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NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communication

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Lecture 40: Random Process (Contd.)

Okay, so we have talked about stationary and regordicity right, what will now show you is something for means some class of signal that is probably wide sense stationary signal will show you demonstrate you that some more property of that signal is still predictable okay, so that is something we will try to appreciate today. So what we mean by that?

Let us say for any signal those entire deterministic signal so far what we have done, we are trying to see what is happening at a frequency domain right, so that was very interesting for us because with that we were means employing filtering and all other things which frequency component it has how to enhance the signal all those in lots of processing and all those were possible is we visualize the signal in the frequency domain.

And whenever we said that frequency domain we want o visualize and it to do a furrier transform, so for furrier transform the input has to be deterministic signal, so far this was all good because I was always dealing with deterministic signal either a pulse or a sinusoidal or something of that nature, but now we have already declare that most of the signals are not deterministic because suppose receiver want to design a filter, he does not know what is coming he has no idea it will be a random either a random pulse or a random signal amplitude modulated how the envelop is varying he has no idea, on top of that there will be noise.

There is no idea about all these things okay because before he receives how can he know what is there in that signal right, so it is random to him but he wants to design a filter for this but filter requires frequency domain characteristics but it sis random signal so now what do we do for this? how do we characterize for a signal what will be the spectrum quality of it, but we know that whenever we wish to go to spectrum we have already devise a strong tool which is called furrier transform, right now because a signal is random we have no way to characterize it, so it is very difficult we cannot do that.

So let us try to see if we can do something whenever we are having a random thing generally how do we measure that or characterize that it is in average sense, so whenever we have different kinds of height in a class if we wish to characterize the height of the class we say average height of the class. So similar thing we will try to do over here, so because it is random process so that means all the signal that will be coming that is random so we will try to characterize it from the average perspective okay.

So we will give some average sense of it, what does that means? So we assume that this signals are power signal, so therefore I will interested in power spectral density right. So what was power spectral density? We have already derived this formula.

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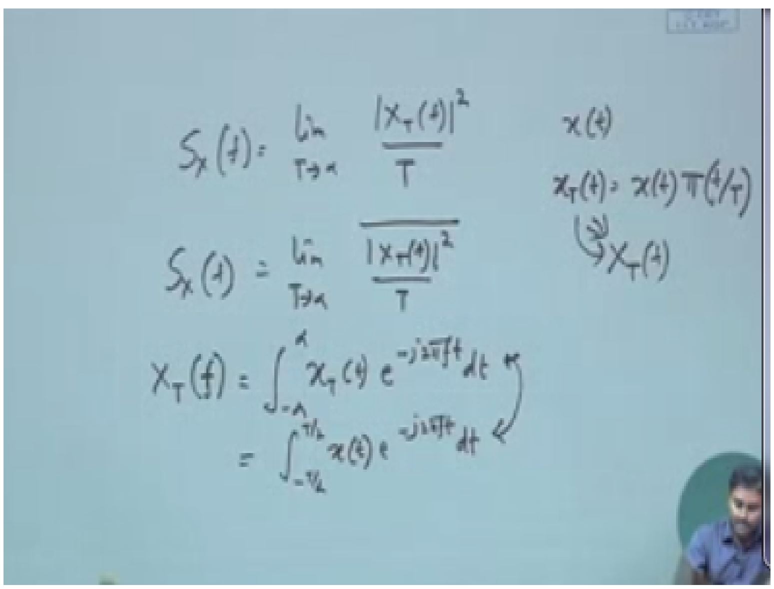

So we have said that power spectral density is something like this right, so this is something we have told that if I have a corresponding x(t) which is truncated, so I create a x(t) where it is nothing but x(t) x a box function of duration T right, so it is multiplied by a box function where it is defined from – t to + t and rest of the things are 0. So it is just basically truncated. So this I can get a corresponding furrier transform as long as x(t) is known to me okay.

So if x(t) deterministic one then xtt also will be also a deterministic one and then what I can do? I can take a corresponding xt(f) okay furrier transform I can do, so it is this furrier transform then divided by T T ∞ that was the definition of my sx f but now my xt is a random process, so what I have to do is I need to if I wish to calculate this power spectral density that must be the average power spectral density.

