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Course
On
Analog Communications

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Lecture 42: Random Process (Contd.)

So we have already started talking about cross correlation function right so that is something we have already done, so what we will try to do now we will get some more property of this cross correlation okay we have defined two things one is cross correlation and then we have given a property of join stationary okay and then we have said what is the property when two signals or two random process will become uncorrelated we have given that okay.

The next part of the definition is when two random process we call them again by definition they are in coherent okay or we should say orthogonal deterministic signal we have already seen now let us see for random process what is the definition of orthogonality or in coherency both are interchangeably used so I have already told it must be if it has to be in coherent or orthogonal it must be first uncorrelated so therefore I must have this property.

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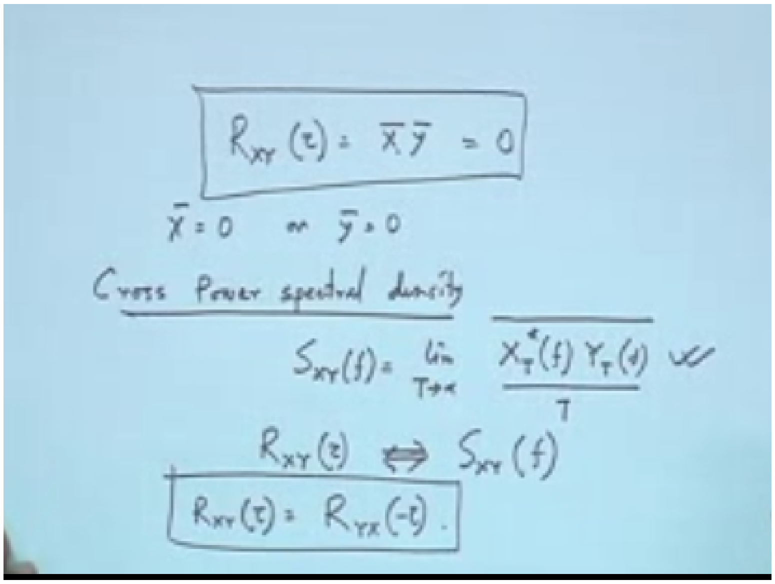

This must be there now among them if one of them is 0 mean okay either $\bar{x}$ is 0 or y bar is 0 okay that happens then this happens to be 0 and those signals are generally termed has incoherent or orthogonal okay so this is just by definition so we are just defining different kind of signals that will be encountering of course accordingly that Rxy that joined correlation will have it is own definition okay or cross correlation will have it is won definition so we are just defining them.

So we are saying that if Rxy τ is just it is Rxy T, T+ τ it is just function of τ then we say it is jointly stationary okay if Rxy τ is equal to x bar . y bar or x bar x y bar then we say it is uncorrelated if Rxy τ = 0 then we say 0 then we say those two signals are incoherent or orthogonal okay so that is by definition and then because we have this we will also have corresponding power spectral analogy okay.

So similarly we can actually put cross power spectral density okay in by definition it is just the fierier transform of this one right so we call it Sxy (f) which is just nothing but limit by dentition we T tends to infinity you trunk it because of basic definition is you trunk the actual signal if it is auto correlation or sorry auto correlation and it is own power spectral density then it is just xtf mod square right.

But here because 2 signals are there so you first so for the first signal okay and then do for the next signal and do are time do in simple average 1/T so that is the definition of this one and if you take this definition again go through the same process of the way we have to win theorem it will be able to show that Rxy τ has long as they are jointly stationary will be able to prove that the Ferrier these two are Ferrier there okay.

So this is something we will be able to show that and similarly we will be able to also show that are Rxy τ is a even symmetric function okay so that is will be but when we say even symmetric it is something that is some hint into it or some trick into it becomes Ryx- τ okay so we are not proving this results it can be done similarly okay it is not very complicated the way we are prove in that similarly if you know the Rxy to τ definition and you just put the things you will see that this has to be the case okay.

So that must be case an immediately by taking this relationship can also show that RSxy f must be equal to Syx – f so this directly comes from here okay so these are the property which are for 2 different random process and they are correlated or cross correlation they can get this properties okay cross power spectral density or cross correlation okay.

