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NPTEL ONLINE CERTIFICTAION COURSE

Course
On
Analog Communication

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Lecture 34: Probability Theory (Contd.)

Okay so in this particular example we have already seen that how do we calculate means if I have the transmitted that prior probability known and then channel what it is doing that is also known to me how do I calculate what is happening at the other end and then whatever is happening at the other end from that we can also start guessing what might have happened in the transmitter side.

So you remember in communication it is, it is the communication actually becomes successful if transmitter and receiver does not have any other form of communication because the very communication you are building up just to give information from transmitter to receiver if you already assume okay for that you need another communication then it just goes on right.

So basically you have to always understand that whenever we are doing communication probably transmitter does not know what will be happening at the receiver and receiver will also have no information about what is happening in the transmitter okay so they will not have this linked information only they will see something which is being transmitted over the channel they will receive that if it is contaminated by the channel.

They have no way to know what exactly has been done by the channel okay so they can only have why we are doing all these things because they can only have a statistical guess nothing else they cannot exactly know in a deterministic way what exactly has happened to my signal okay.

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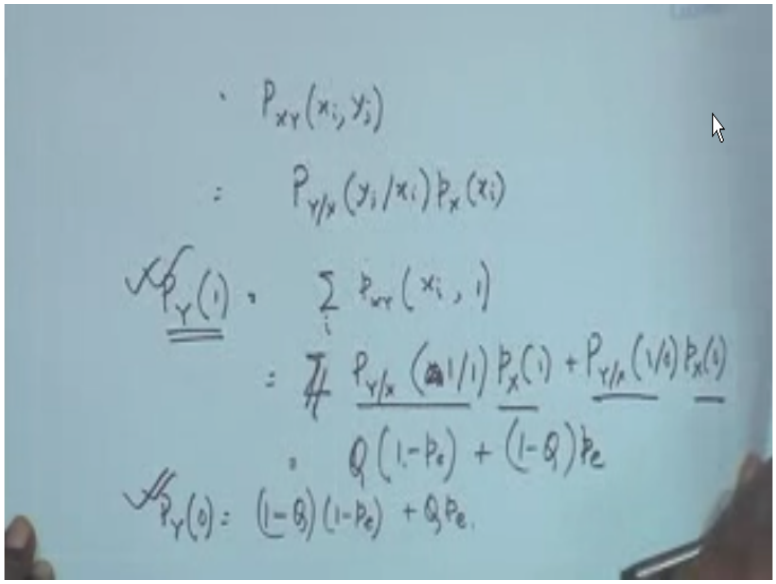

So let us try to see with that example so we have we could define this right P_{XY} X_i Y_j this is something we can define from the information that we have already got so we have already told that this, this and these four we know okay so this P_{XY} X_i Y_j that is actually the joint so from Bayes theorem we can easily evaluate now this must be Py given X, Y_j given X_i, Px Xi okay so this is something I can write this is equal to this why I have written with the condition on X.

That is because I already know that okay I have not written it with a condition on Y because that is something I do not know okay so this is something I know we have also said how to marginalize it now let us try to marginalize and get what do I want to get I do not want to get T X that is already known okay.

I want to get Py so I wish to get Py let us say 1 okay so what do I have to do I have to marginalize this that means I have to sum over X so I will be doing it over i P_{XY} X_i and $Y_{J,}$ I have already told that takes a value 1 so that should be 1 if I put 0 then I will get this done for 0 okay so this is all that I will have to do so I can immediately write it in this format.

So I can write P_Y given X, okay so now it should be why I okay so I will just write Yi given X where right so Y_I and now I have to put this X_i right so X_i I can put anything so Y_i is for me it is actually 1 so I can put Y_i as 1 X can be taken a value of X_i so if I just do a sum I have two possibilities right so I can do it for 1 and then multiply Px1 and I can do it for 0Py given X.

So this should be 1always given 0 Px 0 so I can write this way now you can see I know all this already okay so I can actually calculate this so this is the beauty of it once I know all these things

I know how to calculate the individual probability also which is unknown so I had the prior probability I know all those conditional probability given X that means given that what is transmitted.

So I can now calculate what is the associated probability of Py okay so I can write this as P x1 that is probably px1 is Q and this is 1-PE and then this is 1-Q and this is probably PE so that is how I calculate Py1 similarly you can also see Py0 can be evaluated in a similar manner so that should be 1- Q *1 -P you can just test this and Q* P okay so I get these two fine now I wish to calculate which I was telling that given Y_i want to see what was X okay.

