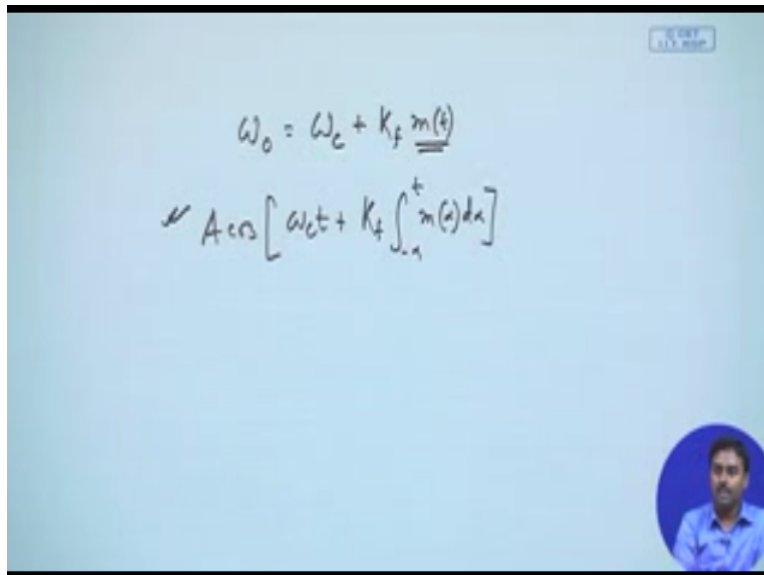


NPTEL
NPTEL ONLINE CERTIFICATION COURSE
Course
on
Analog Communication

by
Prof. Goutam Das
G S Sanyal School of Telecommunications
Indian Institute of Technology Kharagpur

Lecture 51: Frequency Modulation (Contd.)

(Refer Slide Time: 00:23)


$$\omega_o = \omega_c + K_f \underline{m(t)}$$
$$\leftarrow A \cos \left[\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha \right]$$

Okay, so what we have started exploring is the frequency output of the oscillator we are targeting which is we have proven that that should be ω_c into K_f into $M(t)$, we have defined what is ω_c and what is K_f okay. So this is something we have already discussed right. Now FM generation once we have this circuit that means our particular oscillator with an inductor which is fixed and a very cap where it varies with the input that bias voltage okay.

So once we have that we will be getting the output oscillation frequency is accordingly vary okay. So what I will do if you now start giving our input voltage as your $M(t)$ into that very cap immediately the output oscillation that you will be getting that must be FM modulated signal,

because that will be the MT immediately the oscillation frequency suppose it generates a cause some frequency.

So that must be this okay so, and immediately you will get if this is the frequency the phase will be ωCT this is something we have already explored that that should be KF integration minus ∞ to $T M \alpha D \alpha$ that is actually the FM modulated signal. So that gives me a direct modulation where this K F is something which is chosen by that particular very cap right.

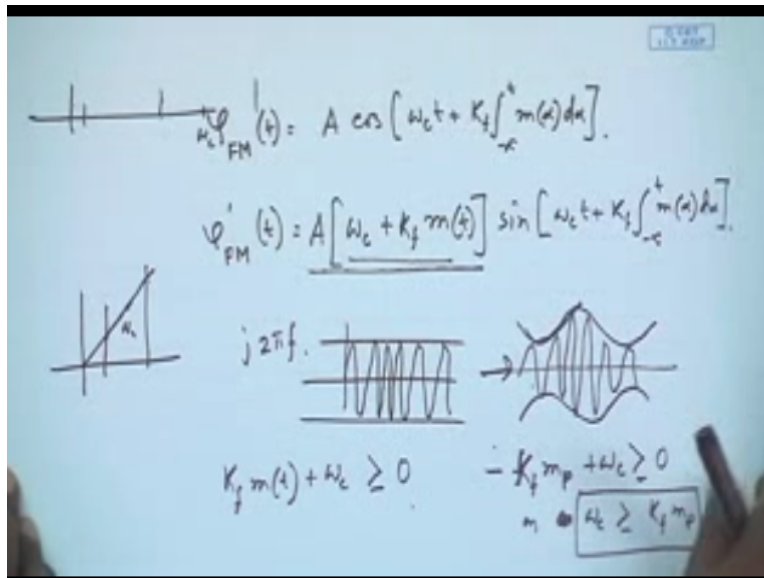
So whatever I choose as my vco I will be getting accordingly the FM deviation and everything, so I can I can choose my parameter accordingly and I will get my FM. So I have to choose accordingly the free running frequency for that FM as well as the K value and that will give me the correct output right. That is called the direct method, so you can see already a lot of complications that were arising from narrow band FM and then with an indirect method to generate wideband FM that goes away.