So therefore that average power spectral density should be is should take average of this, so x t f 2 / t average of that so all I have to do is I have to do the ensemble average of this whole thing okay. So that should characterize in average sense what will be the power spectral density and I need to see whether this is something I can characterize. So that is something will try to do in this class okay.

So let us say I have a particular signal xt furrier transform is this means xtf so therefore I can write the corresponding signal xtt it should be e $^{-j\ 2\pi ft}$ dt right I can write this, because this is just the furrier transform of the truncated signal as long as I have pick the sample okay. So I am ensemble or ensemble averaging I will do later, so this is just a picked sample and then because xt is truncated already I can write that this is –t / 2 to + t / 2 xt e $^{-j\ 2\ \pi f\ t}$ dt I can write this, because it is truncated as long as I take the limit from –t/2 to + t/2 over that duration.

Whether I write xtt or xt it is the same because within that truncation it is the same thing it is just beyond this it is 0 for xtt and it is having some value for xt because I am not going beyond that so I can write this and this and this are equivalent because beyond +t/2 this is 0 so there will be no value, so this and this are equivalent I can write this way okay.

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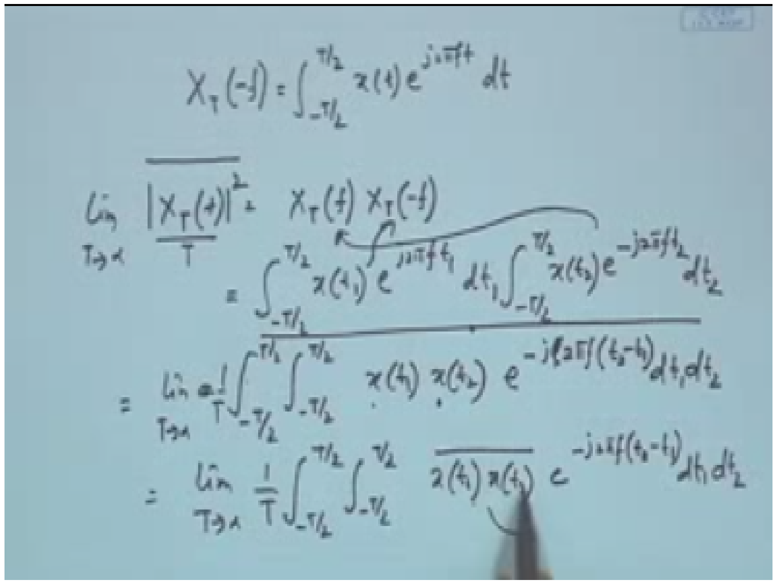

Similarly I can also defined xt – f right so which will be nothing but –t /2 to + t / 2 xt this will be just + if it $e^{+j2\pi ft}$ because it is –f so –f – will be plus dt okay, now what is mod xt f ^{2} that I write as xtf x xt – f y because we know that xt – f is gone complex conjugate of xt f this is something we have proven already, so if I just multiply these two that must be giving me xtf mod ^{2} for a real signal of course so I can write these two.

And because there are two t integration I will take two dummy variable t1 and t2 right, so we just do it –t/2 to + t/2 x t $1e^{j2\pi f}$ t1 so I am just first writing xt – f dt1 – t/2 to + t/2 x t2 $e^{-j\ 2\pi\ f}$ t2 dt2 so this is this one this is this one dty1 and dt2 this is just two dummy variable I have taken it was intergraded over t because both of them are t, so because I will taking double integral means I will be representing them as double integral so I do not want to confuse those two random means those two variable.

So because this and this are independent, so I take that whole integral out so from – t / 2 to + t/ 2 xt2 xt2 $e^{-j2ft2-t1}$ dt1 dt2 right I can write this, now I wish to get that xtf average right which will be nothing but if I just take divide by t take a average and take limit t tends to ∞ if I just do that so what will happen to this side it should be average and there should be 1/t limit t tends to ∞ so I am just modifying it right.

So that should be the overall thing, now this ensemble average is over these values right, these are the random variable that is one integral okay, and that has nothing to do with this t1 and t2 integral, so therefore I can actually exchange these two integral ensemble will be also I integration over this random variable okay that x act t 1 and x act t2 okay, which has nothing to do with this other random variable sorry other variable of integration okay.

