n is also there. So, if you substitute L average this one it will become n square it is 52.

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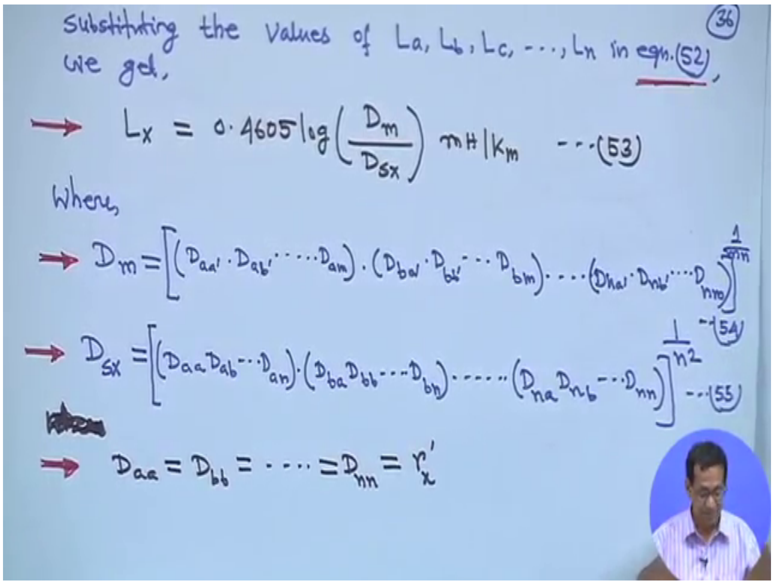

Therefore, now substitute the values of L a L b L c and L n in equation 55, we have computed only L a right. So, if you write this if you compute L a L b L c all sort of like this cannot be made it here only. So, if you can make yourself L b L c like this. So, if you substitute all the values of L a L b L c in equation 52; that means, in this equation you substitute right remember if you substitute L a L b L c all will be function of n into this term.

So, as soon as you submit substitute here that n will be everywhere L a L b L c one n one n will be cancel here only one n will be remained not n square only n will be remaining right. So, that is why if you substitute this equation can be written as 0.4605 log D m upon D s x Milli Henry per kilometer I will define what is this right. So, and as you substitute that this all L a L b L c all function of n; that means, it will not be n square because 1 n will be cancel only n will be there. In that case what will happen that D m is equal to there is D a a dash D a b dash dot, dot, dot, D a m then this is for sub conductor a to all the all the mutual distances of the other group.

Similarly, for sub conductor your b in group X to the distances to all conductors in group Y right that is b a dash b b dash and so on up to your D n a dash D n b dash D n m right and to the power 1 upon m n because 1 upon n I told you that when we will try to find out this one only n will be there if you make all this is function this is n 1, L will be cancelled one n will be there. So, that is why an earlier it was 1 upon n, now it will be multiplied by your 1 upon n into 1 upon m into 1 upon n it will be 1 upon n m right it will be 1 upon m n little bit you try yourself right.

Similarly, for self one it is within that group D a a D a b d and further sub conductor a to all is distance in the group say X right a a b a n similarly from b a f b b d b n up to all product up to D n a, D n b, D n n it will be 1 upon n square because here it was here it was 1 upon m because up that another one; one L will be there to find out your L x this thing to find out the this expression of L x. So, because of that that is one will go to 1 upon n will be there when you will find out the average 1. So, it will be 1 n will be there.

So, it will be 1 upon m n and it will be 1 upon n square right that is why here it is 1 upon m n and here it is 1 upon n square right and this is equation 55, I hope you have understood this one right this is distance of a particular conductor in group X to that all this thing you are what you call conductors in group Y that is why a a dash a b dash a m b a dash b b dash like this to up to the power 1 upon m n 1 upon m n and this is within the group.

