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Course
On Analog communication

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Lecture 10: Fourier Transform (Contd.)

Okay so far we have discussed about Fourier transform so you got some idea about the Fourier transform pair how they are being generated and we have started giving some physical interpretation into this particular after doing Fourier transform.

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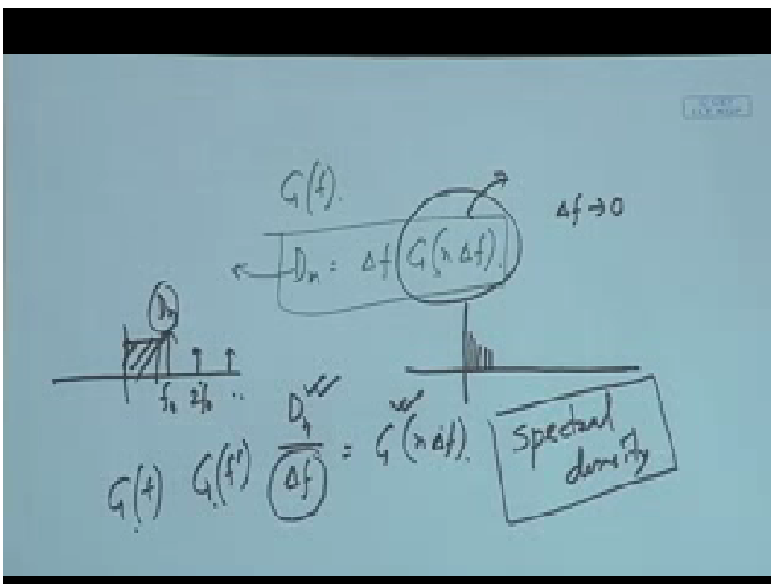

Whatever we get the g(f) so what we have seen that by analogy because the derivation was done through Fourier series so we could see that there was a relationship that Dn was nothing but that Δ F into G and Δ F right so this is something we have got that d1 was a very important parameter in Fourier series so it was actually giving the spectral amplitude as well as phase of every frequency component now we have seen because the Δ X are becoming very small so we actually start getting frequency component at almost every frequency.

So it is becoming continuous in frequency domain earlier it was discrete it was at first harmonics and second harmonics and third harmonics so what is now happening we are almost getting every harmonics but because the Δ F is very small so we are almost getting every frequency component which we have already, so I just see so what was happening as Δ F was tending towards 0 so we were getting very closely packed values okay, so almost continuous values.

So RDR whenever we have plotted spectrum it was like this, so there was some strength at different frequency of 0 to F 0 and soon and the corresponding values was even at the end here what is happening that Dn is just not this G it is actually G into Δ F so it is almost like this box integration okay and on top of that as Δ l F goes to 0 these are becoming closely packed so once they are becoming closely packed basically I am getting a continuous spectrum and the corresponding Dn are getting vanished okay because it is multiplied by Δ s as Δ F becomes smaller and smaller corresponding things are becoming vanished but the Dn might vanish that the G still has a relative strength.

So what it says almost like if G multiplied by the frequency I get the corresponding Dn okay, and we have already seen that integration gives me some value so basically this G by itself is not the spectral component it is defined as our unit spectral because the Δ F has to be taken out of it so it is the overall integration that gives me the spectral component let us say the amplitude that is given by if I just multiply it by Δ F okay so if I just divide this GN by Δ F TN by del F then I get this G value right.

So basically it is this G is nothing but the spectrum component par unit frequency okay so whatever that Δl F is per unit frequency in a very closely packed frequency domain representation I get that at instantaneous frequency what is the path unit frequency spectrum component okay or the spectrum strength so that is why whenever we talk about G it is not the spectra itself it is actually the spectral density earlier.

