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NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog communication

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Lecture 35: Probability Theory (Contd.)

So we have discussed about discreet random variable as well as continuous random variable, so in the last class probably started discussing about the continuous random variable and it is associated PDF and CDF.

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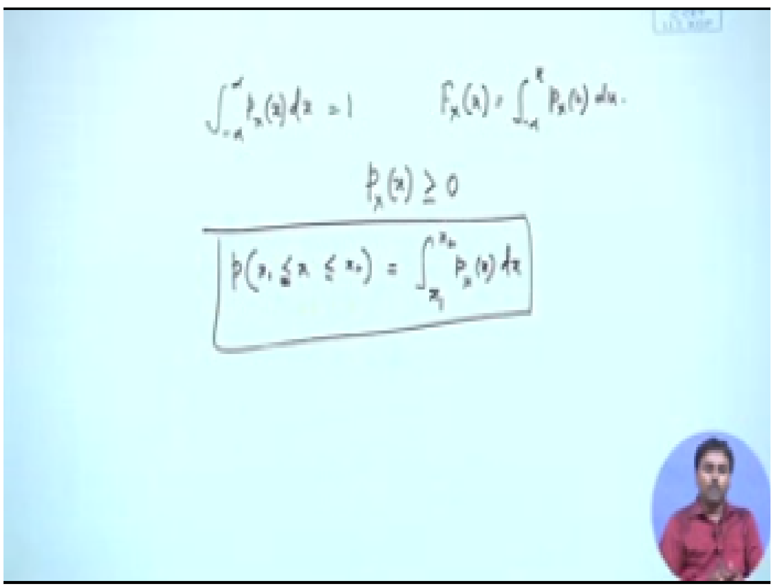

So saw the property that we have already mentioned is something like this 1 is which is a universal theory of probability that if PXX is the PDF of a run a variable X it must be 1 and the CDF and PDF is related like this. Of course its probability so it must be greater than equal to 0. So these 3 things we have already discussed about them we also told what do we mean by this PDF it is an density function remember even it is called PDF its probability density function not distribution. This PXX does not give you probability its value is always or at a particular X is always 0it just provide the density and that is way the other formula that p suppose X not that the quality comes up to be for a particular value to the other value.

If we wish to calculate that our experiment that random variable will lie between this 2 value that has a probability term and that probability we get through integration of this PDF because it is an density function so if you do integration then you get the probability but individual at particular X you do not get anything we also given an example right.

So what will be doing now in communication one of most important PDF or most well known PDF that is being applied you just keep an property of that it just a physical thing we need to just understand some of the property and so the function associated with that so that is called gossoon random variable which is defined this PDF.

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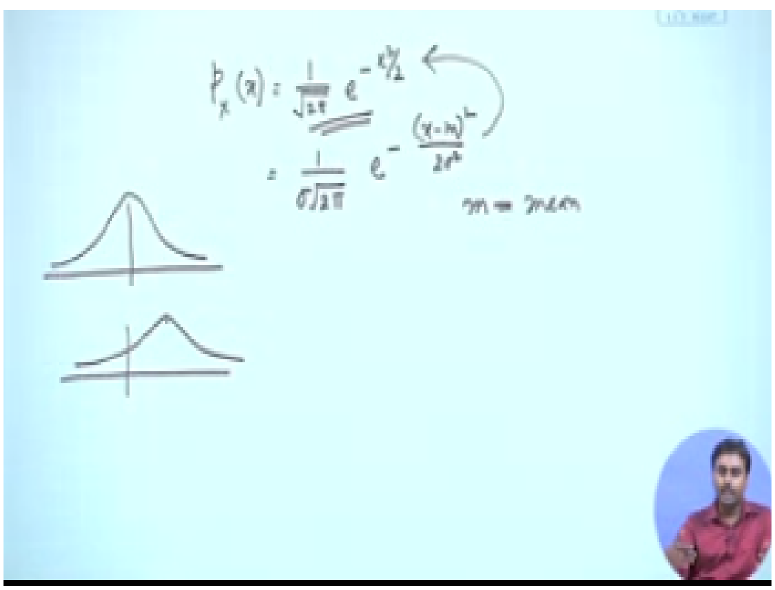