So, D a a D a b, D a n D b a D b b D b n. So, on this will be 1 upon n square right and this is equation 55 where D a d b b up to d n n, they are usually fictitious radius that is r x dash of group X right therefore, this d.

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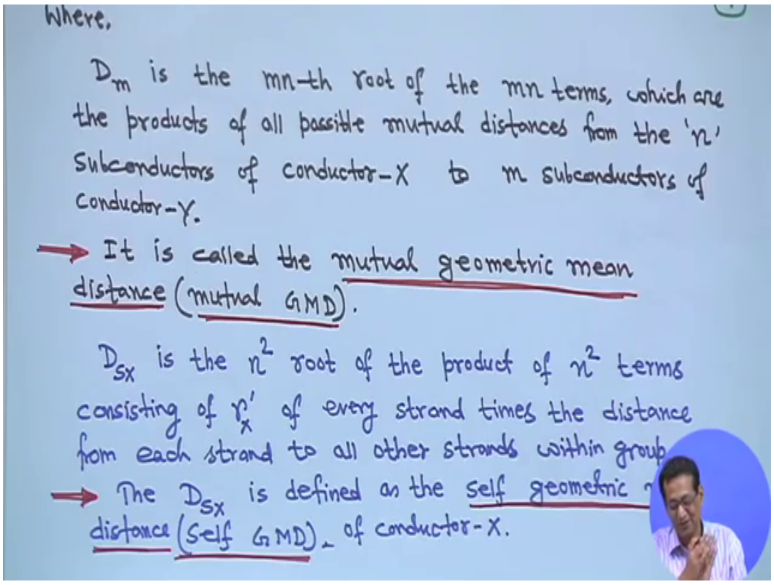

Now, this D m that is your this one; this D m that is D m and D s x right. So, D m is the mn-th root of the m n terms which are the products of all possible mutual distance I told you this are all mutual distances between one conductor in group X to all the conductors of the group Y that is why a a dash a b dash up to m b a dash b b dash b m for b conductors.

Similarly, for nth conductor in group X to other conductors in group Y n a dash n b dash these all are product to the power 1 upon m n right. So, when it is mutual distance is coming in group a m number of conductors are there group Y number of conductors are there. So, total possible possibility is m into n that is why D m is the m nth root of the m n term which are the products of all possible mutual distances from the n sub conductors of conductors X that is group X to m sub conductors of conductor Y right it is call the mutual geometric distance that is mutual GMD, this is called this D m is called that mutual geometric your what you call a distance right. So, it is called mutual GMD.

Similarly, D s x is the n square root of the product of n square term consisting of r x dash that is D a a D b b all are basically fictitious radius r x dash and you have your suppose in a group n number of conductor your conductor sub conductor are there. So, if this is n square root of the product of n square term right because it will be n into n right consisting of r x dash of every stand time the distance from each strand to all other strands within group X the D s x is defined as the self geometric mean distance this is called self GMD of conductor X.

So, one is mutual GMD another is self GMD right. So, for everything we need 2 things needs to find out the inductance one is mutual GMD another is self GMD right I hope you have understood.

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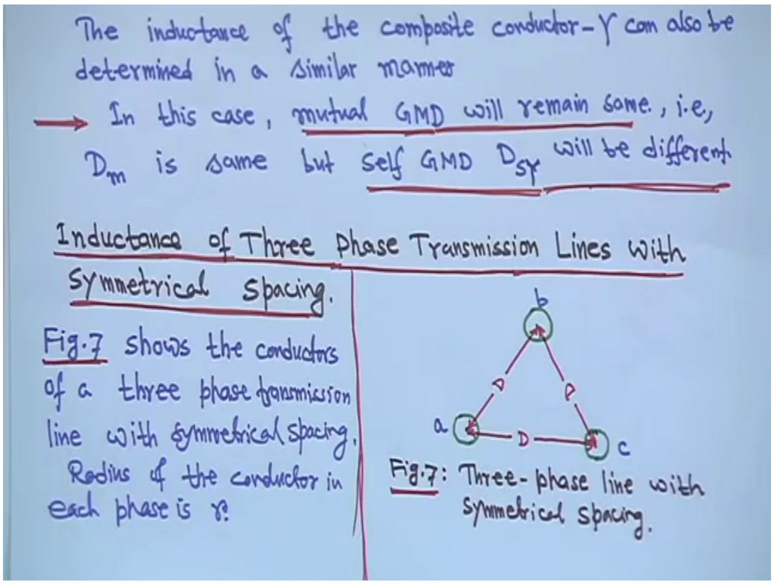