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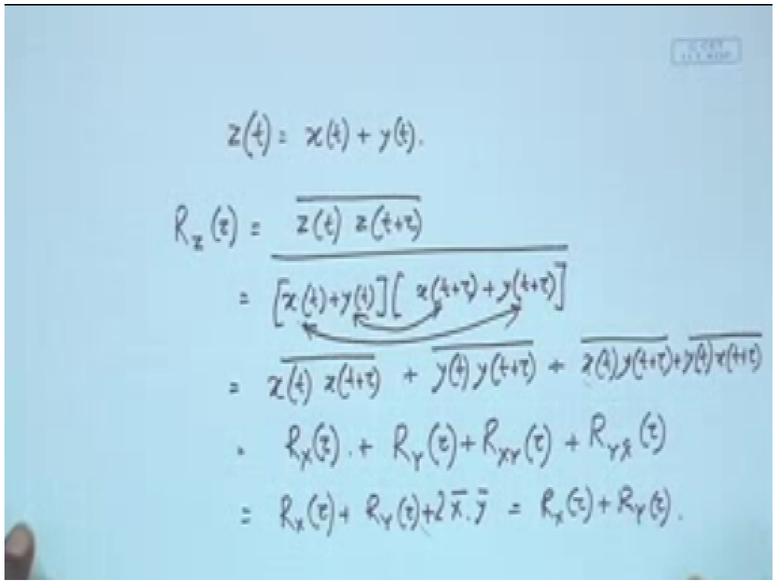

Now with this let us try to see if I have a signal or a random process which is just a summation of 2 random process it is going to be very important because I might have whenever we are receiving something I might have one random process which is associated with the signal itself and another random process which is actually associated with the noise and most of the time my channel is linear so as long has it is linear definitely the overall signal I will be getting that will be just addition of these 2.

So that is why it is called additive channel okay so whenever I will be receiving something I can always by the virtue of this concept that entire channel is linier so I will be able to say that it is just additive the noise will be added on top of that signal so two random process often in my receiver will be additive I need to also how do I characterize that okay so if wish to characterize random process can be characterized by it is auto correlation function so I need to evacuate Rz τ okay which is nothing but z t zt+ τ and symbol average we have the definition now that you replace by xt and yt.

So xt + yt and xt + τ + yt + τ right if first time will be xt, xt + τ and this because it is average it will be distributed over the addition so it should be x t xt+ τ and symbol plus there will be another term related to y so yt yt + τ that is where you will see that y that cross correlation is so important here that term will becoming so you will have multiplication with these 2 and you will have a multiplication with these 3 so xt(y) t + τ and yt x t+ τ so this is nothing but the auto correlation function of x.

So I can write that Rx τ as long as my x and y are all white send stationary at least so it will be just dependent on τ this is also same thing so I can write if these two process x and y are jointly stationary or jointly wide send stationary then I can write this has R xy τ and this will be R because y is taken first so that should be yx τ right so that is something what we are getting okay.

Now let us start putting all those assumption or all those definition that we have given so suppose the process is just not jointly stationary these two process that also uncorrected then immediately what will happen this Rzz will be Rx τ + Ry τ + this should be x bar y bar this also should be y bar x bar or x bar y bar that same so that should be 2 x bar, y bar okay now if they are incoherent or orthogonal now that must be 0 so I get Rx τ put τ = 0 what do I get.

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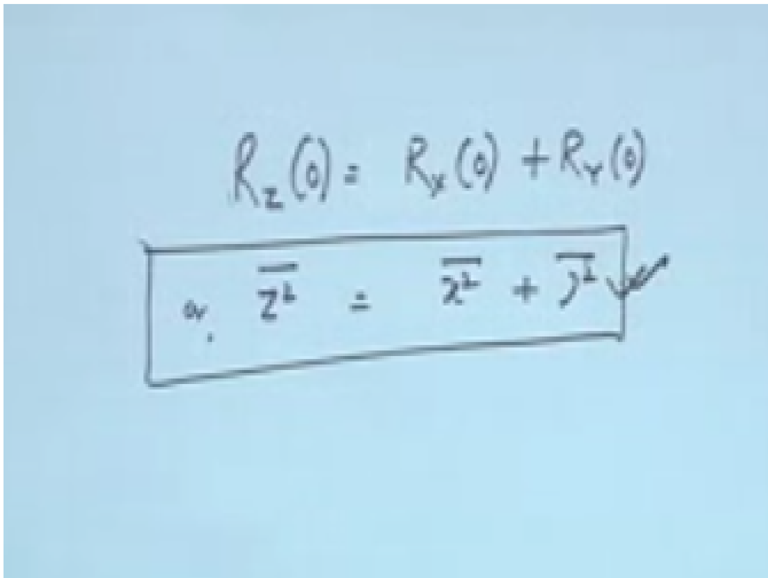