When we are plotting Dn we are saying that frequency component is having this much amplitude spectrum or this is that particular frequency component is having whatever modulus Dn is this much amplitude now we are not saying that we are saying our unit spectrum but because it has to be divided by Δ F this is the amplitude strength so basically what is happening suppose I take a

GF value at a particular frequency and I take another GF at another frequency so it is actually their relative spectral strength actual spectral strength if Δ F is tending towards 0 is 0 for both of them but the relative nature is still being intact GF and G Δ F -will have a relative strength to each other but their actual strength.

If you wish to calculate that must be multiplied by Δ F then only you will be getting the equivalent value with respect to Dn okay, so once I do that and Δ F tends to 0 I do not get anything so that is why whenever we talk about this GF it is actually a spectral density not a spectrum itself whereas in Fourier series whatever we are getting that is actually a spectrum because they are the individual value that we are getting that is a real value and that value just specifies in that frequency component how many how much strength it has whereas here I have all the frequency component in such as continuous.

So because I have every frequency component each individual component like that example I have derived that each in visual component does not have a probability value so if I say 30-degree angle when that pivotal rod will stop and what is the Associated probability that probability value will vanish that you do 0 here also same thing what is the specs spectral strength of 5kilo watt if I say that will be always 0.

The answer will be that that 5 kilo watt be 10 kilo watt be five point five kilo or five point six kilowatt everywhere the spectral actual spectral strength 0 but there is a relative strength between them and the spectral density is well defined which is G is defined and that will have relative nature so G at five might not be same as the at ten so they might be different but if you just ask what's the spectral strength at five and ten it will be all zero because by definition the G which we have evaluated.

Now you can see that that is giving me spectral density not spectrum so this is something why I am repeating again and again because this is very important you need to understand that whenever we are plotting this genius it is not giving me the actual spectrum it is spectral density we are plotting actually spectral density and the difference between our energy signal and period signal in periodic power signal we take Fourier series and each frequency component that we represent over there gives the exact value of the amplitude and corresponding phase of that frequency component here in the spectra so that is why that spectra is discrete spectra it has some values.

Where it is defined and all other values it is not defined and at every value you have a definite a finite spectrum strength or I should say is the amplitude of that particular frequency whereas in a Fourier transform where we are representing energy signal or time bounded signal we have every frequency component so we cannot discriminate that this frequency.

Is there it is a continuous thing with frequency whenever we plot it is a continuous thing and the that means G(f) plot it is not spectra by itself because at a particular frequency the height I will be getting that is actually not the spectral strength because spectral strength is that at that particular frequency the value G(f) into some ΔF which tends to 0 because it is a continuous so the Δ F goes to 0 so basically the spectral strength will still become 0 whereas what we get is that spectral strength divided by Δ F that means spectral density per unit frequency.

How much it has that is something that we get okay so G(f) defines that so that is where these two particular spectrum haves light differences and you have to be very careful about when you are generally talking about these things often we do that mistake and in many books also you'll see that they just represent it as spectrum so they said for a particular time bounded signal I do Fourier transform and then I plot G(f) that is actually my amplitude spectrum it is not or modulus g (f) I draw I get amplitude spectrum it is not it is spectral density it is no way spectrum because the spectrum is g (f) into Δ F which has no value.

It will be 0 everywhere if I try to plot that it is just a density which has some meaning over there in a way it's meaningful because the signal that we are seeing that is a finite energy signal right were now representing it in two frequency domain if in the frequency domain every frequency component has some finite value and then there are frequency component which are infinite in number because it takes continuous values so every frequency value it takes so then overall energy will be infinite.

So it should not be that way it should be each individual component should be still 0 all it is that spectral density has some values so that I integrate from one particular frequency to another see that means I take the area which is actually the energy that will have finite value so the integration will still have a finite value but individual one will not have any value so this is a very important concept which you should be able to always remember okay so whatever it is now what we have done is we have understood what is Fourier series and what is Fourier transform how we get Fourier transform out of Fourier series and what is it is significance so for any signal g(t).

