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Course
On
Analog Communication

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Lecture 53: FM Noise Analysis

Okay, so far I think we have already seen how do we really see the benefit of FM so that is what we started doing we have already finished discussing about FM bandwidth initially people thought its bandwidth benefit we have discarded that and then we could appreciate the circuits that has to be produced for FM generation and demodulation and then we started capturing the advantages of affair mostly all the channel impairments probably FM take care of that in a better fashion compared to its aim counterpart.

So that is what we started discussing we have already given one example where the channel non-linearity we have seen that other modulation schemes like amplitude modulation they are probably not that good to handle channel non-linearity whereas in FM we could see that it can vary means due to the inherent quality of FM it can really handle non-linearity very well. So that is the sole reason why for a high power transmission where you need to really put Class C amplifier which goes into non-linearity.

So for those kind of transmission FM was the de fact modulation scheme because it has a good cancellation mechanism which is inherent to FM, now today what we'll try to see that in presence of interference what happens to FM modulation okay or what kind of protection it has in presence of interference so just to capture that because the full-blown analysis is little bit involved so what we will try to see is that as if FM means we are trying to transmit a FM which is which is either a constant thing or it does not have any modulation it is just the carrier we are transmitting but of course you will see that a sense of the cancellation is already means that will be understood through this analysis.

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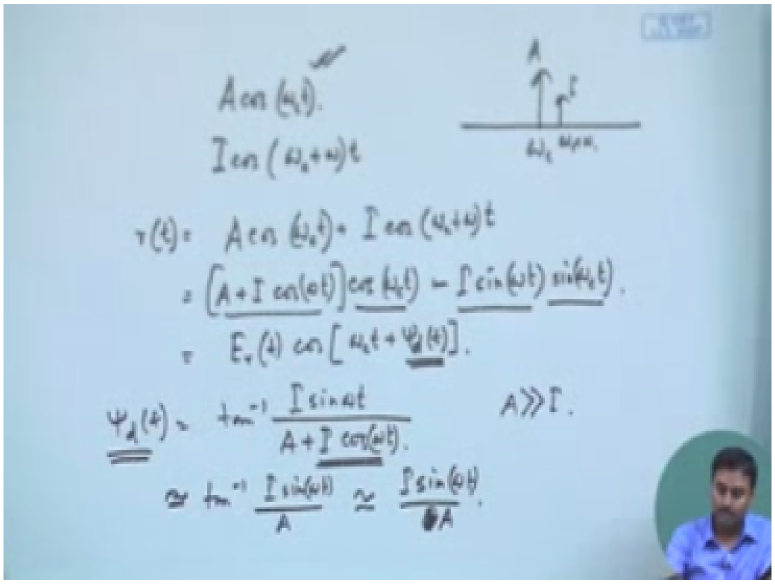

So what we will say we will say unmodulated carrier a cos ω CT is being transmitted so of course if it is FM modulated so there should be some phase term which is for FM it will be our integral of M α D α x K 4 p.m. it will be just empty into K K P probably, so there should be a phase term we are just neglecting that that means unmodulated just the carrier we are sending okay so there should be a phase term we are just neglecting it for our purpose of analysis and you'll see that it is not that much required because that the demodulation will still employ FM demodulation.

So and we will get some insight, so let us say this is the desired carrier and nearby that we have some interferer which has a strength of I and it is nearby so the frequency is α c + ω D okay, so it is just this is the carrier frequency let us say at ω C and there is nearby some strength I this is having strength at ω C + ω so that is the interfere or actually and we have to see what is the effect of this interference and what FM does with this interference so that is something we wish to check.

So basically what will happen? Suppose my received signal will be because in channel I will have this carrier as well as this interfere so it will be addition of these two right so I will have a cos ω CT + I cos ω C + ω T this is what I will be getting I can just open this cos ω C + ω and then I will be getting a plus I cos ω T x cos ω CT just algebraic manipulation - I sine ω T into sine ω CT right this is what I will be getting at the FM and we are thinking that this i is close enough in the band pass filtering zone of FM.

So basically it is a interferer which is within the band so it will not be canceled by the filter so it will still be inside and we want to see if those are the valid interfere and we want to see what is the effect of that interference into the FM modulated signal, so this is what I we are trying to analyze okay, so this can be written as because it is cos ω C sine ω C and this can be represented in a overall co sinusoidal with an envelope and a composite phase okay where of course this ERT should be this square plus this square and square root and we are interested in phase because FM demodulator will actually detect this phase and differentiate it right.