And of course there also there was a problem that narrowband FM it is not if it is not sufficiently narrower then there will be a \tan^{-1} of KF into 80 right. So instead of just KF into eighty terms so that that problem was there whereas here there is no problem like that. So this is called the direct method of FM generation okay. So let us try to see now how do you do FM demodulation okay.

So our next target should be if a modulation is almost done, so we have learned narrowband FM how that can be generated then for wider band FM we have seen two methods one is direct one is indirect. So in the indirect method we have to just put frequency multiplier and we have seen also how to adjust the center frequency as well as frequency deviation, whereas indirect method it is just choosing a proper VCO parameter okay, sorry the direct method it is just choosing a proper VCO parameter and it is very easy.

Just across the very cap you give your input voltage and you will be getting your corresponding FM modulated output at the output of that VCO okay. So let us now try to discuss about FM demodulation again you will see just the mathematics tells us what should be the FM demodulation.

(Refer Slide Time: 04:01)



Suppose I have a ψ FMT and what it is it is a $\cos \omega CT + KF$ right this is my ψ FMT, I want to demodulate it. The first thing that I will do is if you carefully see this if I just differentiate this signal. So let us try to do this what do we get, so simple differentiation you do. So whenever we differentiate it should have that chain rule, so differentiation of cos must be giving me sine and then inside part also has to be differentiated right.

So inside part if I differentiate this must be ωCT must give me ωC , because I am differentiating with respect to T right. And then KF has a constant differentiation and integration actually cancel each other and this will give me MT right, so that is the whole thing that I get after differentiation and cos must be also differentiated, so that must give me sine of ωCT plus KF right.

So what we can see that FM modulated signal if we just differentiate what happens it will generate another sinusoidal, but the whole signal varying part comes into the amplitude okay. Now what we do whatever the sinusoidal, so it will actually look like this suppose ideal differentiator how does that look, that is generally $j2\pi F$ okay, differentiate DDT if you take the corresponding Fourier transform so that looks like this.

So ideal differentiator should be something like this constant means if I put the amplitude part of it so that should look like this it is a linear function of F okay. So if my ωC is somewhere over here and it remains linear over there means if it is an ideal differentiator so I should be expecting

something like this from that, that is the output okay. And after this how the signal will look like, so I had initially FM modulated signal.

So which was having something some modulation, so it was varying with respect to the amplitude of it, it is just the frequency deviation if I just differentiate what happens this frequency variation also comes into amplitude whenever there is a higher variance, variation of frequency. So basically I will have accordingly a higher amplitude and correspondingly if there is a lower variation I will get a lower amplitude right.

So this will be means once I pass it through our differentiator it will look like this, and inside also same pattern will becoming something like this okay, there is something which will be happening. So whatever is happening what we can see that message signal is almost in the envelope. So all I have to now do is envelope detection nothing else as long as I can ensure that envelope detection gives me a signal back as long as this is not doing a zero crossing we have already learned that in a modulation.

So what is the condition that this will not be crossing zero if this remains always positive, so that means I have to write $K_f M_T + \omega_c$ must be greater than zero right, this is something I will have to write. Now what is the minima value of this where it might cross zero that means the minimum of this must also this is true for all T that means the minima of this must be also greater than zero.