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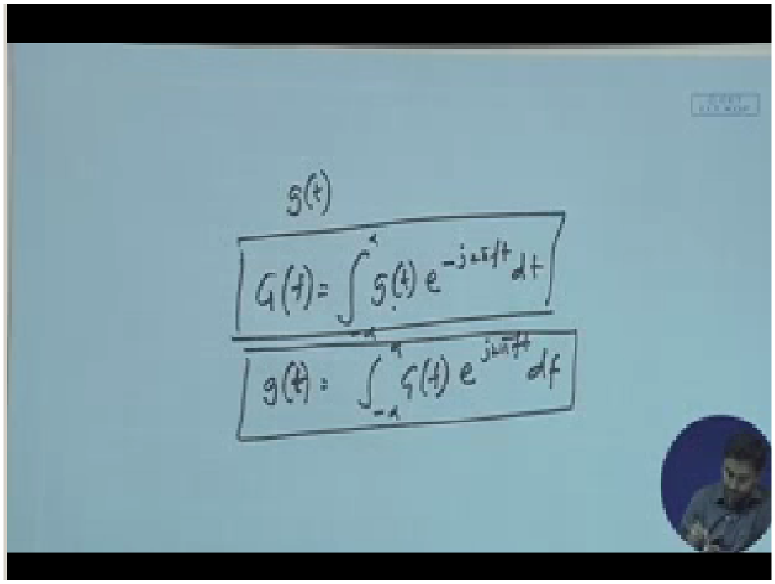

We have also got these two relationship that one is this g (f) which is Fourier transform it is GT to the power - J 2 pi F T DT integration -infinity +infinity and we get inverse Fourier transform which - infinity + infinity g(f) if I + J 2 pi f df so this is in frequency integration we get back our time domain signal this is the time integration from the time signal we get the frequency representation of it okay.

So this is something we have now identified now what we will talk about why we are doing all these things because this is this has a huge implication in communication you will see that every part of communication techniques that has been derived that has some route in Fourier transform or Fourier series you see that and will one by one derive all of them and you will see that this understanding is pretty much essential for our overall understanding of communication technique.

So what we need to now do is try to get some very important property which are particularly in use for communication technique so that something will be now dealing with so before that let us try to see whatever happens in Fourier series that it is a event symmetric spectrum and all those things let us try to derive that part also.

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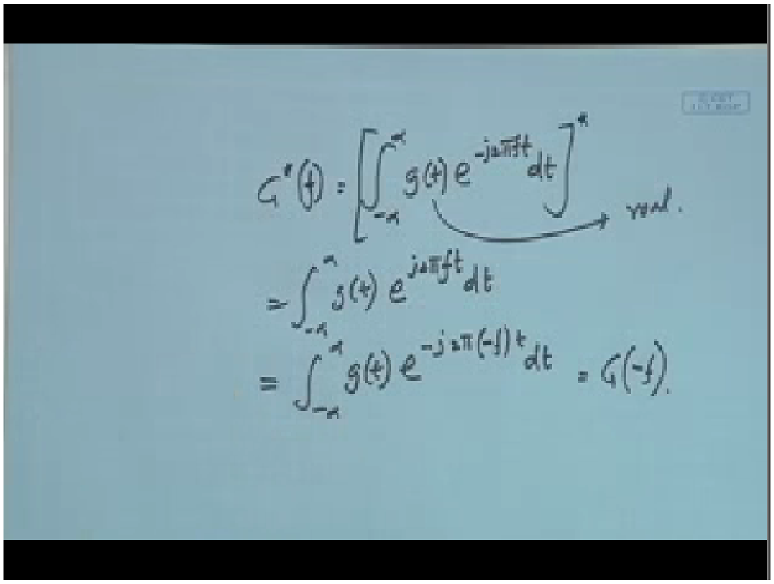

So we are trying to derive this complex conjugate of g (F) what we get so G (f) the expression of g (f) and we will take complex conjugates so that is g (f) it is power -J 2 π ft dt integration -infinity we take a complex conjugate of this now as long as my dt that signal that we are targeting to get Fourier transform is VM if this is real and if I take complex conjugate because integration has nothing to do conjugation.