So therefore I can take the ensemble average inside not problem in that it is just and the limits are not dependent, so I can take them inside, so I can just write this similarly t tends to ∞ this 1/t is remains this remains, so it is xt1 x xt2 average $e^{-j2\pi ft2\ -\ t1}$ dt 1 dt2 right. Now comes the stationary d property, what is this? that is actually the what you call this function is ensemble domain right, so if the process is at least wide sense stationary that means the second order should be just of auto co relation function which depends on the separation of these two so I can write this.

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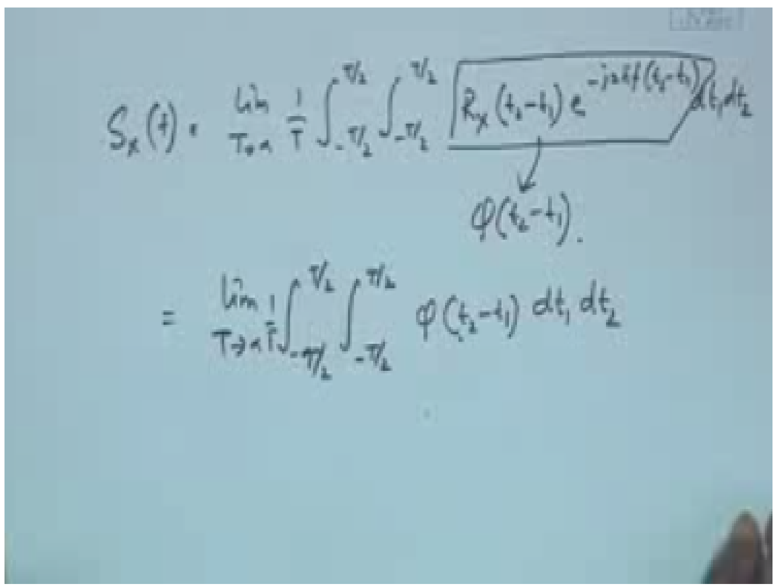

So therefore sx f I can write as limit t tends to ∞ 1/t – t/2 to + t/2 another integration –t/2 to + t/2 this I can write as rx t2 – t1 $e^{-j2\ \pi f\ t2\ -\ t1}$ dt 1dt 2 right I can write this, so this whole thing now as long as they are wide sense stationary this whole thing no becomes our variable of t 2 – t1 so I can defined that as φ t2 – t1 okay, so therefore the whole process becomes limit t tends to ∞ 1/ t – t/ 2 to + t/ 2 again –t /2 to + t/ 2 φ t2 – t2 dt1 dt2 right, so this is what we have bought so far.

Now we will do a trick, so from this double integral we want to convert it to a single integral, that something we wish to do. Why we are doing? Because we want to actually this sx f we want to relate it to the auto co relation function okay, so we have related it some work but we need a little bit more concrete nature of it okay, so let us try to see if we can further simplify it.

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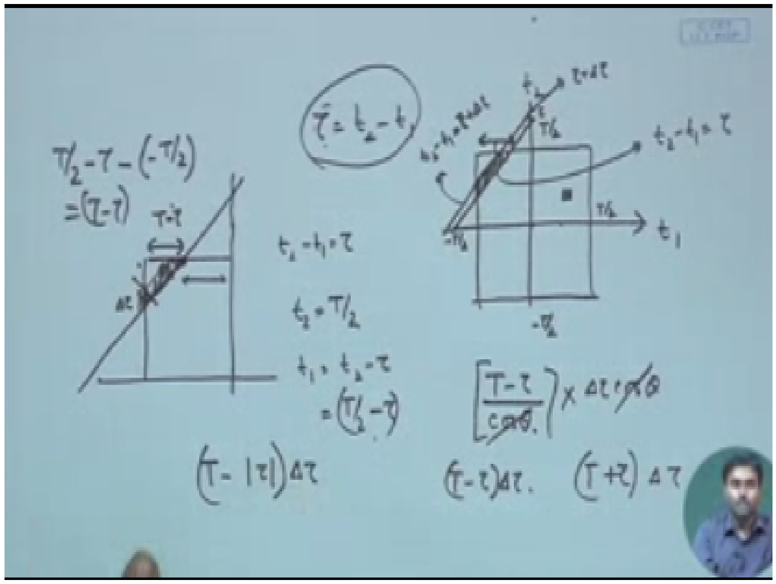

So what we wish to do, this t2 – t1 that should be a variable so defined τ as t2 – t1 okay, let us see how the integration variable changes so this was by t1 and this is my t2 the overall integration if we just see this, this is happening over this box of dt1 dt2 where t1 is varying from –t/2 to + t/2 and t2 is varying from –t/2 to + t/2 so it will be a box of size t cross t okay, so which is something like this.