So minima let us say that is minus or that is - MP let us say okay, so then I can write - K_f into MP where MP is a positive number minus of that is the minimum plus ω_c must be greater than 0 greater than or equal to or ω_c must be greater than equal to K_f into MP. So this is the condition I get what is K_f into MP that is the frequency deviation we have talked about okay. So basically my carrier frequency must be bigger than frequency deviation this is something which always I will be doing.

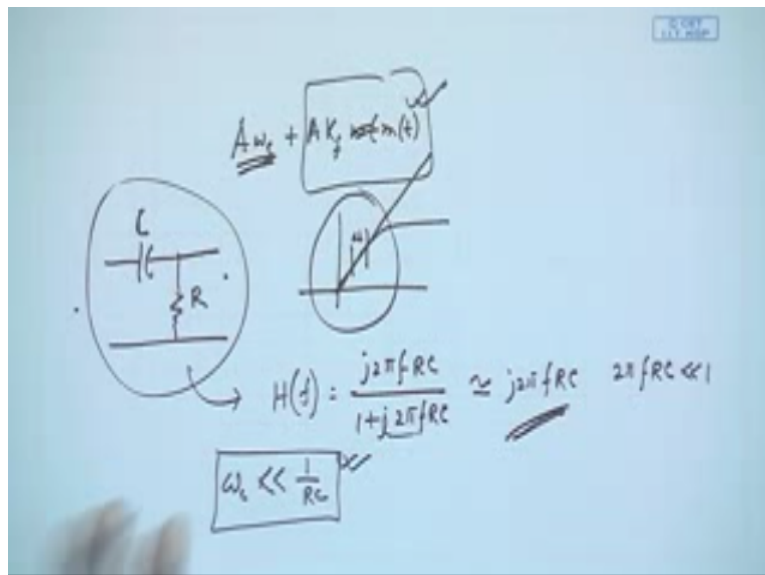
Because if the carrier frequency, because I know from Carson's formula that the bandwidth is more than 2 times of this deviation if my carrier is not even bigger than this particular thing there will be aliasing right, we are actually putting at the carrier ω_c . Now the bandwidth is definitely bigger than my δF twice of δF right. So or $\delta \omega_c$ if this is already bigger than my ω_c then what will happen, this will come even beyond zero and there will be a aliasing.

So definitely I whenever I do FM I will make sure my ω_c is bigger than that ΔF at least at it has to be bigger than $\Delta F + B$ according to Carson's formula right. So it has to be done if that is being always done so this condition will always be there that is prevalent we know that this will be happening if that is the case I know that the envelope will always remain above zero it will not have any zero crossing.

So I actually this is guaranteed that I will never have to cross zero, so if I just take the envelope I will get my signal back. So that makes the FM demodulation pretty simple all you have to do you have to pass it through our ideal differentiator followed by a simple envelope detector the one we have designed for a δ demodulation. So this is what happens and you know that it is guaranteed as long as you make sure that the FM modulated signal that you generate that is not creating aliasing that means ω_c is bigger than at least that $\delta \omega$.

You are pretty sure that your envelope will be above always and if you just track the envelope you get your message signal back. So tracking the envelope will give you $\omega_c + K_f$ into MT this term will be gone because you are just tracking the envelope that sinusoidal variation will be gone.

(Refer Slide Time: 11:18)



So you will just get $A\omega_c + AK_f$ into ω_c sorry $m(t)$, now this is just a DC term you block the DC with a capacitor will get this part which is the message signal. So indirect sorry this method of means this differentiator induced method of FM demodulation that is very simple all you will

have to do is you have to find out our ideal differentiator circuit. Now we will discuss about that that is little bit difficult to find out the ideal differentiator okay.

So generally what people have done means you cannot actually get an ideal differentiator that is not possible, in circuitry every capacitor will put there will be some spurious resistor in that capacitor there will be some other effect and always you will see that it cannot work as an ideal differentiator, so that is not possible there is no nothing called ideal differentiator. What we will do we will employ something like this see our circuit this is actually a high-pass filter right.