For remember whatever we are doing that is for incoherent okay or orthogonal system what is this we have already prove in that should be the power this s what happen for signal which is incoherent or orthogonal okay if two signals two input process are incoherent or orthogonal most of the time this is what is happening okay your signal will be probably even if signal does not have a zero mean you noise definitely will have 0 mean.

So we have told that and they will be actually means uncorrelated okay so that is always true signal cannot be correlated with respect to noise and noise cannot be correlated with respect to they have different source where they are getting created so they cannot be correlated okay so if that is the case then they will be definitely uncorrelated and even if your signal that depends on how your transmitting if that is not having 0 mean but your noise will definitely have 0 mean.

So we have told that it will be incoherent if one of them are having 0 mean so immediately I can say that noise + signal which is coming to me in the receiver will always be incoherent or orthogonal so I can always write this so therefore I can always write so that is a very fundamental thing that for our analysis we can always very safely say that has long they are stationary and uncorrelated and all those properties holds I can always take the overall power that I am receiving it is just addition of noise power + signal power.

This we take has assumption but this s the fundamental bases behind it happens because of these thing if they where correlated if they where means in coherent could not have written this so you have to be very careful when ever your putting this formula you have to also cheek whether all the assumptions that we have already stated or valid for the system as long they are valid you can take this assumption otherwise no okay. So this is something you should always remember okay.

So now we are almost in the mind set or frame work to now analyze we have already analyzed the means low pass noise now in receiver you can always see there are two ways the receiver can function one is I am transmitting base band data okay then probabably it is around 0 the frequency component that is there suppose voice I do not modulate I just transmit it as it is then probably it will be around that 0 and I have to employee a low pass filtering.

So the noise that will be there in the channel or at the receiver it will just go through that low pass filter okay so that case low pass filtering version of noise is good enough but when I do modulation what I will be doing I will be translating my signal I will do modulation I have we have already see that is just translating to a higher frequency so I will take that entire band and put in a higher band.

And at that point if I wish to reject some amount of noise what I have to do I have to employee a band pass filter always you will see your receiver will start with a band pass filter that serves two purpose one is of course it gives you the rejection of all other signals you just want to get your own signal so it just rejects all other signals that is the first thing second thing is it also rejects most of the noise out if band which is not inside your band because inside your band you cannot do anything noise will be added with your signal.

And you have no way to separate them out if you are not doing digital processing okay as long as signals are analogy you have no way to actually suppress them but what you can do outside the band where you are not interested just put a band pass filter it will reject all the noise the way we have seen for low pass filtering that happen so that way any receiver if you have modulation always we will start with a band pass filter as a noise that comes that goes through that band pass filter so we have to first characterize what happens to the noise if it passes through a band pass filter. So that will be your next target okay.