Therefore similarly for conductor Y; similarly for conductor that this thing may be the group Y; so, the inductance of the composite conductor Y can also be determined in a c similar manner. So, in this case as it is one is group X another is group Y. So, there will be no change in mutual GMD it will remain same; that means, dm will remain same right.

So, mutual GMD D m will remain same, but yourself GMD d s Y will be different this D S Y will be calculated same as before the way we have made it for group X that conductor composite conductor X similar here you have to make it here for composite conductor Y not shown here this is up to you little bit this thing up to you right.

So, next one is I hope you have understood little bit practice is necessary right little bit understanding is also necessary. So, next is that inductance of 3 phase transmission line with symmetrical spacing here we have assuming that we have a 3 phase transmission line phase a phase b phase c say and symmetrical spacing d is the distance is a its is conductors are say you are sitting at the corner of a your equilateral triangle. So, this is first you have assumed like this after that we will generalize.

So, this 3 phase line with symmetric spacing d d distance same and the radius is also r r for each conductor r right and this is your. So, this is figure 7. So, figure 7 shows the conductors of a 3 phase transmission line with symmetrical spacing and radius of the conductor we have assume that it is r, right. So, again here we will use the same equation that equation forty six; that means, same equation this equation 46 again we will put this equation if you try to understand again and again if you can understand this equation from this equation you can derive everything

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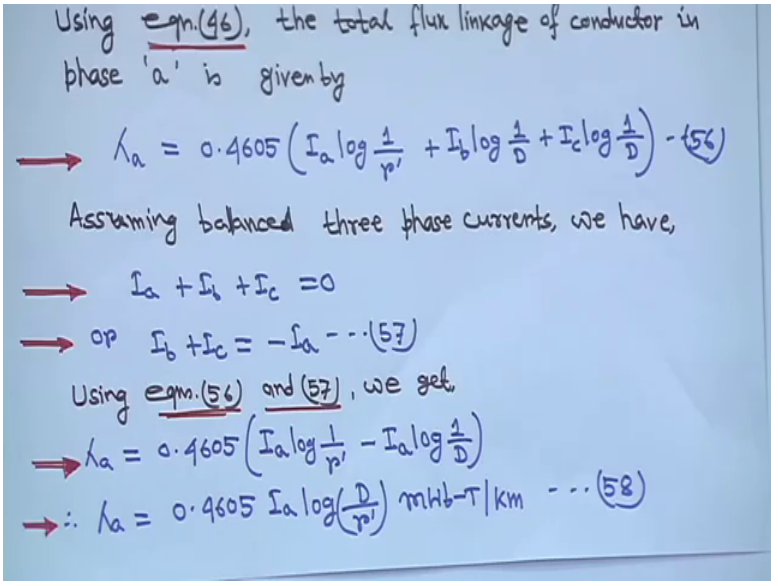

So, this equation 46; the total flux linkages of conductor phase is given by lambda a for phase a only here it is I; I log 1 upon r I suppose for I is equal to a common b common c 3 phases are there a b c suppose I is equal to a I mean i; small i, this a right a b c 3 phases are there. So, lambda a is equal to 0.4605 into I a instead of here it is general one that I a log 1 upon r dash plus I b distance same. So, it will be log 1 upon d plus I c log 1 upon d right.