Where this is + t/2 this is –t/2 and this is –t/2 right, so now we need to also defined in this plan in this particular variable which is τ = t 2 – t1 this is nothing but the straight line of 45 dv slop okay because t2 is exactly t1 + τ where m is one, one means tan θ should be one so that means it has a 45dv slop and it cuts at that is the cutting point at τ, if I take the next line and I increase τ / this becomes τ + Δ τ, so this must be the equation t2 – t1 = τ + Δ τ so that is the equation of this line.

And this line will have equation t2 – t1 = τ okay, so now we wish to convert the integration over this τ how do I do that? So instead of integrating it over suppose at t1 and t2 I have a box right of dt1 and dt2 instead of that box if I just do this integration over this strips okay and then what will happen? If I just this τ if I start varying it so at τ = t it will just be crossing it that is very clear because this is t/2 – t/2 – t/2.

So at τ = t it will just be touching that okay so at τ = t to τ = -t if I just keep changing τ it is just switch through the entire thing all I have to do is because of that whatever strips will be happening I need to take the area of that, over that area I have to do the integration. So that is all I have to do, and if this d τ is sufficiently small why I am doing this because I had a inside variable which is φ t2 – t1 which is already φ τ, so if Δ τ is sufficiently small over that strip φτ will not be changing.

So that is why I can do this integration okay just the area dt1 dt2 has to be change to this particular area as long as I am doing that it should be converted to a single integral right, so I have to then evaluate the area okay because I have already seen the limit of t τ that goes from + t –t to + t okay, so I just have to get the area, so for calculating that area I need to find out this length okay because the angle I know already so immediately I will be getting this value okay so this is something I can always do.

So basically if I just draw it little bit bigger, so let us say this is actually t2 – t2 = τ, so what is this point at this point by t2 is t / 2 so t2 is t/2 what will be t1 that must be this okay so t1 is actually t2 – τ and t2 is t /2 so t/ 2 – τ so that is this value I need this, so that must be because it is already in negative so it must be this value – this value that will become negative so they will give me the distance.

So this value is already t2 – τ so if I wish to get this that must be t t/ 2 - τ – this value what is the value over here – t/ 2 so – of – t/2 that gives me t – τ, so this particular value sorry this particular value is t – τ, if this is θ and what should be this? this should be t – τ / cos θ right, so I have got 1 arm of this I need to also get another arm of this sorry this particular thing, this triangles will be very small and it can be cancelled out okay so that is that will be in consider because that will be $\Delta \tau^2$, so I can neglect that as long as Δ τ is small.

So I just need to get this box okay, so if now you can see basically this is Δ τ right because over here that was the increment τ to τ + Δ τ so this must be Δ τ, what is this? This is already θ, that particular small angle between this lien and this line that is θ okay, so immediately that should be Δ τ cos θ, so this multiplied by Δ τ cos θ must be giving me this whole area this strip area cos θ cos θ gets cancelled so that must be t – τ x Δ τ, that is the actually the strip area and whenever we do integration so at every value of this τ this area has to be considered okay.

So therefore the integration should be okay I should also specify another thing that this was true when I was in the positive half of t τ if I go to the negative half of τ you will be able to similarly prove that this should be t + τ Δ τ this is something can be easily proven similar way okay. So basically I can write that this should be t – mod τ x Δ τ right, I can write this area as this. Once this is the case, now I go back to my calculation.