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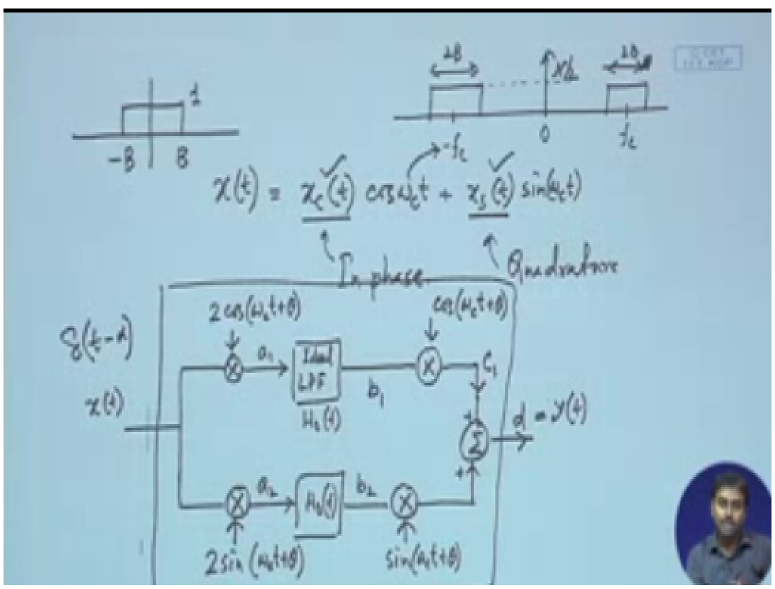

So let us say I have a noise which is a band pass noise okay whose spectrum looks like this it is similar so that is 0 that is fc that is –fc this is that 2B that is the band width okay and because the noise was flat so this particular value is η/2 which ever was there so same noise same power spectral density it is we are still assuming that it is flat over the band of our interest okay so it is still remaining flat it will probably drop some other frequency some other very high frequency but not in the band of our interest.

So as long as we are taking that assumption the noise band pass noise will look like this okay and associated to that this is the spectrum so associated that I will have a noise equivalent random process this xt I will now say something and then we will prove that this what can be done at this xt whenever it has a characteristics of this one that it is a band pass noise can be represented as this.

Where xc and xs will later on prove that they are uncorrelated in coherent random process so they are actually jointly of course jointly stationary that is the first thing so they will be uncorrelated to each other and they are incoherent to each other so we will prove this okay so these 2 are 2 separate random process Xc and Xs this is called the in phase part and this is called the quadrature part of noise.

Whatever I have written I have not prove in that okay so we will try to prove this because then what will happen any noise that comes in once you pass through a band pass filter we will be able to write it this way okay and that will help you because we will also get the property of these 2 things okay and that will actually help you will in the process we will characterize both of them okay.

Xc and Xs what is the associated auto correlation function what is there cross correlation function what is there cross correlation function and then what are the relationship between them as well as the prospective density so characteristics density but we are first we are saying that Xt can be represented like this okay where ωc is related to fc 2Πfc okay so this will be always possible so let us try to see what are these things.

So for that I will draw one circuit okay just to prove this we will see we will appreciate after I analyze this but initially just take the circuit has it okay so there is multiplier so Xt whatever the input signal I have this is a band pass signal right that is a it, exits in a band I multiply this with 2 cos ωct + θ where θ is a know random variable okay I get a1 over here I pass it through a ideal low pass filter of frequency response HOF where HOF looks like this.

It is ideal filter form – B to + B transfer function it is 1 okay I pass it through it I call this b1 and the I again multiply this with cos ωct + θ remember I am just creating a hypothetical circuit but this circuit is realizable because these 2 are having same phase that means it is means actually generated through same oscillator okay so that is possible in a circuit further this is c I have a adder over here which creates d and the other arm I will have similar multiplier but now I will multiplying with sin.

This is should be a2 again ideal similar low pass filter which has same transfer function so it will be 2 another multiplier where I multiply again with the sin add these two and I get the okay so I can write d as yt that is my output so what will I try to see is the transfer function of this particular system that I am creating okay so I will first try to evaluate what is the transfer function of this really you will understand.

Why I am doing that all those things will be very clear okay is I am just first trying to get the transfer function of this whole system how do I get a transfer function I first need to get the impulse response of it so basically I exceed this with a δt and try to see what will be the corresponding out and when creating this transfer function have to also make sure that this is linear time impalement.

So therefore what I will do instead of just putting a δt I will give a delay to that Δ okay so basically I will put a δt – α okay so I will just give delay so that I want to test that output it is just a delayed same I want delay to the output it is just function of that entire t – α that is what is happening okay so that is something I want to test then it will be linear time in variant okay.

