Power System Analysis
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Lecture - 08
Resistance & Inductance (Contd.)

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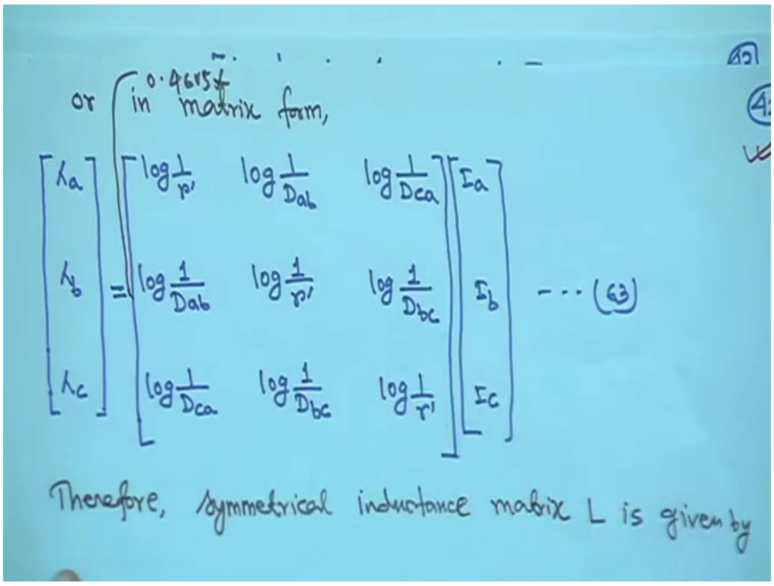

So, just have a look on this that flux linkages for phase a, phase b, phase c, lambda a, lambda b is equal to 0.4605 into this whole matrix right into I a, I b, I c this is equation 63. Therefore, symmetrical inductance matrix L is given by right. So, just hold on by this expression L is equal to 0.4605 then put them in the matrix form in the milli Henry or kilometre.

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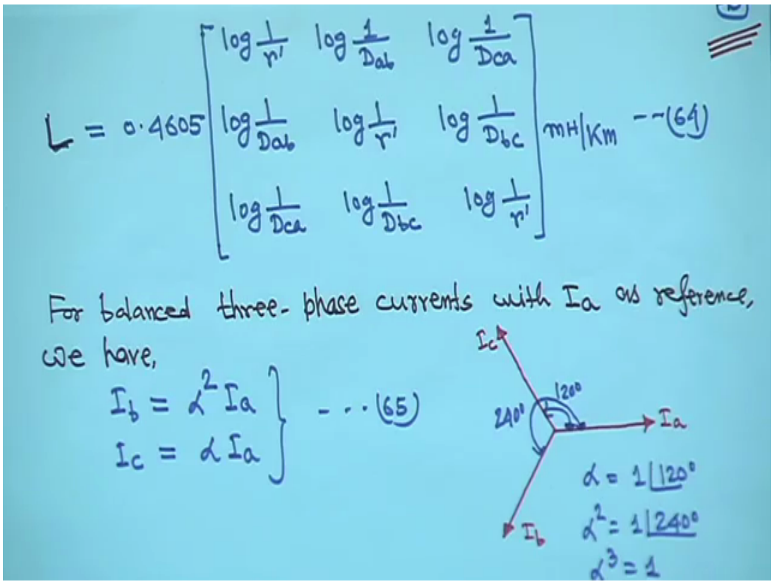

Now, for balance 3 phase currents with I a as reference, this I a as reference I a I b I c they are 120 degree C difference. So, alpha is 1 angle 120 degree hence alpha square will be 1 angle 240 and alpha 3 is equal to 1 right. You know this therefore, we can write that I c is equal to right actually it is leading I by 120 degree. So, I c is equal to alpha I a that this one right and I b actually leading by 240 degree that is your alpha square therefore, I b is equal to alpha square into I a this is equation 65 right.

So, this expression for I b I c right if you substitute here, if you substitute in this equation just hold, on if you substitute here I b expression for I b I c are in equation 63 Then what we will get that is we are writing using equation 63 and 65. So, what we will get we will get L a is equal to lambda a upon I a 0.4605 log 1 upon r dash plus alpha square log 1 upon D ab plus alpha log 1 upon D ca this is equation 66.

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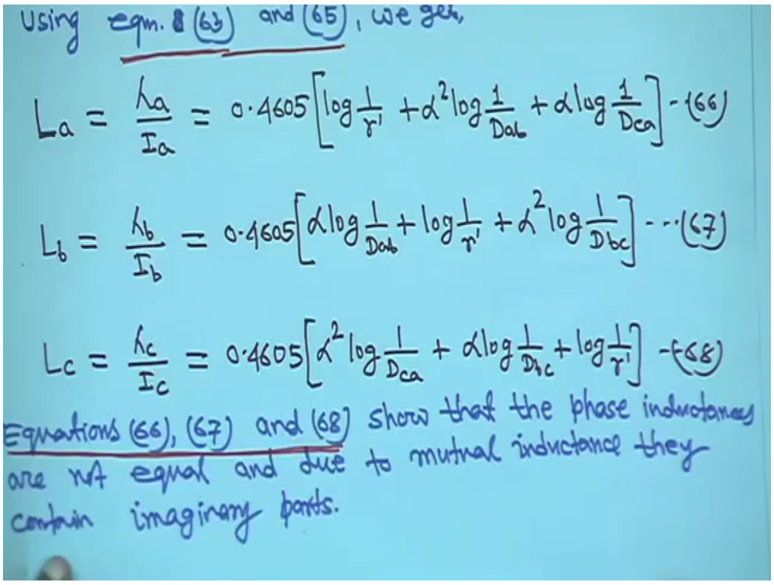