So conjugation will come inside then it will be distributed over these two g (t) is complex conjugate because it is real so that remains VT so we get minus infinity plus infinity (t) and the complex conjugate of this so that should be it is work plus J 2 π ft dt okay, so what is this let us rewrite this in a different fashion so we write g(t) it is the power - J 2π(-f)t dt okay, now if you see this is almost like a Fourier transform g (t) into to the power - J 2 π there is something in t dt.

So this is actually if it was f it could have been gf because it is minus s this must be G (-f) so if you now see it is almost giving similar result as Fourier series so d-n if you do complex conjugate of d+n you get g-n v-m same thing is happening for a particular frequency F if I do complex conjugate of gf I get the negative of that so basically it says that spectrum also if I take amplitude spectrum it will be when symmetric and if I take same spectrum if g(f) is complex then I will get because the complex conjugate is just negative so I will always get odd symmetry in phase.

So this will also have if you clearly see g(f) if it is complex it will have amplitude as well as it will of phase so it will always be having two spectrum one is for amplitude one is for the Faye's okay so we are saying there is a spectrum this is again you have to remember that it is a local term what you use generally but it is no longer as long as we are talking about energy signal we are not talking about spectrum its spectrum density so now let us try to derive few Fourier transform pairs okay.

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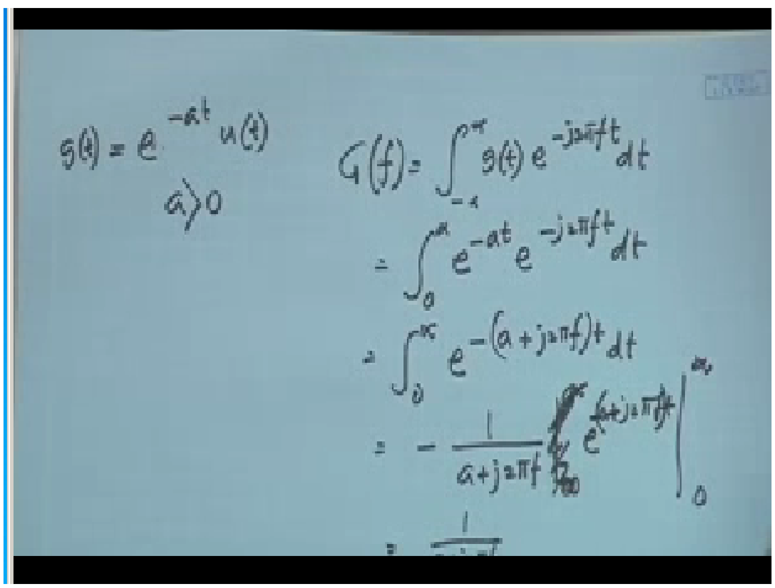

So let us say if I have a signal e^{-at} u (t) we have already defined UV it sits up to from - ∞ to 0 it is 0 and then from 1 onwards sorry 0 onwards it is actually it is value is 1 okay, so this signal we want to get the Fourier transform of that that a> 0 let us do it in a very simple manner so we want suppose this is g(t) because we want to evaluate G(f) which is nothing but -∞ to + ∞ or g (t) into e –j 2π ft dt okay now GT because of u Tit is defined from 0 to ∞ only so this integration - ∞ it will be all 0 - ∞ 0 it will be 0so it should be evaluated from 0 to ∞ now GT becomes e - ^{at} 80 because u T>0 it is always 1 so it will just give me this value e –(a+j2πf) dt now let us evaluate this so it is just exponential will be power 1 a +j2πf into T DT.

So right putting value 0 and ∞ at ∞ if I put = ∞ this goes to means 0 so we don't have a value at 0 I will be getting see 40 of course it should be over to second that should be there 0 ∞ so if I ∞ this becomes 0 right so this is the integration one by that coefficient it is the power - J of course that a should be there so if I put T = ∞ what happens is goes to 0 because e to the power- ∞ so that goes to 0 so that whole value will be Z well when I put t =0 so this becomes 1 so it remains as this so because it is - so that - will be canceled so this becomes a+j2πf so as you can see for a signal.