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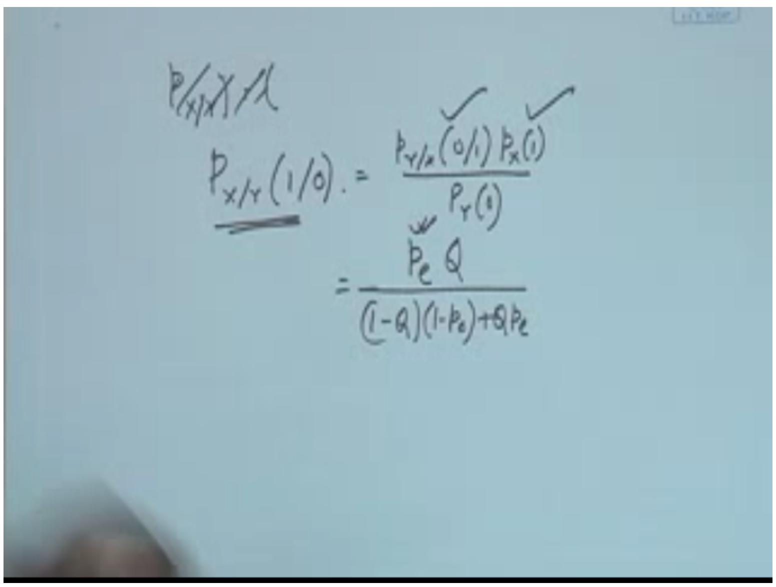

So my evaluation should be PX given Y okay let us say sorry I wish to calculate this, this specifically means that I have received at the receiver 0 what is the probability that 1 has been transmitted so that means I have received an erroneous thing okay certainly whatever was transmitted was 1and I have now received 0 so I am trying to evaluate this probability after receiving okay.

So we can just put Bayes theorem again so Px given Y can be calculated as Py given X, 0 1 P x1 given Py0 so this I can do okay and immediately this is already known which is actually PE this is known this is Q and P y0 I have already evaluated which is 1- Q *1-P+ Q *P so I get this so this is the way we calculate this particular part so whenever we have something received we are trying to guess what is the probability that something else was transmitted okay for our case something else is just another one so if we just calculate that that must be our error probability okay.

So that is the probability that given I have received 1 what is a probability that I have probably received an erroneous thing that something else was transmitted similarly I can do it forgiven I have received 0 what is the probability that I have received probably an erroneous thing that one was transmitted I have received this so this must be your error at the receiver side and he will try to minimize these things.

So all you will be doing is once you get this now you have to see what has to be minimized okay so something which can be minimized is PE that is the channel error so you will have to see how to manipulate that so that we get overall error probability minimize so this is something people do often okay so given these things now let us try to define another thing which will actually lead us towards a continuous random variable definition.

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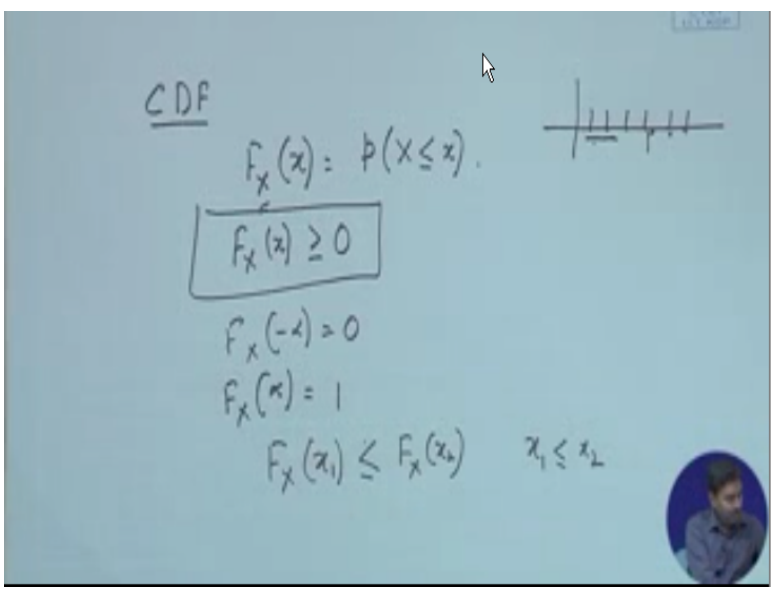

So we will define something called cumulative distribution function so that is called CDF so what is CDF our CDF definition is something like this we call it capital F so capital F_X X so X is a random variable this X is something we will talk about that so this is nothing but probability that this random variable takes a value less than equal to that particular thing I have defined okay.