Similarly, lambda b is equal to L b rather L b is equal to lambda b upon I b is equal to 0.4605 then into alpha log 1 upon D ab plus log 1 upon r dash plus alpha square log 1 upon D bc this is equation 67. Then L c is equal to lambda c upon I c is equal to 0.4605 into alpha square log 1 upon D ca plus alpha log 1 upon D bc plus log 1 upon r dash this is equation 68.

So, equation 66 67 68 it shows this show that this inductance L a L b L c are not really quantity right that that are not equal, then first thing they are not equal due to mutual inductances they contain because they contain imaginary parts. So, these are not real quantities right. So, that means all L a L b L c they will be different.

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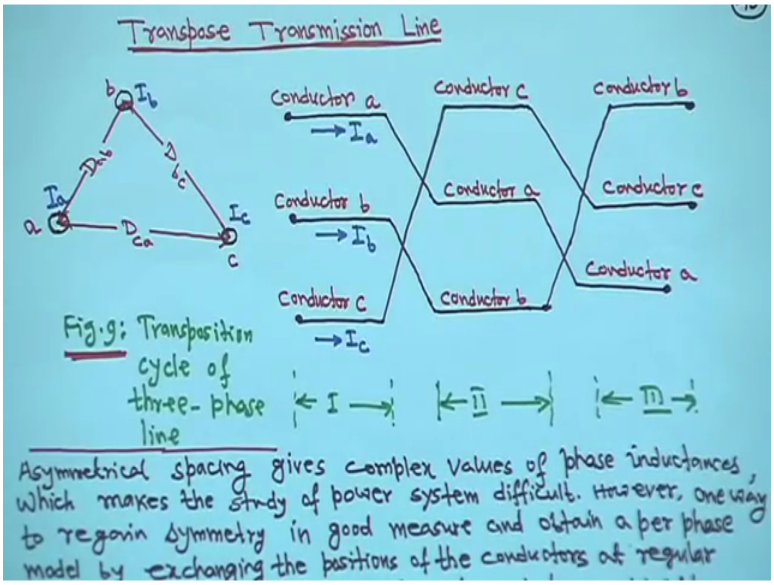

So, to make the inductance of phase a phase b phase c more or less same right we go for a methodology for transmission line that is called transpose transmission line right. So, in that case suppose you have a conductor suppose this is this way triangular configuration is taken say this is a, this is b and this is c phase current in conductor phase a that is I, a phase b it is I b and phase c that current is I c at distance phase a and b as usual it is D ab, between phase b and c it is D bc and between a and c we were writing D ca.

Now, this is that transmission cycle, cycle of 3 phase line this is section 1, this is section 2 and this section 3. Now what will happen that you take for example, you take fast this is conductor a, this is conductor b, conductor c. So, consider the section first. So, this is conductor a, conductor b and conductor c as usual, current through conductor I a through conductor b I b and through conductor c I c.

Now what will happen? This is the first of section 1 say now in the next section what we make it section 2 that it will be in cyclic order right; that means, conductor a was suppose to be here, but instead of that it is occupying the position of conductor b. So, it is coming here conductor a and similarly if we do not take transmission conductor a should have been here also in section 3, but ultimately it is taking the position of conductor c.

So, conductor a conductor a; that means, in 3 different section that means 1 cycle that because 3 phases. So, therefore, it is changing it is in (Refer Time: 05:36) position of the conductor is being changed.

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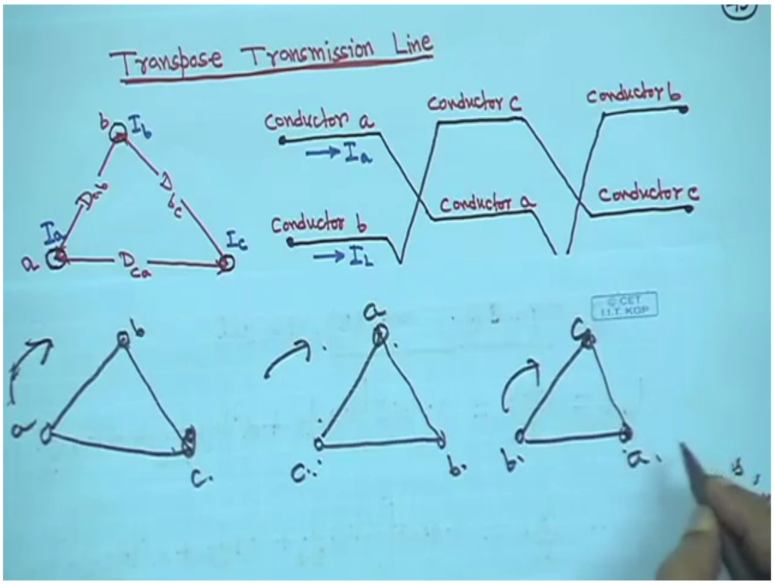

So, it is a b c, now if you take like this that your suppose it is a, it is b and it is your it is your c. So, this is your this is a b c this is a this is b this is c. Now in the next section that conductor position actually is a changed; that means, what that it will be again I am drawing it for you. So, say this is here now you rotate it in clockwise. So, this is c that initially it was c, c is occupying position of a. So, it is coming here c, a is occupying position b. So, it is going to here and b is occupying its position your what you call c’s position. So, here it is b. So that means, here you take in clockwise direction in clockwise direction right clockwise direction.