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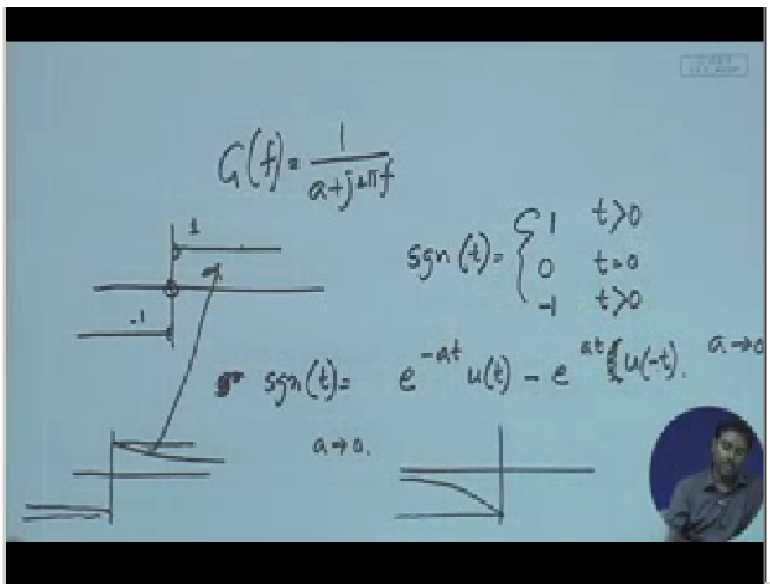

G(f) =1by a+j2πf it is a complex signal okay, so we have evaluated this good the next part is we will try to evaluate some other things so let us try to see if we can evaluate another very important signal so we will use this part to evaluate that that is called Σ function so what is actually a Σ function a signal function is something like this it is almost like UT but it has something in the negative so basically it is 1 in the positive form it is -1 in the negative half so it is defined as this says if I say Σ of T that is actually+ 1 whenever t>0 at this point it gets a value 0 so this is open so the discontinuous function of course at t=0 and this is- 1 t> 0so if you wish to do a Fourier transform.

This signal function you will see that we will have a technique called single sideband modulation where signal function is a very important function you will see and that is why we are doing this Fourier transform so it is very important that we understand the Fourier transform of this thing how do we evaluate this Fourier transform so let us try to see directly you won't be able to evaluate this if you just put it this way and you will see all kinds of ∞coming up directly we won't be able to evaluate.

This but we can evaluate in a clever way we can represent this Σ function as this it is actually e –at u(t)-e at u(-t) into you sorry so if you see what is happening it is actually combination of these two first is e –at u(-t) what does that means e-at looks like this so UT means it is defined only in the positive half right and e-at means at T = 0 it is 1 and from there it is exponentially decaying okay, now in this one if I put a tends to and a is just tending towards 0 if I just put that what will happen this particular value will almost become one.

And this will almost mimic the positive half because his will be the slope of this exponential decay will remain as it is so it will for every value because a tends to almost 0 so this will always become one so this will be very slowly decaying and it will almost mimic these things ok and this part it is u-t that means it is defined over here this range because u T is defined from 0 to infinity therefore u-t is defined from -∞ to T right so u- t means it is defined over here and I have put e8t ok so as at ends to 0.

This will again become a value 1 for every value of T okay, but what is happening because there is a -so it will always attain a - and it will be something like this okay, that is slowly decaying so at if I just put some value of a some finite value of a okay so then e to the power at will be as T increases this will be decreasing almost similar like this here from - 1 it will be actually increasing and at t=∞ probably it will go to 0 so it will at the negative this one it will start from-1 and it will go to 0 but if a tends to 0 this will almost become flat so this particular function as a tends to 0 almost mimics the signal so what we will do now to get fourier transform Σ we will take this representation and use the previous result to evaluate the Fourier transform of Σ function.