So in the real axis suppose for our that rolling of died we have events which are 1 2 3 4 5 6 whenever we say that X is less than equal to 4 that happens to be a CDF up to 4 when we say X is less than equal to 5 that half we are CDF of parameter 5 so you keep doing that so that means we will have to add all this probability till that point in the real axis so we will start from probability minus infinity.

And we will go up to that point X so this is that definition okay so immediately what we can say there are few things which we can immediately say about this $F_X X$ can we say this that is always true because whatever happens it is just a probability and probability must be we have already discussed that probability must be greater than equal to 0 so this must be the case that is the first thing at minus infinity.

So that means this sum goes up to minus infinity okay and it starts at minus infinity so it could not take anything any probability value so this must be 0at infinity what it should be at infinity means I have taken all the probability that means all possible probability values for all the random outcomes I have already some them and we know that summation of all those probability must be 1 so this is a function which starts at 0 at a minus infinity level in the real axis and goes up to plus 1 okay.

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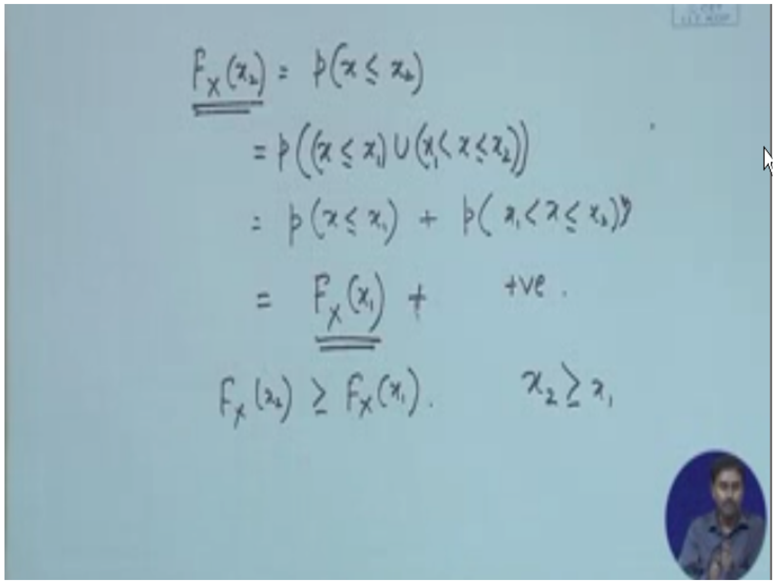

Now let us see the nature of the function so we will now next prove this part that this is always less than as long as this happens okay so we will be trying to prove this so this is will be with this we will be able to say that it is a monotonically increasing function okay so let us see how do we prove this so let us write F_X X2 as probability by definition X must be less than equal to x2 this I can write as PX less than equal to x1 so this Union X from this I can always write that X less than equal to x2 means that I take those X which goes up to because I know x2 is greater than X_1.

So I go up to X 1 and then the rest of the set I take so these two set I can always write it's the probability that this Union this now these two set by definition they are disjoint because whatever is covered within this is not covered within this okay so they are disjoint so we have already proven that probability of union of two disjoint events must be summation of probability. So this must be probability X less than equal to X 1 plus probability that your X remains between these two now what is this by definition this is f_X x1 this is a probability value whatever it is that must be always positive so this is always positive so what is happening this particular value is some value plus some positive term.

So this must be greater than equal to this at least this can be 0 so I can from this definition immediately I can write F_X x2 is always greater than equal to F_X x1 for all x2 by definition we have taken X 2 is greater than otherwise I could not have written this like x2 greater than equal to x1 so for all these things this must be happening so I can always write that this is just monotonically increasing function okay.

It will never have a depth so it will never go down it will just increase 0 it will start it will go up to one so all these things we have proven so this particular function some characterization we have got now okay now let us try to define something which will take us towards the continuous part okay so let us say now I have a random variable which is continuous what do I mean by that let us give an example so we have already given an example in one of our earlier class.