So this phase will be that must be tan inverse as we know this divided by this whole thing so this divided by this so that should be I sine ω T divided by a + I cos ω T now if it is an interferer within the band we expect that the interference probably will be smaller in strength because if interference is almost of similar strength with the signal itself probably nobody can do anything with that it will it will still be present so we will assume that because it's interference so of course a must be much greater than I okay.

So this is a interference in picture also we have almost demonstrated similar thing, so if that is the case then I see because it is I cos ω T this will never be bigger than I so this can be neglected compared to a and then this will be so this becomes tan inverse I sine ω T / a again I/a that is a very small number so this can still be approximated so these are all approximation so tan inverse theta will be just tan θ I sine ω T / θ right.

So that is becoming my phase the FM over all FM phase okay, so if I modulated signal and the phase part of that so when we D modulate it what do we do we take the phase and we actually differentiate it whatever we get that should be sorry that is not theta that should be so we differentiate it that will be the frequency and that must be my signal that is how FM demodulation goes it detects the phase and then differentiate it to get this right. So that we have already understood because in the phase it will be integration and then you differentiate it you get the overall phase so here also because it is a FM demodulation change so we will be doing employing same thing so this particular phase will be differentiating and whatever will be getting that is my receive signal.

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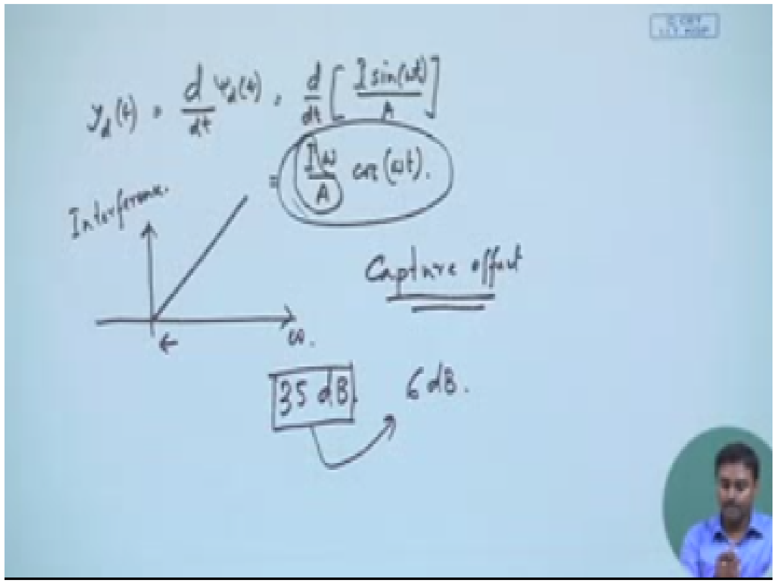

So what we can say that D/DT of π DT let us call that as Y DT that must be the demodulated signal, so that must be so D DT of π D whatever we have calculated that approximate value I sine ω T / K oaky, so if I differentiate it will be I ω / a cos ω T right this is what we get so basically we have what we have done we have tried to see the interference we have just said that we are probably transmitting a FM signal but we are just considering its unmodulated FM that means the just the carrier we are just for testing purpose just a carrier we are sending and nearby that carrier within the band of FM I have an interferer which is at ω C + ω okay.

So these two signal are in composite falling my receiver I am just trying to detect the face the way FM will be doing and say is if you differentiate that should be the frequency variation and that we call as a detection okay, so this is what I will be detecting right so that is my phase and now let us see what happens okay if I just try to plot so this is actually see actual my FM modulated one was having nothing so in the detection I should get a zero thing okay because there was no modulation so it must be a zero so it should be getting zero but it is getting this thing.

So that is my interference in this case interference getting transformed after doing means FM demodulation so what is that let us try to see that first of all there is a factor of I by a okay, so we have already said if a is bigger than I so there is already means the interference is becoming a times lesser okay so when it is interfering with FM because of FM demodulation process it is already becoming a times lesser as long as is higher probably it will be already lesser that is one thing second thing is it is getting multiplied with ω so if you see that ω is the separation right.