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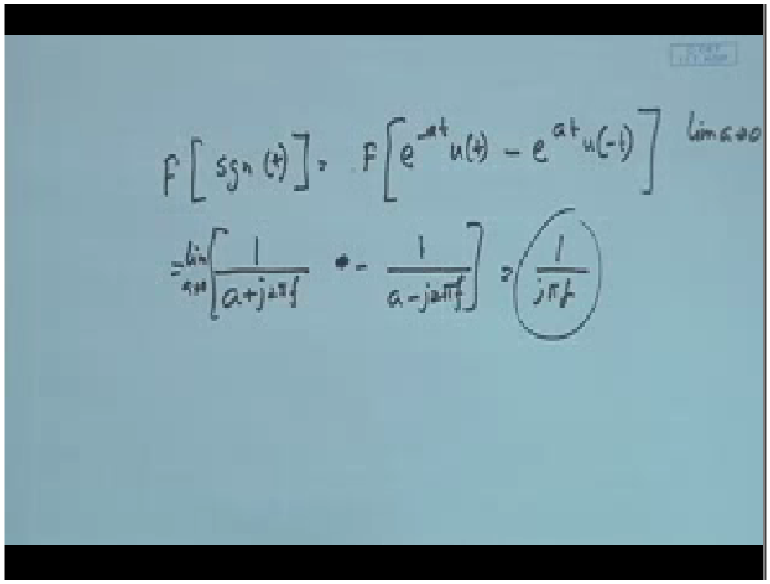

So let us try to seem y Σ function has been already represented so Fourier transform of Σ T must be Fourier transform of e-at u(t) –e at u(-t) right so this is our Fourier transform now Fourier transform of at-ut that we have already derived going to a - UT is 1 by a +J 2 π F so this and Fourier transform if a function is having a linear combination Fourier transforms as it is distributed that is very easy to prove just put the Fourier transform formula and you can see that if g(t) is represented as C 1 XT + C 2 XT.

The integration gets distributed so basically corresponding Fourier transform you can individually take so I can do a Fourier transform of this one which is nothing but 1a+j2πf similarly you can evaluate this one also it will power a-t so that is very easy if you just take this integration and replace – t/t okay, just take a dummy variable called J = - t it will just become this and then if you just try to evaluate that what you will get is this one take that as homework you will be able to evaluate this from the fundamental principle as we have derived this one similarly you will be able to derive for e –j2π a t u – t so that is pretty easy.

So if you do that you will be getting this okay, and we have to also put now that particular part comes limit a tends to 0 so here we'll have to put limit a tends to 0 whenever we put this we immediately get one by J PI s because a goes to 0 so we have this and we get one by J 2 1 by because 2 is there so 1/2 + 1/2 it will be 1 ,1 by J PI s we get this so signal function that's a very important result even though it is a discontinuous function it has a Fourier representation and Fourier transform correspondingly so Σ function has a Fourier transform of 1 by J 2 π F okay.

So this is again another important property now the one we will be describing probably that's the most important property of Fourier transform which has been used heavily for communication this is called frequency shifting property.

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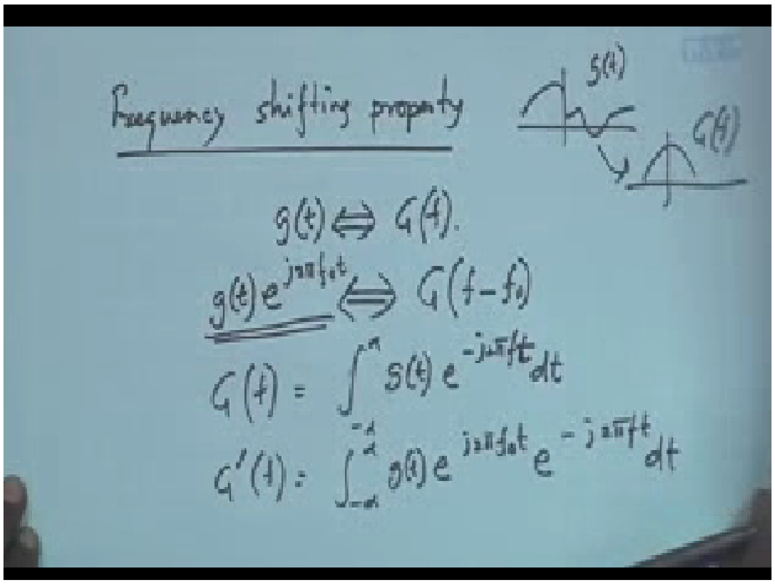