Otherwise it should have been D a b D b c like that that we will see later this is equation 56, but in this case distance are equal. So, things are easy right. So, lambda is equal to 0.4605 I a log 1 upon r dash plus I b log 1 upon r d plus I c log 1 upon d this is 56 assuming that balance 3 phase current we are assuming therefore, I a plus I b plus I c is equal to 0.

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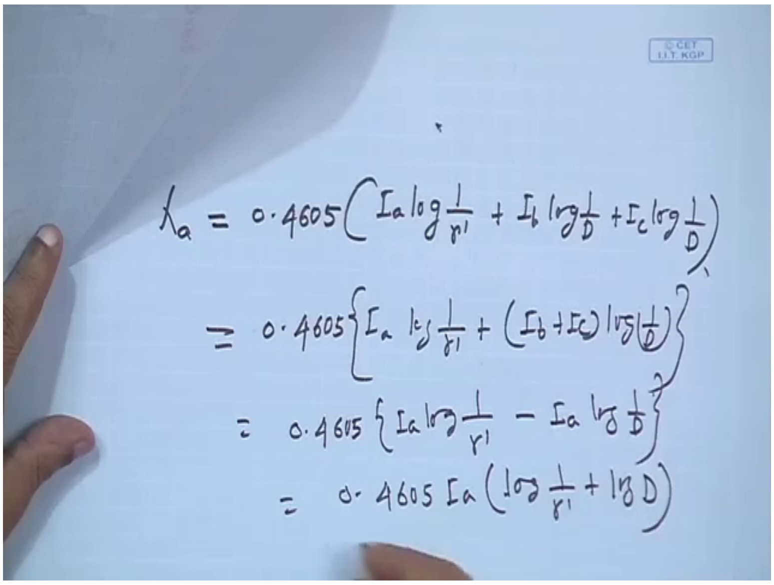

Therefore I b plus I c is equal to minus a; that means, this equation that just hold on this equation lambda a is equal to 4605; I a log 1 upon r dash plus I b log 1 upon d plus I c log distance same this is from equation fifty 6 right is equal to 0.4605 then I a log 1 upon r dash right plus I b plus I c you outside you write log 1 upon d right and I b plus I c is balance condition. So, I b plus I c is equal to minus I a you put here minus I a log 1 upon r dash minus I a log 1 upon d.

So, this one you can write 0.4605 right I a common right log 1 upon r dash now minus is 1 upon d.

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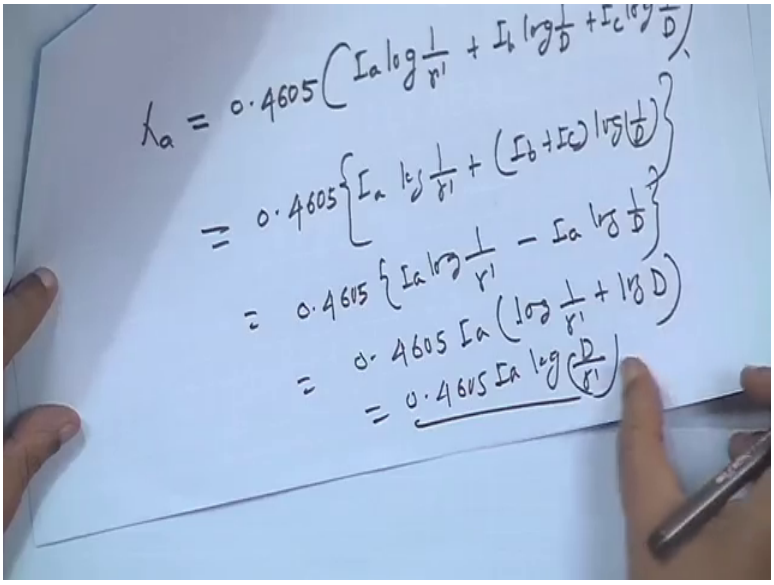

So, we are writing log d. So, minus as become plus is equal to is equal to 0.4605 I a log d upon r dash; right so, that that thing that is the thing that we are substituting for using equation this 56 and 57, this 56 and 57 right. So, we are writing that just now I shown this expression right. So, we can write lambda is equal to 0.4605 I a log d upon r right it is it is milli weigh, but tons per kilometre. So, this is equation 58.