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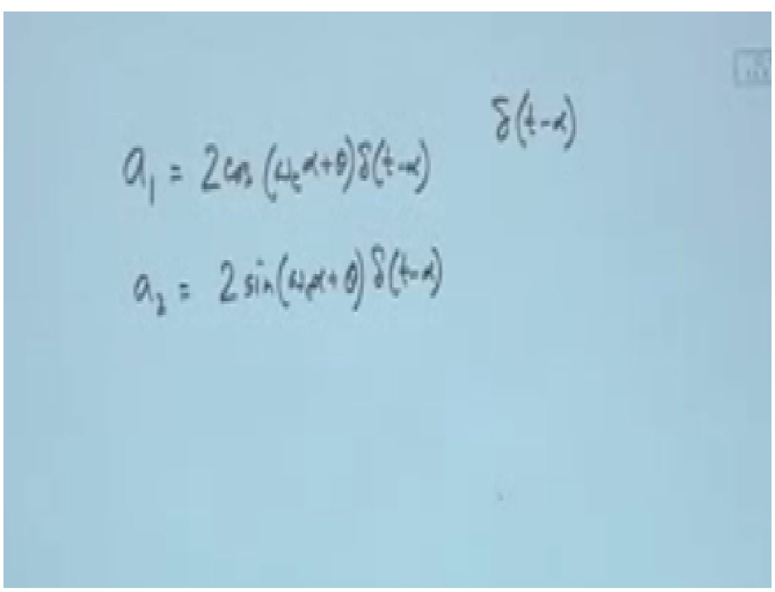

So let us try to characterize this whole thing so at a1 what do I get I have a δ x by cos so any function multiplied by δ is just pick the functional value at that point so if I multiplying $\delta t - \alpha$ that means at $t = \alpha$ it w3ill pick the functional value and it will put a δ function on over there so therefore it will be if you see over here a1 is nothing but this cos is multiplied by $\delta t - \alpha$ so therefore it will just get multiplied and t will be replaced by α.

So I will just get a1 has 2 cos $\omega c\ \alpha + \theta\ \delta\ t - \alpha$ right this is what I should expect at a1 similarly a2 must be 2 sin $\omega c\ \alpha + \theta\ \delta\ t - \alpha$ right so these 2 thing get a1 a2 value now this is just a δ function pass through a I just again come back to the same picture.

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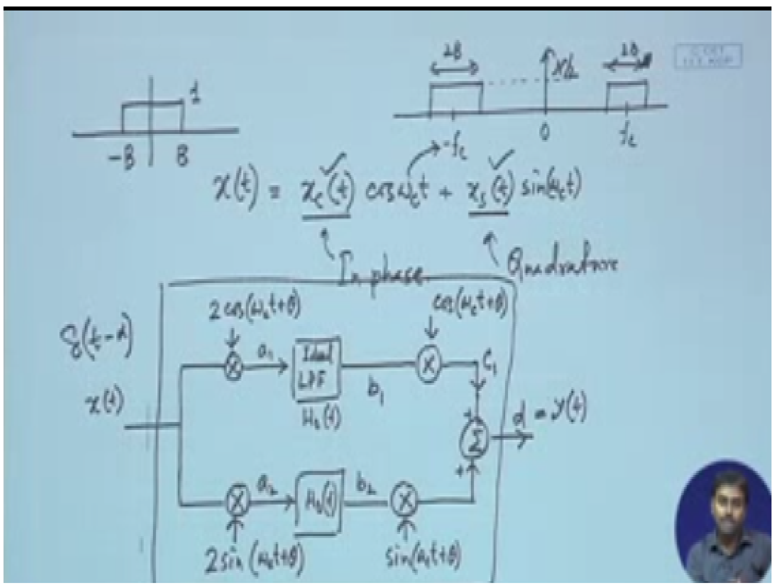

This a1 passes through a ideal low pass filter which has a corresponding transfer function as h0t so at δ function now is the input to a h0t what do I expect at the output it should be convoluted with h0t and h0t convoluted with a δ function must give me the same function we know that any function convoluted with δ will give me that same function okay.