So how the high-pass filter will look like it will it will actually look like this, but there is a region generally high pass filter we are not bothered about this roll-off, generally we want to neglect that roll-off in high pass filter we are most more concerned about the where the filter response is flat, but this is further for the FM demodulation, we are actually bothered more about this roll-off.

So we want to see where exactly it remains linear that is where it almost behaves like an ideal differentiator. So basically we have to target a particular frequency zone or we have to design our high pass filter in such a way that in the frequency of interest which is this ωC and around that $\delta F +$ and $\delta F -$ right. So that region it remains linear okay, so that is something we will have to find out.

So let us for our ideal this particular filter let us try to see that what is the transfer function. So that should be $J2 \pi FRC$ if you just put it accordingly. So this is what we get okay HF will be just this okay. Now this can be approximated as $J2 \pi FRC$ if this $2 \pi FRC$ is much less than 1, because then this term will be neglected, so there will be only 1 so I get this is almost like a ideal differentiator.

So this will happen if this condition is valid, so now I can get a condition on my ωC okay. So what I can do this $2 \pi FC$ let us put it as ωC , so ωC must be much, much less than $1/RC$ okay. So this is the condition I get, if I choose it accordingly then the ωC will be falling in this region where it looks like ideal differentiator. So all I will have to do is, I have to choose a RC corresponding to my ωC that has been put over there.

Accordingly I put my RC so maybe I can fix C and then try to find out what should be my R. And then try to find out a RC value which is much, much bigger than this ωC , then I know that

around that ω_c it remains linear, because this particular transfer function characteristics will be valid that approximation also will be valid, and it will almost behave like a ideal low-pass filter sorry high-pass filter sorry ideal differentiator right.

And then if I just pass my signal through this at the output I will get a differentiated output at that particular frequency. So this is all that I will have to do it is simple enough all we will have to do is we have to design our RC accordingly and then try to get a particular differentiation circuit. Once this is being done I know that the means envelope tracking will be very simple, whatever we have done at a.m. that has to be mimicked over here okay. This is one way of means demodulating FM.

(Refer Slide Time: 16:22)

FM Demod using PLL

$$\sin(\omega_c t + \theta_i(t))$$

$$\theta_i(t) = K_f \int_{-\infty}^t m(x) dx + \frac{\pi}{2}$$

$$A \sin\left[\omega_c t + K_f \int_{-\infty}^t m(x) dx + \frac{\pi}{2}\right]$$

$$\theta_o(t) = K_f \int_{-\infty}^t m(x) dx + \frac{\pi}{2} - \theta_i(t)$$

$$e_o(t) = \frac{1}{c} \dot{\theta}_o(t) = \frac{1}{c} K_f m(t) - \dot{\theta}_i(t) \approx \frac{1}{c} K_f m(t)$$

There is another way that is FM demodulation using PLL. So if you remember that for PLL we have also used a VCO right that was how we have designed TL. So for FM modulation we have already used VCO, now for demodulation also we will be just using PLL and PLL also as VCO in it. So basically FM modulation demodulation both will have key component as the VCO so once you have VCO you can do both modulation demodulation if we can appreciate this particular circuitry.

So let us try to see how FM the modulation can be done with a PLL, so this is another application of PLL which is coming out earlier we have seen that for carrier tracking probably PLL is very good and with that target only, because we are talking about phase lock loop which

was with the target of carrier locking, carrier phase, and frequency locking we have discussed about that effectively.

So now we will try to see the other application of PLL which is FM demodulation, so let us say the PLL generate a sine wave means its input this one to which it gets locked that is a sinusoidal. So that is something like $\sin(\omega_c t + \theta)$ okay where θ is the input. So what we will do, we will actually give this FM signal if I modulated signal to the PLL input. So what will happen to this θ that must be whatever phase it is getting so FM is a $\cos(\omega_c t)$ plus the FM modulated part that KF into 80 okay.