What it is let us try to see suppose I have a signal g(t) already know the Fourier transform this so I have a known signal let us say that is g(t) and I also know its Fourier transform let us say this is here this is something we know already so this pair we have already evaluated for some means by means of Fourier transform we have validity so this is given suppose now we are trying to prove that if this signal g(t) is multiplied by a exponential it will ej2πf 0t if that happens corresponding.

Fourier transform will be just that is why it is called frequency shifting property so it will be just the spectrum will remain the same it will just in the frequency domain translated by a amount F 0 so this is something we wish to prove that is a very simple way of proving this so suppose we know G F =- ∞ +- ∞ g (t) e –j2πftdt so this is something we know now we want to take Fourier transform of this one so therefore we have to do let us denote that as G dash if that should be - ∞ to + - ∞ g(t) into e j2πft e-j2πft dt so that should be the Fourier transform.

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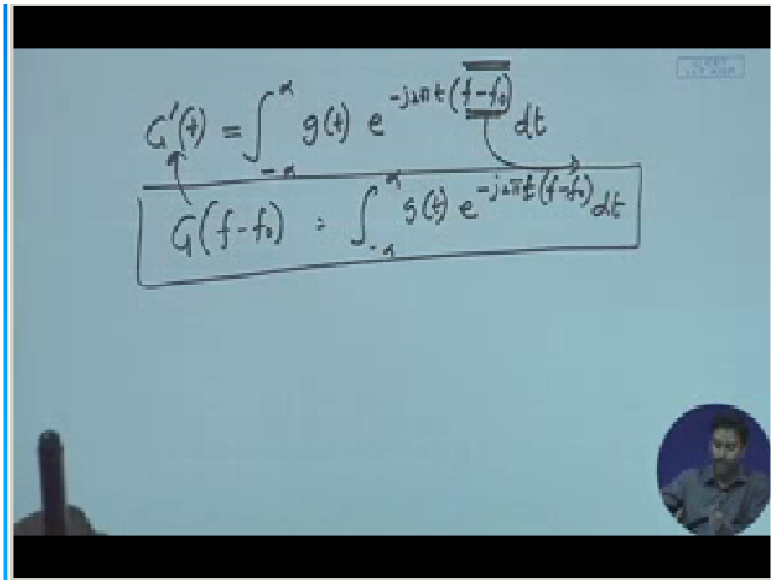

Let us try to evaluate this so - ∞ +∞ g(t) e-j2πt(f-f) dt so if I take just this common so we get now if we assume see what was happening my Fourier transform if g(t) has a Fourier transform is the ej2πt(f-f)dt then I get g(f) so if I just replace F by F - f 0 or a new variable so therefore from that analogy I can get this is = to G of F - F 0 which is nothing but g(t)e-j2πt(f-f)dt so this is something I get so immediately I can write that G - f is nothing but this so I can see that if I know C GT into the e-j2πt(f-f)dt that is g(f) therefore g(t) to the ej2πt(f-f) be g(f) -F 0 this is very obvious.

Because it is a just a function of whatever is there over here so it is becoming this back function so what we are seeing that if we multiply a signal with and exponential ej2π 0t that corresponding frequency F 0 is just translate the spectrum to that particular part okay, so this is pretty understandable now what will we will try to see why exactly this is a very fundamental property for communication in the next class.