We have told suppose I have a P voted rod and just I give exact a random force it starts freely rotating around the point where it is pivoted okay and then it will stop somewhere it completely where it will stop it completely depends on what kind of force I insert and the amount of force that I can give that is a continuous any force I can give okay so accordingly he can stop between that 0 and 2π so it is a pivoted rot which is getting rotated and stopping somewhere at θ so this θ which is a random variable.

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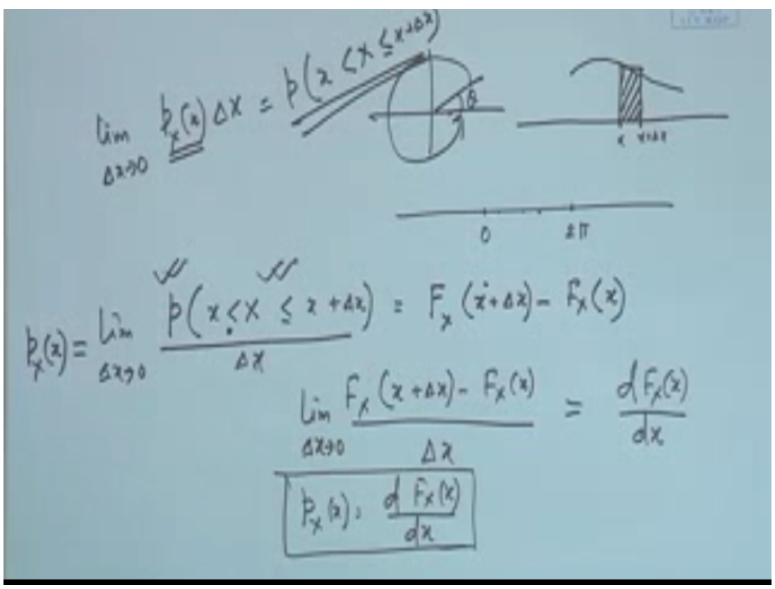

Now where it will stop so that can be anywhere between 0 to 2π and he can take any value so this random variable θ now it is not taking a discrete value it can take any value 0 to 2π any value you can think of or any real number you can think of between 0 to 2π it can take that so this is a typical example of a random variable which is continuously okay so that means if I just try to similarly map it to real axis between 0 to 2π.

Now it can take any value okay so the definition of PMF we have defined that is not valid over here let us try to see can we give some other definition over here so let us say that I have this random variable which is taking a value between X and X+Δ so from that previous prove let us take X as X 1 and this X+Δ X as X 2I can immediately write this as just do not give this equality probably so this I can write as F_X X+ΔX- F_X X this is alright because we have started defining it with FX x2 this is my x2 is equal to we have already proven.

That should be F_X x1 plus this probability that it lies between x1 and x2 okay so this particular probability value that it lies excluding of X from X to X+Δ X that is lying on that interval that probability is defined by this okay so now what I do we actually divide this by ΔX okay so I can I can or this side you can divide this, this F_X x+Δx- F_X X divide this by ΔX and take a limit ΔX tends to 0 what do I get I will be getting this is differentiation of this F_X.

According to the basic definition of differentiation so eventually what I have done is I have divided this part by ΔX and then try to evaluate this so if this is that probability this was just the separation okay so if I just try to relate this to this I can always write that probably this part multiplied by ΔX and taking ΔX tends to 0 right so this particular part whatever it is okay.

So that must be this part right now we all we have done is just divided by ΔX and taken limit ΔX tends to 0 define this as some px X which is defining it so this divided by ΔX which is defining this as px X so I immediately get PX to be differentiation of this one right that definition we get and in that process I was more interested towards this, this probability so what that is I can immediately see this px X if I multiply with ΔX and take the limit that ΔX tends to 0.

That must be my this Px that is the probability so what is eventually happening suppose I have a particular probability mapping on the real axis and I have said this is my X and this is my X+T ΔX okay and I am trying to find out this particular part which is a probability that it will be lying between this and this so that overall probability I am trying to report as del X goes towards zero what will happen the variation of that okay will become vanishing there will be no variation okay.