Therefore, next section it is coming c a b c a b right. Similarly for the last 1 again you rotate it this is your conductor like this 3 conductor. So, b will occupy the position of here it is coming to b will be here c was here now c will be here, now a was here now a will take the position of b in the next; that means, it will be again in clockwise a b you what you call b c a right here it was c a b it starting from here it is b c a look at that, it b c a.

So, from a b c this is start from here a b c a b c next just be c is occupying position of a, a is occupying its position at b, b is occupying its position at you are what you call c s position b is coming here that starting from here only c a b, c a b similarly third one also it is b c a right. So, this is this is actually is called what you call the transposition of that your what you call conductor and that is and if you say that is at regular interval in reality if it is a regular interval then average inductance will be more or less same.

Now, what is happening that asymmetrical spacing gives complex values of phase inductance. So, which we have seen just now right which makes the study of power system no difficult; however, one way to remain region symmetry in good measure and obtain a per phase model by exchanging the position of the conductors at the regular intervals along the lines as that these conductors occupies the your original position of every other conductor right. So, this way we will go for transposition such that 1 cycle; that means, 3 section this one, this one and this one.

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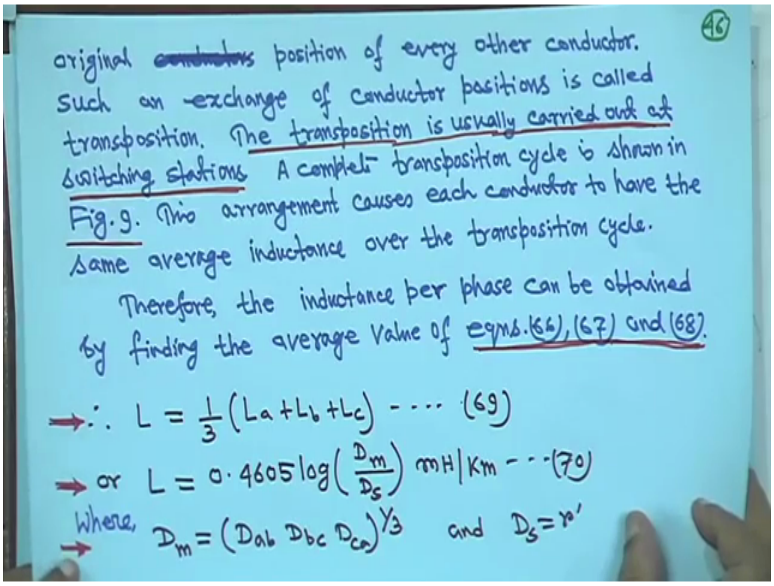

So, in that case what will happen that such exchange of the conductor position is called the transposition right. So, transposition is usually carried out at switching stations when; that means, its substation right. So, a compute cycle is shown in figure 9, figure 9 means this one, this one this is your this is figure 9 figure 9, just now I explain this is figure 9.

So, this arrangement causes each conductor to have the same average inductance over the transposition cycle because we have conductors every in 3 section or conductors they are position is getting change. So, expected that average inductance will remain same for all the 3 phases right therefore, to get the average inductance will make L is equal to one-third then L a plus L b plus L c right this is equation 69. So, or L is equal to 0.4605 log D m upon D s millihandy per kilometre this is equation 70 where all this things we have defined before where D m is equal to D ab into D bc into D ca to the power one-third and D s here is equal to just r dash that fits D sas radius.

So, this will be more or less you are what you call all the phases inductance will remain as L. So, that is the advantage of transposition.

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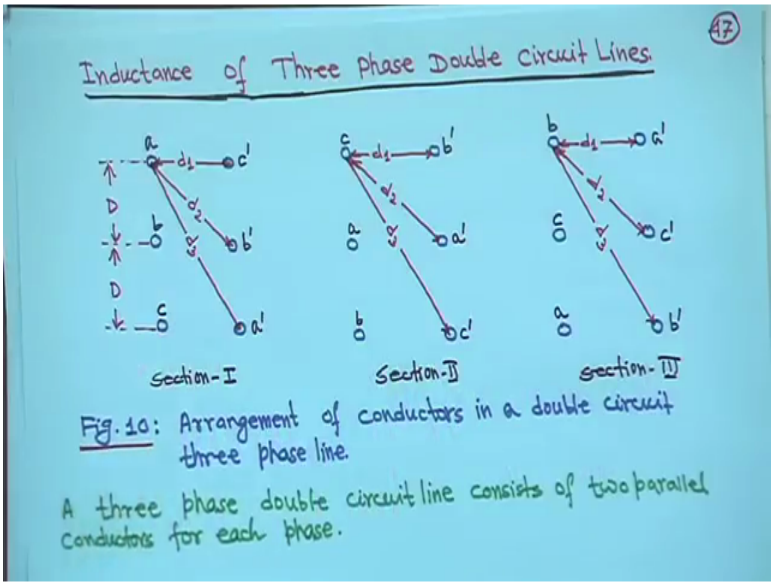