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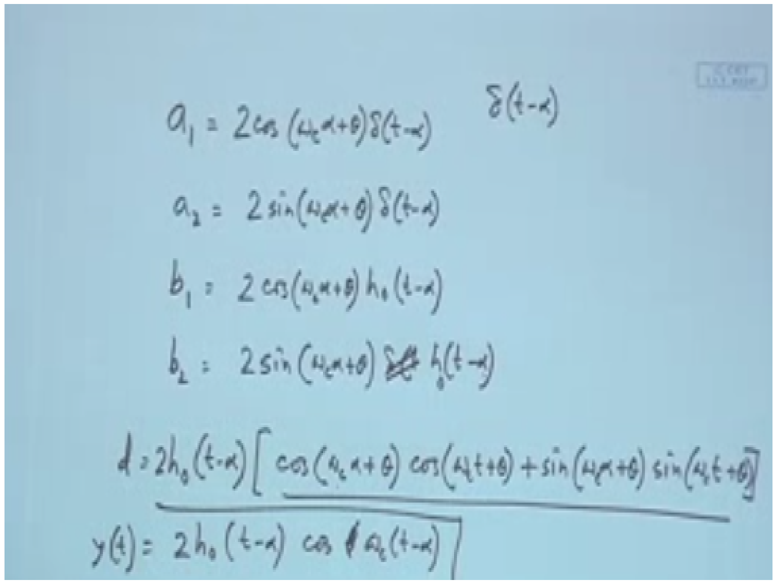

So therefore if I just try to calculate d1 at must be same this is a constant for it right so 2 cos ωc α + θ now *δ* will be convoluted so it should be h0t – α similarly b2 should be 2 sin ωc α + θ *δ* sorry h0t - α after that what we are doing so from b1 and b2 we want to get c1 and c2 then add these 2 w will get yt right so what is c1 that is just multiplication of this and c2 just multiplication of this so therefore overall d if I write that is nothing but this h0t - α will be there in both the cases.

t- α I will have 2 everywhere so 2 I can take out and this is nothing but cos ωc α + θ x cos it will be multiplied by ωct + θ + sin ωc α + θ x sin ωct + θ right see why I have created that whole circuit because I wanted a cos a-b formula nothing else this whole circuit was created because I wanted that cos a-b okay y cos a-b immediately you can see if just do cos a- b so it will be ωct + θ.

Θ get cancelled it will be just ωct- α h0t – α x cos ωc t –α that makes it a linear time in variant system because it is just a function of this yt what I get because I have put a *δ* with a delay of α then what I get it is just function of that t – α so entire gets similarly delayed right so it is time in variants so I could prove my point by that okay.

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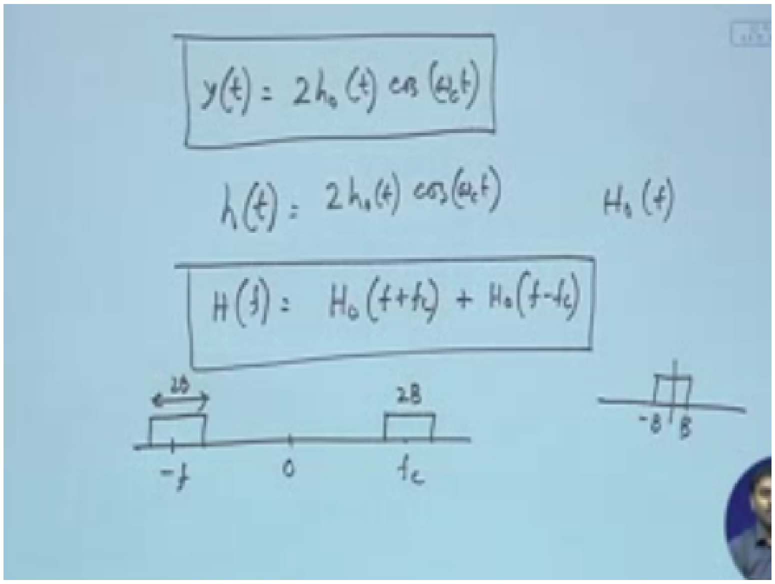

So over all if I just say that my yt if I just exceed with δ t it should be 2h0 t x cos ωct right I can just when I put δ into it so that must be my impulse response therefore because I have put δ t as my x and I have got this so what is this is nothing put let us try to just see this is actually this s happens to be my ht now which is 2 h0t because it is exceed by δt and I have got yt has this.