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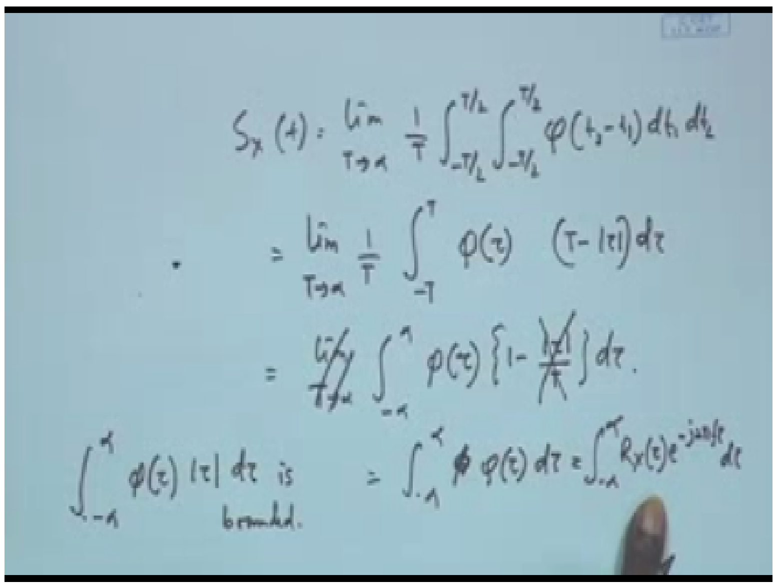

So sx f was actually limit I will write it one more time 1/t – t t / 2 to + t/ 2 again – t /2 to + t/2 φ t 2 – t1 dt 1 dt2 this integration now can be converted limit t tends to ∞ 1/t this becomes a single integral now over τ so the area for that integration is t – mod τ sorry Δ τ that becomes dt d τ, and this is already φ τ integration and the limits goes from – t to + t that is what we have seen so – t to + t right.

So this is the overall integration limit t tends to ∞ I take the t inside because that has nothing to do with the integration so if I just take that what will happen? Because it goes to ∞ so I can even write that to be ∞ so basically I can take this limit I might write this to – ∞ to + ∞ φ τ, and now this will be one – mod τ / t of course here that limit t tens to ∞ should be there okay.

Now this is something where we have to specify that this as t goes to ∞ probably this might be canceled out but we should be very careful about that we have to say this thing that as long as this particular integration φ τ mod τ d τ is bounded most of the signal that will be considering this will be bounded once this is bounded divided by t okay and t tends to ∞ that goes to 0. So therefore I will be this terms get cancelled I will be left with – ∞ to + ∞ φ τ t τ, which is nothing but – ∞ to + ∞.

Now replace φ τ it was actually rx τ $e^{-j\ 2\pi f\ \tau}$ dτ can you identify this form this is nothing but the furrier transform of xr τ, that is a fundamental theorem of random process that whenever we have a random process which is characterized as either wide sense stationary or stationary I will be having a rx τ calculated or t2 – t1 calculated if I know that, I can always relate means do a furrier transform of that.

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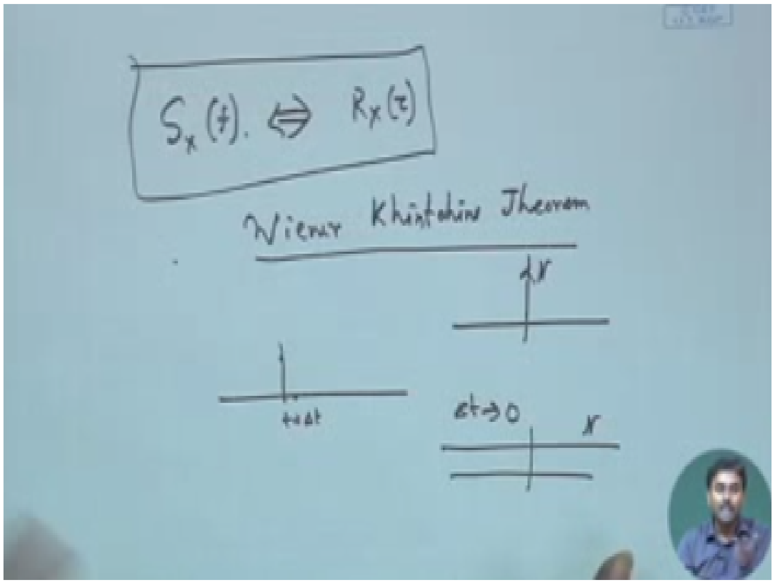

So that means rx τ if I just do furrier transform I get back my sx f which is defined in average way, so this is the average power spectral density because we told for a random thing random signal I can never calculate the actual power spectral density, because every signal will have its own power spectral density but what we can characterize is that average power spectral density.

And we have got now a very strong formulas where it is characterize that if I know that signal is stationary it would not have happened if the signal was not stationary.