So, next one is inductance of 3 phase a double circuit line right. So, in this case that again its section 1, section 2 and section 3 right. So, here it is a b c I mean it is actually double circuits line; that means, in each phase there are 2 conductors right. So, a b c here also other conductors in phase a dash then b dash then c dash right. So, and distance this a c dash is D 1; that means, b b dash is also D 1, c c dash is also c a c to a dash is also D 1.

Similarly, your a to b dash is equal to D 2. That means, b to a dash is also is D 2 because from symmetric right and a 2 a dash is D 3. That means, c dash to c is also D 3 right and between a and b distance between phase a phase this conductor a and conductor b it is D similarly conductor c dash b dash right and b to c it is D and a to c it will be D plus D, so 2 D.

Now, same as before same as before if we change this, what you call this is that section 1, this is section 1 right now in this section 2 it is c a b, c a b. So, here also it is change that c a b. So, c to c dash, a to a dash and b to b dash right. So, this is c a b here also c a b, but 2 conductors are there in each phase that is why c to c to c dash a to a dash and your b to b dash.

Similarly, in the section 3 it is b c a right, just now we explain this, therefore, here also b c a. So, b to b dash then a to a dash and c to c dash; that means, it is double circuits line also then section 1, section 2 and section 3 this will be the configuration. So, this is actually figure 10 the arrangement of conductors in a double circuit 3 phase line. Now a 3 phase double circuit line consist of 2 parallel conductors for each phase, I told you that a and a dash there are 2 conductors there in parallel similarly in b b dash similarly for c and c dash.

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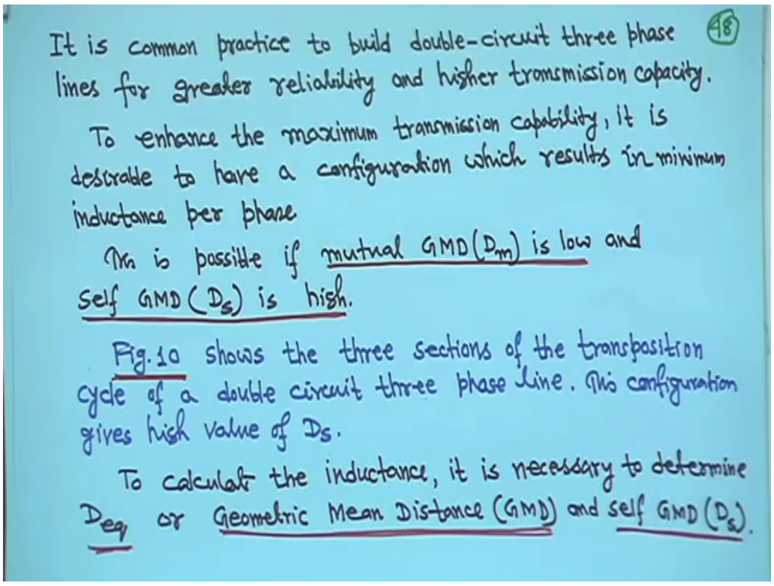

Generally, it is a common practice right to build double circuit 3 phase lines of greater reliability and higher transmission capacity because in each space if you have a 2 conductors in parallel right first thing is for they are, there is they have to shy the resistance will be half in each phase that is the thing. Second thing is that inductance means the reactance, so 2 parallel conductors in each phase means the reactance inductance will be half hence the reactance also will be the half right. Therefore, to end that means, if it is so that it actually increase the maximum transmission capability.

So, to enhance the maximum transmission capability it is desirable to have a configuration which results in minimum inductance per phase we want inductance should be minimum because for transmission system resistance is quite low I mean very low compared to the your reactance therefore, we want that inductance should be less hence the reactance also will be less.

So, this is possible if mutual GMD is low and self GMD is high we have seen know just I am taking all the that term, that whenever that expression is coming that log of function of log of D m by D s right.

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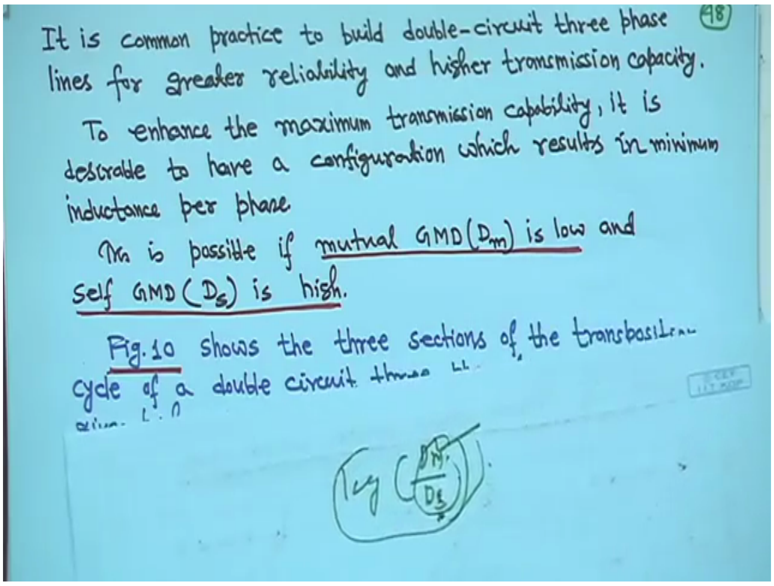