This first be my impulse response of that whole system which is this okay so if I just take it to the frequency domain so what will be my Hf that must be so cos I am multiply so this is just a frequency translation of this h0t if that is H0F so H0F goes to + fc and –fc there will be a ½ due to that cos and that ½ with this 2 gets canceled right so I get H0f + fc + H0 f- fc so if my H0 was something like this – B to B what is hf I thing but this gets translated to + fc and –fc this 0 + fc – fc that is 2B how does it look.

Is just like a band pass filter so therefore with all these things what I have created is a ideal band pass filter so what was my x that was within that band okay or band pass noise we are talking about band pass noise if x is that because x was actually my band pass noise if I pass through a band pass filter which is having equivalent band so this yt be exaltedly same as xt because it will just pass everything so that is already a band limited band pass signal passing through the same band will just give me same thing nothing extra will be coming out.

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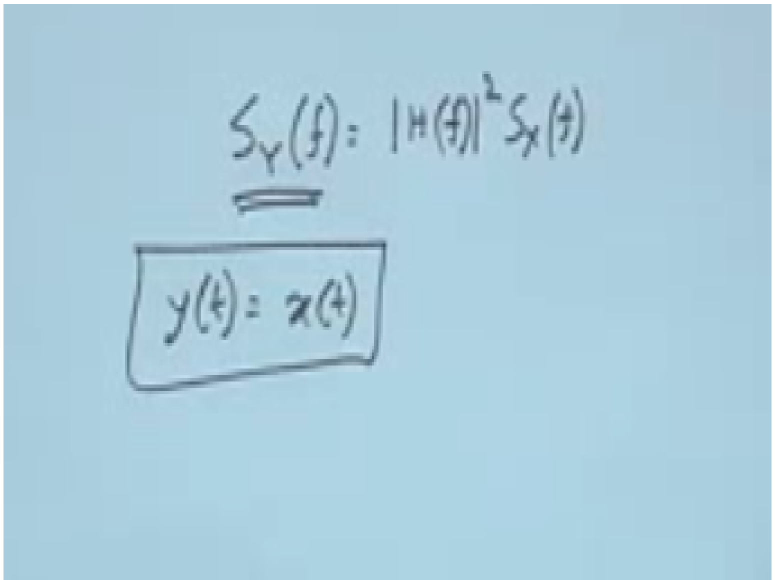

So therefore I can always write that my Syf what that should be that should be mod we have already prove in that Syf output should be Sxf into mod Hf^2 right but my Syf is exactly equivalent to my xf right so therefore I can immediately write my yt is nothing but xt is something I know already Π design okay.

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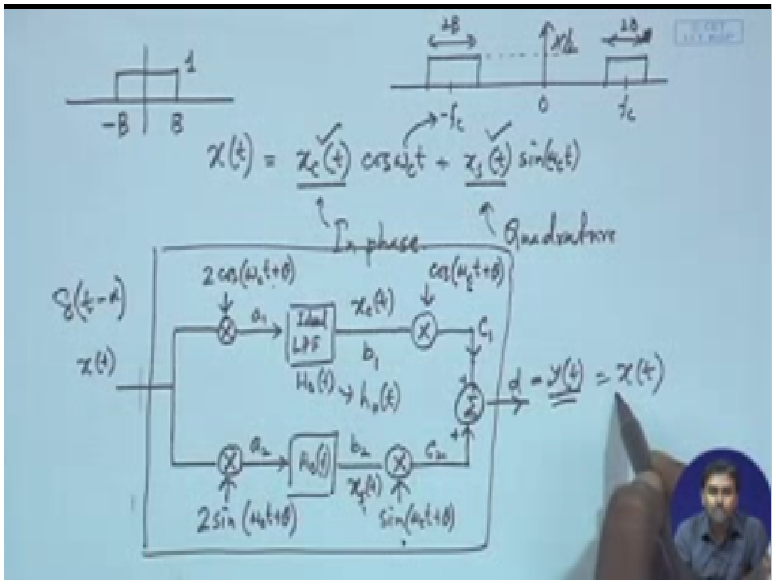

So now if I just take back that same thing okay so this is now becoming my xt now let us stress back what is xt, xt is something over here let us say that is b1 I call that as xct whatever it is and I cal this as xst this multiplied by cos and this multiplied by sin added should be means giving me back xt.