That is one definition or it can be another definition where you have these so as you can see if I just put sigma= to 1 and m = 0 I get back these one so that gossoon distribution well the m is call the mean that will see how the mean can be calculated random variable so then we will understand and the sigma is mean the stranded diversion. So as long as the variance is 1 and your m=0 we get back to the definitions so this is the typical gossoon distribution it if the mean is 0 it look like this.

The famous bell car or if the mean is m so it will centered around m it look like this as we increase the sigma what will happen this will become more and more flatter and the top will come down because the area under this will always be 1. And as you decrease the sigma this the top will up and up and the this will become more sharper. So that is typical Gaussian distribution. So that will try to see this Gaussian Distribution has been related to some other function, like they call it either Complementary function or Error function or the Cue function.

So we will see what are those functions and how it is related so that is something we will try to define because most of the times may be not much in analogue communication but when we will be going into digital communication we will see that this cue function error function those are the main functions which keeps coming back. For the noise analysis or communication analysis with noise.

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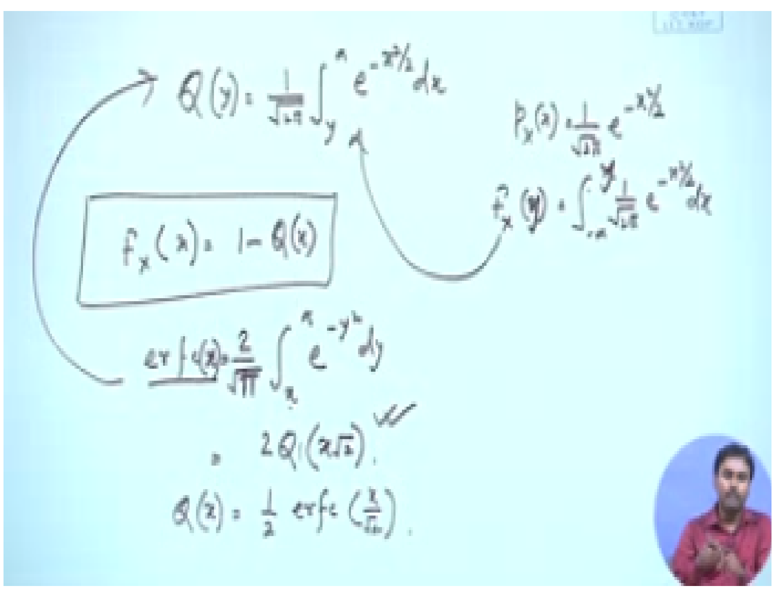

Okay so let us, try to define a function called Q(Y). This is by definition this given as one by two pie some Y to infinity it is the power minus X square by two dx. This can be easily related to our that PDF of CDF such so if Gaussian distribution we have already written as this PXX is 1 by √2τit is power –x square by 2. So of course, Fx(x) will become – infinity to x and the same thing right. So this thing with the dummy variable let us say 1 by √2τ power or if we just put it with variable Y so this will become this one –x square by 2dx.

So immediately we can see that, this and this are complementary okay! So I can write Fx(x) = 1-Qx so this is always true. Remember it is a distribution so the integration of that from – infinity + infinity must be one. So that is what we have applied over here that – infinity + infinity is 1-infinity to Y if I just take that out this will automatically become Y to infinity Right! So that’s the way they are related

The other function is Error Function or ERFC that is defined as this 2 by √τ so these are just by definition, these functions are defined so this is called error complementary function okay. That i’s by definition is this , this complementary function is of x of course so the integration it is 2 by √τ so it goes from x to infinity it is called –y square immediately you can see what I need is to define it with respect to the Gaussian or Q function

So if I go back to the q Function this has the relationship with the Q Function but this is x square by 2 so I have to replace it, y must be replaced as x by √2 right! So if I do that immediately I will be getting that particular function and correspondingly what will happen this, become 2q(√τ) so that just by doing that replacement and then changing the integration so Y if you replace it with another dummy variable take U so put it as U / √2.