If you can reduce D m right and if you can increase D s, so it is D m is D m is low and at the denominator is D s D s is high means it is a, it is a 1 upon it is your what you call. So, overall D s is high D m is low D s is high then overall this D m by D s ratio will be much low hence this quantity will be lower.

So, that if you make, so inductance will be less that is why if mutual GMD is low and self GMD is high then only it is possible right therefore,. So, the figure 10 I showed you the 3 sections of the transmission cycle of a double circuits 3 phase line. Now to calculate the inductance it is necessary to determine D eq or geometric mean distance GMD first you have to find out right and then self GMD both the things you have to find it out right first GMD and then self GMD.

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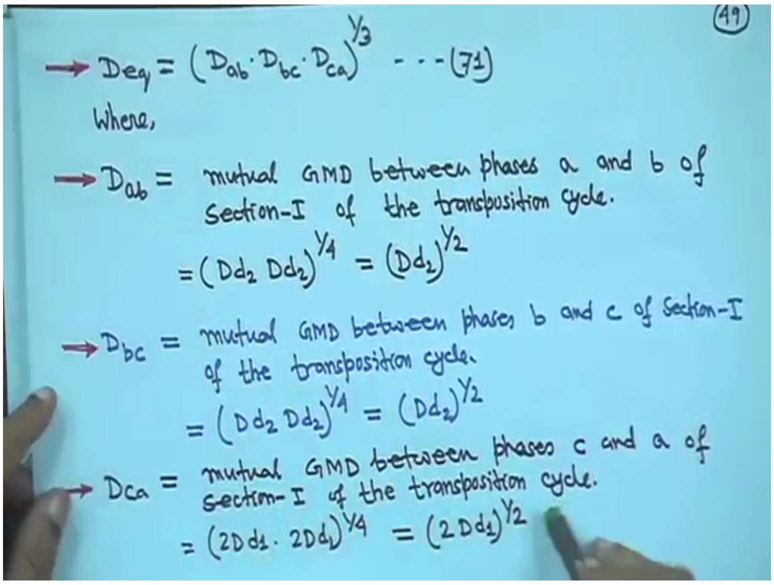

Now, to do this I have to go to this figure again. Now you know earlier also we have seen that D eq will be D ab into D bc into D ca to the power one-third that is, so will say equation 71. So, D ab; that means, that mutual GMD between phase a and b of section 1 on the transposition cycle. So, if you if you see that D ab that is your a to b right, so it will be your what you call that first you take taking in between a b and a b dash right basically it will be in between phase a or D ab means phase a and phase b. So, only you have to consider your a a b and a dash b dash.

So, D ab D ab is D right and then your what you call D ab dash actually it will be something like this if I explain it will be clear just hold on, right just hold on. Suppose your D just one I am showing you suppose you have that you want to find out D ab.

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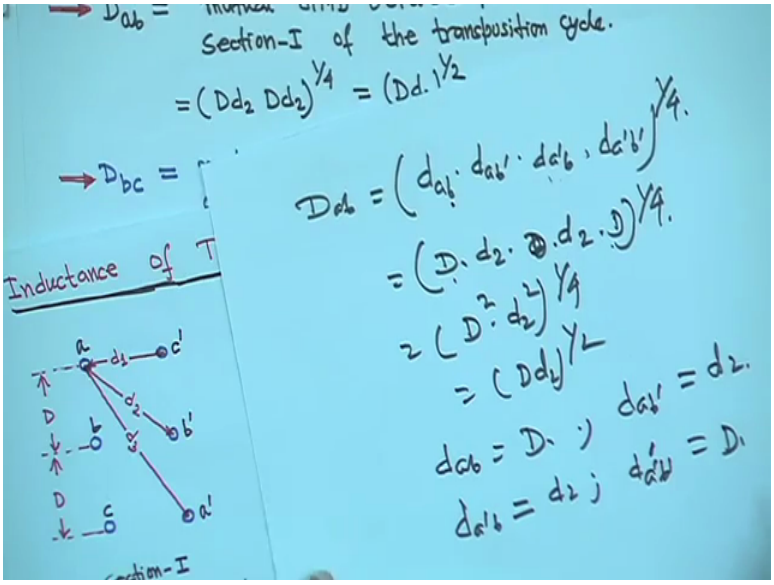

D a b is equal to right. So, is equal to your I am making in my way D ab is equal to first you take a to b, say it say I am giving you this is your small D suffix a b right, then into D a to b dash so D a to b dash into. Again this one is coming D a dash b because it will be in phase a ad b only phase a to conductors phase b to conductors right.