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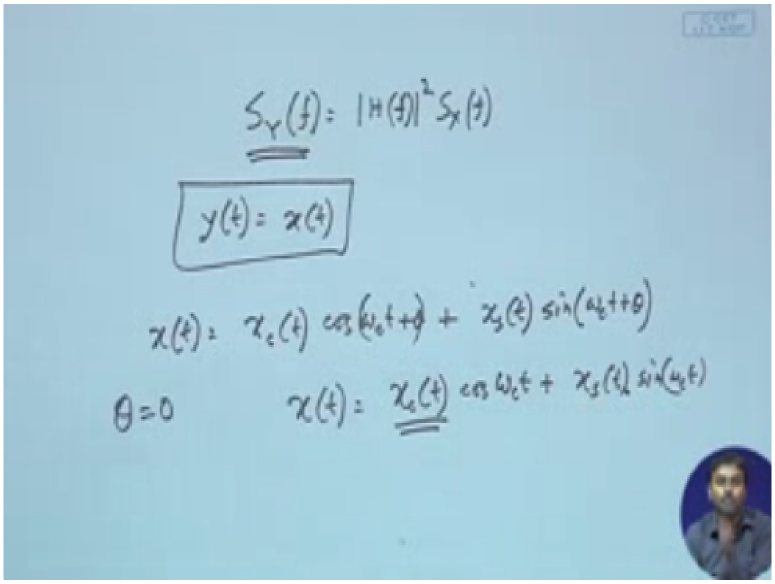

So therefore I can always write xt as something I do not want that is which is x ct multiplied by cos ωct according to the design of my over all circuit and xst sin ωct + θ this I true for nay value of θ see even if put θ = 0 it must be still true so I can always write therefore xt is actually nothing but xct cos ωct + xst sin ωct where xct is nothing but my signal at b1 and xst is nothing but my signal at b2 now if yi just ask what is this signal this was after passing through a low pass filter.

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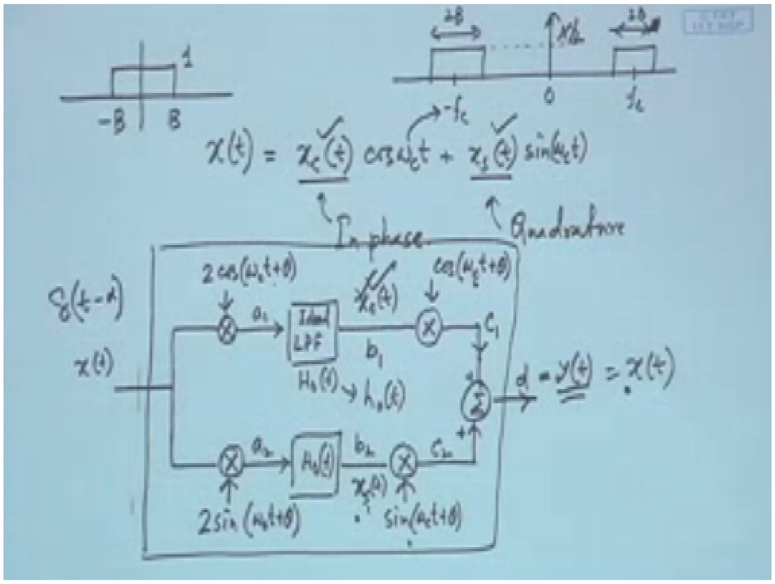

So this must be a pass band signal or base band signal so it is a low pass equivalent signal and this is also a low pas equivalent signal so the band pass signal or band pass noise is nothing but means represented by in phase component and quadirature component which are just loss equivalent signal that is something we are improve in now term so what we will try to so in the next class is to see the characterize of those xct and xst.

What are they because we already through the circuit we know they also have relationship if we just see the other half of the circuit will be able to get some relationship between xct and xt and from there we will try to characterize what they are what are the they are specific properties we will be able to characterize those things so in the next class properly w will try to do that okay thank you.