Then this will be immediately Usq/2 and here that here that 2 will be coming into the limit and then you will see that it will become x√2 and there will be a two factor which will be cancelled out okay. So this is how it is and we can also write this Qx can be a swell as written as half the reverse thing okay. So here the Error FC or ERFC was written as a function of Qx and we are just doing the reverse thing same thing

Just a manipulation of should I put in the this one x/√2 or x√2 ok! so if you just put that automatically you can manipulate and you can get this relationship ok! This is just to know how this Error function or Error Complementary Function or Complementary Error Function or this Q function how they are related to the Gaussian PDF Ok!

Because most of the time we will see that these functions are means numerically valuated for different values oh X so therefore Gaussian can be easily mapped to this functions ok! So that is why it is often required. So it is good to know these things okay. This was just for general information so that whenever you are handling Gaussian Function probably you will know these things.

Now what we will be doing what we have already done for discrete random variable that joined distributed condition distribution conditional probability all those things how do you define and characterize them in the continuous version so that is something we will start. So first we will start with the joined distribution.

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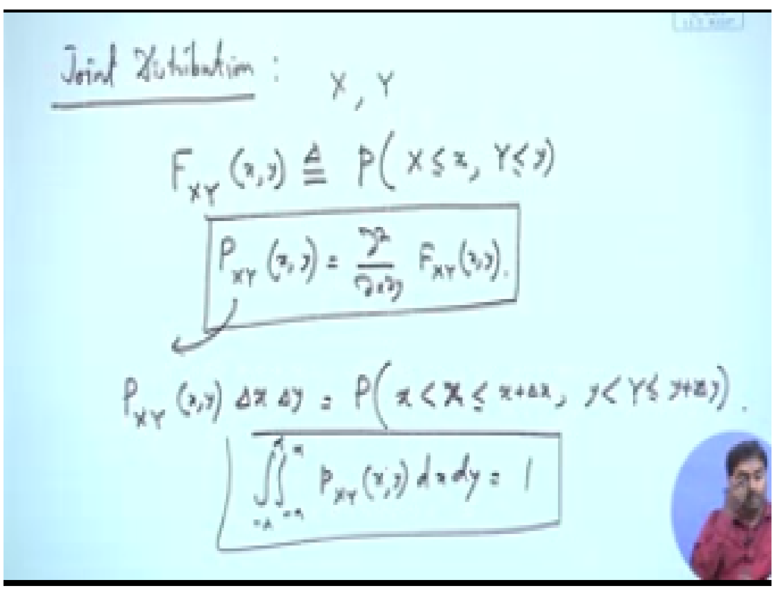

So that means we have two random variable which takes continuous value one is X and another one is Y okay. So what I need to know is this Fx(x,y) = which is by definition written as probability that a random variable x ≤ x this x specified x and the random variable Y ≤ to that specified y okay. So this is called the joint CDF Ok! So whenever we talk about joint CDF that means it is actually means there are underlined two random variable

And whenever we specify that joint CDF with two functional output X and Y then we are actually saying that it is the probability that one of the variable X capital x that is always less than equal to some x specified over here and the other variable Y capital Y les than equal to this particular y okay. So whenever this is happening if you just calculate the overall probability that with what probability this particular thing phenomena is occurring then we get this one okay.

And with the same definition we can define because it is joined so there should be a double differentiation with respect to CDF to get the PDF right. So Pxy with the similar mathematical thing which we have already proven CDF and PDF are related to differentiation and integration.

So we can just write it as double derivative with respect to x and y. so this is the corresponding density joined as re-function.

So this is actually the joint density function that we were talking about okay. So this actually tells you what it tells I will list that so this like the single variable one Pxy(x,y)its density so I have to multiply with the corresponding delta means change of random variable so this define a probability that my x or the random variable x is greater than x and less than equal to x plus del x so it lies between x and x plus del x and the other one jointly both the things are happening so the random variable y that lying between y and y+ del y.