Because then the whole process will not be proven if the signal is stationary or at least wide sense stationary that means up to second order it is stationary, if that happens then the auto co relation function if I can somehow evaluate you just take a furrier transform you get the average power spectral density. And once you get the average power spectral density now you can even though that was a random process you have the average power spectral density with that you can design the system okay that is the advantage you get.

Otherwise you could not have done this, so this is the famous wiener Khintchine theorem. That is the WK theorem we all know about, so this is that fundamental property of a random process, it is says that as long as and that is why we were saying so much of things about our stationary or wide sense stationary random process because as long as random process is stationary or wide sense stationary we know tha5t we can always derive its average power spectral density which will be used for characterize in signal okay.

So that means if I have a random signal I should not no worry I just have to test whether it is at least wide sense stationary, that means second order statistics is still stationary as long as I know that I know I still have a means to go to the spectrum earlier I was little bit afraid that if the signal is random I have no way to go to the spectrum I do not know how to explode the spectral property of the system of the signals okay. But now with this strong relationship I know that as long as I can evaluate somehow the auto co relation function I will be able to characterize the corresponding average spectrum okay.

So this is a fundamental theorem and this will be used. Now let us after knowing this let us talk about something which is very fundament al to noise analysis, let us try to see what this noise, so noise is something which is completely random to s that mean if I absorb something at the next time I do not know what will be happen, that is why we call that noise okay. because that is completely random and therefore in the next time instance even though it is infinite symbol is small just at t I absorb some sample value even at $t + \delta t$ let us over here even if δt tens to 0 I have no way to say where that will be that can take any random value.

This particular power process we characterize as noise okay, so that is to our mind that is probably the noise okay we will see that as a special characteristics but if that is the noise then what is the let us try to ask because if this is noise this is the random process I need to first try to understand what will be the corresponding auto co relation function okay. I can immediately see because at this time seeing something I am not able to at the next time what will be happening I am not able to tell that means it is completely uncorrelated.

So time we just progress little bit I must not any co relation, so the co relation value should be 0 so what I can say but signal with itself has full co relation will be very high co relation because that I know exactly that signal it will be only thing is that I just shift the time even a little bit I know that there will be no correlation absolutely 0 co relation. So if this is the case when the auto co relation function should ideally look like a δ function, so that is noise fouls.

Whenever we characterize noise we say it does not have any co relation to next time instants, and immediately single function comes to our mind that is the δ function which some strength of that δ, let us say that is ε okay, so with that in mind immediately what will be the corresponding furrier transform that should be the power spectral density, we know δ furrier transform is dc, so this is the power of strength it, so this is actually for noise this is the power spectral density, what does this says.

That it has equivalent component at every frequency or equivalent amplitude of power at every frequency and that is why probably we say it is wide noise that means all equivalently all frequency components are present and it will be wide noise because it is the most random noise that we know which has no correlation what so ever even if I know some means absorb some sample value over here at a particular instance of time the next time instance I cannot predict anything about it.

So whenever the signal is like that I have a auto correlation function which is δ, so immediately you can see why this particular thing is so important is that I have a sense of creating auto co relation function from the description of that noise that I have in mind the most random noise that I know and then immediately from there I know the spectral property of that noise because of wiener Khintchine theory, so that is the strength of wiener Khintchine theory.

So once I know that random process even though the signal can be anything, see the signal can be of any nature I do not even know even though the signal is like this I have a very nice spectral property of it that is the fundamental process over here. So immediately I know the noise characteristics and this is actually called the white noise okay.

So once we have that spectral property of noise now we can actually start dealing with this noise that what this noise will do to my signal and how I should restrict this noise, so immediately you know that the filtering is the big thing in noise means combating noise whenever your signal is just filter that portion out because rest of the noise will be immediately surprised if you do not apply filtering see even if you do not have any other signal still you are actually taking lot of noise to the receiver and that we will contaminate your signal.

Whereas if you surprise it lot of power signal power will be relate a signal power to this auto co relation as well as the spectral property of it but you will see that lot of extra power noise power which is contaminating the signal are unnecessarily coming to your receiver, so that is why filter has a big role whenever you are combating noise, so what will do next we will try to appreciate all this things to see how noise analysis can be dealt with in a communication system or particularly analog communication system okay. Thank you.