So, it will D a dash b and into it will be D a dash b dash. Now this D ab your D ab is equal to D this distance is D therefore, it will be capital D rather capital D right and your D ab dash D ab dash is equal to small D 2. So, it is D 2 right, then D a dash b. So, D a dash b again it is D d a dash b, so again it is D and D a dash b dash and here it will be power should be 1 upon 4 because 1 2 3 4, so it will be your 1 by 4 right. And D and D a dash b dash D dash your D a dash b while it as gone D a dash b D a dash b is equal to D 2 sorry D a dash b, D a dash b this one, this is a b dash and a dash b they are same symmetrical right they are same a dash b.

So, should be is equal to your D 2 right and D a dash b dash a dash b dash; that means, this is equal to D to the power 1 by 4. That means, it is coming that is your D into D d square and D 2 square to the power 1 by 4 is equal to D d 2 2 half right. That means, your if you have you do not should not have any confusion things are simple that is your D ab actually D ab is equal to capital D then your D ab dash D ab dash is equal to it is D 2, then your D a dash b D d a dash b that is equal to your a dash b that is equal to your D 2 and D a dash b dash D a dash b dash, D a dash b dash that is also equal to capital D.

So, see how many how many mutual possibilities are there 1 2 3 4. So, it will be 1 upon 4. So, ultimately it is coming D d 2 a 2 to the power half right; that means, that is why I have written here that your D d 2 into 1 by 4 is equal to D d 2 to the power half. So, D ab, so D ab and D bc they will remain as same thing because a a and b happening here, b and c a and b happening in this reason only in this reason, similarly your b and c b and c it is happening in this reason. So, same symmetrical therefore, D bc D ab is equal to D bc D d 2 this 2 are same, this 2 are same.

Now, when you think about D ca that is that; that means, here it is a D ca right a c dash a dash c, D ca in that case it will be something like this now you take your D ca right. So, in that case what will happen that that your D ca, capital D ca is equal to look at this one right.

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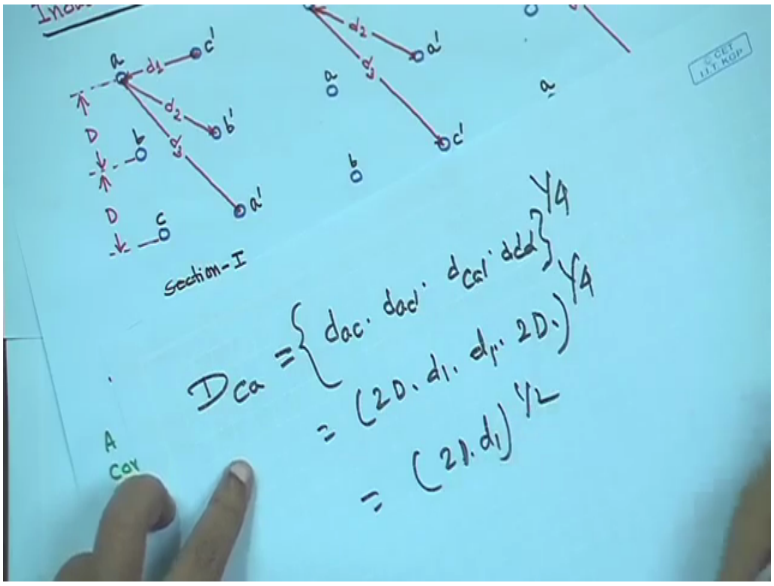

So, it is first I have to take your D a c D a c right then your D your what you call a c dash D a c dash then your D your while it as gone a h a c see here then D ca dash right and next is next one is your a 2 a 2 D a c, then D a c dash, then D ca same thing right and your D ca dash.

So, in that case your same thing it will be D ca is equal to your 2 D d a c will be 2 D this total distance D a c will be D 1 D your c here it is D ca c and D a c dash a dash right. Then D ca, D ca c a dash D while it is called c a dash it is also D 1 and it is also 2 D right to the power 1 by 4 to the power 1 by 4.

So, ultimately it will be 2 D into D 1 right to the power half. So, here also it is coming 2 D d 1 to the power half right so; that means, this way I mean when you will compute all this things that D ab D bc D ca they are should not be any confusion right, just you consider all the possibilities b to in if a conductors in 2 phases.

So, ultimately D ca will be 2 D d 1 to the power half right therefore, this D ab D bc D ca all this expressions all this expressions you put in equation 71.

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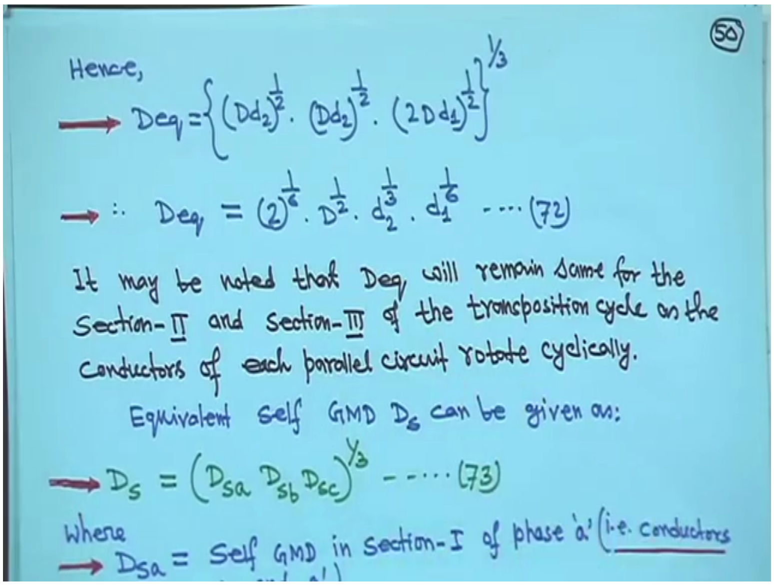