So that is the definition of the PDF right. So PDF is the density function as long as we multiply by the change okay. So we get the area under that and our assumption is this del x and del y are sufficiently small so that things are not changing over there ok! So this actually characterizes the probability that x lies at the vicinity of x that means x to x+ del x and y lies in the vicinity of y. that means y+ y to y+ del y right! So this is the definition which we have seen already for the single variable right. It is just the extension.

So immediately we also know because this is the probability. So if we just integrate over all possible values of x and y I must be getting 1 because that is the sudden event because I am searching through all values of x all values of y. So I will be getting something definitely. So if I just integrate it from –infinity to +infinity this Pxy (x,y) d x d y that must be giving new one. So this is again the fundamental postulate of fundamental of probability theory.

Because it is probability so if we just integrate it over the whole region of interest I must be getting one okay. So this will be always happening so let us try to see from joint distribution in our other case this discrete random variable we have defined the marginal distribution. So let us try to see whether that is happening over here.

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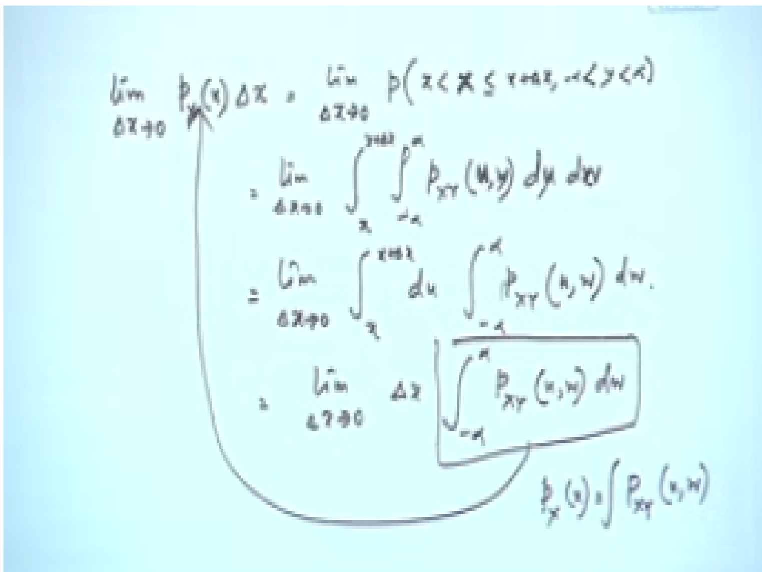

Similar definition of marginal distribution, so let us characterize this thing that this is my marginal distribution okay. So that means that I have made it free of y I knew that there was a experiment which was going on were two variables were random debarring x and y, so but I m interested in any value for y I do not care what is the probability density function of x that is what I am interested in.

So if I do this that tells me the probability that x lies between x and x+ del x so that is the probability that I am targeting! So I can write this as limit del x tends to 0 that is the random variable but remember I also had y in the definition but now I do not care about y so I am saying that y must be lying anywhere. So from –infinity to +infinity okay, by definition I can write this I am only interested in x that I want to restrict my x between x and x+ del x so that is my px(X) x del x where del x tend to 0.

But I have only the joint distribution so that also specifies y so y I do not want to keep any restrictions so I can take any value from –infinity and +infinity okay. So this is the probability I am targeting if I wish to only get x as marginal distribution right. Let us try to evaluate this thing, so I can now write it to be limit del x tends to 0. So now let us write it from the definition of PDF so y goes from –infinity to +infinity.

Therefore the integration this pxy this must be integrated overall y that is the first task! – infinity to +infinity for x I should be integrating it from x to x+ del x right. So this integration should be x to x+ del x right. So I am integrating for y from –infinity to + infinity and for x I am integrating from x to x+ del x of course you can always argue that okay, it is excluding that value of x but by the definition of integration I do not have to take that.