So at that point I can say safely that it is actually whatever that Px X if I just plot it over X and it is the area under that part which defines my probability that it will lie between X and X+ΔX so this probability is nothing but whatever this particular thing I have got by differentiation of this

CDF cumulative distribution function if I multiply that with a ΔX that almost means ΔX tends to 0.

So this P_xX will not be varying this would be almost constant so that x ΔX is almost getting the area under that curve okay so which is the definition now we still have not defined what this P_x X is but we can now see that there is a particular function p_x X whenever we take it to the continuous domain this is just a differentiation of the CDF if CDF is defined the way we have defined I that it is less than something okay.

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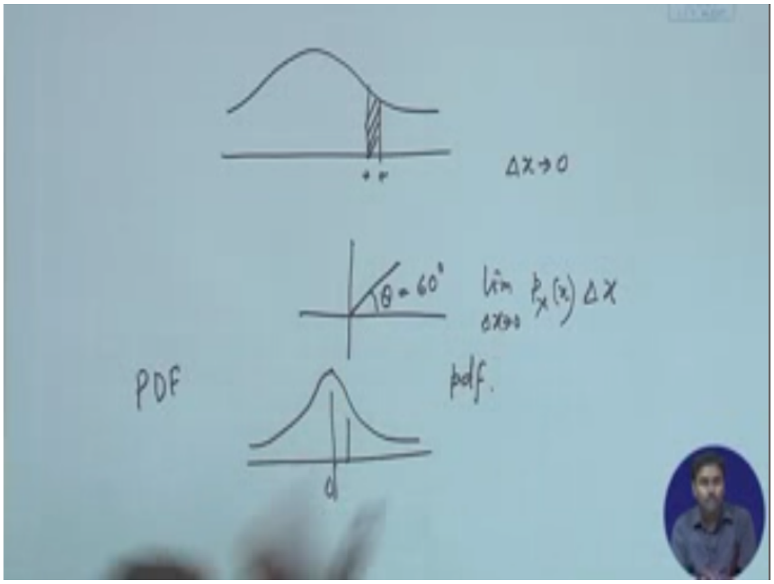

So that can be defined so that probability that CDF if I differentiate it I get whatever value I get if I plot it with respect to X so if I just plot it with respect to X and then choose a particular X and if I get the area under it for a very small ΔX that must give me the probability that it is lying between X and X +Δ X okay right so that is the probability that it will be now what do I get from there suppose let us say ΔX goes towards zero.

So this will be almost coming to this point so almost I am going back to a probability of a value of a random value like our means discrete random variable what will be that value irrespective of p_xX if p_xX is bounded we know that T xx must be bounded as long as that is happening what do we get p_xX into Δ X ΔX towards tends towards 0 so I get an individual probability 0 so in a continuous variable like that example that I give a random force and it just keeps rotating and fixed at a particular θ okay.

So from the very definition frequency definition or favorable cases let us try to think about this that it will be stopping exactly at θ equal to 60^0 what do I get about the associated probability how many favorable outcomes are there only one that is the 60^0 okay how many overall possible outcomes are there how many angles I can get infinite so what is my probability it should be 1 by infinity.

So that must be0 so individual angles I must get a probability value of 0 so remember that is why we take an example of this particular thing knowing that many of you might be knowing probability theory and given the similar explanation for Fourier series to Fourier transform the same thing we are doing or dealing with again its PMF or discrete random variable.

And we will be calling that as PDF and remember we will not be calling that deliberately add as distribution function that should be a density function it is, it is whenever we say PDF it is actually probability density function not distribution it is it has nothing to do with distribution, distribution is not that p_x X distribution is multiplied by this and then limit the ΔX tends to 0.

So this is actually the probability that gives me probability what P_X X does not give me probability it just gives me almost like a density that as if at that point if I now multiply with del x sufficiently ΔX small so that this does not get changed and then I will be getting the area will be defining the probability okay.

So as if the probability divided by this ΔX gives me this okay as long as Δ X is sufficiently small so that is why it is becoming density that per unit on that real axis per unit that random variable change what is the probability so that is why it is density it is not probability so individually at a particular point the probability will be always 0 but there will be a relative nature of this and that tells you where exactly the probability values are higher and where exactly the probability values are lower.

So suppose I have just drawn a PDF like this then I will be able to say that around zero it has a higher probability I would not be able to say specifically that at 0this is the probability that will be 0 at one this is the probability that will be all zero but I can say around zero this is the probability and this is having higher probability around zero whereas as I go away from zero it will have lower probability so from the nature of this curve.