So if I just plot it with respect to my frequency ω and then try to plot this interference how it will be at ω = 0 it is 0 and then there is a linear curve okay, so of course you can always say that there is a cos ω T term but okay if this costs ω T and because I do not plot it with respect to T I will take the probably worst case scenario, so it is the highest value of this is one, so it will be just I Omega by 8 so I am plotting the highest value of interference that's what we are interested in that how much interference at the pick will be coming to me.

So that at most will be this which is I / a x ω so it goes linear with ω so this is the interference here remember deliberately we have taken out the signal so we should expect after FM demodulation zero nothing but we are getting something and we are calling that as interference so that that was just to make the overall analysis simplified if you give FM then there will be more involved term, so we just wanted to simplify that we are just capturing the interference effect.

So what we can see that as ω is closer this is a very interesting thing the interference term is smaller so as the interfering signal comes closer to you are more afraid that because the interference is closer probably more interference I will beginning but FM has a cancellation part okay due to its modulation and demodulation it cancels that so basically if the interferer is closer to it overrides it in a better fashion as the interferer goes away from it of course within the band of FM if it goes beyond that you will your filter will cancel it but within the band of it if it goes little farther probably a little bit more interference will be coming to you okay.

So this is what will be happening so what we can see already that FM has a nice cancellation property of interfere so any interference that is coming in the band of FM closer it is with the FM carrier more it will be canceled not only that it also gets overridden by the carrier strength so if the carrier strength is enough the interfere are if that is much smaller than that so overall worst case interference will already be overridden by this FM so this is particularly termed as capture effect is called as capture effect of FM, so that means what it does actual FM signal if you transmit that is your desired signal so whenever you are demodulating it will actually have a overriding factor over all the close by interference so it captures that and actually exert more power in terms of the modulated output okay.

So that is of good property because we want that we want interference cancellation and FM automatically due to its demodulation inherent in a demodulation technique it cancels them out that is a very nice property so what people have seen over here that in a.m. if there is a nearby interferer that must be almost 35DB less then only you get proper signal to interference ratio okay whereas for FM you can go as far up to 6DB less okay.

So that is a big advantage so almost 29 DB extra interference still you are good enough almost both the receivers will behave equivalently, so that that is the advantage of FM we were talking about that FM we have are now shown two advantages one was already with the nonlinear channel that FM cancels it out very nicely due to its inherent demodulation and modulation technique and now we could show that if there are interfere or even within the band FM actually over writes them which is called termed as capture effect and it means gives huge advantage over the interference whenever you are transmitting FM.

So next what we will try to do is we will try to show another big advantage of FM that is the noise cancellation which is almost similar to this part but probably the analysis will be little more involved so we will try to capture the overall interference sorry overall noise effect in FM okay so for that we will go back to our noise analysis in similar fashion, so what we did in our noise analysis it was a simple thing we have drawn the receiver transmitter module and then we means for a man Alice's also we have done always try to capture the figure of Merit what does that means if I recollect that means that if you transmit the same signal in same or insert or put say same amount of power as you are putting in FM in the baseband and try to see what is the performance.

So this is the benchmarking one that is the reference one try to see if I do not do FM I just insert same amount of power that FM has to put okay, if a modulated signal has to put so I put the same amount of power and try to see what is the performance if I just transmit it in baseband without doing any modulation and on the other side any demodulation only that filtering will be there low-pass filtering, so with that we get a reference SNR okay, and then what we do with the same power because now the powers are equivalent.

So we actually transmit the FM modulated signal and then in the channel as expected noise will be added after that the entire demodulation of FM will go on then we will get probably the signal and noise will go through this demodulation process and finally we will get some expression for signal and noise, and that will be the actual signal-to-noise ratio if we get the power of these two and then after FM demodulation we try to see what is the SNR and then we compare these two snr compared to the references nr how better or how words this is and that gives me gives us a ratio which is called the figure of in it.

So you have already calculating evaluate it figure of Merit for aim modulation the SBS sees SB SC so we have done all those things and we could see that they are d SB SC and s SB SC they are having figure of Merit of 1 that means it has no advantage with respect to bed equivalent baseband transmission whereas a.m. was even worse it means with a tone modulation we have proven that the highest it can get is 1 / 3 okay, it will be always less than that. So now we will try to see with FM what happens okay. So let us first try to draw the receiver chain that is the first task probably so I have FM.