As long as I am integrating if you just divide it into small small area the last one you can take it out. So this is what happens right. So this was according to the definition of probability theory this is a probability and we always know that an probability is given by the integration of it is PDF from particular value that we have specified over here for x this is the value we have specified so that is where we are integrating okay. Y that is the value over which we are integrating

But we have to do another thing because this is in the limit so maybe we can just define it as U and W so two dummy variable. We should not mix the limit with the internal integration variable. That is all right, so Pxy u v we are integrating from for y this and x this okay, the assumption is del x tends to be 0 so within that interval this is not changing as it is varying it is not changing so what I can do is, I can actually separate out this integration

I can still write del x tends to 0 I can take x to x+ del x this where ever x variability is there that means du and y –infinity + infinity Pxy(u,w) dw right! I can write it this way. Why I could separate this because I know that this particular thing is constant whit respect to x. So therefore, x integration and y integration can be separated out. I will have nothing within this. So this happens to be del x because integrating du from x to x+ del x so that should be del x

So I can write limit del x tend to 0. This is del x and this – infinity to + infinity Pxy (u,w)dw. Now match these two, I get a definition of this, so immediately I can write my Px (x) is nothing but the integration with respect to the other variable. So I take the same this one same joint distribution I integrate it with respect to y from – infinity to + infinity so that is called the marginalization. So a particular if it is a joint distribution I want to marginalize it so marginalize it with respect to the other variable. I get the marginal distribution with respect to whatever I am targeting

Here we are targeting for x and the we have to marginalize it respect to y. same thing will be happening if we wish to target Py(Y) then we have to marginalize with respect to x. so that is how the joint distribution and the marginal distribution are related.

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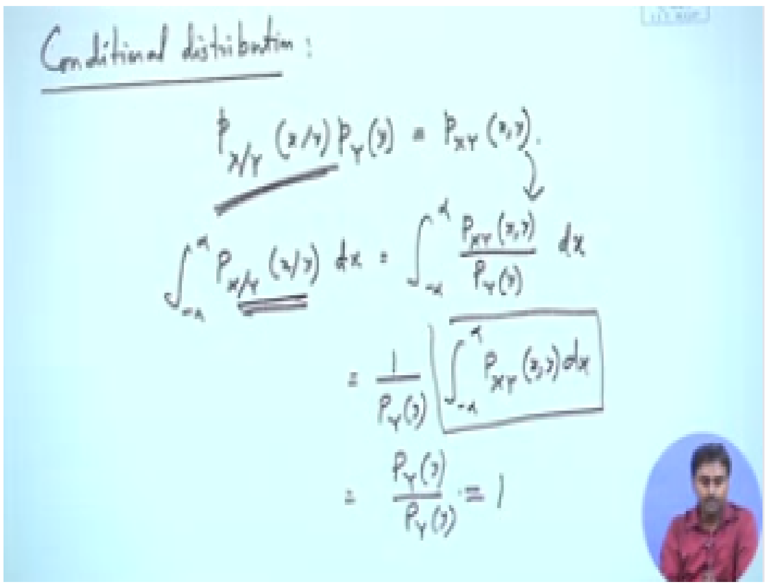

Now let us see the conditional distribution, so the base theorem still prevails I can always write Px/y (x/y)Py (y)=Pxy(x,y) we have proven this. So the conditional distribution is somehow related to the joint distribution. Here we should not talk about any distribution we are just saying that conditional probability is just related to the joint probability for a particular targeted value of x y. that is how we are discussing about these things.

Now what we can write, we can prove that this is also forming a PDF. What does that mean? That means, if I integrate it with respect to suppose x or – infinity + infinity I must be getting one. Because it is a finally this is nothing but given random variable y = y this is the probability of x=x. this is a density function but if I take it multiply with del x and limit del x tends to 0 we can say that x lies between x+ del x. I will be able to talk about that probability

So if that is the case if I integrate that function with respect to x because x is the variable over here y s the condition so that condition I am not bothered about if I integrate it with respect to x, I must be getting 1 , if this is a this should be a PDF or probability density form. So I should be proving that. Let us see – infinity + infinity Px/y(x/y)dx so that is what I want to evaluate so I can write this as from this formula Pxy(X,Y)/Py(y) dx I can write it this way.