So that means that θ must have this KF into 80 plus because its cause and PLL input take sine, so there should be a $\pi/2$ phase shift so therefore, my θ should be KF integration $-\infty$ to T $M \alpha D \alpha$ plus some $\pi/2$. If this becomes θ , so immediately I can put my FM input should be a sign $\sin(\omega_c t)$. So I can put a sign $\sin(\omega_c t + \theta)$ which is KF integration $-\infty$ to T $M \alpha D \alpha$ plus $\pi/2$ right.

That immediately becomes cos, so it is $A \cos(\omega_c t)$ plus this that is actually FM signal. So therefore if I wish to put FM signal to PLL input and if we correspond it we get my θ as this one right, this is fine. Let us say the output phase error that is generated due to the locking that is θ_e of course this should be small enough we will see that. So basically therefore, what is θ output T which is being generated after the VCO means the PLL output.

That should be this input phase by minus this θ right, so input phase is KF integration $-\infty$ to T $M \alpha D \alpha + \pi/2$ and - this θ right. So that is what it will lock to and it will get this output phase, but what we also know that this θ if we have if we now try to see the error signal that is being generated by PLL, so this that is θ_e that is nothing, but the differentiation of this output phase.

So and with a factor $1/C$ which is the factor of PLL right. So $1/C$ differentiation of θ or T now let us try to differentiate it so it should be $1/C$ and if θ we have to differentiate. So this term will be gone, I will have a differentiation of this one and I will have differentiation of this one. So differentiation of that one is KF and if I just differentiate integration differentiation cancels each other so I get M right.

And I will get differentiation of this now that is exactly the error frequency, because phase differentiation is the error frequency PLL if it is properly locked then frequency error must be

zero. So this I can almost say it should be $1/C \cdot K_F \cdot M \cdot T$ which is very good because that is actually the message signal with some constant factor which I do not bother.

So basically at the error of PLL I get FMD modulated signal if I give at the input of PLL FM modulated signal that is whatever analysis we have done for PLL that actually directly comes from means it directly comes from there. If we means we just take a PLL circuit earlier we are not bothered about the error signal of PLL we are not bothered about that.

Now in this particular demodulation what we will try to do, we will try to suppose at that time we are bothered about the VCO generated output okay. Now we are actually looking into the error signal that is being generated after the PLL, after the loop filter problem. So what we have to do is we will give as input the FM modulated signal we do not actually tracking we are not tracking anything.

So we are not worried about the VCO generated output we are not worried about now we are not worried about that, we will take that from the loop filter whatever is coming out we will take that and we could prove mathematically again you can see, why we are able to use this, because mathematically this is getting proved. So it is the circuit almost operating the way it is defined its transfer function and everything operating on the signal this is what it is giving.

So once you prove that mathematically you know that with this circuit I can do these things. So we can see that the error signal is actually becoming the differentiation of that particular thing okay. So differentiation sorry differentiation of this output and that becomes happens to be proportional to my message signal. So that means at the error I am getting my demodulated signal right.

So what we have so far done is by we employed two methods of FM one was direct method one was indirect method indirect method B could be used VCO indirect method we had to do a lot of things, a lot of multiplier part lot of non linear and nonlinear circuit and all those things. And then for demodulation also we could see there are two methods one is through differentiation followed by envelope detector which is simple enough.

So for differentiation you will probably have to employ a linear part of a high pass filter okay. And the other part is just use PLL for demodulating FM signal, so these are two things that can be done to demodulate FM signal okay. So far we have analyzed about FM bandwidth and how

we can generate or demodulate FM modulated signal and FMD modulated signal okay. So this is something we have analyzed.

But first of all we need to understand that why we should study FM or why one should employ FM so this is something we will now be exploring. So we will try to see or try to appreciate that FM has some good characteristics initially people who would have thought probably it is the bandwidth efficient protocol sorry, bandwidth efficient modulation technique.