So, in this case if you put it will be D eq is equal to D d 2 to the power half into D d 2 to the power half into 2 2 D capital D d 1 to the power half whole to the power 1 upon 3. So, after simplification D eq will 2 to the power 1 by 6 then D to the power half then D 2 to the power one-third and D 1 to the power 1 6, this is equation 7 2. So, all the cycles only in all the cycles that conductor actually changing their position, so D eq will remain same for other 2 sections also I means this 2 sections also that D eq will remain same. So, even after transposition D eq will not change it will remain same as section 1 right. So, section 2 D eq and section 3 D eq they will remain same.

Now, after this one as to find out the equivalent self GMD D s, so it can be given as D s is equal as D sa self GMD for phase a D sb self GMD for phase b and D sc means self GMD for phase c to the power one-third this is equation 73. So, I have written here for phase a D sa is equal to self GMD in section 1 of phase a, that is that conductors a and a dash only.

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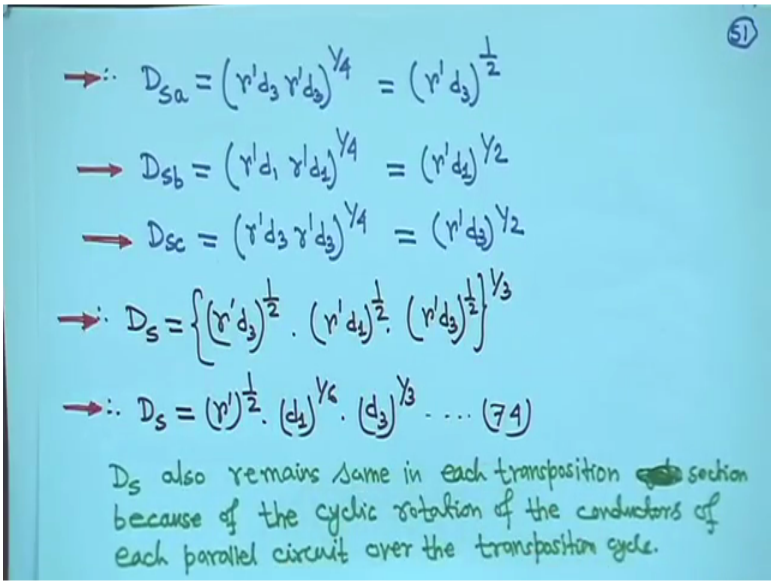

So, in that case in that case D sa, if you look into look into this, if you look into this one right for it is your only for phasor. So, in that case 1 will come I mean if you write like this D sa if you write like this, so one will I am putting it this way say D a then D a a dash right into your D a a dash into your D d a into D a dash then D a dash a put this way D a dash a into D or a dash D a dash right to the power 1 by 4.

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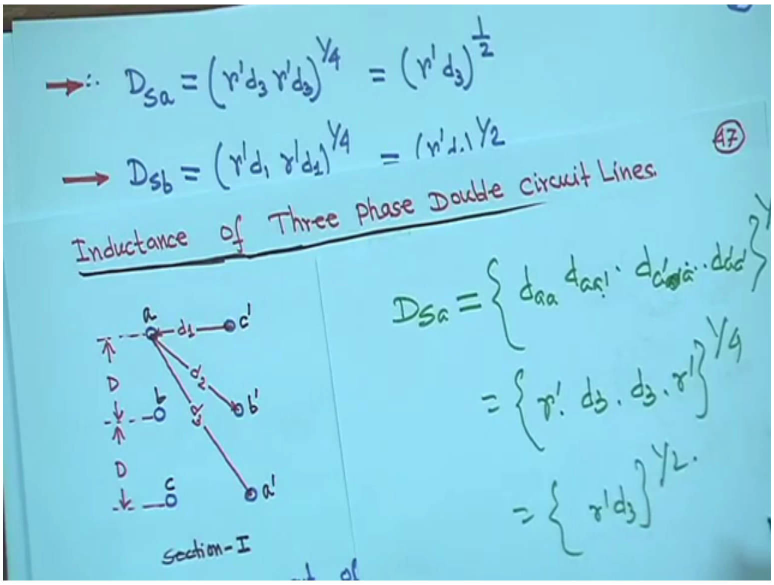

Now, assuming this conductors all this phase phases they have the same radius they have the they have the same radius; that means, the D a a will be your edges right then D a a dash D a to a dash it will be your D 3 then D a dash a D a dash a again it will be D 3 and again D a dash a dash. That means, again r dash to the power 1 by 4. That means, this one will be r dash r dash means r dash square and D 3 this is square; that means, this one actually will become r dash D 3 right to the power half. So, this way you will find out yourself GMD of sa.