Just coming from this formula, its Px given y is nothing but pxy divided by py. This is the integration over x so y has nothing to do with it so I will take that out. What is this? This is just we have proven now that's the marginal distribution. A marginalizing with respect to x. therefore I will be left with y so that must be Py(y) divided by Py(y) = 1. So you could see that this is actually a PDF. Whatever we get because we have just from the base theorem directly got it.

Now we can see it is actually a conditional PDF. So we can evaluate this PDF. Similarly, for the other things also, y/x similar things will be getting. Now from here conditional we came to the independency. So same thing we will do over here.

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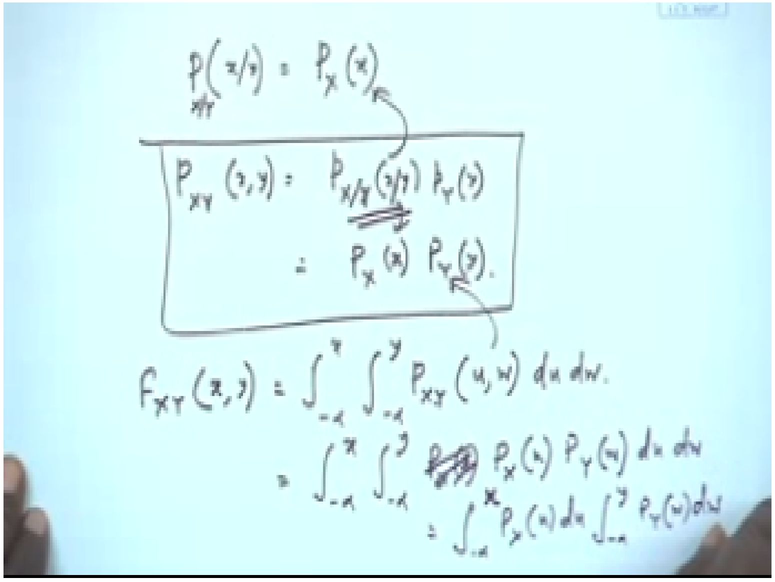

Px given y, x will be independent of y if this is just does not depend on y. So whatever the value of y given y, I don't care! I will still get the xx if this is happening then I will know that it is a independent event means x doesn't depend on the occurrence of y. y can happen, might not happen still my x will be following the same thing. It is like the tossing the coin twice. If it is a fairly uuencoded biased coin then will happen? The second toss the outcome does not depend on first toss what is happened. Whether that was head or tail I do not care, I still have similar probability value of the second toss.

And if that is the case now pxy definition what it was according to the base theorem this was px given y pyy this was the definition. Now this is according to this, so we immediately see if they are independent then the PDF also gets multiplied or separated out in the multiplication form. That is a very interesting theorem of probability theory. So if two random events are independent then I can always write this otherwise not because otherwise you would not be able to write this thing.

And immediately we can also get into this, this is actually the CDF of the joint distribution so this I can write as from – infinity to x and – infinity to Y, Px y (u,w) two dummy variable I have taken du dw. So I can write this now the good part is as they are independent they can be separated out so I can write this. Because they are separated I can now separate these two integrations. The integration with respect to variable u and w can be separated. So I get – infinity + infinity PXu du sorry – infinity + infinity to x and – infinity y Py w dw which is nothing but the fx and fxy so this will be, if I write this will be equal to fxx and fyy even their CDF will get multiplied form.

So so far these are the things which we could means it is almost the similar thing that we have derived for discrete random variable we have derived all those things for continuous time random variable. So in the next one we will try to see is what is the statistical property or measurement property that we can extract and how we relate them to probability okay. That is something we will try to see in the next class. Thank you.