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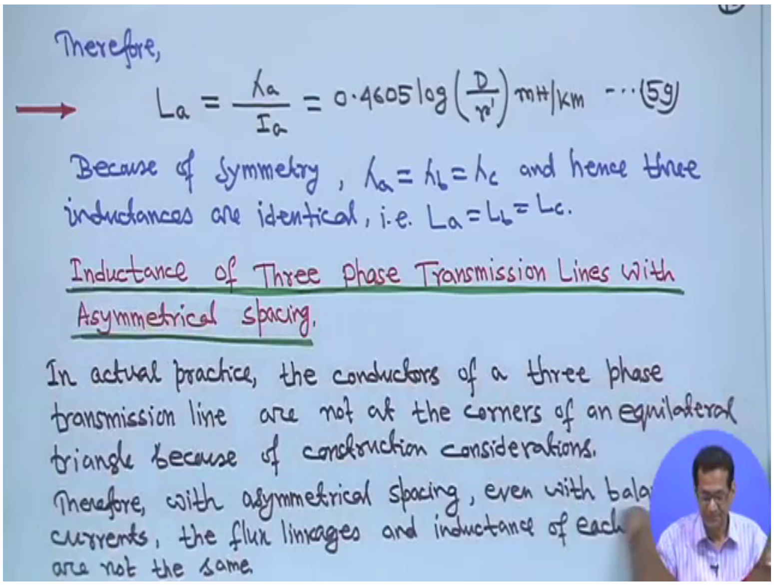

Therefore inductance L a is equal to lambda a upon I a. So, for I a will not be there now. So, is equal to 0.4605 log d upon r dash Milli Henry per kilometre this is equation fifty nine right. So, because of symmetry right because of symmetry lambda a is equal to lambda b is equal to lambda c hence it is an equilateral triangle actually and hence this inductances are identical that is L a is equal to L b is equal to L c right. So, they are same because it is symmetrical spacing.

So, I hope up to this up to this it is it will be this thing understandable to all of you I been a this calculation of this inductance.

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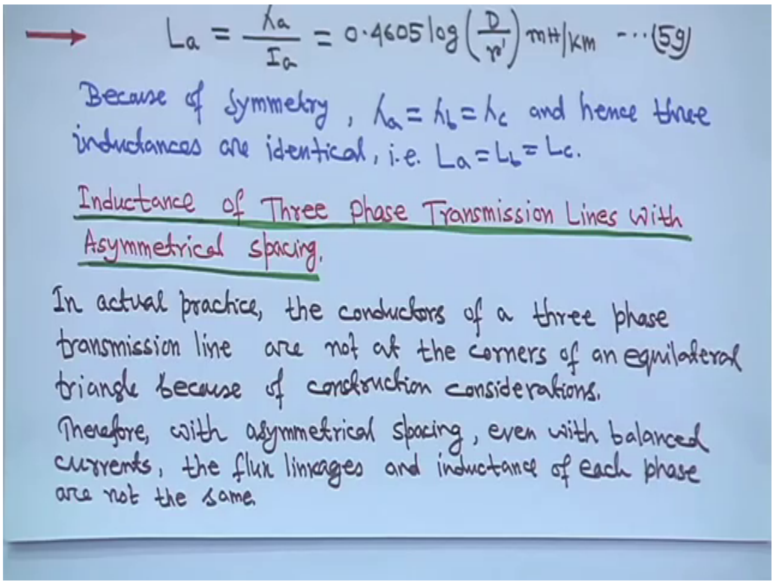

And after this what we will do now inductance of 3 phase transmission lines with asymmetrical spacing now that your; what you call that it is not symmetrical now right therefore, we asymmetrical spacing even with balance currents the flux linkages is an inductance of each phase are not the same simply write the spacing is not symmetrical it is asymmetrical spacing.