Similarly, for phase b also same thing will come r dash D 1 to the power half and similarly when we will find D sc in the similar way right it will become your r dash D 3 all that in 4 possibilities will come, but it will become r dash square D 3 square. So, ultimately r dash D 3 to the power half. So, D sa, D sb and D sb all these are 3 self GMD you have got and then you substitute that that expression D s is equal to that, you are in this expression that you substitute here in this expression equation 73 you substitute D sa D sb and D sc to the power one-third here you substitute; if you substitute then you will get this expression D s is equal to write everything whole thing to the power one-third and then simplify.

So, D s is equal to r dash to the power half then D 1 to the power 1 upon 6 and D 3 to the power 1 upon third this is equation 74. D s also remains same in each transposition cycle because that position change, but there will be no change of D s also right D m and D s there will be not change in other cycle right, so over the transposition cycle. So, they will remain same. So, you need not calculate your D m and D s for other 2 cycles.

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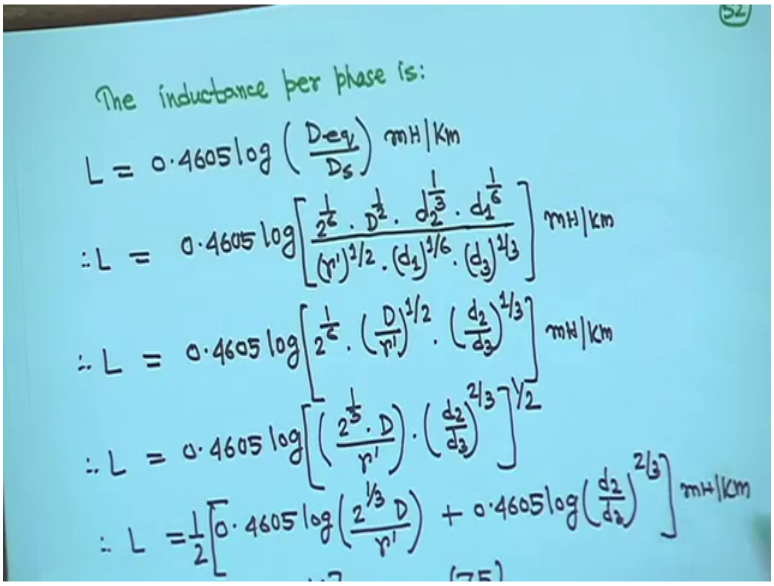

So, once you got D m and D s so that means, after that that inductance per phase the formula you know L is equal to 0.4605 log D eq, D eq upon D s millihandy per kilometre. So, D eq and D s you substitute here you substitute here and it will be millihandy per kilometre. So, upon simplification it is 0.4605 log 2 to the power 1 upon 6 then D upon r dash whole to the power half into D 2 upon D 3 to the power one-third millihandy per kilometre.

Further simplification 0.4605 log 2 to the power one-third into D upon r dash into D 2 upon D 3 whole to the power 2 by third then overall to the whole to the power again half. So, this is log, so this half will come this side right. So, half in bracket we are putting 0.4605 log of 2 to the power one-third into D upon r dash plus 0.4605 log D 2 upon D 3 two-third to the power two-third millihandy per kilometre.

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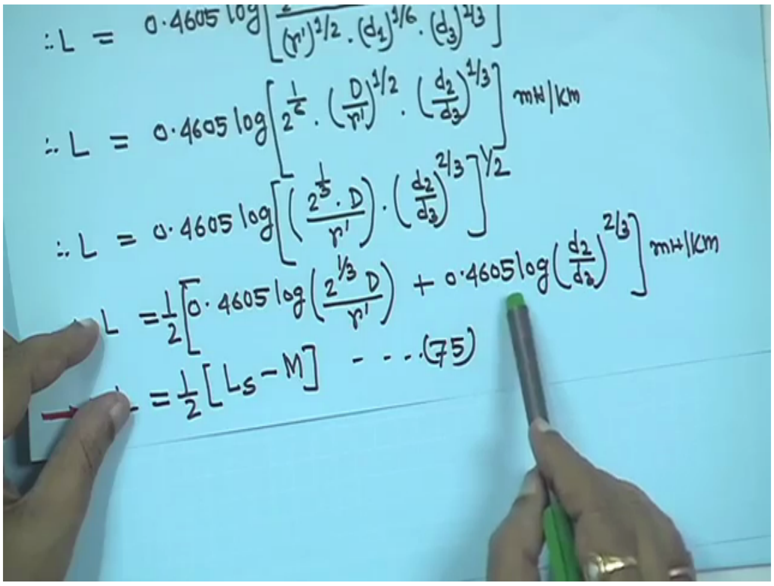

So, this one is equal to again can be written as L is equal to half in bracket L s minus m this is equation 75. This way this expression we are writing half L s minus m right minus m we are writing. That means, this part is L s right half this part is L s L s is equal to 0.4605 log 2 to the one-third into capital D by r dash this is your L s and this mutual it is minus taken m. So, m will be 0.4605 log it will be then D 3 by D 2, because this minus sign is here. So, it will be D 3 by D 2 to the power two-third millihandy per kilometre.