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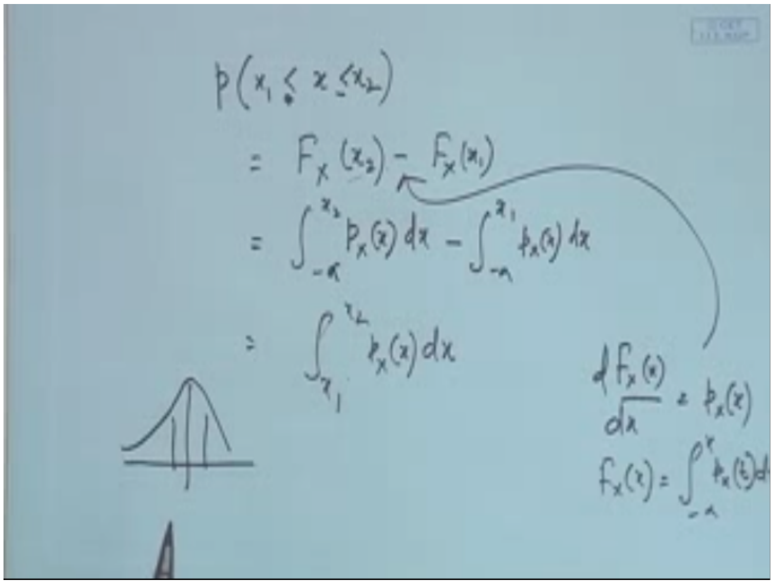

I will be always getting that hint so that is one thing and the second thing that I wanted to discuss is something like this I can always write P of suppose so this particular part we have already seen that can be written as F_X x2 minus F_X x1 this is something we have already proven so what is this, this I can write from minus infinity to plus x2 this FX x2 we have already shown that derivative of F_X X is equal to the PDF.

Therefore from the relationship of derivative and integration we can say FXs is just the integration of P_x X so I can write that probability distribution function going up to X2 must be interrelated this way minus similarly I can write right so very nicely because I have this definition so F_X X DX was P_x X.

Therefore from the conjugate definition of differentiation and integration I can always write this okay so I will be able to write F_X X must be integration minus infinity to X px suppose T DT so this is something I could write so here also I can write that way immediately from this I can write it from x1 to x2 again from the definition of integration with respect to limit.

I can write this so what it says that if I target instead of a single value a particular zone on that red random axis or real axis where the random variable is defined I can always integrate that PDF from that particular value to whatever value I wish to evaluate it so I will be getting the corresponding probability so if you just think about that people dead rod if I just say instead of specifying a 60^0 angle I say between 0 to 60^0 it will be lying what's the probability associated probability.

So then there is a ratio between 0 to 60^0 is a solid 60^0 angle and what is the overall angle that is 360^0 so 60^0 /360^0 should be my probability which is 1/6 okay so whenever we talk about a range because it is density I integrate it and I get a probability associated with a range but individual value will be all 0 and that is why we were talking about this thing right so in this range if you integrate the value will be higher associated probability will be higher.

And if you integrate in this range if even if the ranges are same probability the integrated value will be smaller and the associated probability will be smaller so you can always say I have more likelihood to get this random variable whereas I have less likelihood towards getting this random variable that has lower probability so PDF actually tells you that information okay what gives you that information and this is how they are linked so will be always able to evaluate whenever we need to evaluate.

That it is the probability between this to this we will be able to calculate that okay so with this definition probably we have now gone into the definition of continuous random variable and its associated functions like CDF remember CDF is still called cumulative distribution function because that is to the distribution up to x1what is the overall probability that is still a probability where as PDF that is not probability that is a density.

So often people do this mistake even in Fourier series and transform also same thing happens whenever you go to transform its spectral density remember that wherever whenever you are talking about Fourier series in each frequency will be getting a particular amplitude value or overall energy value or power value I should say okay so there is a definite power value whereas for Fourier transform you do not have a power value.

It is spectral density actually and there also you have seen that it is almost similar concept you have if you integrate from some particular value to some particular value you get instead of probability there you get some amount of power or energy okay so with this definition will stop today and then what we will do next is try to see some more things which will actually get us toward random process so some more things on probability theory thank you.