So, even if that current is balance right, but flux linkages is an inductance of each phase will not be the same. So, in that case suppose you have a 3 phase line this is phase a phase b and this is phase c sorry phase a this is phase b this is phase c right.

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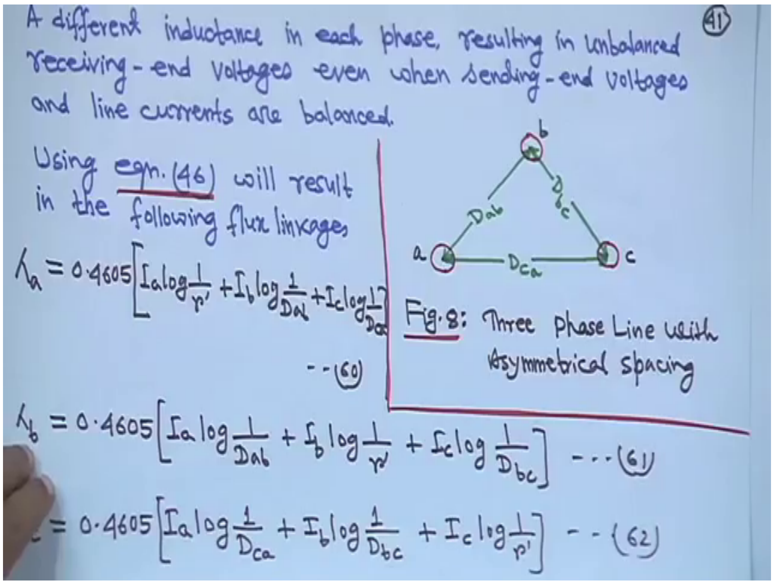

So, distance between the phase a and b is D a b phase a and c is D c a and b and c is D b c D b c right. So, now, that D a b D b c D c a are different. So, using that same formula that equation this 46 again and again we will use this one right use this equation again and again then everything is easy using this equation 46 right you can write lambda a for the flux linkages of the conductor in phase a is equal to 0.4605 I a log 1 upon r dash plus I b log D a b 1 upon D a b a to be plus I c log 1 upon D c a that distance is a to c well I think a b b c c a basically a to c is a c we are writing D c a. So, I c log 1 upon D c a this is equation 60.

Similarly, for phase b you can write lambda b is equal to 0.4605 then I a then log 1 upon D a b right because b to a I a distance is a to b D a b I a log D a b as soon as you are coming an I b plus 1 upon I b that it will be log 1 upon r dash I b into and when it is plus I c log 1 upon D b c right. So, between this next is this is equation 61.

Similarly, lambda c is equal to 0.4605 I a log 1 upon D c a right this is now for lambda c plus I b I b log 1 upon D b c right that is 1 upon D b c plus I c log 1 upon r dash when I a I b I c is coming 1 upon r dash right this conductor may have different radius also r 1 r 2 r 3 in this case this will happen r a dash this will happen r b dash this will happen r r c dash, but you have assume they are same and just I am using this formula only right.

So, this generalized formula you try to understand again and again I am telling this right so; that means, this is equation 62. So, you have got lambda a lambda b and lambda c this flux linkages of phase a phase b phase c conductors right now or in matrix form if you put or in matrix form if you put then it will be lambda a lambda b and lambda c.