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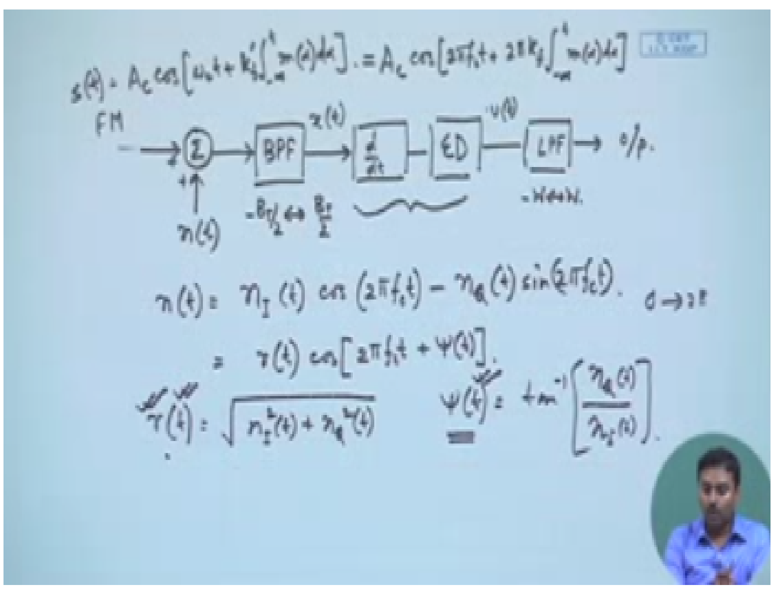

That is the summation this is the channel so basically FM modulated signal I will be putting over here which is my signal which is just FM modulated signal that means I can write it as AC cos ω CT + KF if it is represented in ω domain and 2 π K F if it is represented in F domain okay, so accordingly KF $-\infty$ to t m α D α that is it okay, so this has an equivalent representation if I if I do it for frequency domain so I can write it AC cos 2 π fct + in instead of kf let us call this as K of – so here we wi ll be writing 2 π K F.

So this is in frequency domain F domain we should call and this is an angular frequency domain ∞ domain okay so that is my fm so this is what will be inserted over here and from here in the channel there will be noise which will be added over here right and after that after the channel the FM demodulation starts, so the first part of demodulation should always be a band pass filter, so this is the band pass filter of FM now this band pass filter is no way related to just the frequency of the modulating signal it is the FM band okay.

So we call that as from - BT / 2 to + BT / 2 so BT is the overall FM band suppose which comes from the Carson's formula okay, so we take that it has a part of the Δ f it has a part of B so it is actually 2 x ΔF + B right so that is what we have understood, so this is that particular part okay so that band pass filter should be put so that the entire FM band comes into my receiver after that so we call this signal as XT okay this should have noise part as well as the FM modulated signal after that what will be putting will be putting a FM demodulation.

So the simplest one we know is a differentiator followed by a envelope detector right that is one of the FM d modulator we have seen it is just you differentiate it the entire frequency variation comes into amplitude as well and then you do envelope detection we have already proven in the last class that with envelope detection will be always getting our signal back right.

So that is the whole purpose of this is probably the main part of FM demodulation okay, so after that let us say we get a VT after this in FM demodulation this will be clearly understood from the noise analysis that we need to also employ a low-pass filtering you will see that later on why that is required.

So this low-pass filter is just of the message signal band so that goes from - W to + W and after this we get our output okay this low-pass filter will just facilitate us in terms of noise cancellation and you will see that FM only gives this facility all other modulation scheme does not give this facility and that along with some other things which we will be exploring has gives the FM age over other modulation scheme.

So we will come to that okay, so this is the overall receiver architecture right so let us see how the demodulation happens so I have a signal which is s T of course but I also have a noise which is band pass noise so I can always write it in terms of in-phase and quadrature component, so I can write it as n I T cos 2 π FCT here I can means the representation does not depend on whether

I write plus or minus so I can take any one of them, so for our advantage which will be clear in the next phase of derivation we take – so it is equivalent it is just a representation whether you take plus or minus it spectrum wise there is no difference okay.