But that is not the case we have already proven that this is not probably as bandwidth efficient as any of the modulated signal. But there are some advantage and this is something which we will be exploring next. So the first thing is we have already discussed that there are any modulated signal that will be transferring that has some effect when it is passed through a channel right.

So what are those effects, the first effect we have discussed is if the channel is nonlinear okay. So that is the first thing which happens, so we have also seen that if we have A modulated signal and if there is a non-linearity there will be detrimental effect this is something we have already appreciated, and we have seen that okay.

So what we now wish to see that for FM is there any effect or is it very means very much superior in terms of modulation that any non-linearity in the channel can be just rejected by FM okay. So that is something we wish to see so let us try to appreciate that.

(Refer Slide Time: 25:36)

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \dots + a_n x^n(t)$$

$$x(t) = A \cos \left[\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha \right]$$

$$y(t) = C_0 + C_1 \left\{ \cos \left[\omega_c t + K_f \int_{-\infty}^t m(\alpha) d\alpha \right] \right. \\ \left. + C_2 \cos \left[2\omega_c t + 2K_f \int_{-\infty}^t m(\alpha) d\alpha \right] + \dots \right.$$

So let us say we have an arbitrary non-linearity, so why T is some $A_0 + A_1 X^1T$, so X^1T , X^2T is the input $A_2 X^2T$ some n^{th} order non-linearity in the channel. Now what will happen I will be launching FM signal, so FM signal that means my X^1T should be a $\cos \omega_c T + K_f \int_{-\infty}^t m(\alpha) d\alpha$, so whatever discussion we are doing that is equally valid for FM as well as PM right.

So both are equivalent that that is something we have proven instead of integration you will be putting just empty over here okay. So if I just put this X^1T what will be my Y^1T there is something we have just done for our direct modulation right. So what will happen if I just put it over here I will be just seeing something like $C_0 + C_1$ into sorry, cos of course this A can be taken inside C so do not worry about that.

So it should be $\cos \omega_c T + K_f \int_{-\infty}^t m(\alpha) d\alpha + \cos C_2 \cos 2 \omega_c T$ plus $K_f^2 K_f$ and so on, up to a next term, but what has happened. Now this FM modulated thing after it passes through this particular channel will probably get all these extra higher terms and I my receiver what generally that will have at the means at the front end of receiver we have already talked about that it wants to neglect the effect of noise.

So it will have a band pass filter where that band pass filter will be centered at Ω_c and the band will be just FM that, so if I just pass it through that band pass filter these things all will be neglected. So all those higher frequency term where this is getting contaminated, because the

frequency deviation is getting twice, thrice and all those things they will be all canceled out, what will happen I will only have this particular thing.

So even if I have channel non-linearity I do not bother about it, because FM automatically due to that band pass filter will cancel out that effect of non-linearity and it will get pure FM modulated signal even after passing through a nonlinear channel. So that is a very big advantage which FM has or FM has that edge over amplitude modulated signal, because in amplitude modulated signal if you multiply it that $M^2 T$ term will be coming out.

Because it is in the amplitude, so that multiplication will create a multiplication in the signal also and then that will create problem for you whereas that is not happening over here okay. So that is a big advantage which we will have whenever we have a channel which is slightly nonlinear. So in that channel if you just put FM that is more protected compared to your any form of a modulated signal okay.

So this is the first thing where we could get some advantage of FM we will see if there are interference see whatever I have told that is actually creating a source of interference also. So if the channel is nonlinear and I have here probably at ωC my FM signal, but at $2 \omega C$ there might be some others FM signal where due to these things in the channel they will be all created.

So this will create interference to them however small that non-linearity is this a_2 a_3 coefficients are small they might be smaller, but they will still create some effect at those frequencies. So those are treated as interference, so what we will see in the next class that in presence of interference how FM survives okay.

So that is something we will try to appreciate in the next class. And then at the end we will also try to do noise analysis and we will be able to prove that in any time in any channel FM is much better in terms of noise cancellation compared to any of its AM counterpart okay. So that is something we will be proving and with that we will probably end our discussion of FM okay thank you.