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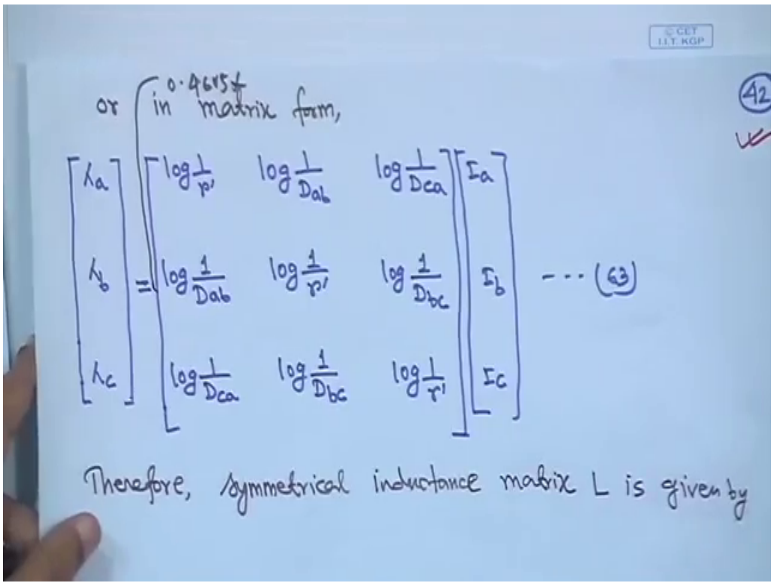

And it will log 1 upon r dash log 1 upon D a b log 1 upon D c a log 1 upon D a b log 1 upon r dash log 1 upon D b c log 1 upon D c a log 1 upon D b c log 1 upon r dash into I a I b I c this is your 63 therefore, symmetrical inductance matrix L is given by right because everywhere you will you will divide for lambda a for lambda a you will divide it by I a for lambda b you will divide it by I b and for lambda c you will divide by I c such that L a L b L c all will get. So, that is why in matrix form if you write only this matrix will be there.

So, L is equal to if you do. So, L is equal to hold on one thing here a lambda a lambda b I have missed one thing that 0.4605 I have missed it right it should be multiplied by this should be multiplied in making it here somewhere this should be multiplied by 0.4605, I have missed it actually right it should be multiplied

So, because here it is 0.4605; 0.4605; 0.4605, so, I have missed it.

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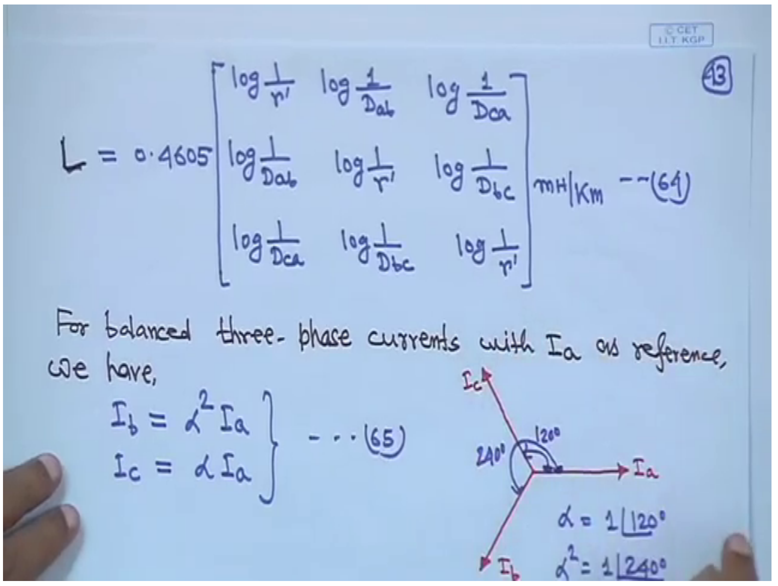

Therefore, L is equal to L is equal to L means L a L b L c right is equal to 4605 log 1 upon r dash log 1 upon D a b log 1 upon D c a log 1 upon D a b log 1 upon r dash log 1 upon D b c log 1 upon D c a log 1 upon D b c log 1 upon Milli Henry per kilometre.