And represent in addition wise also there is no difference okay, so this can be written because the central frequency of my band pass filter is this FC therefore this amount of noise which is from -BT to + BT over that band or - BT / 2 to + BT by2 that means BT that is the FM bandwidth or that band the overall noise will be coming through the receiver and that is the band pass representation of that particular signal okay, this can further be written as RT as any summation of addition or subtraction of two sinusoidal can be represented as another sinusoidal so this can be written as RT cos 2 π FCT + KY t right.

So this is something I can write whereof course RT you know already it should be root over ni T^2 plus NQ TS^2 that is the RT and Shai T must be tan inverse this divided by this so it should be NQ so as long as these two ni and NQ we have probably not talked about these part and due to time constraint probably we would not be able to prove that but this is a very important derivation in random process that if these two we have already proven that they are independent okay.

So that is something we have proven in fact we have proven they are orthogonal also, so as long as that is happening and they are Gaussian okay so that is something if we assume generally that is what happens then we can always prove that this RT actually follows a Rayleigh distribution so this is something which will be can be proven I am not going into the details of that because that means take us to somewhere else so due to time constraint we are just taking that. So it is a Rayleigh distributed one and the corresponding chai T also can be proven that that is uniformly distributed between 0 to 2 π okay.

So this is something which will be happening you know random pose so these are also a separate random process which follows like this ni and NQ or overall NT that follows a Gaussian distribution it can be proven that these are also random process which follows this follows Rayleigh distribution this follows a uniform distribution between 0to 2 π okay, so that something can be proven, so we are just leaving it over here but probably you'll see that's notthat much required for our derivation okay.

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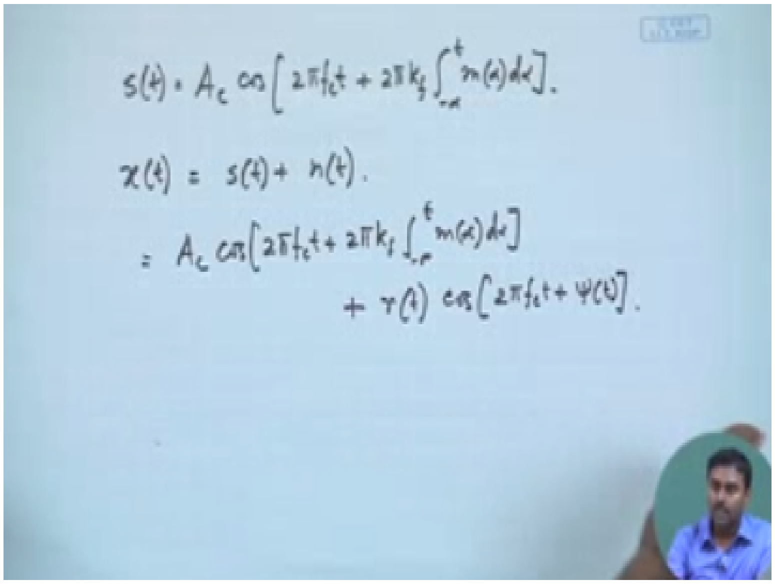

So anyway we have got this so immediately what we can write is our st that is something we know already it is AC cos 2 π FCT that's the transmitted 1+ 2 π K F in terms of frequency if we represent - ∞ to T M α D α so that is the FM modulated signal so overall signal that is XT after the band pass filter will be this st+ empty right, so st happens to be SE cos 2 π FCT + 2 π KF integration - ∞ to t m α T α so this is the part and the noise also has its equivalent representation which I can write as RT cos 2 π FCT + sy t okay, that is how we have represented at the noise that is the after the band pass filter so then this is a band pass noise and this is the actual FM.

Because FM the band pass filter just exactly allows the entire FM signal so FM signal remains undisturbed okay, so this is what we get after the band pass filter now what we will have to do is next we have to see if this is the signal plus noise what my discriminator will do or probably I have not given that term so this entire part that differentiation followed by envelope detection that is called termed as discriminated in FM demodulation, so that is called overall a discriminator circuit so what this entire circuit will do that is something we have to see because I earlier whatever we have seen that was easier because we are directly differentiating this term but now we have the noise which is a random process again.

So therefore that direct differentiation will not work probably we have to do it means borrowing the theory of a random process ok so that is something we'll have to do and equivalently we will have to calculate the overall power spectral density of the noise that we get after the discriminator that will be the next task that will have to do to analyze the FM in presence of noise okay, so in the next class probably we will discuss in details how we can analyze FM in presence of noise okay, thank you.